30/06/2021

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1 \\ a_{n+1} = a_n e^{-|a_n|} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

f e φ

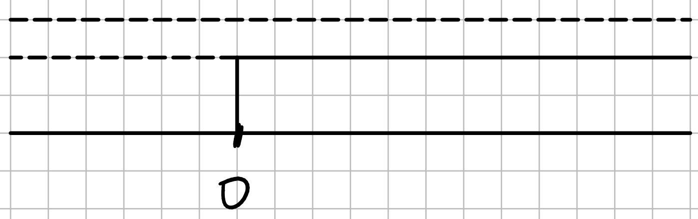
$$f(t) = te^{-|t|} \qquad \varphi(t) = f(t) - t = te^{-|t|} - t = t\left(e^{-|t|} - 1\right)$$

Studiamo φ (e quindi i punti fissi)

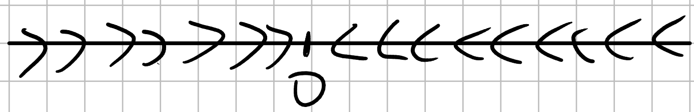
$\varphi(t) \geq 0$

$t \geq 0 \quad e^{-|t|} - 1 \geq 0 \to e^{-|t|} \geq 1 \qquad -|t| \geq \log 1 \to$

$$\to \begin{cases} -t \geq 0 \xrightarrow{t<0} & t \geq 0 \quad \nexists t \\ t \geq 0 & t < 0 \quad \nexists t \end{cases}$$



$\varphi(t) \geq 0 \iff t \leq 0$



Il limite è 0

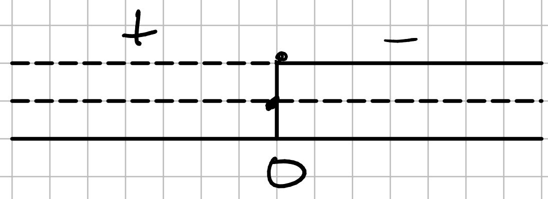
Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1 \\ a_{n+1} = a_n e^{-a_n^2} \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$f(t) = te^{-t^2}$

$\varphi(t) = f(t) - t = t(e^{-t^2} - 1)$

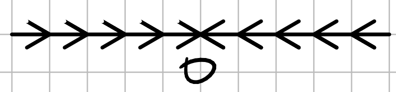
$\varphi(t) \geq 0$



$t \geq 0$

$e^{-t^2} - 1 \geq 0 \qquad e^{-t^2} \geq 1 \quad \Rightarrow \quad -t^2 \geq 0 \quad \Rightarrow \quad t^2 \leq 0 \qquad t = 0$

$\varphi(t) \geq 0 \iff t = 0 \Rightarrow$ Punto fisso



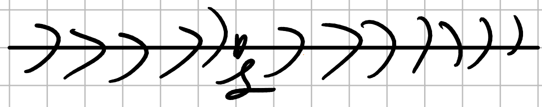
Il limite Tende a 0

$$\begin{cases} a_0 = \lambda \\ a_{n+1} = a_n^2 - a_n + 1 \quad \forall n \in \mathbb{N} \end{cases}$$

$f(t) = t^2 - t + 1$

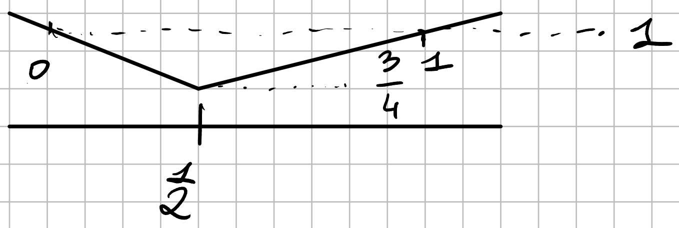
$\varphi(t) = t^2 - 2t + 1 = (t-1)^2 \qquad \varphi(t) \geq 0 \quad \forall t$

