30/06/2021

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1 \\ a_{n+1} = a_n e^{-|a_n|} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

f e φ

$f(t) = te^{-|t|}$

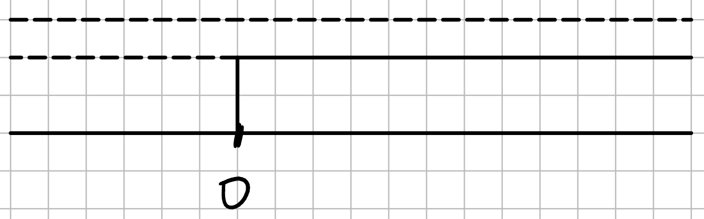
$$\varphi(t) = f(t) - t = te^{-|t|} - t = t\left(e^{-|t|} - 1\right)$$

Studiamo φ (e quindi i punti fissi)

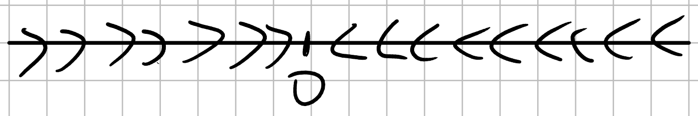
$\varphi(t) \geq 0$

$t \geq 0 \qquad e^{-|t|} - 1 \geq 0 \rightarrow e^{-|t|} \geq 1 \qquad -|t| \geq \log 1 \rightarrow$

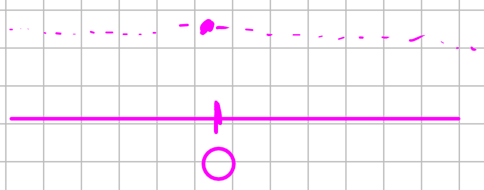
$$\rightarrow \begin{cases} -t \geq 0 \quad {\scriptstyle t\leq 0} \quad t \geq 0 & t = 0 \\ t \geq 0 \quad\quad\;\; t < 0 & \nexists\, t \end{cases}$$



$\varphi(t) \geq 0 \iff t \leq 0$



Il limite è 0



Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1 \\ a_{n+1} = a_n e^{-a_n^2} \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

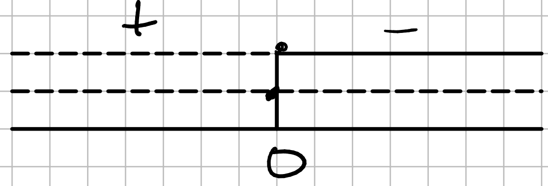
$f(t) = te^{-t^2}$

$\varphi(t) = f(t) - t = t(e^{-t^2} - 1)$

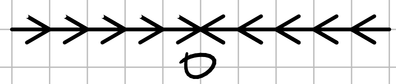
$\varphi(t) \geq 0$

$t \geq 0$

$e^{-t^2} - 1 \geq 0 \qquad e^{-t^2} \geq 1 \quad \Rightarrow \quad -t^2 \geq 0 \quad \Rightarrow \quad t^2 \leq 0 \qquad t = 0$



$\varphi(t) \geq 0 \iff t = 0$ → Punto fisso



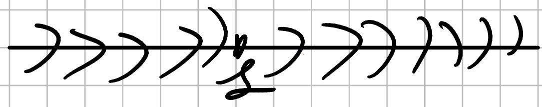
Il limite Tende a 0

$$\begin{cases} a_0 = \lambda \\ a_{n+1} = a_n^2 - a_n + 1 \quad \forall n \in \mathbb{N} \end{cases}$$

$f(t) = t^2 - t + 1$

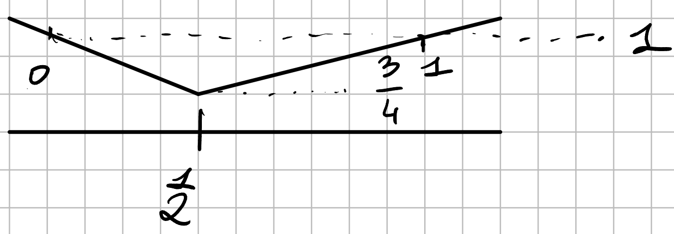
$\varphi(t) = t^2 - 2t + 1 = (t-1)^2$ $\qquad \varphi(t) \geq 0 \quad \forall t$

