$f = \log\left|\frac{x}{x+1}\right| - \frac{x}{2}$

C.E. $x+1 \neq 0 \rightarrow x \neq -1$

$\left|\frac{x}{x+1}\right| > 0 \rightarrow x \neq 0$

$]-\infty, -1[\cup]-1, 0[\cup]0, +\infty[$

$\lim_{x \to -\infty} f(x) = +\infty$

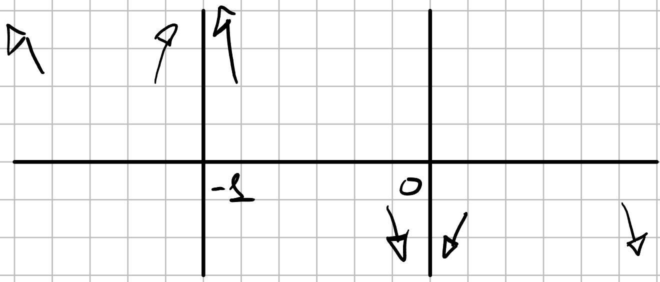
$\lim_{x \to -1^-} f(x) = +\infty$

$\lim_{x \to -1^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = -\infty$

$\lim_{x \to +\infty} f(x) = -\infty$



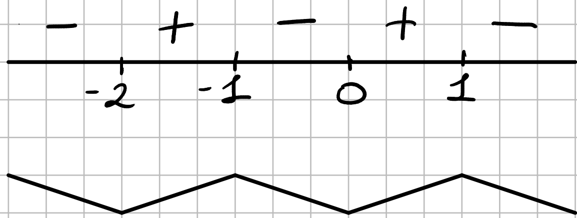
$(\log|t|)' = \frac{1}{t}$

$\exists f'$

$f'(x) = \frac{x+1}{x} \frac{x+1-x}{(x+1)^2} - \frac{1}{2} = \frac{1}{x(x+1)} - \frac{1}{2}$

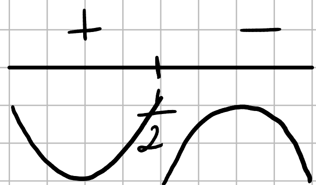
$f'(x) \overset{?}{\geq} 0$

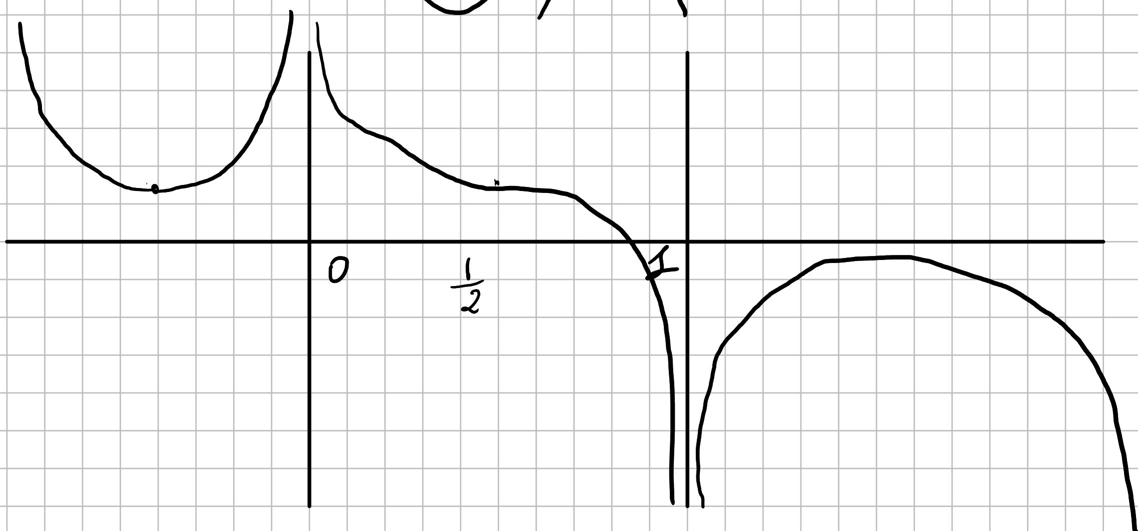
$\frac{2 - x(x+1)}{2x(x+1)} \geq 0 \rightarrow \frac{-x^2 - x + 2}{2x(x+1)} \geq 0$



