Derived rules

Lambert Meertens

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$$\frac{\vdash cx : \textsc{context} \quad cx \vdash T : \textsc{type}}{cx \vdash \lambda v{:}T.\, v : T \rightarrow T} \quad (\textsc{dexId}[\![cx; v; T]\!])$$

$$\frac{\vdash cx : \textsc{context}}{cx \vdash \lambda v{:}\,\mathsf{Bool}.\, v : \mathsf{Bool} \rightarrow \mathsf{Bool}} \quad (\textsc{dexIdBool}[\![cx; v]\!])$$

$$\frac{\vdash cx : \textsc{context}}{cx \vdash \mathsf{true} : \mathsf{Bool}} \quad (\textsc{dexTrue}[\![cx]\!])$$

$$\frac{\vdash cx : \textsc{context} \quad cx \vdash T : \textsc{type}}{cx \vdash \lambda v{:}T.\, \mathsf{true} : T \rightarrow \mathsf{Bool}} \quad (\textsc{dexConstTrue}[\![cx; v; T]\!])$$

$$\frac{\vdash cx : \textsc{context} \quad cx \vdash T : \textsc{type}}{cx \vdash \forall_T : (T \rightarrow \mathsf{Bool}) \rightarrow \mathsf{Bool}} \quad (\textsc{dexForallF}[\![cx; T]\!])$$

$$\frac{\vdash cx : \textsc{context} \quad cx \vdash T : \textsc{type} \quad cx \vdash \lambda v{:}T.\, e : T \rightarrow \mathsf{Bool}}{cx \vdash \forall v{:}T.\, e : \mathsf{Bool}} \quad (\textsc{dexForAll}[\![cx; v; T; e]\!])$$

$$\frac{\vdash cx : \textsc{context} \quad cx \vdash T : \textsc{type} \quad cx \vdash e : T}{cx \vdash (\lambda v{:}T.\, \mathsf{true})\; e : \mathsf{Bool}} \quad (\textsc{dexConstTrueApp}[\![cx; v; T; e]\!])$$

$$\frac{\vdash cx : \textsc{context}}{cx \vdash \mathsf{true}} \quad (\textsc{dthTrue}[\![cx]\!])$$

$$\frac{\vdash cx : \textsc{context} \quad cx \vdash T : \textsc{type} \quad cx \vdash e : T}{cx \vdash (\lambda v{:}T.\, \mathsf{true})\; e \equiv \mathsf{true}} \quad (\textsc{dthConstTrueApp}[\![cx; v; T; e]\!])$$

$$\frac{\vdash cx : \textsc{context} \quad cx \vdash T : \textsc{type} \quad cx \vdash \lambda v{:}T.\, e : T \rightarrow \mathsf{Bool} \quad cx \vdash \forall v{:}T.\, e \quad cx \vdash e' : T}{cx \vdash e[v/e']} \quad (\textsc{dthForAll}[\![cx; v; T; e; e']\!])$$

1 dexId

$\textsc{dexId}[\![cx; v; T]\!] :=\quad cx \vdash \lambda v{:}T.\, v : T \to T \quad \Leftarrow \quad \vdash cx : \textsc{context},\ cx \vdash T : \textsc{type}$

Pick variable name $v' \notin \mathcal{V}(cx)$.

(1.0) $cx \vdash \lambda v{:}T.\, v : T \to T$

by $\textsc{exAbsAlpha}[\![cx; \gamma; T; \mathsf{true}; T \to T; v']\!]$ from (1.1)

(1.1) $cx \vdash \lambda v'{:}T.\, v' : T \to T$

by $\textsc{exAbs}[\![cx; v'; T; v'; T]\!]$ from (1.2)

(1.2) $cxv1T \vdash v' : T$

by $\textsc{exVar}[\![cx; v'; T]\!]$ from (1.3)

(1.3) $\vdash (cx, \mathsf{var}\ v' : T) : \textsc{context}$

by $\textsc{cxVdec}[\![cx; v'; T]\!]$ from (1.4) and (1.5)