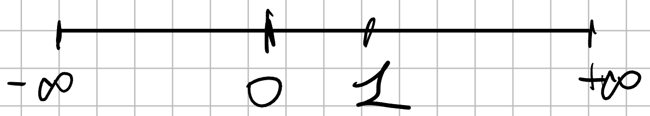
$t = 1$ Punto fisso



$f'(t) = 2t - 1$

$f'(t) > 0 \qquad t > \frac{1}{2}$





$t^2 - t + 1$

$\lim_{t \to -\infty} f(t) = +\infty$

$\lim_{t \to +\infty} f(x) = +\infty$

$f(0) = 1 \qquad f(1) = 1$

$f(\frac{1}{2}) = \frac{3}{4}$

$f(]-\infty, 0[) =]1, +\infty[$

$f(]0, 1[) =]\frac{3}{4}, 1[$

$f(]1, +\infty[) =]1, +\infty[$

Se $\lambda \in]-\infty, 0[\Rightarrow$ Il limite è $+\infty$

Se $\lambda \in]0, 1[\Rightarrow$ " 1

Se $\lambda \in]1, +\infty[\Rightarrow$ $+\infty$

$\lambda = 0 \Rightarrow 1 \qquad \lambda = 1 \Rightarrow 1$

09/09/21

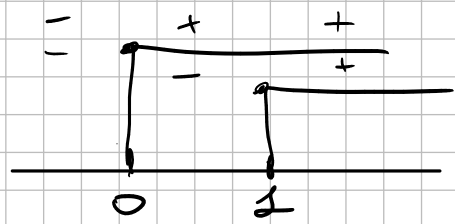
Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1/3 \\ a_{n+1} = \dfrac{a_n^2 + 2a_n}{3} \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

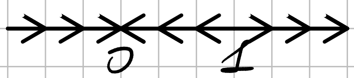
$$f(t) = \frac{t^2 + 2t}{3} \qquad \varphi(t) = \frac{t^2 - t}{3}$$

Studio Punti fissi

$\varphi(t) \geq 0 \quad t(t-1) \geq 0$



$\varphi(t) \geq 0 \iff t \leq 0 \vee t \geq 1$ Punti fissi: 0 e 1

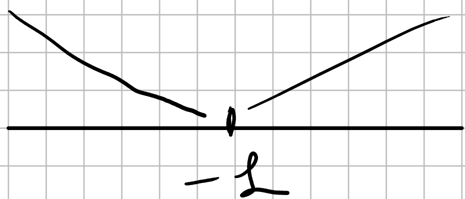


La funzione potrebbe
- convergere a 0
- divergere a $+\infty$

Studio Monotonia di f

$$f'(x) = \frac{2t+2}{3} = \frac{2}{3}(t+1)$$

$f'(x) > 0 \iff t > -1$



$f(0) = 0 \qquad f(1) = 1$

$f(]0,1[) =]0,1[$

Quindi, siccome $a_0 = \frac{1}{3}$

Il limite è 0

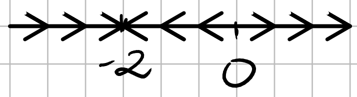
27/04/2021

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1/3 \\ a_{n+1} = \dfrac{a_n^2+7a_n}{5} \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$f(t) = \dfrac{t^2+7t}{5}$ $\qquad$ $\varphi(t) = \dfrac{t^2+2t}{5}$

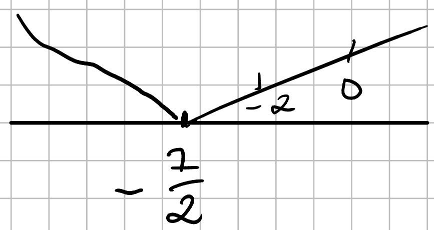
$\varphi(t) \geq 0 \qquad t(t+2) > 0 \quad \Rightarrow \quad t < -2 \vee t > 0$



