10/02/2022

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \sqrt[3]{|a_n|^3 + 1} \end{cases}$$

$f(t) = \sqrt[3]{|t|^3 + 1}$ $\qquad$ $\varphi(t) = \sqrt[3]{|t|^3 + 1} - t$

① Studio della monotonia della Successione

$\varphi(t) > 0$

$\sqrt[3]{|t|^3 + 1} - t > 0 \Rightarrow |t|^3 + 1 > t^3$

$$|t|^3 - t^3 + 1 > 0 \qquad \begin{cases} -2t^3 + 1 > 0 \\ t < 0 \end{cases} \cup \begin{cases} 1 > 0 \\ t \geq 0 \end{cases} \qquad \forall t$$

$$\begin{cases} -t^3 > -\frac{1}{2} \\ " \end{cases}$$

$$\begin{cases} -t > -\frac{1}{\sqrt[3]{2}} \\ " \end{cases} \Rightarrow t < \frac{1}{\sqrt[3]{2}}$$

$$t < 0 \quad \cup \quad t \geq 0$$

$$\varphi(t) \geq 0 \quad \forall t$$

$\frac{1}{\sqrt[3]{2}}$

La successione potrebbe
- convergere a $\frac{1}{\sqrt[3]{2}}$
- divergere a $+\infty$

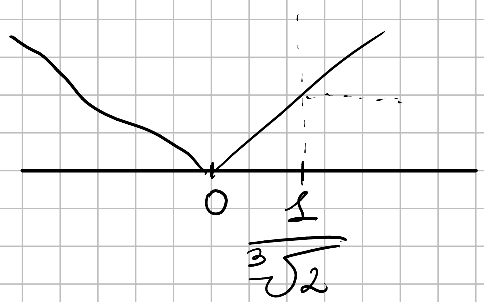
$$f'(t) = \frac{1}{\cancel{3}}(|t|^3+1)^{-\frac{2}{3}} \cdot \cancel{3} \cdot |t|^{\cancel{2}} \cdot \frac{t}{\cancel{|t|}}$$

$$= \frac{t \cdot |t|}{\sqrt[3]{(|t|^3+1)^2}} > 0$$

$$f'(t) > 0 \iff t > 0$$

$$f\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{1}{\sqrt[3]{2}}$$

$$f(0) = 1$$



$$\lim_{x \to -\infty} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$f\left(\left]-\infty, \frac{1}{\sqrt[3]{2}}\right[\right) = \left]1, +\infty\right[$$

$$f\left(\left]\frac{1}{\sqrt[3]{2}}, +\infty\right[\right) = \left]\frac{1}{\sqrt[3]{2}}, +\infty\right[$$

Se

$\lambda = \frac{1}{\sqrt[3]{2}} \to$ la successione è costante.

$\lambda > \frac{1}{\sqrt[3]{2}} \to a_n$ tende a $+\infty$ $\forall n \in \mathbb{N}$

$\lambda < \frac{1}{\sqrt[3]{2}} \to a_n$ tende a $+\infty$ $\forall n \in \mathbb{N}, n > 2$