(1.4) $\vdash cx : \textsc{context}$

assumed given

(1.5) $cx \vdash T : \textsc{type}$

assumed given

2 $\textsc{dexIdBool}$

$\textsc{dexIdBool}[\![cx; v]\!] := \quad cx \vdash \lambda v{:}\,\mathsf{Bool}.\, v : \mathsf{Bool}{\to}\,\mathsf{Bool} \quad \Leftarrow \quad \vdash cx : \textsc{context}$

(2.0) $\quad cx \vdash \lambda v{:}\,\mathsf{Bool}.\, v : \mathsf{Bool}{\to}\,\mathsf{Bool}$

by $\textsc{dexId}[\![cx; v; \mathsf{Bool}]\!]$ from (2.2) and (2.1)

(2.1) $\quad cx \vdash \mathsf{Bool} : \textsc{type}$

by $\textsc{tyBool}[\![cx]\!]$ from (2.2)

(2.2) $\quad \vdash cx : \textsc{context}$

assumed given

3 $\textsc{dexTrue}$

$\textsc{dexTrue}[\![cx]\!] := \quad cx \vdash \mathsf{true} : \mathsf{Bool} \quad \Leftarrow \quad \vdash cx : \textsc{context}$

(3.0) $\quad cx \vdash \mathsf{true} : \mathsf{Bool}$

by $\textsc{exEq}[\![cx; \lambda\gamma\colon \mathsf{Bool}.\, \gamma; \mathsf{Bool}\!\rightarrow \mathsf{Bool}; \lambda\gamma\colon \mathsf{Bool}.\, \gamma]\!]$ from (3.1)

(3.1) $\quad cx \vdash \lambda\gamma\colon \mathsf{Bool}.\, \gamma : \mathsf{Bool}\!\rightarrow \mathsf{Bool}$

by $\textsc{dexIdBool}[\![cx; \gamma]\!]$ from (3.2)

(3.2) $\quad \vdash cx : \textsc{context}$

assumed given

4 DTHTRUE

$\textsc{dthTrue}[\![cx]\!] := \quad cx \vdash \mathsf{true} \quad \Leftarrow \quad \vdash cx : \textsc{context}$

(4.0) $cx \vdash \mathsf{true}$

by $\textsc{thRefl}[\![cx; \lambda\gamma{:}\,\mathsf{Bool}.\,\gamma]\!]$ from (4.1)

(4.1) $cx \vdash \lambda\gamma{:}\,\mathsf{Bool}.\,\gamma : \mathsf{Bool}\!\rightarrow \mathsf{Bool}$

by $\textsc{dexIdBool}[\![cx; \gamma]\!]$ from (4.2)

(4.2) $\vdash cx : \textsc{context}$

assumed given

5 $\textsc{dexConstTrue}$

$\textsc{dexConstTrue}[\![cx; v; T]\!] :=$
$cx \vdash \lambda v{:}T.\,\mathsf{true} : T \to \mathsf{Bool} \quad \Leftarrow \quad \vdash cx : \textsc{context},\ cx \vdash T : \textsc{type}$

Pick variable name $v' \notin \mathcal{V}(cx)$.

(5.0) $cx \vdash \lambda v{:}T.\,\mathsf{true} : T \to \mathsf{Bool}$

by $\textsc{exAbsAlpha}[\![cx; v; T; \mathsf{true}; T \to \mathsf{Bool}; v']\!]$ from (5.1)

(5.1) $cx \vdash \lambda v'{:}T.\,\mathsf{true} : T \to \mathsf{Bool}$

by $\textsc{exAbs}[\![cx; v'; T; \mathsf{true}; \mathsf{Bool}]\!]$ from (5.2)

(5.2) $cx, \mathsf{var}\ v' : T \vdash \mathsf{true} : \mathsf{Bool}$

by $\textsc{dexTrue}[\![cx, \mathsf{var}\ v' : T]\!]$ from (5.3)

