The role of quantum coherence viewed from synpatic-type information-processing

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I. INTRODUCTION

Artificial emulations of biological neurons have been realised, using for example, highly doped TMD materials with hysterisis property, atomic switches and classical aspect of spiking Josephson junctions and so on, without quantum ingredients explicitly manifested on the level of functionality. On the other hand, the established study of quantum machine learning is the use of unitary quantum gates for solving mathematical problems encoded for machine learning tasks from computer science. Henceforth, it is not a direct emulator of biological neural networks.

The present work is in the context of the previously mentioned several implementations of artificial systems that exhibit features of biological synapses. The primary question is what possible roles can quantum properties play in such context. This is certainly an open question since classical variables, for example, membrane voltage or action potential, etc., and classical nonlinear dynamics, are in use for the main stream description of the physical behaviours of biological neurons as well as their information processing manifestations. Here we describe a toy model for emulating synaptic faciliation/depression by the use of a simple quantum system. The purpose of such study is to make use of the established the synaptic signal processing for learning more about the nature of quantum resource. Why should one expolre the nature of quantum resources, namely, coherence and entanglement, from a synaptic-signal-processing point of view? The answer is simple.

Synaptic signal processing protocols are conceptually apparent different from the information processing scheme realised by the gate-based-quantum-computers and the nature of quantum coherence and entanglement has already been largely exploited using the latter context. For the latter, the input signal is encoded in the initial state of the multi-qubit system. The precisely programmed unitary evolution of the multi-qubit system is part of the processing procedure. The output of the signal processing is inferred from quantum measurements applied upon the final evolved state of the multi-qubit system. In short, both the input and the output are encoded in some time-independent quantum states. Only the programmed unitary evolution is a real-time dynamical process.

However, in synaptic signal processing protocols, the input is a time sequence of action potential spikes and the output is also a time-changing stream of post-synaptic currents or a time-evolving flux of neurotransmitter release. The input, the output and the processing procedures are all going on the run. Therefore, already on the level of encoding the input information into the state of quantum matters, we have a departure from the readily established conventional quantum computer scenario. Although one can take the while time sequence of spikes and encode this sequence into some static state of a multi-qubit system, this would not help us to gain a different understanding of the nature of quantum coherence in terms of signal processing. This very important difference between the principal scenarios used in biological synapses and man-made gate-based quantum computers thus should serve a good reason and motivation for looking into the quantum possibility of processing signals on the run. Physical systems that manifest dynamics of quantum nature usually show characteristic time scales on the orders of atto- to nano-seconds while the synaptic activities are occuring on the time scales of a few to a few hundred milli-seconds. Therefore, if miniatured quantum systems can emulate synaptic dynamics, then they are potentially advantageous of realising bio-mimic functionalities on a much faster time scales while occupying much smaller spaces.

II. A THREE-LEVEL TOY SYNAPSE

The basic activity of a synapse goes like the following. The pre-synapatic part of the synapse receives a train of action potential splikes generated on the axon-initial-segment that travel down the axon all the way to the synpase. These action potential spikes change the local electrical properties of that part of the neuron there which then causes prepared vesicles to release neurotransmitters. The neurotransmitters are then diffused to contact another neuron involved in this synapse. In simple input-output language, the input is a time sequence of spikes and the output, to the lowest level of imitation, is the release of neurotransmitter.

To mimic a synapse, the basic criterium is to be able to produce phenomena that can be called synaptic plasticity. Here we only consider the simplest of all kinds of synaptic plasticity, namely, the faciliation and depression exhibited in short-term plasticity. In the facilitation phenomena, the neutrotransmitter release gets more and more intensive during the input spiking train and the depression phenomena do the opposite. Whether a synapse is facilitating or depressing is found to be dependent on the initial probability of neurotransmitter release [Synaptic computation, L.

F. Abbott & Wade G. Regehr, 2004].

Here we try with a simple three-level system with level labels 0, 1, and 2. The three levels are consecutively coupled like a tight-binding model with three sites. There is no direct coupling between the 0th site and the 2nd site. The 0th site is initially loaded with population 1. This initial population is then spike-wisely transferred to the 1st site which then is transferred to the destination site 2. The coupling between 0 and 1 is turned on and off in time, in an attempt to mimic the input sequence of spikes. The release of neurotransmitter is then corresponded to the transfer of population from site 1 to site 2. The on-site energies and the couplings are made time-dependent to generate synapse-like activities without involving any nonlinear dynamics.

A. Steered population transfer

A three-level system is given by the simple Hamiltonian,

$$H(t) = \sum_{i=0}^{2} \varepsilon_i(t) |i\rangle\langle i| - J_0(t) [|0\rangle\langle 1| + |1\rangle\langle 0|] - J_1(t) [|1\rangle\langle 2| + |2\rangle\langle 1|]. \tag{1}$$

To generate phenomena in analogy to facilitation/depression, one needs to set up the time dependence of the parameters, namely, $\varepsilon_i(t)$, $J_i(t)$, accordingly, henceforth no further complication of the model involving mutual interaction is needed. The 0th level is termed the injector site. The 1st level is the middle site and the last level $|2\rangle$ is the receiver site of the injected population. We analyse only selected spans of time appropriate for identifying the required phenomena.