Candidati: -2 e $+\infty$

$f'(t) = \dfrac{2t+7}{5}$

$f'(t) > 0 \qquad 2t+7 > 0 \qquad t > -\dfrac{7}{2}$



$\lim_{x \to +\infty} f(x) = +\infty$

$f(]0, +\infty[) =]0, +\infty[$

a_n tende a $+\infty$ $\forall n \in \mathbb{N}$

25/01/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \sqrt[4]{a_n^2+2} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

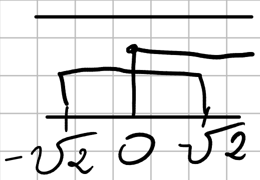
al variare del parametro reale λ.

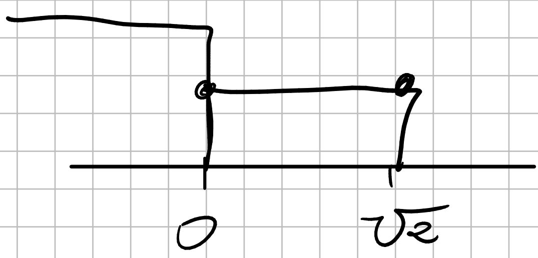
$f(t) = \sqrt[4]{t^2+2}$ $\qquad \varphi(t) = \sqrt[4]{t^2+2} - t$

$\varphi(t) \geq 0 \quad \sqrt[4]{t^2+2} \geq t$

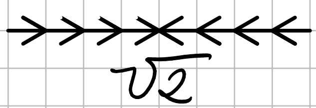
$$\begin{cases} t^2+2 \geq 0 \\ t \geq 0 \\ t^2+2 \geq t^4 \end{cases} \cup \begin{cases} t^2+2 \geq 0 & \forall t \\ t < 0 & t<0 \end{cases} \Rightarrow t<0$$

$$\begin{cases} \forall t \\ t \geq 0 \\ -\sqrt{2} \leq t \leq +\sqrt{2} \end{cases} \Rightarrow 0 \leq t \leq \sqrt{2}$$





$t \leq \sqrt{2}$



Limite: $\sqrt{2}$ $\qquad \forall \lambda$

$-t^4 + t^2 + 2 \geq 0$ $\qquad \Delta = 1 + 8 - 9$

$y = t^2$ $\qquad y = \frac{-1 \pm 3}{-2} \begin{cases} 2 \\ -1 \end{cases}$ $\qquad -1 \leq y \leq 2$

$-1 \leq t^2 \quad \forall t$

$t^2 \leq 2 \quad -\sqrt{2} \leq t \leq \sqrt{2}$

10/02/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \sqrt[3]{|a_n|^3 + 1} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

al variare del parametro reale λ.

$$f(t) = \sqrt[3]{|t|^3 + 1} \qquad \varphi(t) = \sqrt[3]{|t|^3 + 1} - t$$

① Studio della monotonia della successione

$$\varphi(t) > 0$$

$$\sqrt[3]{|t|^3 + 1} - t > 0 \Rightarrow |t|^3 + 1 > t^3$$

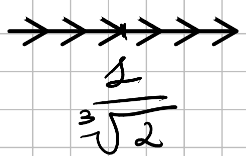
$$|t|^3 - t^3 + 1 > 0 \qquad \begin{cases} -2t^3 + 1 > 0 \\ t < 0 \end{cases} \cup \begin{cases} 1 > 0 \\ t \geq 0 \end{cases} \qquad \forall t$$

$$\begin{cases} -t^3 > -\frac{1}{2} \\ '' \end{cases}$$

$$\begin{cases} -t > -\frac{1}{\sqrt[3]{2}} \\ '' \end{cases} \Rightarrow t < \frac{1}{\sqrt[3]{2}}$$

