30 / 06 / 2021

Esercizio 2. Studiare la funzione definita dalla legge

$$\frac{1}{x \log x}$$

e tracciarne un grafico qualitativo.

$$f(x) = \frac{1}{x \log x}$$

① Dominio

$x \neq 0$ $\quad$ $x > 0$

$\log x \neq 0 \to x \neq 1$

$]0, 1[\cup]1, +\infty[$

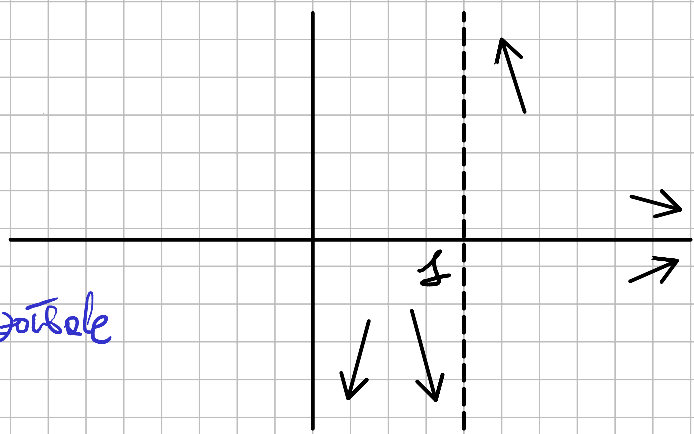
② Limiti nei punti di frontiera

$$\lim_{x \to 0^+} f(x) = -\infty$$

$$\lim_{x \to 1^-} f(x) = -\infty$$

$$\lim_{x \to 1^+} f(x) = +\infty$$

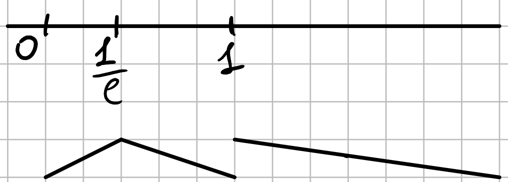
$$\lim_{x \to +\infty} f(x) = 0 \to \text{Asintoto orizzontale}$$



③ Derivata prima e monotonia

$$f'(x) = -\frac{\log x + x\frac{1}{x}}{(x \log x)^2} = -\frac{\log x + 1}{(x \log x)^2}$$

$$f'(x) > 0 \iff 1 + \log x > 0 \to \log x < -1 \to x < \frac{1}{e}$$



④ Derivata Seconda e Convessità

$$f''(x) = -\frac{\frac{1}{x}(x\log x)^2 - (\log x + 1)(x \log x)2(\log x + 1)}{(x \log x)^4}$$

$$= -\frac{\frac{1}{x}(x \log x)^2 - 2(\log x + 1)^2(x \log x)}{(x \log x)^4} = \frac{(x \log x)\left[\frac{1}{x} x \log x - 2(\log x + 1)^2\right]}{(x \log x)^4}$$

CALCOLI SBAGLIATI !

$$= \frac{(x\log x)\left[\log x - 2(\log x + 1)^2\right]}{(x \log x)^4}$$

$$f''(x) > 0 \iff (x\log x)\left[\log x - 2(\log x+1)^2\right]$$

$0 < x < 1$

$(x \log x) < 0 \quad \forall x$

$\log x - 2(\log x + 1)^2 < 0$

$\log x - 2(\log^2 x + 2\log x + 1) < 0$

$\log x - 2\log^2 x - 4\log x - 1 < 0$

$-2\log^2 x - 3\log x - 1 < 0$

$y = \log x \qquad 2y + 3y + 1 = 0 \quad \Rightarrow \Delta = 9 - 8 = 1 \Rightarrow \quad y = \frac{-3 \pm 1}{4} \left\langle \begin{matrix} -1 \\ -\frac{1}{2} \end{matrix} \right.$

$y < -1 \vee y > -\frac{1}{2}$

$\log x < -1 \vee \log x > -\frac{1}{2}$

$x < -\frac{1}{e} \vee \ x > -\frac{1}{\sqrt{e}}$

$\Downarrow$

$0 < x < 1$

Quindi $f''(x) > 0$

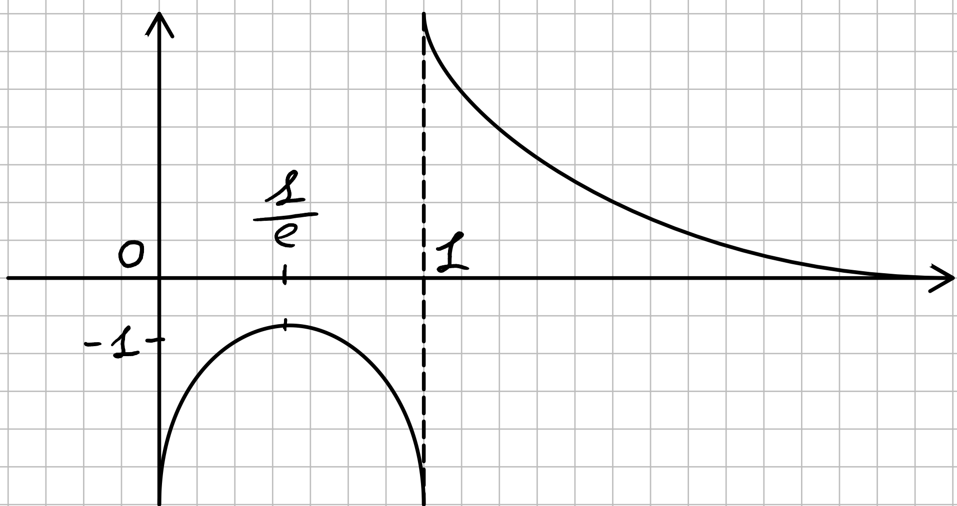
$\iff x \in \left]0, 1\right[$

$x > 1$

$x \log x > 0 \quad \forall x$

$\log x - 2(\log x + 1)^2 > 0$

$\nexists x$



27/07/2021

Esercizio 2. Studiare la funzione definita dalla legge

$$f(x) = x\sqrt{1-|x|}$$

e tracciarne un grafico qualitativo.