1. *Description of the time-dependence of the model parametres*

The initial population is loaded onto the injector site $|0\rangle$. We set $\varepsilon_0(t) = \varepsilon_1(t)$ for all-time for the ease of injection. During the injection phase, $J_0(t)$ is turned on to some nonzero positive value J_{on} for a time periond τ_{on} and then it is turned off to zero for some other length of time τ_{off}. It can then be turned on again and again. So we have a sequence of τ_{on}'s and τ_{off}'s. When $J_0(t)$ is during the on-phase, we let $J_1(t)$ be zero during that same interval of time. When $J_0(t)$ is during the off-phase, $J_1(t)$ is turned on to some nonzero value J_T. For simplicity, we let J_{on} be a constant of time during each on-phase and all the on-phases of $J_0(t)$ share the same turned-on-value J_{on}. The same simplification of the time-dependent protocol of $J_1(t)$ is also applied such that $J_1(t)$ takes either the value J_T or 0, depending on at the time t the injection coupling $J_0(t)$ is off or on.

We set $\varepsilon_2(t) = \varepsilon_R$ as a constant reference value which can be zero without loss of generality. As we have known from neuroscience study that facilitating synapse is characterised by a low initial release probability. This would correspond to a configuration of the energy levels such that the transfer between the middle and the receiver sites is less efficient. This is supposed to be implemented by $\varepsilon_1(t = 0)$ being far away from ε_R. Facilitation of population transfer in the course time is expected by moving $\varepsilon_1(t)$ gradually toward ε_R. For depression, we then set initially that $\varepsilon_1(t = 0) = \varepsilon_R$ and we move $\varepsilon_1(t)$ gradually away from ε_R. In fact, the phenomena of facilitation and depression can already show up without involving the 0th site, just as a two-level system. One prepares the initial population on site 1 and the level energy $\varepsilon_1(t)$ is moved accordingly, then facilitation and depression appear. This is demonstrated in E:\Forschung\mathcaword_3\q-neurons-cal\2lvl-leak-2-B.nb by letting $\varepsilon_1(t)$ be a constant over a fixed length of time τ_ε and change that constant over the next same length of time, etc., toward or away the reference level ε_R.

The result with truely three-level system is more interesting. A case study is presented and summarised in E:\Forschung\filetex\q-neurons\q-model-synapse\3lvl-tmsch-1\test-1\data-pres.ppt. The source code is in We let τ_{on} be a small fraction of τ_ε, the length of time during which $\varepsilon_1(t)$ takes a certain constant value. We denote $n_2(t)$ as the population on the receiver site. By the facilitation setup, $n_2(t)$ shows increment in time that corresponds to the off-phase of $J_0(t)$. The latter increment is more than the earlier during the course of repeated injection. However, in the depression setup, $n_2(t)$'s increment corresponding to population transfer from the middle to the receiver site does not show clear correlation with the injection event. After the population has reached the middle site, the population in $n_2(t)$ will first increase and then decrease, as a manifestation of Rabi oscillation. This increase-decrease cycle appears consecutively several times in time. The depression is shown by that the latter cycles show less and less increments than the earlier cycles. The above phenomenology hints that facilitation is more frequently revealed if the injection events come more frquently while too-frequent injection may interfere the show-up of depression effect. This implies readily that facilitation acts as a high-pass filter while depression acts as a low-pass filter.

More importantly, the way depression is revealed is much less correlated to the injection events has an implication. This says that depression is not a "direct" opposite of the facilitation. The fact that depression in coherent popolation

transfer is not a "direct" opposite of the facilitation should be a special effect of quantum coherent transfer. Already through this rudimentary toy model, by the attempt of mimicking a synapse, we learn something special about quantum coherence.

2. *quantum dynamics equation*

The Schroedinger equation $H(t)|\psi(t)\rangle = i\hbar\left|\dot{\psi}(t)\right\rangle$ with the wavefunction expansion

$$|\psi(t)\rangle = \sum_{i=0}^{2} u_i(t)|i\rangle, \tag{2}$$

is rewritten into

$$i\hbar\begin{pmatrix}\dot{u}_0(t)\\ \dot{u}_1(t)\\ \dot{u}_2(t)\end{pmatrix} = \begin{pmatrix}\varepsilon_0(t) & -J_0(t) & 0\\ -J_0(t) & \varepsilon_1(t) & -J_1(t)\\ -J_1(t) & 0 & \varepsilon_R\end{pmatrix}\begin{pmatrix}u_0(t)\\ u_1(t)\\ u_2(t)\end{pmatrix}. \tag{3}$$

If population leakage is considered such that population can leak from the site 2 to some continuum, the equation can be modified into

$$i\hbar\begin{pmatrix}\dot{u}_0(t)\\ \dot{u}_1(t)\\ \dot{u}_2(t)\end{pmatrix} = \begin{pmatrix}\varepsilon_0(t) & -J_0(t) & 0\\ -J_0(t) & \varepsilon_1(t) & -J_1(t)\\ -J_1(t) & 0 & \varepsilon_R - \hbar\gamma/2\end{pmatrix}\begin{pmatrix}u_0(t)\\ u_1(t)\\ u_2(t)\end{pmatrix}, \tag{4}$$

with some leaking rate γ. This can be derived by assuming a wide band approximation to the continuum.