$\exists f''$

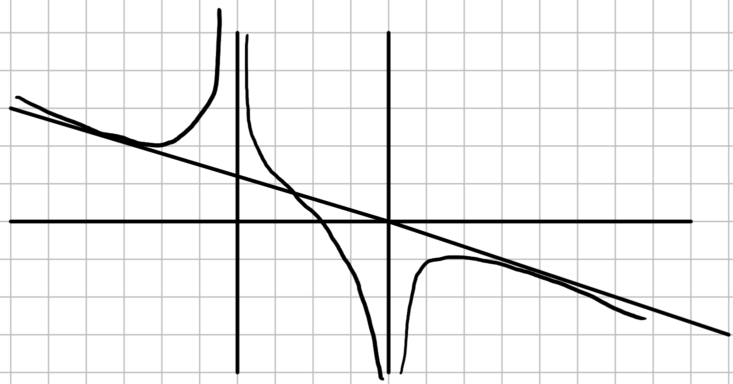
$f''(x) = -\frac{2x+1}{(\quad)^2} \geq 0$





Asintoto Obliquo $m = \lim_{x \to +\infty} \frac{f(x)}{x} = -\frac{1}{2}$ (con De l'Hôpital)

$$q = \lim_{x \to +\infty} f(x) + \frac{1}{2}x = \lim_{x \to +\infty} \log\left|\frac{x}{x+1}\right| = 0$$



$$\begin{cases} a_0 = 1 \\ a_{n+1} = \frac{1}{3}(a_n + 1) \end{cases}$$

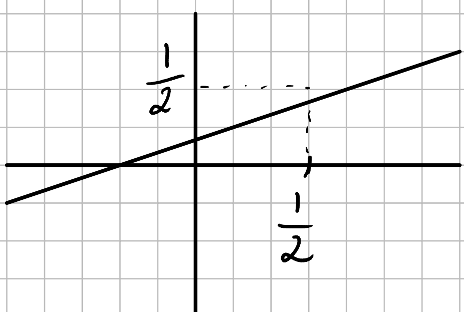
$$\varphi(x) = \frac{1}{3}(t+1) - t = \frac{1-2t}{3}$$

$$\gg\gg\gg \; \frac{1}{2} \; \ll\ll\ll$$

Limite: $\frac{1}{2}$

Se avessimo proseguito

$$f(x) = \frac{1}{3}(t+1)$$



$$f(]\tfrac{1}{2}, +\infty[) =]\tfrac{1}{2}, +\infty[$$

$$f(]-\infty, \tfrac{1}{2}[) =]-\infty, \tfrac{1}{2}[$$

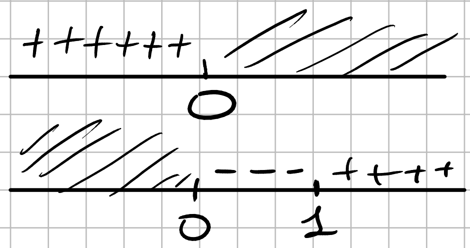
$$\begin{cases} a_1 = 2 \\ a_{n+1} = 2a_n^2 - |a_n| \end{cases}$$

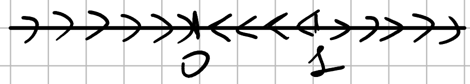
$$\varphi(t) = 2t^2 - |t| - t$$

$$\varphi(t) \overset{?}{>} 0$$

$t<0 \rightarrow$

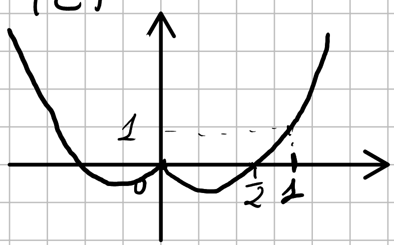
$t>0 \rightarrow$





PARI

$$f(t) = 2t - |t|$$



$f(]1, +\infty[) =]1, +\infty[$ Siccome $2 \in]1, +\infty[$, il limite della successione è $+\infty$

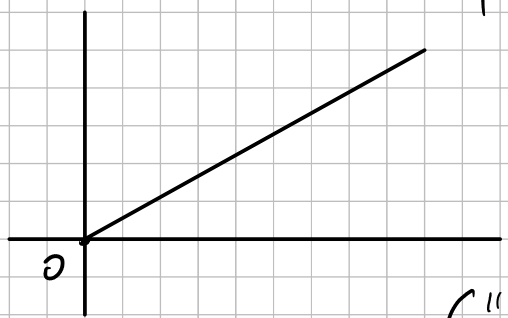
Grafico

$$f(x) = x^2 \arctan|x|$$

$f(x)$ è pari, si studia con $x>0$

$$f(x) = x^2 \arctan x$$

$$\lim_{x \to +\infty} f(x) = +\infty$$



$$f'(x) = \underbrace{2x}_{>0}\, \underbrace{\arctan x}_{>0} + \underbrace{x^2 \frac{1}{1+x^2}}_{>0}$$

$$f''(x) = 2\arctan x + 2x \frac{1}{1+x^2} + \frac{2x(1+x^2) - x^2(2x)}{(1+x^2)^2}$$

$$f''(x) > 0$$

Asintoto Obliquo $\lim_{x \to +\infty} \frac{f(x)}{x} = +\infty$ Non ce n'è

$$z^2 + |z| = z$$