Dominio $1-|x| \geq 0 \Rightarrow |x| \leq 1$

$-1 \leq x \leq 1 \qquad x \in [-1, 1]$

Si nota subito che la funzione è dispari $(f(-x) = -f(x))$

Si studia per $x > 0$ $\quad x\sqrt{1-x} = x(1-x)^{\frac{1}{2}}$

Non sono presenti asintoti $\qquad f(0) = f(1) = 0$

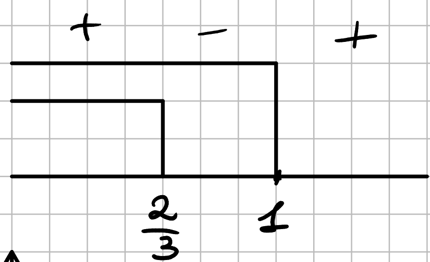
Derivata Prima $\qquad \exists f'(x) \quad x \neq 1$

$$f'(x) = \sqrt{1-x} - \frac{1}{2}x(1-x)^{-\frac{1}{2}} = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}} =$$

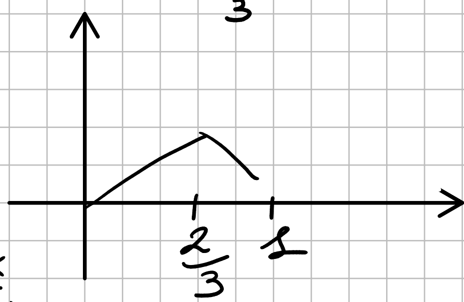
$$= \frac{2\sqrt{1-x}^2 - x}{2\sqrt{1-x}} = \frac{2-2x-x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}$$

$f'(x) > 0 \qquad 2-3x > 0 \Rightarrow x < \frac{2}{3}$

$\qquad 2\sqrt{1-x} > 0 \Rightarrow x < 1$



$x < \frac{2}{3} \vee x > 1$



$\exists f''(x) \quad x \neq 1$

$$f''(x) = \frac{-3(2\sqrt{1-x}) + \frac{2-3x}{\sqrt{1-x}}}{(2\sqrt{1-x})^2} = \frac{-6\sqrt{1-x} + \frac{2-3x}{\sqrt{1-x}}}{4(1-x)^2} =$$

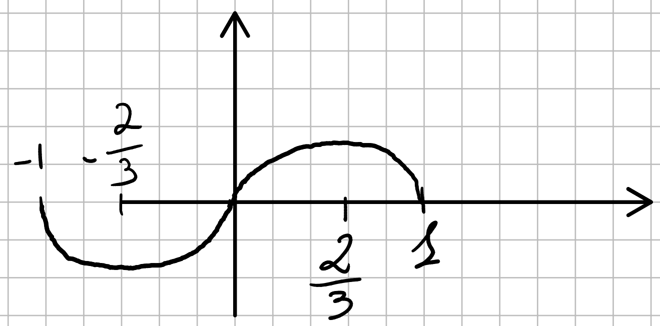
$$= \frac{\frac{6+2-3x}{\sqrt{1-x}}}{4(1-x)^2} = \frac{8-3x}{4(1-x)^2\sqrt{1-x}}$$

$f''(x) > 0$

$8 - 3x > 0 \Rightarrow x < \frac{8}{3}$

$4(1-x)^2 \sqrt{1-x} > 0 \Rightarrow 4\sqrt{1-x} < 0 \Rightarrow \sqrt{1-x} < 0 \quad \nexists x$

$f''(x) < 0 \quad \forall x$



$\lim_{x \to 1} f'(x) = \infty$

09/09/2021

Esercizio 2. Determinare gli eventuali estremi relativi e gli estremi assoluti della funzione definita dalla legge
$$\sqrt[3]{x^2(x-4)}.$$

Dominio

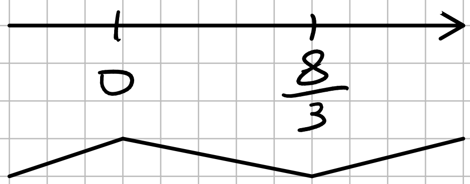
$\mathbb{R}$

$$f(x)=\left[x^2(x-4)\right]^{\frac{1}{3}}$$

$$f'(x)=\frac{1}{3}\left[x^2(x-4)\right]^{-\frac{2}{3}}\left[2x(x-4)+x^2\right]=$$

$$=\frac{1}{3}\frac{2x(x-4)+x^2}{\left[x^2(x-4)\right]^{\frac{2}{3}}}=\frac{3x^2-8x}{3\sqrt[3]{(x^2(x-4))^2}}$$

$$f'(x)>0 \iff 3x^2-8x>0 \Rightarrow x(3x-8)>0 \begin{cases} x=0 \\ x=\frac{8}{3} \end{cases} \quad 0<x \vee x>\frac{8}{3}$$



