$\boxed{1}$ Trovare l'integrale generale delle seguenti equazioni differenziali:

$$y' = e^x \ (y-2)$$
$$y' - 2xy = x$$
$$y'' - y' - 2y = e^{2x} \ (x+3)$$
$$y'' + 3y' - 4y = 2x \ e^{3x}$$
$$y'' - 8y' + 16y = e^{-x}$$
$$y'' - 2y' + y = e^x \ (x+3)$$
$$y'' - 9y = x + 1$$
$$y'' + 2y' - 8y = e^x \ (x^2+1)$$
$$y'' + 2y' - 15y = (2x+1) \ e^x$$
$$y'' + 3y' - 4y = x^2 \ e^x$$
$$y'' + y' = x - 6$$
$$y'' + 4y = \cos 2x - \sin 2x$$
$$y'' + 2y = 4 \sin \sqrt{2} \ x$$
$$y'' - 2y' - 3y = e^x \ (\cos x - 3 \sin x)$$

1) $y' = e^x (y-2)$

$y' - e^x y = -2e^x$

EO: $y' - e^x y = 0$ $\qquad A(x) = \int a(x)dx = -e^x + K, \ K \in \mathbb{R}$

Sol: $c \cdot e^{e^x}, \ c \in \mathbb{R}$

EC:

$\bar{y} + c \cdot e^{e^x}$ $\qquad \bar{y} = K \cdot e^{e^x}$

$\int f(x) \cdot e^{A(x)} dx = \int -2e^x \cdot e^{-e^x} dx = -2\left(\int e^{-t} dt\right)_{t=e^x} =$

$= -2\left(-e^{-t}\right)_{t=e^x} = 2e^{-e^x} + c, \ c \in \mathbb{R}$ $\qquad K = 2e^{-e^x}$

Eq. generale:

$$2e^{-e^x}\cdot e^{e^x} + ce^{e^x}, \quad c\in\mathbb{R}$$

$$2+ce^{e^x}$$

2) $y'-2xy=x$ (è lineare e e V.S)

V.S.

$y'=x(2y+1)$

$X(x)=x \quad (a,b)=\mathbb{R}$

$Y(y)=2y+1 \quad (c,d)=\mathbb{R}$

1ª Cost.

$$H=\{h\in\mathbb{R} : Y(h)=0\}=\left\{-\frac{1}{2}\right\}$$

2ª Cost.

$$y'=x(2y+1) \qquad y\neq -\frac{1}{2}$$

$$\frac{y'}{2y+1}=x$$

$$\exists\int\frac{y'}{2y+1}\,dx=\frac{1}{2}\ln|2y+1|$$

$$\exists\int x=\frac{1}{2}x^2$$

$$D\left[\frac{1}{2}\ln|2y+1|\right]=D\left[\frac{1}{2}x^2\right] \qquad \ln a=b \to a=e^b$$

$$\frac{1}{2}\ln|2y+1|=\frac{1}{2}x^2+K$$

$$|2y+1|=e^{x^2+2K} \qquad (\alpha,\beta)\subseteq\mathbb{R}$$

$$2y+1=\begin{cases} e^{x^2+2K} & \text{se } y\geq -\frac{1}{2} \\ -e^{x^2+2K} & \text{se } y<-\frac{1}{2}\end{cases}$$

$$y=\begin{cases} \dfrac{e^{x^2+2K}-1}{2} & \text{se } '' \\ \dfrac{-e^{x^2+2K}-1}{2} & \text{se } ''\end{cases}$$

Lineare

$$y'-2xy=x$$

EO: $y'-2xy=0$

$$\int a(x)=-x^2$$

$$c\,e^{x^2},\ c\in\mathbb{R}$$

EC

$$\int x\cdot e^{-x^2}dx=-\frac{1}{2}\int 2x\,e^{-x^2}dx=-\frac{1}{2}e^{-x^2}+c,\ c\in\mathbb{R}$$

$$-\frac{1}{2}e^{-x^2}\cdot e^{x^2}+ce^{x^2},\quad c\in\mathbb{R}$$

$$-\frac{1}{2}+ce^{x^2},\quad c\in\mathbb{R}$$

3)