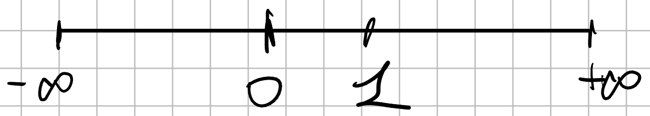
$t = 1$ Punto fisso



$f'(t) = 2t - 1$

$f'(t) > 0 \qquad t > \frac{1}{2}$





$t^2 - t + 1$

$\lim_{t \to -\infty} f(t) = +\infty$

$\lim_{t \to +\infty} f(x) = +\infty$

$f(0) = 1 \qquad f(1) = 1$

$f(\frac{1}{2}) = \frac{3}{4}$

$f(]-\infty, 0[) =]1, +\infty[$

$f(]0, 1[) =]\frac{3}{4}, 1[$

$f(]1, +\infty[) =]1, +\infty[$

Se $\lambda \in$ $]-\infty, 0[$ $\to$ Il limite è $+\infty$

$]0, 1[$ $\to$ " 1

$]1, +\infty[$ $\to$ $+\infty$

$\lambda = 0 \to 1 \qquad \lambda = 1 \to 1$

03/09/21

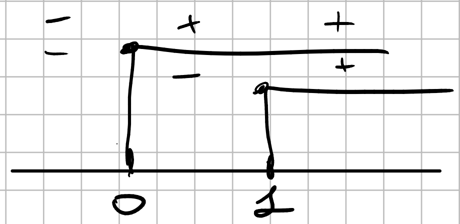
Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1/3 \\ a_{n+1} = \dfrac{a_n^2 + 2a_n}{3} \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$$f(t) = \frac{t^2 + 2t}{3} \qquad \varphi(t) = \frac{t^2 - t}{3}$$

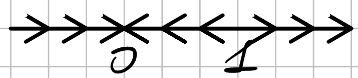
Studio Punti fissi

$\varphi(t) \geq 0 \quad t(t-1) \geq 0$



$\varphi(t) \geq 0 \iff t \leq 0 \vee t \geq 1$

Punti fissi: 0 e 1

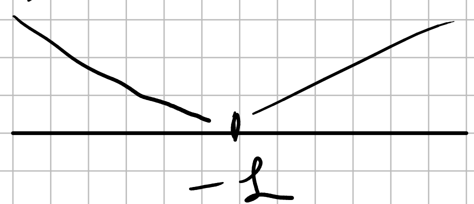


La funzione potrebbe

- convergere a 0
- divergere a $+\infty$

Studio Monotonia di f

$$f'(x) = \frac{2t+2}{3} = \frac{2}{3}(t+1)$$



$f'(x) > 0 \iff t > -1$

$f(0) = 0 \qquad f(1) = 1$

$f(]0,1[) =]0,1[$

Quindi, siccome $a_0 = \frac{1}{3}$

Il limite è 0

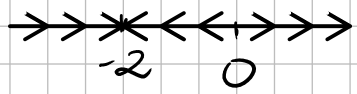
27/04/2021

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 1/3 \\ a_{n+1} = \dfrac{a_n^2+7a_n}{5} \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$$f(t) = \frac{t^2+7t}{5} \qquad \varphi(t) = \frac{t^2+2t}{5}$$

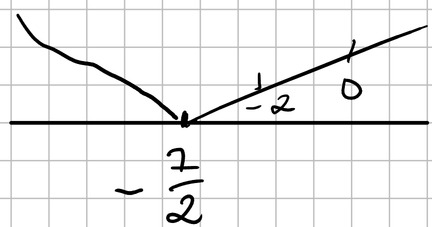
$$\varphi(t) \geq 0 \qquad t(t+2) \geq 0 \quad \to \quad t<-2 \vee t>0$$



Candidati: -2 e $+\infty$

$$f'(t) = \frac{2t+7}{5}$$

$$f'(t) > 0 \qquad 2t+7>0 \qquad t > -\frac{7}{2}$$



$$\lim_{x \to +\infty} f(x) = +\infty$$

$$f(]0,+\infty[) =]0,+\infty[$$

a_n tende a $+\infty$ $\forall n \in \mathbb{N}$

25/01/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \sqrt[4]{a_n^2 + 2} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

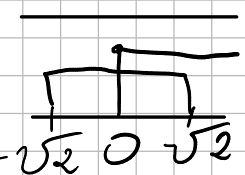
al variare del parametro reale λ.

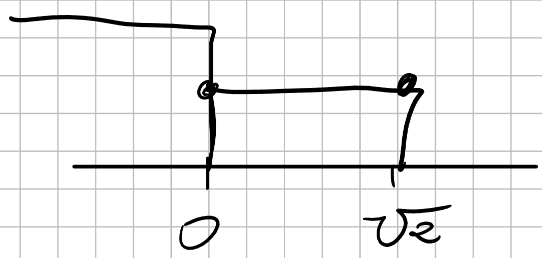
$f(t) = \sqrt[4]{t^2+2}$ $\qquad \varphi(t) = \sqrt[4]{t^2+2} - t$

$\varphi(t) \geq 0 \qquad \sqrt[4]{t^2+2} \geq t$

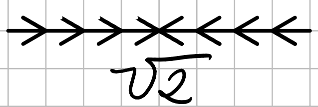
$$\begin{cases} t^2+2 \geq 0 \\ t \geq 0 \\ t^2+2 \geq t^4 \end{cases} \cup \begin{cases} t^2+2 \geq 0 & \forall t \\ t < 0 & t<0 \end{cases} \Rightarrow t<0$$

$$\begin{cases} \forall t \\ t \geq 0 \\ -\sqrt{2} \leq t \leq +\sqrt{2} \end{cases}$$





$t \leq \sqrt{2}$



Limite: $\sqrt{2} \qquad \forall \lambda$

10/02/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \sqrt[3]{|a_n|^3 + 1} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

al variare del parametro reale λ.