Estremi Relativi

$f(0)$ max

$f\left(\frac{8}{3}\right)$ min

$$\lim_{x\to+\infty} f(x)=+\infty$$

$$\lim_{x\to-\infty} f(x)=-\infty$$

Estremi Assoluti

sup $+\infty$

inf $-\infty$

27/09/2021

Esercizio 2. Determinare gli eventuali estremi relativi e gli estremi assoluti della funzione definita dalla legge

$$\sqrt{\frac{x}{1+x^2}}$$

Dominio $x \geq 0$

$\exists f'(x) \quad x \neq 0$

$$f'(x) = \frac{1}{2} \frac{1}{\sqrt{\frac{x}{1+x^2}}}\left[(1+x^2) - x(2x)\right] = \frac{1-x^2}{2\sqrt{\frac{x}{1+x^2}}}$$

$$f'(x) > 0 \iff 1-x^2 > 0 \Rightarrow -1 < x < 1 \Rightarrow 0 < x < 1$$

$$\lim_{x \to +\infty} f(x) = 0$$

$$f(0) = 0$$

$f(1)$ massimo relativo e assoluto

25/01/2022

Esercizio 2. Dare la definizione di punto di estremo relativo per una funzione. Determinare **poi** gli eventuali punti di estremo relativo per la funzione definita dalla legge

$$\sqrt{\left|\frac{x-1}{x+1}\right|}$$

nel suo campo di esistenza.

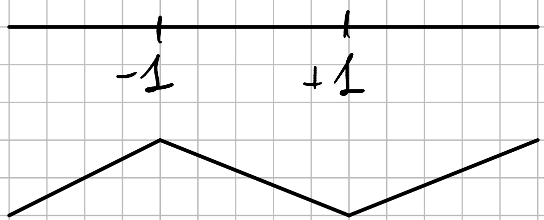
Dominio $\mathbb{R}\setminus\{-1\}$

$\exists f'(x) \quad x\neq 1$

$$f'(x)=\frac{1}{2\sqrt{\left|\frac{x-1}{x+1}\right|}}\cdot\left|\frac{x-1}{x+1}\right|\cdot\frac{\cancel{x+1}}{x-1}\cdot\frac{\cancel{x+1}\ \cancel{-x}+1}{(x+1)^{\cancel{2}}}=$$

$$=\underbrace{\frac{1}{\sqrt{\left|\frac{x-1}{x+1}\right|}}}_{>0}\cdot\underbrace{\left|\frac{x-1}{x+1}\right|}_{>0}\cdot\frac{1}{x^2-1}$$

$f'(x)>0 \iff x^2-1>0 \to x^2>1 \to x<-1 \vee x>1$



-1 non è massimo (non appartiene al limite)

Derivata prima in $x=1$

$\lim_{x\to 1^+} f'(x) = +\infty\cdot 0$ Forma Indeterminata

$$\underbrace{\frac{1}{\sqrt{\left|\frac{x-1}{x+1}\right|}}}_{\downarrow\ +\infty}\cdot\underbrace{\frac{\cancel{x-1}}{x+1}}_{\downarrow\ \frac{1}{2}}\cdot\underbrace{\frac{1}{(x+1)\cancel{(x-1)}}}_{\downarrow\ \frac{1}{2}}$$

$x\leq -1 \vee x\geq 1$

$\lim_{x\to 1^+} f'(x) \to +\infty$

$$\lim_{x\to 1^-} f'(x) =$$

$$= \lim_{x\to 1^-} \underbrace{\frac{1}{\sqrt{\left|\frac{x-1}{x+1}\right|}}}_{\downarrow\ +\infty} \cdot \underbrace{\frac{\cancel{x-1}}{-x-1}}_{\downarrow\ -\frac{1}{2}} \cdot \underbrace{\frac{1}{(x+1)\cancel{(x-1)}}}_{\downarrow\ \frac{1}{2}}$$

$$\lim_{x\to 1^-} f'(x) = -\infty$$

$f(x)$ non è derivabile in $x=1$

$$\lim_{x\to -\infty} f(x) = 1$$

$$f(1)=0$$

Quindi

$x=1 \to$ Minimo Assoluto

Non esiste massimo assoluto

$\sup f = +\infty \quad \inf f = 0$

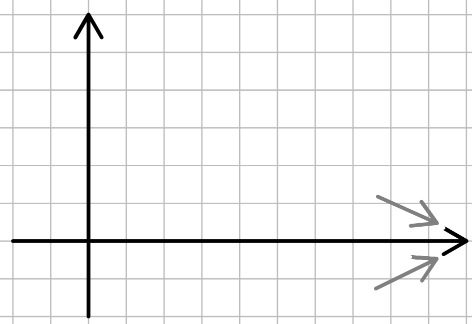
10/02/2022

Esercizio 2. Studiare la funzione definita dalla legge

$$\varphi(x) = \log\left(1 + e^{-|x|}\right)$$

e tracciarne un grafico qualitativo.

Dominio $\quad 1 + e^{-|x|} > 0 \quad \forall x \in \mathbb{R}$



La funzione è pari $\quad f(x) = \varphi(x) \quad x > 0$

$$\lim_{x \to +\infty} f(x) = \log(1) = 0$$

Studio Monotonia

$\exists f'(x)$

$$f'(x) = \frac{1}{1+e^{-x}} \cdot e^{-x} \cdot (-1) = -\frac{\cancel{e^{-x}}}{\cancel{e^{-x}}\left(\frac{1}{e^{-x}} + 1\right)} = -\frac{1}{e^x + 1}$$

$f'(x) < 0 \quad \forall x$

$\exists f''(x)$

$$f''(x) = \frac{\cancel{e^x + 1}}{(e^x + 1)^{\cancel{2}}} = \frac{1}{e^x + 1} \qquad f''(x) > 0 \quad \forall x$$

Come studiare in $x = 0$?

