Dominio $\mathbb{R}$

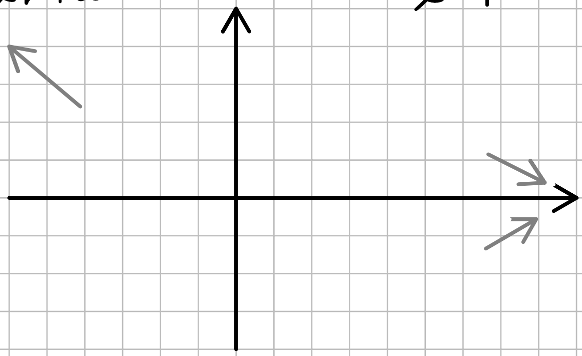
$f(x) = \sqrt{|x^2-4|} - x$

$\lim_{x\to-\infty} f = +\infty$

Potrebbe essere un asintoto obliquo

$\lim_{x\to+\infty} f = +\infty - \infty \sim \lim_{x\to+\infty} \sqrt{|x^2|} - x = 0$

Asintoto Orizzontale



$\lim_{x\to-\infty} \frac{f(x)}{x} = -2$

$\lim_{x\to-\infty} f(x) + 2x = 0$

C'è asintoto $y = -2x$

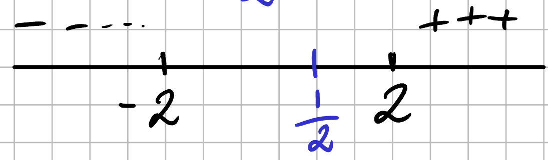
$\exists f'(x) \quad x^2-4 \neq 0 \Rightarrow x \neq \pm 2$

$$f'(x) = \underbrace{\frac{1}{2\sqrt{|x^2-4|}}}_{>0} \cdot \frac{x^2-4}{|x^2-4|} \cdot 2x - 1$$

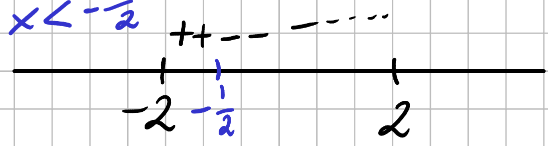
$$f'(x) > 0 \iff 2x \frac{x^2-4}{|x^2-4|} - 1 > 0$$

$$\begin{cases} 2x \dfrac{x^2-4}{\underbrace{x^2-4}_{>0}} - 1 & x < -2 \vee x > 2 \Rightarrow \\ -2x \dfrac{x^2-4}{x^2-4} - 1 & -2 < x < 2 \end{cases}$$

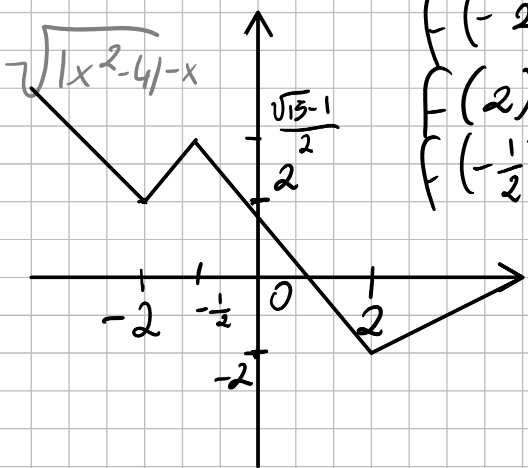
$x > \frac{1}{2}$



$x < -\frac{1}{2}$



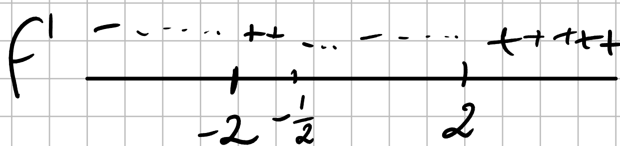
$\frac{\sqrt{15}}{2} - \frac{1}{2}$



$f(-2) = 2$

$f(2) = -2$

$f\left(-\frac{1}{2}\right) = \frac{\sqrt{15}-1}{2}$



$f'(x) > 0 \iff x \in \left]-2, -\frac{1}{2}\right[\cup \left]2, +\infty\right[$

$\exists f'(-2)$? No

$\lim_{x \to -2^-} f(x) = -5$

$\lim_{x \to -2^+} f(x) = 3$

$x = -2$ $\to$ Punto angoloso

$\exists f'(2)$? No

$\lim_{x \to 2^-} f(x) = -5$

$\lim_{x \to 2^+} f(x) = 3$

$x = 2$ Punto Angoloso

$\exists f''(x)$ (± 2 sono già esclusi)

$$f''(x) = 2\frac{x^2-4}{|x^2-4|} + 2x\left\{\frac{2x|x^2-4| - \cancel{(x^2-4)}\frac{|x^2-4|}{\cancel{x^2-4}}\cdot 2x}{|x^2-4|^2}\right\}$$

$$= 2\frac{x^2-4}{|x^2-4|}$$

$f''(x) > 0 \iff x^2-4 > 0$ $\quad x < -2 \vee x > 2$

$f(1) = \sqrt{3} - 1$