$$t < 0 \quad \cup \quad t \geq 0$$

$$\varphi(t) \geq 0 \quad \forall t$$

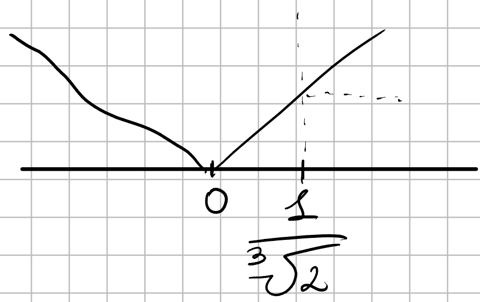


La successione potrebbe
- convergere a $\frac{1}{\sqrt[3]{2}}$
- divergere a $+\infty$

$$f'(t) = \frac{1}{\cancel{3}}\left(|t|^3+1\right)^{-\frac{2}{3}} \cdot \cancel{3}\,|t|^{\cancel{2}} \cdot \frac{t}{\cancel{|t|}}$$

$$= \frac{t \cdot |t|}{\sqrt[3]{\left(|t|^3+1\right)^2}} > 0$$

$$f'(t) > 0 \iff t > 0$$



$$f\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{1}{\sqrt[3]{2}}$$

$$f(0) = 1$$

$$\lim_{x \to -\infty} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$f\left(\left]-\infty, \frac{1}{\sqrt[3]{2}}\right[\right) = \left]1, +\infty\right[$$

$$f\left(\left]\frac{1}{\sqrt[3]{2}}, +\infty\right[\right) = \left]\frac{1}{\sqrt[3]{2}}, +\infty\right[$$

Se

$\lambda = \frac{1}{\sqrt[3]{2}} \Rightarrow$ la successione è costante

$\lambda > \frac{1}{\sqrt[3]{2}} \Rightarrow a_n$ tende a $+\infty$ $\forall n \in \mathbb{N}$

$\lambda < \frac{1}{\sqrt[3]{2}} \Rightarrow a_n$ tende a $+\infty$ $\forall n \in \mathbb{N}, n > 2$

13/04/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \dfrac{a_n}{a_n^2+1} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

al variare del parametro reale λ.

$$f(t) = \frac{t}{t^2+1} \qquad \varphi(t) = \frac{t - t(t^2+1)}{t^2+1} = \frac{-t^3}{t^2+1}$$

$$\varphi(t) \geq 0 \qquad t^3 \leq 0 \qquad t \leq 0$$

$$t^2+1 > 0 \qquad \forall t$$

$$\varphi(t) \geq 0 \iff t \leq 0 \qquad \rightarrow\rightarrow\underset{0}{\times}\leftarrow\leftarrow$$

06/07/2022

Esercizio 3. Determinare il limite della successione

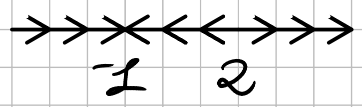
$$\begin{cases} a_1 = 3 \\ a_{n+1} = a_n^2 - 2 \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

$f(t) = t^2 - 2$ $\qquad \varphi(t) = t^2 - t - 2$

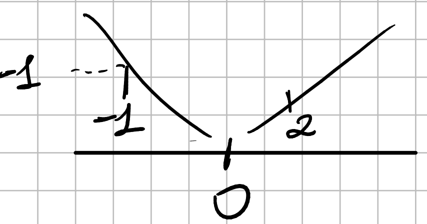
$\varphi(t) \geq 0 \qquad t^2 - t - 2 \geq 0 \qquad \Delta = 1 + 8 = 9$

$t = \dfrac{1 \pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$

$\varphi(t) \geq 0 \iff t < -2 \vee t > 1$



$f'(t) = 2t \qquad f'(t) > 0 \iff t > 0$



$f(]2, +\infty[) =]2, +\infty[$

$\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = +\infty$

Il limite è $+\infty$

28/07/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 2 \\ a_{n+1} = a_n^2 + |a_n| - 1 \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$f(t) = t^2 + |t| - 1$