13/04/2022

$f(x) = \sqrt{|x^2-4|} - x$

Dominio $\mathbb{R}$

$|x| - x = \begin{cases} 0 & x > 0 \\ -2x & x < 0 \end{cases}$

$\lim_{x \to -\infty} f = +\infty$

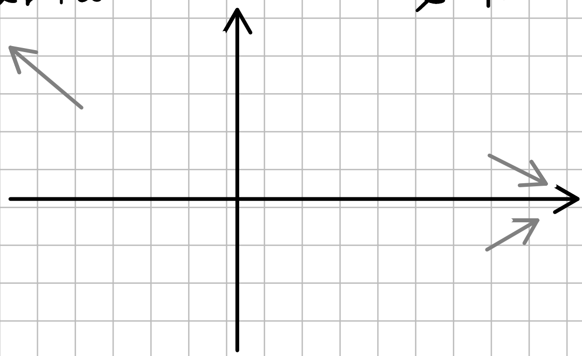
Potrebbe essere un asintoto obliquo

$\lim_{x \to -\infty} \frac{f(x)}{x} = -2$

$\lim_{x \to -\infty} f(x) + 2x = 0$

$\lim_{x \to +\infty} f = +\infty - \infty \sim \lim_{x \to +\infty} \sqrt{|x^2|} - x = 0$

Asintoto Orizzontale



C'è asintoto $y = -2x$

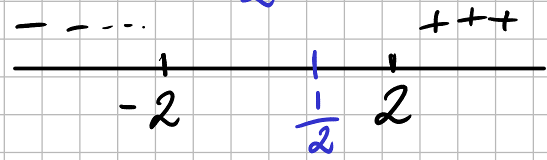
$\exists f'(x) \quad x^2 - 4 \neq 0 \Rightarrow x \neq \pm 2$

$$f'(x) = \underbrace{\frac{1}{2\sqrt{|x^2-4|}}}_{>0} \cdot \frac{x^2-4}{|x^2-4|} \cdot 2x - 1$$

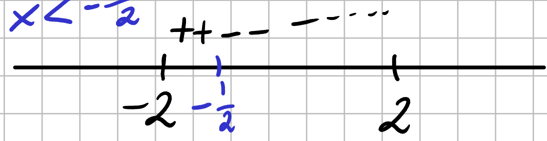
$$f'(x) > 0 \iff 2x \frac{x^2-4}{|x^2-4|} - 1 > 0$$

$$\begin{cases} 2x \frac{x^2-4}{\underbrace{x^2-4}_{>0}} - 1 & x < -2 \lor x > 2 \Rightarrow \\ -2x \frac{x^2-4}{x^2-4} - 1 & -2 < x < 2 \end{cases}$$

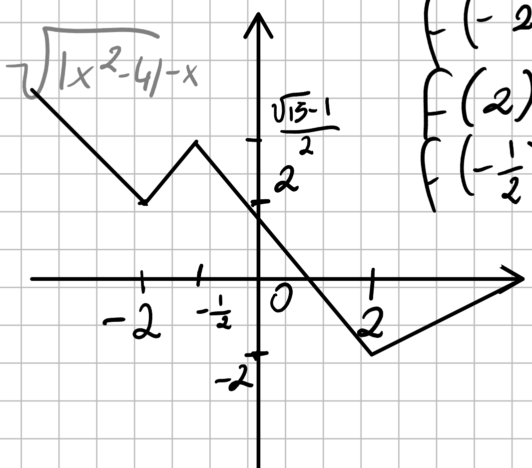
$x > \frac{1}{2}$



$x < -\frac{1}{2}$



$\frac{\sqrt{15}}{2} - \frac{1}{2}$

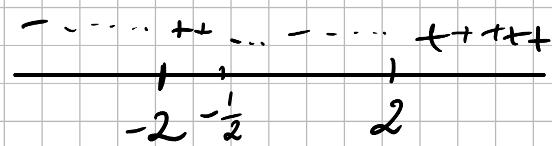


$f(-2) = 2$

$f(2) = -2$

$f\left(-\frac{1}{2}\right) = \frac{\sqrt{15}-1}{2}$

f'



$f'(x) > 0 \iff x \in \left]-2, -\frac{1}{2}\right[\cup \left]2, +\infty\right[$

$\exists f'(-2)$? No

$\lim_{x \to -2^-} f(x) = -5$

$\lim_{x \to -2^+} f(x) = 3$

$x = -2$ → Punto angoloso

$\exists f'(2)$? No

$\lim_{x \to 2^-} f(x) = -5$

$\lim_{x \to 2^+} f(x) = 3$

$x = 2$ Punto Angoloso

$\exists f''(x)$ (± 2 sono già esclusi)

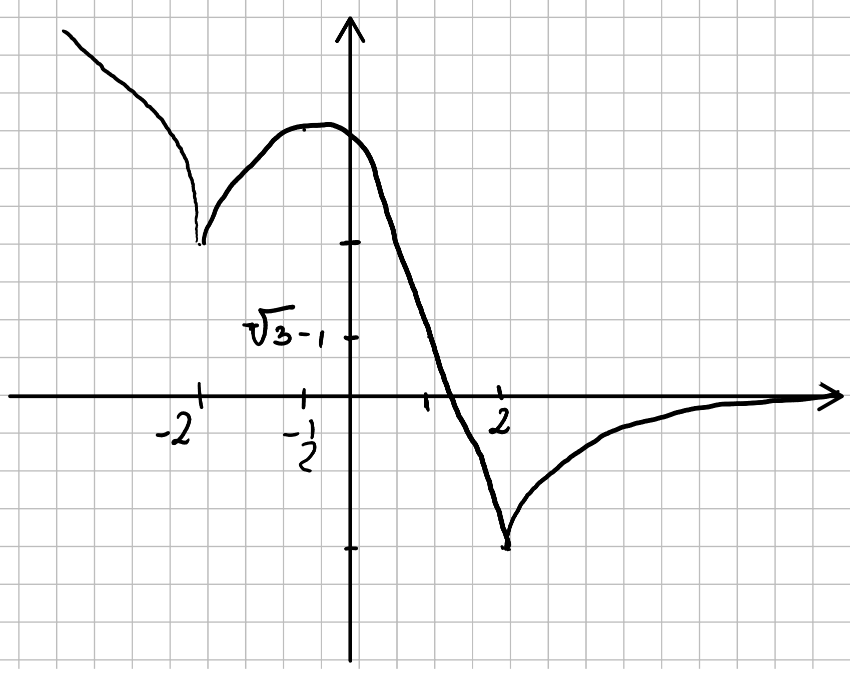
CALCOLI SBAGLIATI ! GUARDA 10/06/2023 xOPP