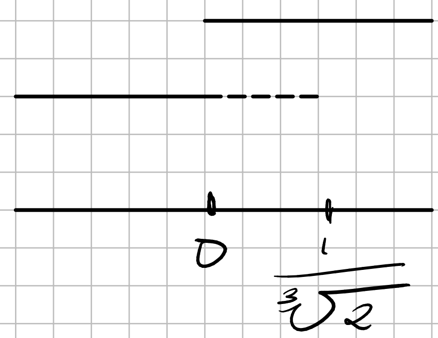
$f(t) = \sqrt[3]{|t|^3 + 1}$ $\qquad$ $\varphi(t) = \sqrt[3]{|t|^3+1} - t$

$\varphi(t) \geq 0 \qquad \sqrt[3]{|t|^3+1} \geq t$

$|t|^3 + 1 - t^3 \geq 0$

$t \geq 0: \quad \forall t$

$t < 0: \quad t^3 \leq \frac{1}{2} \Rightarrow t \leq \frac{1}{\sqrt[3]{2}}$



$\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow$

$\forall t$

Il limite è $+\infty$

13/04/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \dfrac{a_n}{a_n^2+1} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

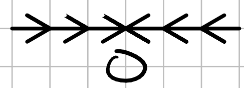
al variare del parametro reale λ.

$$f(t) = \frac{t}{t^2+1} \qquad \varphi(t) = \frac{t - t(t^2+1)}{t^2+1} = \frac{-t^3}{t^2+1}$$

$\varphi(t) \geq 0$ $\qquad t^3 \leq 0 \qquad t \leq 0$

$\qquad t^2+1 > 0 \qquad \forall t$

$\varphi(t) \geq 0 \iff t \leq 0$



06/07/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 3 \\ a_{n+1} = a_n^2 - 2 \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

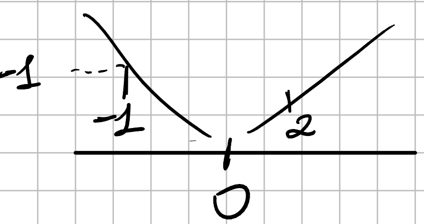
$f(t) = t^2 - 2$ $\qquad$ $\varphi(t) = t^2 - t - 2$

$\varphi(t) \geq 0$ $\qquad$ $t^2 - t - 2 \geq 0$ $\qquad$ $\Delta = 1 + 8 = 9$

$t = \dfrac{1 \pm 3}{2} < {2 \atop -1}$

$\varphi(t) \geq 0 \iff t < -2 \vee t > 1$

$f'(t) = 2t$ $\qquad$ $f'(t) > 0 \iff t > 0$



$f(]2, +\infty[) =]2, +\infty[$

$\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = +\infty$

Il limite è $+\infty$

28/07/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = 2 \\ a_{n+1} = a_n^2 + |a_n| - 1 \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$f(t) = t^2 + |t| - 1$

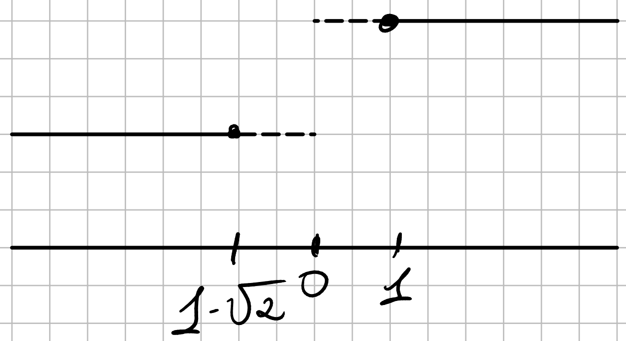
$$\varphi(t) = t^2 + |t| - 1 - t = \begin{cases} t^2 - 1 & t \geq 0 \\ t^2 - 2t - 1 & t < 0 \end{cases}$$

$\varphi(t) \geq 0$

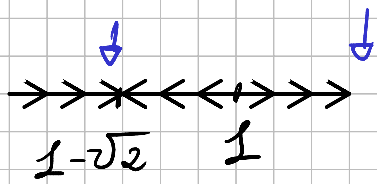
$t \leq -1 \vee t \geq 1 \qquad t \geq 0$