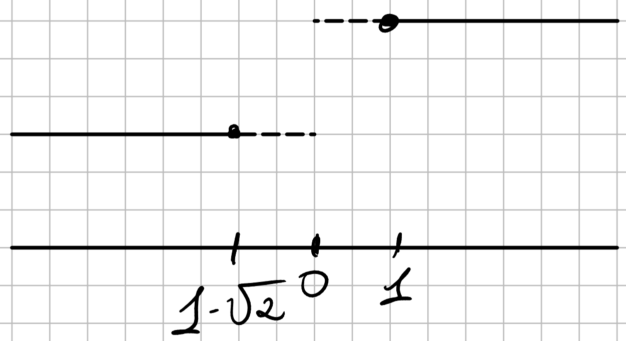
$$\varphi(t) = t^2 + |t| - 1 - t = \begin{cases} t^2 - 1 & t \geq 0 \\ t^2 - 2t - 1 & t < 0 \end{cases}$$

$t \leq -1 \vee t \geq 1 \qquad t \geq 0$

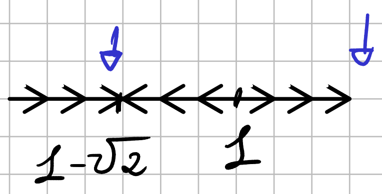
$\varphi(t) \geq 0$

$\Delta = 4 + 4 = 8 = 2\sqrt{2} \qquad t = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2} \quad t < 0$

$t \leq 1 - \sqrt{2} \vee t \geq 1 + \sqrt{2}$



$\varphi(t) \geq 0 \iff t \leq 1 - \sqrt{2} \vee t \geq 1$



Candidati: $1 - \sqrt{2}$, $+\infty$

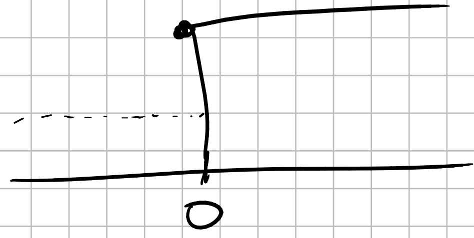
$\exists f'(t) \quad t \neq 0$

$$f'(t) = 2t + \frac{|t|}{t} = \begin{cases} 2t + 1 & t \geq 0 \\ 2t - 1 & t < 0 \end{cases}$$

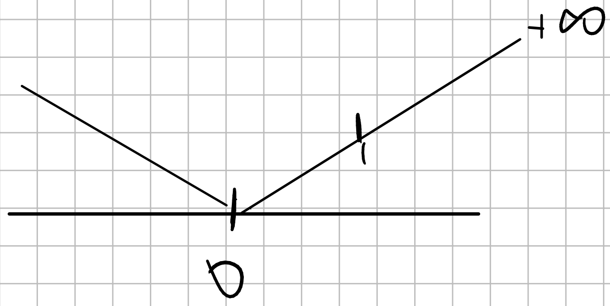
$f'(t) > 0$

$t > -\frac{1}{2}$ $t \geq 0$

$t > \frac{1}{2}$ $t < 0$



$f'(t) \geq 0 \iff t \geq 0$



$\lim_{x \to +\infty} f(x) = +\infty$

$f(]1, +\infty[) =]1, +\infty[$

Limite: $+\infty$

26/09/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \dfrac{1+a_n}{1+a_n^2} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

al variare del parametro reale λ.

$$F(t) : \frac{1+t}{1+t^2} \qquad \phi(t) : \frac{1+t}{1+t^2} - t$$

$$\frac{1+t-t(1+t^2)}{1+t^2}$$

$$\frac{1+\cancel{t}-\cancel{t}-t^3}{1+t^2}$$

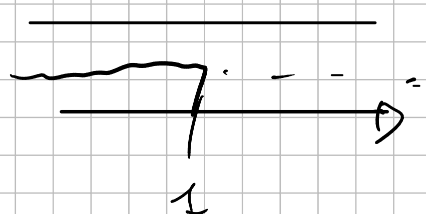
$$\phi(t) = -\frac{t^3+1}{1+t^2}$$

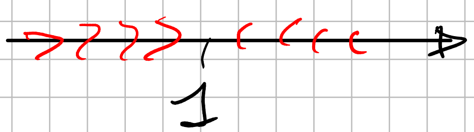
$$\phi(t) \geq 0 \Rightarrow -\frac{t^3+1}{1+t^2} \geq 0 \Rightarrow -t^3+1 \geq 0 \qquad 1+t^2 \geq 0$$

$$t^3 \leq 1 \qquad t^2 \geq -1$$

$$t \leq 1 \qquad \forall t$$

$$\phi(t) \geq 0 \iff t \leq 1$$





Il limite tende a 1

26/09/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \sqrt{1+|a_n|} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

al variare del parametro reale λ.