$$f''(x) = 2\frac{x^2-4}{|x^2-4|} + 2x\left\{\frac{2x|x^2-4| - (\cancel{x^2-4})\frac{|x^2-4|}{\cancel{x^2-4}}\cdot 2x}{|x^2-4|^2}\right\}$$

$$= 2\frac{x^2-4}{|x^2-4|}$$

$f''(x) > 0 \iff x^2-4 > 0 \qquad x < -2 \vee x > 2$

$f(1) = \sqrt{3} - 1$



Esercizio 2. Studiare la funzione definita dalla legge

$$f(x) = \sqrt{|x| + x^2}$$

e tracciarne un grafico qualitativo.

Dominio $x \neq 0$

$]-\infty, 0[\cup]0, +\infty[$

Le Funzione è pari

Studiamo f per $x > 0$ $\sqrt{x+x^2}$

$\lim_{x \to 0^+} f(x) = 0$ $\lim_{x \to +\infty} f(x) = +\infty$

$x=0$ Asintoto Verticale

Cerchiamo Asintoto Obliquo $\lim_{x \to +\infty} \frac{f(x)}{x} = \frac{+\infty}{+\infty}$ Serve $f'(x)$ per De L'Hopital

Studio Monotonia

$\exists f'(x)$ $x \neq 0$

$$f'(x) = \frac{1}{2\sqrt{x^2+x}} 2x = \frac{x}{\sqrt{x^2+x}}$$

$\lim_{x \to +\infty} \frac{1}{\sqrt{x^2+x}} = 0$

Non esiste

$f'(x) > 0 \iff x > 0$

$f'(x) \quad x=0$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{\sqrt{x^2+x}} \sim \frac{x}{\sqrt{x}} = \lim \frac{1}{\frac{1}{2\sqrt{x}}} = \lim 2\sqrt{x} = 0$$

$\lim_{x\to 0^-} f(x)$ Non esiste

Studio Convessità

$$\exists f''(x) = \frac{\sqrt{x^2+x} - x\frac{1}{\sqrt{x^2+x}}\cdot 2x}{x^2+x}$$

$$f''(x)>0 \iff \sqrt{x^2+x} - \frac{2x^2}{\sqrt{x^2+x}} > 0$$

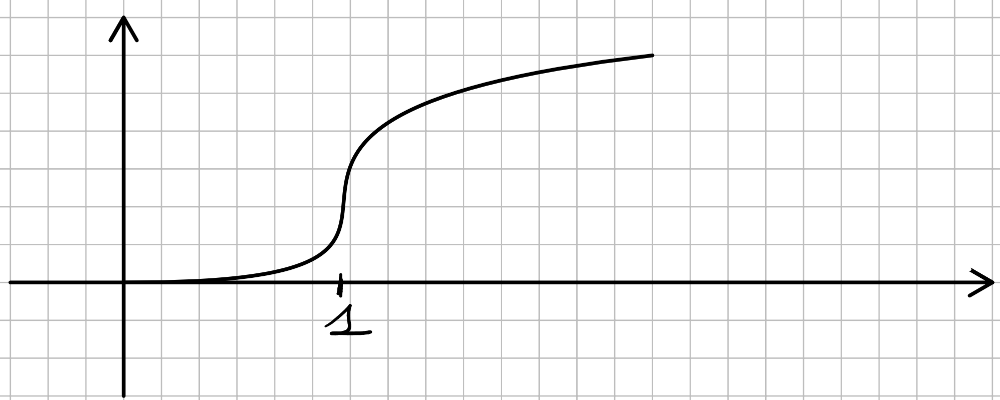
$$\frac{x^2+x-2x^2}{\sqrt{x^2+x}} = \frac{-x^2+x}{\sqrt{x^2+x}} > 0$$

$$x(-x+1) > 0$$

$$x=0 \vee x=1$$

$$0<x<1$$

$$f''(x)>0 \iff 0<x<1$$



Come studiare in $x=0$?

06/07/2022

Esercizio 2. Studiare la funzione definita dalla legge

$$\sqrt{|x|+x^2}$$

e tracciarne un grafico qualitativo.