$$y''-y'-2y=e^{2x}\,(x+3)$$

EO $\quad y''-y'-2y=0$

$$\lambda^2-\lambda-2=0$$

$$\lambda=\frac{1\pm3}{2}=\begin{cases}-1=\lambda_1\\ 2=\lambda_2\end{cases}$$

lin. ind. $\quad y_1=e^{-x} \qquad y_2=e^{2x}$

EC. $\quad y''-y'-2y=e^{2x}(x+3)$

$$h=2=\lambda_2 \rightarrow s=1$$

$$\bar{y}=x^s\cdot p(x)\cdot e^{hx}=x(Ax+B)e^{2x}=(Ax^2+Bx)e^{2x}$$

$$\bar{y}'=(2Ax+B)e^{2x}+2(Ax^2+Bx)e^{2x}=$$
$$=(2Ax^2+2Ax+2Bx+B)e^{2x}$$

$$\bar{y}''=(4Ax+2A+2B)e^{2x}+2(2Ax^2+2Ax+2Bx+B)e^{2x}=$$

$$(4Ax^2+4Ax+4Bx+2B)$$

$$=(4Ax^2+8Ax+4Bx+2A+4B)e^{2x}$$

$$y''-y'-2y=e^{2x}(x+3)$$

$$\Downarrow$$

$$(4Ax^2+8Ax+4Bx+2A+4B)\cancel{e^{2x}}-$$
$$(2Ax^2+2Ax+2Bx+B)\cancel{e^{2x}}-$$
$$(2Ax^2+2Bx)\cancel{e^{2x}}=\cancel{e^{2x}}(x+3)$$

$$\Downarrow$$

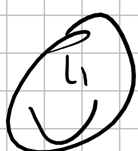
$$(6A)x+(2A-3B)=x+3$$

$$\begin{cases} 0=0 \\ 6A=1 \\ 2A+3B=3 \end{cases} \rightarrow \begin{cases} 0=0 \\ A=\frac{1}{6} \\ \frac{1}{3}+3B=3 \rightarrow B=\frac{8}{9} \end{cases}$$

$$\bar{y}=e^{2x}\left(\frac{1}{6}x^2+\frac{8}{9}x\right)$$

Int. generale:

$$e^{2x}\left(\frac{1}{6}x^2+\frac{8}{9}x\right)+k_1e^{2x}+k_2e^{-x}, \qquad k_1,k_2\in\mathbb{R}$$



4)

$$y'' + 3y' - 4y = 2x\, e^{3x}$$

E.O. $\quad y''+3y'-4y=0$

$$\lambda^2+3\lambda-4\lambda=0 \qquad \Delta=9+16=25$$

$$\lambda=\frac{-3\pm 5}{2} \begin{cases} 1 \\ -4 \end{cases}$$

Int. generale di E.O: $K_1 e^{x}+K_2 e^{-4x} \quad K_1,K_2\in\mathbb{R}$

E.C. $\quad y''+3y'-4y=2x e^{3x}$

$2x \to 1°$ grado $\to p(x)=Ax+B$

$h=3 \neq 1, \neq -4 \qquad s=0$

$$\bar{y}=(Ax+B)e^{3x}$$

$$\bar{y}'=Ae^{3x}+3(Ax+B)e^{3x}=e^{3x}(3Ax+A+3B)$$

$$\bar{y}''=3e^{3x}(3Ax+A+3B)+e^{3x}\cdot 3A=e^{3x}(9Ax+6A+9B)$$

$$\cancel{e^{3x}}(9Ax+6A+9B)$$
$$+\cancel{e^{3x}}(9Ax+3A+9B)$$
$$-\cancel{e^{3x}}(4Ax+4B) \qquad =2x\cdot\cancel{e^{3x}}$$

$$\begin{cases} 14A = 2 \\ 9A + 14B = 0 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{7} \\ B = -\frac{9}{7 \cdot 14} = -\frac{9}{98} \end{cases}$$

$$\bar{y} = \left(\frac{1}{7}x - \frac{9}{98}\right) e^{3x}$$

Int. gen.

$$\left(\frac{1}{7}x - \frac{9}{98}\right)e^{3x} + K_1 e^{x} + K_2 e^{-4x}, \quad K_1, K_2 \in \mathbb{R}$$

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