$$f(t) = \sqrt{1+|t|} \qquad \varphi(t) = \sqrt{1+|t|} - t$$

$$\varphi(t) \geq 0 \qquad \sqrt{1+|t|} \geq t$$

$$\begin{cases} 1+|t| \geq 0 \quad \forall t \\ t \geq 0 \\ 1+|t| \geq t^2 \Rightarrow t^2 - t - 1 \leq 0 \end{cases}$$

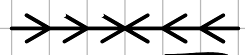
$$\Delta = 1+4 = 5$$

$$t = \frac{1 \pm \sqrt{5}}{2}$$

$$0 \leq t \leq \frac{1+\sqrt{5}}{2}$$

$$\begin{cases} 1+|t| \geq 0 \quad 1-t \geq 0 \Rightarrow t \leq 1 \\ t < 0 \end{cases}$$

$$\varphi(t) \geq 0 \iff t \leq \frac{1+\sqrt{5}}{2}$$



$$\frac{1+\sqrt{5}}{2}$$

Limite: $\dfrac{1+\sqrt{5}}{2} \quad \forall \lambda$

31/01/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 2 \\ a_{n+1} = a_n^3 + a_n^2 - 1 \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$f(t) = t^3 + t^2 - 1$

$\varphi(t) = t^3 + t^2 - t - 1$

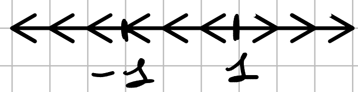
$\varphi(t) \geq 0 \qquad t^3 + t^2 - t - 1 > 0$

	1	1	-1	-1
1		1	2	1
	1	2	1	0

$(t-1)(t^2+2t+1) > 0$

$(t-1)(t+1)^2 > 0 \Rightarrow t > 1$

Punti fissi: $-1, 1$



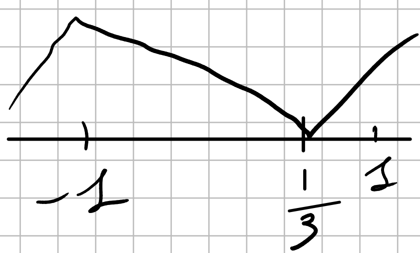
La successione può tendere a $-\infty, -1, +\infty$

$f'(t) = 3t^2 + 2t - 1$

$\Delta = 4 + 12 = 16$

$t = \dfrac{-2 \pm 4}{6} \begin{cases} -1 \\ \frac{1}{3} \end{cases}$

$f'(t) > 0 \iff t < -1 \vee t > \frac{1}{2}$



$f(1) = 1$

$f(]1, +\infty[) = f(]1, +\infty[)$

Il limite è $+\infty$

17/02/2023

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 2 \\ a_{n+1} = a_n |e^{a_n} - 1| \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$f(t) = t|e^t - 1|$ $\qquad\qquad$ $\varphi(t) = t|e^t - 1| - t$

$\varphi(t) \geq 0$

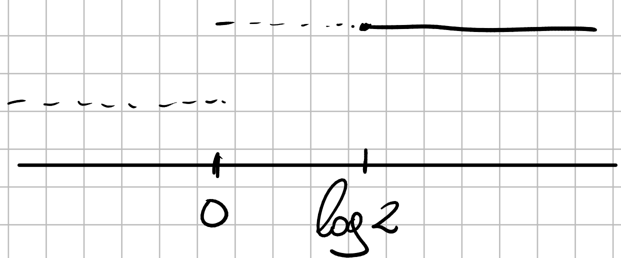
$t \geq 0$

$t(|e^t - 1| - 1) \geq 0$ $\qquad$ $|e^t - 1| - 1 \geq 0$

$$\begin{cases} e^t - 2 \geq 0 \to e^t - 2 \geq 0 \to t \geq \log 2 \\ e^t - 1 \geq 0 \to t \geq \log 1 \to t \geq 0 \end{cases}$$