Dominio $\mathbb{R}$

La funzione è pari $\forall x>0$

$\lim_{x\to+\infty} = +\infty$ → Ricerca Asintoto Obliquo

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+x}}{x} = 1 = m$$

$$\lim_{x\to+\infty} \sqrt{x^2+x} - x = \frac{1}{2} = q$$

$$\frac{x^2+x-x^2}{\sqrt{x^2+x}+x} = \frac{x}{x\sqrt{1}+x} = \frac{1}{2}$$

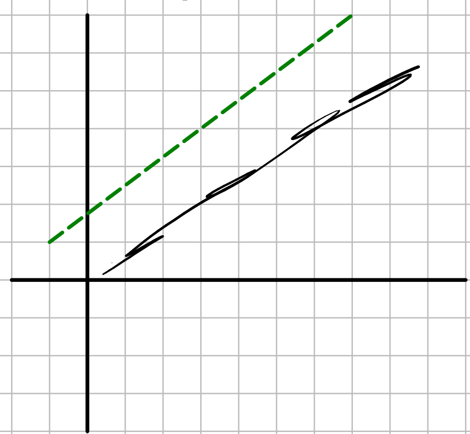
Monotonia

$\exists f'(x),\ x \neq 0$

$$f'(x) = \frac{2x+1}{2\sqrt{x^2+x}}$$

$f'(x)>0 \iff 2x+1>0 \to x>-\frac{1}{2} \to \forall x$

$f(0)=0$



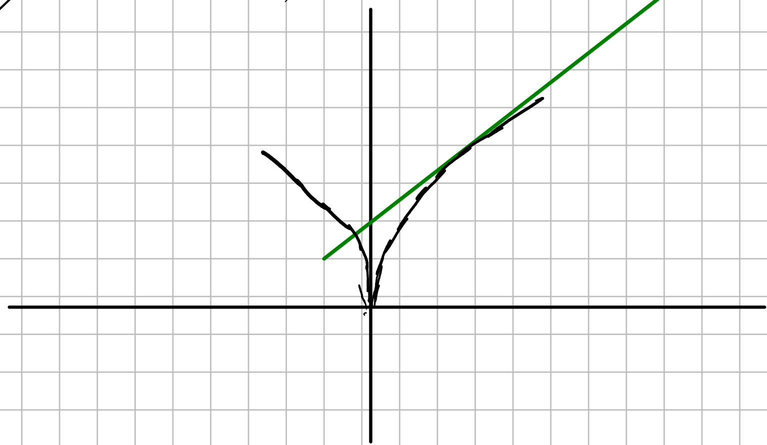
Convessità

$$\exists f''(x) = \frac{4\sqrt{x^2+x} - (2x+1)\frac{2x+1}{\sqrt{x^2+x}}}{x^2+x}$$

$$f''(x) > 0 \iff 4\sqrt{x^2+x} - \frac{(2x+1)^2}{\sqrt{x^2+x}} > 0$$

$$\frac{4(x^2+x) - (2x+1)^2}{\sqrt{x^2+x}} > 0$$

$$\cancel{4x^2} + \cancel{4x} - \cancel{4x^2} - 1 - \cancel{4x} > 0 \qquad \nexists x$$



$$f'(0) = \pm\infty$$

27/07/2022

Esercizio 2. Studiare la funzione definita dalla legge

$$f(x) = |x| + \log(1-x^2)$$

e tracciarne un grafico qualitativo.

N.B. Il simbolo log indica il logaritmo in base e.

Dominio $1-x^2>0$

$\Leftrightarrow x^2<1 \Rightarrow -1<x<1$

La funzione è pari $x>0$

$$\lim_{x\to 1^-} f(x) = 1 + \log(0^+) = 1-\infty = -\infty$$

Asintoto Verticale

$$\lim_{x\to 0^+} f(x) = \log(1) = 0$$

Monotonia

$$\exists f'(x) = \frac{|x|}{x} + \frac{1}{1-x^2}(-2x) = \frac{x}{x} - \frac{2x}{1-x^2} = \frac{1-x^2-2x}{1-x} = -\frac{x^2+2x-1}{1-x}$$

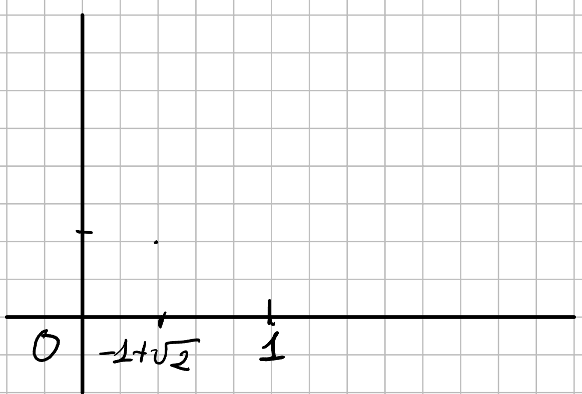
$$f'(x)>0 \Leftrightarrow \frac{x^2+2x-1}{1-x}<0 \qquad 1-x>0 \ \forall x$$

$$\Delta = 4+4 = 8 \qquad x = \frac{-2\pm2\sqrt{2}}{2} \to \begin{cases} -1+\sqrt{2} \\ -1-\sqrt{2} \end{cases}$$

$$\underbrace{-1-\sqrt{2}}_{<0} < x < -1+\sqrt{2}$$

$\hookrightarrow 0<x<-1+\sqrt{2}$

$$f(-1+\sqrt{2}) = -1+\sqrt{2}+\log(1+1+2-2\sqrt{2})>0$$



Convessità

$$\exists f''(x) = \frac{(2x+2)(1-x) - (x^2+2x-1)(-1)}{(1-x)^2}$$

$$f''(x) \geq 0 \iff \cancel{2}x - 2x^2 + 2 \cancel{-2x} + x^2 + 2x - 1 > 0$$

$$-x^2 + 2x + 1 > 0$$

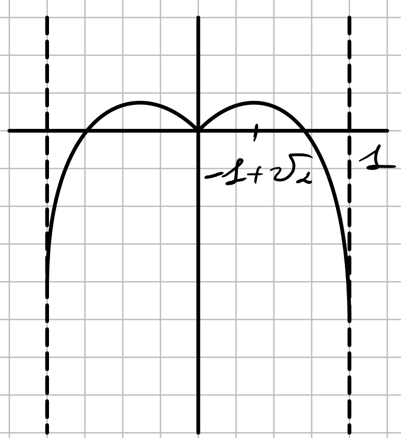
$$\Delta = 4 + 4 = 8$$

$$x = \frac{-2 \pm 2\sqrt{2}}{-2} \quad \begin{cases} 1+\sqrt{2} \\ 1-\sqrt{2} \end{cases}$$

$$x < 1-\sqrt{2} \vee x > 1+\sqrt{2}$$

$$\nexists x$$

Derivata nel punto zero: $\lim_{x \to 0^+} f'(x) = 1$



05/04/2022

Esercizio 2. Studiare la funzione definita dalla legge
$$f(x) = -2x + \sqrt{|x-1|}$$
e tracciarne un grafico qualitativo.