(5.3) $\vdash (cx, \mathsf{var}\ v' : T) : \textsc{context}$

by $\textsc{cxVdec}[\![cx; v'; T]\!]$ from (5.4) and (5.5)

(5.4) $\vdash cx : \textsc{context}$

assumed given

(5.5) $cx \vdash T : \textsc{type}$

assumed given

6 dexForallF

$\textsc{dexForallF}[\![cx; T]\!] :=$
$\quad cx \vdash \forall_T : (T \to \mathsf{Bool}) \to \mathsf{Bool} \quad \Leftarrow \quad \vdash cx : \textsc{context},\ cx \vdash T : \textsc{type}$

Pick variable name $v' \notin \mathcal{V}(cx)$.

(6.0) $cx \vdash \forall_T : (T \to \mathsf{Bool}) \to \mathsf{Bool}$

by $\textsc{exAbsAlpha}[\![cx; \psi; T \to \mathsf{Bool}; \psi \equiv \lambda v{:}T.\,\mathsf{true}; (T \to \mathsf{Bool}) \to \mathsf{Bool}; v']\!]$ from (6.1)

(6.1) $cx \vdash (\lambda v'{:}T \to \mathsf{Bool}.\,(v' \equiv \lambda v{:}T.\,\mathsf{true})) : (T \to \mathsf{Bool}) \to \mathsf{Bool}$

by $\textsc{exAbs}[\![cx; v'; T \to \mathsf{Bool}; v' \equiv \lambda v{:}T.\,\mathsf{true}; \mathsf{Bool}]\!]$ from (6.2)

(6.2) $cx, \mathsf{var}\ v' : T \to \mathsf{Bool} \vdash (v' \equiv \lambda v{:}T.\,\mathsf{true}) : \mathsf{Bool}$

by $\textsc{exEq}[\![cx, \mathsf{var}\ v' : T \to \mathsf{Bool}; v'; T \to \mathsf{Bool}; \lambda v{:}T.\,\mathsf{true}]\!]$ from (6.3) and (6.4)

(6.3) $cx, \mathsf{var}\ v' : T \to \mathsf{Bool} \vdash v' : T \to \mathsf{Bool}$

by $\textsc{exVar}[\![cx; v'; T \to \mathsf{Bool}]\!]$ from (6.8)

(6.4) $cx, \mathsf{var}\ v' : T \to \mathsf{Bool} \vdash \lambda v{:}T.\,\mathsf{true} : T \to \mathsf{Bool}$

by $\textsc{dexConstTrue}[\![cx, \mathsf{var}\ v' : T \to \mathsf{Bool}; v; T]\!]$ from (6.8) and (6.5)

(6.5) $cx, \mathsf{var}\ v' : T \to \mathsf{Bool} \vdash T \to \mathsf{Bool} : \textsc{type}$

by $\textsc{tyArr}[\![cx, \mathsf{var}\ v' : T \to \mathsf{Bool}; T; \mathsf{Bool}]\!]$ from (6.6) and (6.7)

(6.6) $cx, \mathsf{var}\ v' : T \to \mathsf{Bool} \vdash T : \textsc{type}$

by Theorem 4.58 ('Metaslang logic is monotonic') from (6.8) and (6.9)

(6.7) $cx, \mathsf{var}\ v' : T \to \mathsf{Bool} \vdash \mathsf{Bool} : \textsc{type}$

by $\textsc{tyBool}[\![cx, \mathsf{var}\ v' : T \to \mathsf{Bool}]\!]$ from (6.8)

(6.8) $\vdash (cx, \mathsf{var}\ v' : T \to \mathsf{Bool}) : \textsc{context}$

by $\textsc{cxVdec}[\![cx; v'; T \to \mathsf{Bool}]\!]$ from (6.11) and (6.9)

(6.9) $cx \vdash T \to \mathsf{Bool} : \textsc{type}$

by $\textsc{tyArr}[\![cx; T; \mathsf{Bool}]\!]$ from (6.12) and (6.10)

(6.10) $cx \vdash \mathsf{Bool} : \textsc{type}$

by $\textsc{tyBool}[\![cx]\!]$ from (6.11)