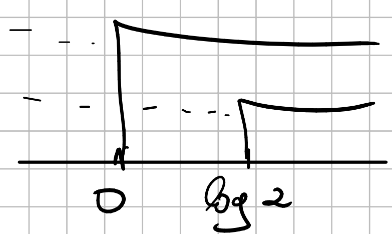
$\cup$

$$\begin{cases} -e^t \geq 0 \to e^t \leq 0 \to \nexists t \\ t < 0 \end{cases}$$

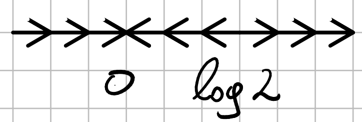


Punto fissi:

$0, \log 2$



$t < 0 \vee t > \log 2$



Possibili limiti:

$0, +\infty$

$$f(t) = \begin{cases} te^t - t & t \geq 0 \\ t - te^t & t < 0 \end{cases}$$

$$f'(t) = \begin{cases} e^t + te^t - 1 & t \geq 0 \\ 1 - e^t - te^t & t < 0 \end{cases}$$

$$f'(t) > 0 \rightarrow \begin{cases} e^t(1+t) > 1 & t \geq 0 \\ e^t(1+t) < 1 & t < 0 \end{cases}$$

$e^t > 1$ $\quad$ $1 + t > 1$ $\quad$ $e^t > 1$ $\quad$ $1 + t > 1$

$\Downarrow$ $\quad$ $\Downarrow$ $\quad\quad\quad\quad$ $\Downarrow$

$t = 0$ $\quad$ $t = 0$ $\quad$ $t > 0$ $\quad$ $t > 0$

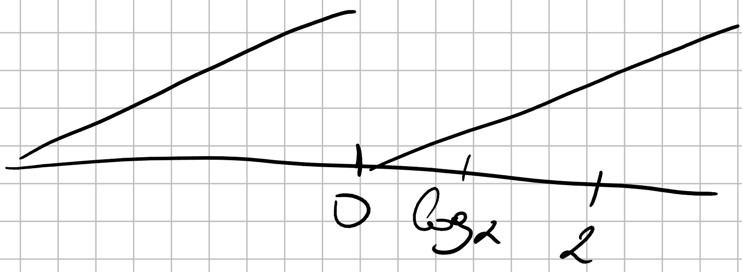
$\Downarrow$

$t \neq 0$ $\quad\quad\quad\quad$ $t < 0$

$\Downarrow$

$t > 0$

$$f'(t) > 0 \Leftrightarrow \forall t,\ t \neq 0$$



$\lim_{t \to +\infty} f(t) = +\infty$

$f(]\log 2, +\infty[) =]\log 2, +\infty[$

Limite : $+\infty$

04/04/2023

Esercizio 3. Determinare il limite della successione

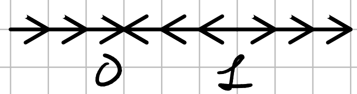
$$\begin{cases} a_1 = 2 \\ a_{n+1} = 2a_n^2 - |a_n| \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$$\varphi(t) = 2t^2 - |t| - t = \begin{cases} 2t^2 - 2t & t \geq 0 \\ 2t & t < 0 \end{cases}$$

$$\varphi(t) > 0$$

$$\begin{cases} 2t^2 - 2t > 0 & t \geq 0 \quad 2t(t-1) > 0 \to t > 1 \\ 2t^2 > 0 & t < 0 \quad \forall t \end{cases}$$

$$\varphi(t) > 0 \iff t < 0 \vee t > 1$$



$$f'(t) = \begin{cases} 4t + 1 & t \geq 0 \\ 4t - 1 & t \leq 0 \end{cases}$$

$$\begin{cases} t > -\frac{1}{4} & \to t \geq 0 \\ t > \frac{1}{4} & \to \nexists t \end{cases} \to t \geq 0$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

Limite: $+\infty$