Dominio: $\mathbb{R}$

$$\lim_{x\to-\infty} f(x) = +\infty + \sqrt{+\infty} = +\infty$$

$$\lim_{x\to+\infty} f(x) = -\infty + \infty \text{ IND} = \lim_{x\to+\infty} \frac{|x-1| - 4x^2}{\sqrt{|x-1|}+2x} = \lim \frac{-4x^2+x-1}{\sqrt{x-1}+2x} =$$

$$= \frac{-\infty}{+\infty} \text{ IND} = \frac{-8x+1}{\frac{1}{2\sqrt{x-1}}+2} = \frac{-\infty}{2} = -\infty$$

Asintoti Obliqui

$$\lim_{x\to-\infty} \frac{f(x)}{x} = -2 + \frac{1}{2\sqrt{|x-1|}} = -2 = m$$

$$\lim_{x\to-\infty} f(x) + 2x = +\infty \notin \mathbb{R}$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{-4x^2+x-1}{x(\sqrt{x-1}+2x)}$$

$$= \lim_{x\to+\infty} \frac{-8x+1}{(\sqrt{x-1}+2x) + x\left(\frac{1}{2\sqrt{x-1}}+2\right)} =$$

$$= \lim_{x\to+\infty} \frac{-8}{\underbrace{\left(\frac{1}{2\sqrt{x-1}}+2\right)}_{0} + x\left(\underbrace{\frac{1}{4\sqrt{x-1}}}_{0}+2\right) + \left(\underbrace{\frac{1}{2\sqrt{x-1}}}_{0}+2\right)}$$

$$= \lim_{x\to+\infty} \frac{-8}{2+\infty+2} = 0 = m$$

Non esistono asintoti

Monotonie

$\exists f'(x) \quad x \neq 1$

$$f'(x) = -2 + \frac{1}{2\sqrt{|x-1|}} \cdot \frac{|x-1|}{x-1} =$$

con

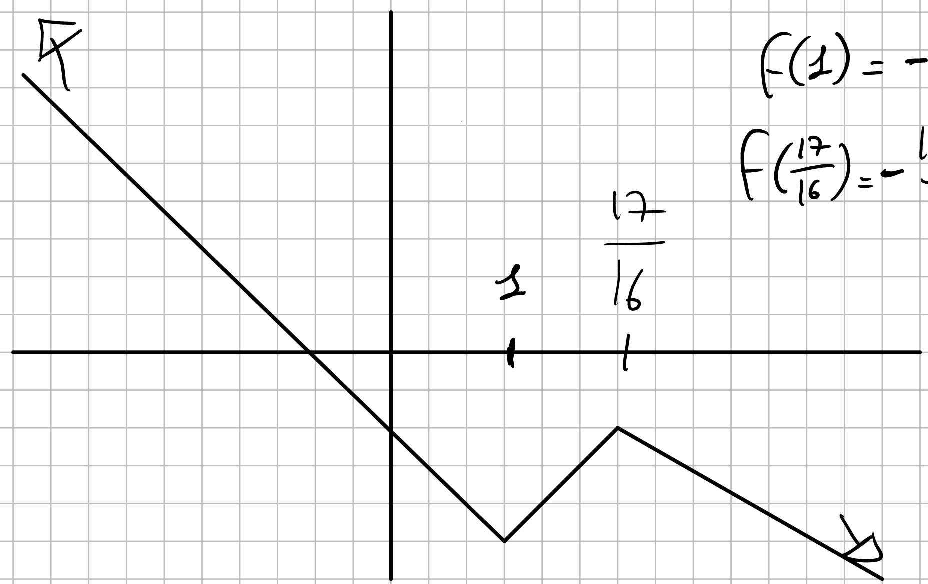
$$= \frac{-4\sqrt{|x-1|} + \operatorname{sgn}(x-1)}{2\sqrt{|x-1|}} = \begin{cases} -2 + \dfrac{1}{2\sqrt{x-1}} & x > 1 \\ -2 - \dfrac{1}{2\sqrt{1-x}} & x < 1 \end{cases}$$

$f'(x) > 0$

$x > 1$: $-4\sqrt{x-1} + 1 > 0 \Rightarrow \sqrt{x-1} < \frac{1}{4} \Rightarrow x < \frac{17}{16}$