(6.11) $\vdash cx : \textsc{context}$

assumed given

(6.12) $cx \vdash T : \textsc{type}$

assumed given

7 dexForall

$\textsc{dexForAll}[\![cx; v; T; e]\!] :=$
$$cx \vdash \forall v{:}T.\, e : \mathsf{Bool} \quad \Leftarrow$$
$$\vdash cx : \textsc{context},\ cx \vdash T : \textsc{type},\ cx \vdash \lambda v{:}T.\, e : T \to \mathsf{Bool}$$

(7.0) $\quad cx \vdash \forall v{:}T.\, e : \mathsf{Bool}$

by $\textsc{exApp}[\![cx; \forall_T; T; \mathsf{Bool}; \lambda v{:}T.\, e]\!]$ from (7.1) and (7.4)

(7.1) $\quad cx \vdash \forall_T : (T \to \mathsf{Bool}) \to \mathsf{Bool}$

by $\textsc{dexForallF}[\![cx; T]\!]$ from (7.2) and (7.3)

(7.2) $\quad \vdash cx : \textsc{context}$

assumed given

(7.3) $\quad cx \vdash T : \textsc{type}$

assumed given

(7.4) $\quad cx \vdash \lambda v{:}T.\, e : T \to \mathsf{Bool}$

assumed given

8 dexConstTrueApp

dexConstTrueApp$[\![cx; v; T; e]\!]$:=

$$cx \vdash (\lambda v{:}\,T.\,\mathsf{true})\, e : \mathsf{Bool} \quad \Leftarrow \quad \vdash cx : \text{CONTEXT},\ cx \vdash T : \text{TYPE},\ cx \vdash e : T$$

(8.0) $cx \vdash (\lambda v{:}\,T.\,\mathsf{true})\, e : \mathsf{Bool}$

by exApp$[\![cx; \lambda v{:}\,T.\,\mathsf{true}; T; \mathsf{Bool}; e]\!]$ from (8.1) and (8.4)

(8.1) $cx \vdash \lambda v{:}\,T.\,\mathsf{true} : T \to \mathsf{Bool}$

by dexConstTrue$[\![cx; v; T]\!]$ from (8.2) and (8.3)

(8.2) $\vdash cx : \text{CONTEXT}$

assumed given

(8.3) $cx \vdash T : \text{TYPE}$

assumed given

(8.4) $cx \vdash e : T$

assumed given

9 $\textsc{dthConstTrueApp}$

$\textsc{dthConstTrueApp}[\![cx; v; T; e]\!] :=$
$$cx \vdash (\lambda v{:}\, T.\, \mathsf{true})\, e \equiv \mathsf{true} \quad \Leftarrow \quad \vdash cx : \textsc{context},\ cx \vdash T : \textsc{type},\ cx \vdash e : T$$

(9.0) $cx \vdash (\lambda v{:}\, T.\, \mathsf{true})\, e \equiv \mathsf{true}$

by $\textsc{thAbs}[\![cx; v; T; \mathsf{true}; e]\!]$ from (9.1)

(9.1) $cx \vdash (\lambda v{:}\, T.\, \mathsf{true})\, e : \mathsf{Bool}$

by $\textsc{dexConstTrueApp}[\![cx; v; T; e]\!]$ from (9.2), (9.3) and (9.4)

(9.2) $\vdash cx : \textsc{context}$

assumed given

(9.3) $cx \vdash T : \textsc{type}$

assumed given

(9.4) $cx \vdash e : T$

assumed given

10 dthForall

$\textsc{dthForAll}[\![cx; v; T; e; e']\!] :=$

$$cx \vdash e[v/e'] \quad \Leftarrow \quad \vdash cx : \textsc{context},\ cx \vdash T : \textsc{type},\ cx \vdash \lambda v\colon T.\, e : T \to \mathsf{Bool},\ cx \vdash \forall v\colon T.\, e,\ cx \vdash e' : T$$