$\Delta = 4 + 4 = 8 = 2\sqrt{2} \qquad t = \dfrac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2} \quad t < 0$

$t \leq 1 - \sqrt{2} \vee t \geq 1 + \sqrt{2}$



$\varphi(t) \geq 0 \iff t \leq 1 - \sqrt{2} \vee t \geq 1$



Candidati: $1 - \sqrt{2}$

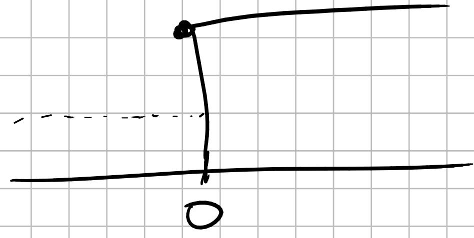
$+\infty$

$\exists f'(t) \quad t \neq 0$

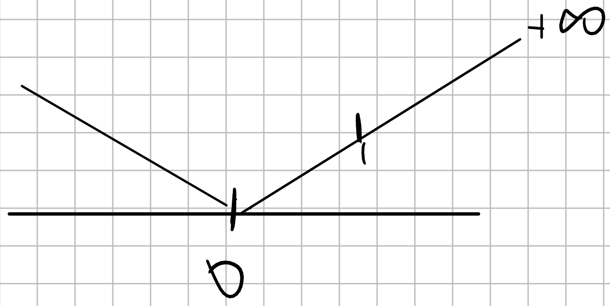
$$f'(t) = 2t + \frac{|t|}{t} = \begin{cases} 2t + 1 & t > 0 \\ 2t - 1 & t < 0 \end{cases}$$

$f'(t) > 0$ $\quad$ $t > -\frac{1}{2}$ $t \geq 0$

$t > \frac{1}{2}$ $t < 0$



$f'(t) \geq 0 \iff t \geq 0$



$\lim_{x \to +\infty} f(x) = +\infty$

$f(]1, +\infty[) =]1, +\infty[$

Limite: $+\infty$

26/09/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \dfrac{1+a_n}{1+a_n^2} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

al variare del parametro reale λ.

$$F(t): \quad \frac{1+t}{1+t^2} \qquad \phi(t): \frac{1+t}{1+t^2} - t$$

$$\frac{1+t-t(1+t^2)}{1+t^2}$$

$$\frac{1+\cancel{t}-\cancel{t}-t^3}{1+t^2}$$

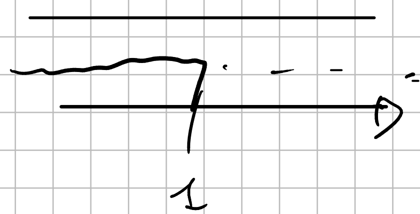
$$\phi(t) = -\frac{t^3+1}{1+t^2}$$

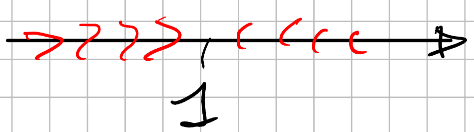
$$\phi(t) \geq 0 \Rightarrow -\frac{t^3+1}{1+t^2} \geq 0 \Rightarrow -t^3+1 \geq 0 \qquad 1+t^2 \geq 0$$

$$t^3 \leq 1 \qquad t^2 \geq -1$$

$$t \leq 1 \qquad \forall t$$

$$\phi(t) \geq 0 \iff t \leq 1$$





Il limite tende a 1

26/09/2022

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \sqrt{1+|a_n|} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

al variare del parametro reale λ.

$$f(t) = \sqrt{1+|t|} \qquad \varphi(t) = \sqrt{1+|t|} - t$$

$$\varphi(t) \geq 0 \qquad \sqrt{1+|t|} \geq t$$

$$\begin{cases} 1+|t| \geq 0 \quad \forall t \\ t \geq 0 \\ 1+|t| \geq t^2 \to t^2 - t - 1 \leq 0 \end{cases}$$

$$\Delta = 1+4 = 5$$

$$t = \frac{1 \pm \sqrt{5}}{2}$$

$$0 \leq t \leq \frac{1+\sqrt{5}}{2}$$

$$\begin{cases} 1+|t| \geq 0 \quad 1-t \geq 0 \to t \leq 1 \\ t < 0 \end{cases}$$

$$\varphi(t) \geq 0 \iff t \leq \frac{1+\sqrt{5}}{2}$$

$$\frac{1+\sqrt{5}}{2}$$

Limite: $\frac{1+\sqrt{5}}{2} \quad \forall \lambda$