$x < 1$: $-\sqrt{1-x} > \frac{1}{4} \Rightarrow \sqrt{1-x} < -\frac{1}{4} \Rightarrow \nexists x$

$$f'(x) > 0 \iff x \leq 1 < \frac{17}{16}$$



$f(1) = -2$

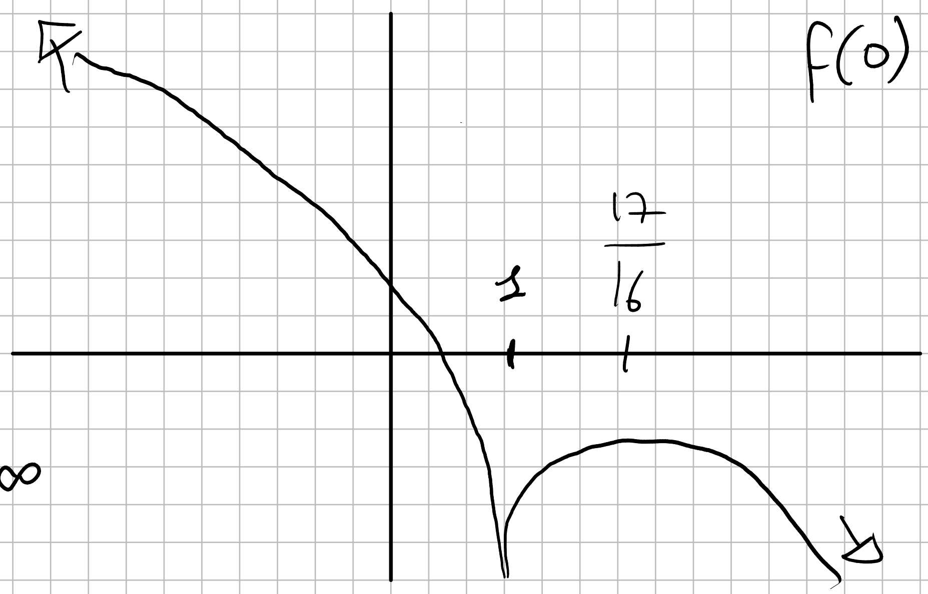
$f\left(\frac{17}{16}\right) = -\frac{17}{8} + \frac{5}{2\sqrt{2}} < 0$

Convessità

$$f''(x) = \begin{cases} \dfrac{2 \cdot \dfrac{1}{\sqrt{x-1}}}{(2\sqrt{x-1})^2} = \dfrac{-2}{\sqrt{x-1}\,(2\sqrt{x-1})^2} & x>0 \\[2ex] -\dfrac{-2 \cdot \dfrac{1}{\sqrt{1-x}}(-1)}{(2\sqrt{x-1})^2} > 0 = \dfrac{-2}{\sqrt{1-x}\,(2\sqrt{x-1})^2} \end{cases}$$

$f''(x) > 0 \quad \nexists\, x$

$f''(x) > 0 \iff x < 1$



$f(0) = 1$

$\lim_{x \to 1^+} f'(x) = +\infty$

$\lim_{x \to 1^-} f'(x) = -\infty$

26/09/2022

Esercizio 2. Studiare la funzione definita dalla legge

$$f(x) = x^{1/3} + x^{-2/3}$$

e tracciarne un grafico qualitativo. $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x^2}}$

Dominio

$\mathbb{R} \setminus \{0\}$

$]-\infty, 0[\cup]0, +\infty[$

Limiti Punti di Frontiera

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to 0^-} f(x) = +\infty$

Asintoto orizzontale

$\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to 0^+} f(x) = +\infty$

Asintoti Obliqui

Asintoto Verticale: $x=0$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x^{\frac{1}{3}} + x^{-\frac{2}{3}}}{x} = \lim \frac{x^{\frac{1}{3}}\left(1 + \frac{1}{x}\right)}{x^1} = \lim x^{-\frac{2}{3}}\left(1 + \frac{1}{x}\right)$$

$$= \frac{1}{\sqrt[3]{x^2}}\left(1 + \frac{1}{x}\right) = 0 = m$$ Non esiste asintoto obliquo

$\lim_{x \to +\infty} \frac{f(x)}{x} = 0 = m$ Non esiste asintoto obliquo

Monotonia

$$x^{1/3} + x^{-2/3}$$

$$f'(x) = \frac{1}{3} x^{-\frac{2}{3}} - \frac{2}{3} x^{-\frac{5}{3}} = \frac{1}{3} \frac{1}{\sqrt[3]{x^2}} - \frac{2}{3} \frac{1}{\sqrt[3]{x^5}} =$$

$$= \frac{1}{3}\left(\frac{1}{\sqrt[3]{x^2}} - \frac{2}{\sqrt[3]{x^5}}\right)$$

$$f'(x) > 0 \iff \frac{1}{\sqrt[3]{x^2}} - \frac{2}{\sqrt[3]{x^5}} > 0 \to \frac{\sqrt[3]{x^5} - 2\sqrt[3]{x^2}}{\sqrt[3]{x^2} \cdot \sqrt[3]{x^5}} > 0$$

$$\to \sqrt[3]{x^2 \cdot x^5} = \sqrt[3]{x^7}$$

31/01/2023

Esercizio 2. Studiare la funzione definita dalla legge

$$f(x) = e^{-x^2} - x - 1$$

e tracciarne un grafico qualitativo.

Dominio $\mathbb{R}$

$$\lim_{x\to+\infty} f(x) = -\infty \qquad \lim_{x\to-\infty} f(x) = +\infty$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \frac{\cancel{x}\left(\frac{e^{-x^2}}{x} - 1 - \frac{1}{x}\right)}{\cancel{x}} = 0 - 1 - 0 = -1 = m$$

$$\lim_{x\to+\infty} f(x) + x = e^{-x^2} - 1 = -1$$

Asintoto Obliquo: $y = -x - 1$ a $+\infty$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = 1 = m$$

$$\lim_{x\to-\infty} f(x) - x = e^{-x^2} - 2x - 1 = +\infty$$

Non esiste Asintoto Obliquo a $-\infty$

Monotonia

$$f'(x) = e^{-x^2}(-2x) - 1$$

$$f'(x) > 0 \iff -2x\,e^{-x^2} - 1 > 0 \to -xe^{-x^2} > \frac{1}{2} \to xe^{-x^2} < -\frac{1}{2}$$

$\to$