(10.0) $cx \vdash e[v/e']$

by $\textsc{thSubst}[\![cx; \mathsf{true}; \mathsf{true} \equiv e[v/e']\,]\!]$ from (10.1) and (10.2)

(10.1) $cx \vdash \mathsf{true}$

by $\textsc{dthTrue}[\![cx]\!]$ from (10.14)

(10.2) $cx \vdash \mathsf{true} \equiv e[v/e']$

by $\textsc{thSymm}[\![cx; e[v/e']; \mathsf{true}]\!]$ from (10.3)

(10.3) $cx \vdash e[v/e'] \equiv \mathsf{true}$

by $\textsc{thTrans}[\![cx; e[v/e']; (\lambda v\colon T.\, e)\, e'; \mathsf{true}]\!]$ from (10.4) and (10.7)

(10.4) $cx \vdash e[v/e'] \equiv (\lambda v\colon T.\, e)\, e'$

by $\textsc{thSymm}[\![cx; (\lambda v\colon T.\, e)\, e'; e[v/e']]\!]$ from (10.5)

(10.5) $cx \vdash (\lambda v\colon T.\, e)\, e' \equiv e[v/e']$

by $\textsc{thAbs}[\![cx; v; T; e; e']\!]$ from (10.6)

(10.6) $cx \vdash (\lambda v\colon T.\, e)\, e' : \mathsf{Bool}$

by $\textsc{exApp}[\![cx; \lambda v\colon T.\, e; T; \mathsf{Bool}; e']\!]$ from (10.16) and (10.18)

(10.7) $cx \vdash (\lambda v\colon T.\, e)\, e' \equiv \mathsf{true}$

by $\textsc{thTrans}[\![cx; (\lambda v\colon T.\, e)\, e'; (\lambda \gamma\colon T.\, \mathsf{true})\, e'; \mathsf{true}]\!]$ from (10.8) and (10.13)

(10.8) $cx \vdash (\lambda v\colon T.\, e)\, e' \equiv (\lambda \gamma\colon T.\, \mathsf{true})\, e'$

by $\textsc{thAppSubst}[\![cx; \lambda v\colon T.\, e; e'; \lambda \gamma\colon T.\, \mathsf{true}; e']\!]$ from (10.16), (10.9) and (10.12)

(10.9) $cx \vdash \lambda v\colon T.\, e \equiv \lambda \gamma\colon T.\, \mathsf{true}$

by $\textsc{thSubst}[\![cx; \forall v\colon T.\, e; \lambda v\colon T.\, e \equiv \lambda \gamma\colon T.\, \mathsf{true}]\!]$ from (10.17) and (10.10)

(10.10) $cx \vdash \forall v\colon T.\, e \equiv (\lambda v\colon T.\, e \equiv \lambda \gamma\colon T.\, \mathsf{true})$

by $\textsc{thAbs}[\![cx; \psi; T \to \mathsf{Bool}; \psi \equiv \lambda \gamma\colon T.\, \mathsf{true}; \lambda v\colon T.\, e]\!]$ from (10.11)

(10.11) $cx \vdash \forall v\colon T.\, e : \mathsf{Bool}$

by $\textsc{dexForAll}[\![cx; v; T; e]\!]$ from (10.14), (10.15) and (10.16)

(10.12) $cx \vdash e' \equiv e'$

by $\textsc{thRefl}[\![cx; e']\!]$ from (10.18)

(10.13) $cx \vdash (\lambda\gamma{:}T.\,\mathsf{true})\,e' \equiv \mathsf{true}$

by $\textsc{dthConstTrueApp}[\![cx; \gamma; T; e']\!]$ from (10.14), (10.15) and (10.18)

(10.14) $\vdash cx : \textsc{context}$

assumed given

(10.15) $cx \vdash T : \textsc{type}$

assumed given

(10.16) $cx \vdash \lambda v{:}T.\,e : T \to \mathsf{Bool}$

assumed given

(10.17) $cx \vdash \forall v{:}T.\,e$

assumed given

(10.18) $cx \vdash e' : T$

assumed given