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Determination of the Orbits of Near-Earth Asteroids from Observations at the First Opposition

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Abstract—Observations at the first opposition are used to determine the orbits of 16 near-Earth asteroids with two or more observed oppositions. The orbits are improved by the differential method. This paper considers two modifications of the improvement technique, which are compared to the classical method based on the principle of the least square method (LSM). The first modification uses the principle of least absolute deviations (LAD). In the second modification, the differences $O - C$ (between the observed and calculated positions) are transformed to fit into a new coordinate system whereby the axes go parallel and perpendicular to the asteroid's apparent path (the modified differential method (MDM)). The orbits determined from one opposition by the classical LSM, LAD, and MDM are compared to a more accurate orbit calculated by the LSM from all the available oppositions. The calculations show that in 13 cases out of 16, the asteroid orbits calculated by LAD are more accurate than those calculated by the classical LSM. The improvement by the modified differential method, which includes the $O - C$ transformation, does not produce any perceptible increase in accuracy when compared to the orbits calculated by the classical method.

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INTRODUCTION

The accepted method for estimating the initial values determining the orbit of an asteroid is the differential technique based on the least square method (LSM). The orbit obtained thus is usually referred to as nominal and is statistically the best in terms of representing the observations (the weighted sum of the squared residual errors is minimal) under the assumption that the observation errors underlying the orbit improvement have a normal distribution.

However, observations of near-Earth asteroids have some specific features; in particular, they are carried out in a broad range of geocentric distances to the asteroids (from a few hundredths or even thousandths of an astronomical unit to several astronomical units). Because of the asteroids' quick apparent motions during their close approaches to the Earth, their observations may have substantial errors. LSM solutions are sensitive to the so-called cluttered observation sets, when the observation array contains a series of observations with strongly deviated errors. The discarding procedures by the three-sigma rule, including those based on the analysis of a "Student's type" ratio (Linnik, 1962), could combat these errors; however, they are not always effective for recently detected asteroids with small observation arcs. Consequently, the orbits calculated from observations at the first opposition are of rather low accuracy. Makarova (2000) compared the least absolute deviations (LAD) method and LSM on observations of selected small planets and inferred that LAD could be helpful in solving the problem.

This paper considers two modifications of the differential method of orbit improvement, the purpose of which is an increase in the accuracy. In the first modification, the asteroid orbits are determined by LAD, i.e., by minimizing the weighted sum of absolute errors, not squared errors as in the LSM. In the second modification of the orbit improvement technique, the differences $O - C$ (between the observed and calculated positions) are transformed to fit into a new rectangular coordinate frame whose origin is at the calculated position and axes are tangent lines parallel and perpendicular to the asteroid's apparent path. This system may be expected to have larger errors on the parallel axis and smaller errors on the perpendicular axis. However, this situation can be dealt with by assigning the relevant weights so that the LSM could be used.

The accuracy of these methods was assessed by the examples of orbit improvement for real near-Earth asteroids (NEAs).

DATA ON THE ASTEROIDS

In order to assess effectiveness of the suggested methods, 16 asteroids were selected by the criteria of orbit reliability (≥2 observed oppositions and ≥250 observations) and availability of observations at the first opposition, whereby the asteroid had an apparent angular velocity of 0.2″ per second or greater. Moreover, we prioritized asteroids with a fairly large number of modern positional CCD observations.

Table 1. Data on 16 potentially hazardous asteroids

Number	H, magn.	ρ, AU	ϑ, arcsec/s	N_{opp}	Arc, year	N_{all}	N_1	$N_{0.2}$
8566	16.5	0.147	0.2	7	1996–2007	368	243	10
10563	16.9	0.156	0.3	14	1993–2008	469	116	35
37655	17.7	0.045	0.7	7	1994–2003	297	88	42
54509	22.7	0.012	0.8	5	2001–2005	507	190	172
85770	20.6	0.112	0.2	10	1999–2008	400	94	4
137120	18.1	0.039	0.4	4	1999–2006	286	252	70
138175	20.3	0.043	0.3	9	2000–2008	588	171	54
138971	18.4	0.045	0.3	7	2002–2007	1092	292	48
139359	16.7	0.098	0.4	4	2001–2006	409	279	99
141432	20	0.046	0.4	6	2003–2008	272	112	42
141614	19.4	0.100	0.2	7	2002–2008	296	169	16
154276	17.6	0.084	0.3	4	2002–2005	577	555	111
162000	19.3	0.033	0.3	2	2003–2005	302	287	14
163373	19	0.067	0.3	3	2002–2004	343	312	94
184266	19.4	0.014	1.1	4	2004–2008	573	443	54
185851	18.2	0.048	0.4	5	2000–2008	874	397	35

When setting the apparent angular velocity threshold at 0.2″, we were guided by the fact that the average exposure time of a NEA for modern telescopes is several seconds to several dozens of seconds; therefore, its CCD image might be not point-like but extended along the apparent path (or the images of the comparative stars could be stretched if the telescope is guided on the NEA). As a result of the attenuated images, the largest errors can be expected along the apparent path.

The information on the observations and absolute magnitudes H is given in Table 1. The first column presents the number of an asteroid; the second gives its magnitude H; the third shows the asteroid's minimum geocentric distance at the first opposition ρ; the fourth presents the asteroid's maximum angular velocity at the first opposition ϑ; the fifth shows the number of observed oppositions N_{opp}; the sixth gives the observation interval in years; the seventh and eighth present the total number of observations and number of observations at the first opposition N_{all} and N_1, respectively; and the ninth gives the number of observations at which moment the asteroid's angular velocity was 0.2″ per second or greater $N_{0.2}$.

LEAST ABSOLUTE DEVIATIONS

The accepted differential method of orbit improvement uses the least squares technique; i.e., the vector of initial orbital parameters E is found by minimizing a target function of the form:

$$F(E) = \sum_{i=1}^{N} w_i (O_i - C_i(E))^2, \tag{1}$$

where N is the number of observations; w_i is the weight of observation i; O_i are the components of the N-dimensional observation vector (position, reflected-signal delay, and Doppler shift); and C_i are the calculated positions, which are a function of the target parameters of E (the state vector) if we can determine the coordinates and components of the velocity or vector of six Kepler elements.

This study uses least absolute deviations to improve asteroid orbits. This method has been shown by experiments to be generally less sensitive to clogged observation series. Its idea is to minimize the sum of absolute errors of conditional equations, i.e., a target function of the form:

$$F(E) = \sum_{i=1}^{N} w_i |O_i - C_i(E)|. \tag{2}$$

The literature contains two procedures for determining the minimum of a function which is a sum of absolute parameters: linear programming and the Weiszfeld algorithm (iteratively reweighted least squares method). A detailed discussion of the latter is given in (Mudrov and Kushko, 1971). As applied to the orbit improvement problem, the main idea is to minimize, a function of two vector arguments E_1 and E_2, as follows, instead of the target function (2):

$$\Phi(E_1, E_2) = \sum_{i=1}^{N} w_i \frac{(O_i - C_i(E_1))^2}{|O_i - C_i(E_2)|}. \tag{3}$$

Table 2. Comparison of the more accurate orbit with those determined by the LSM and LAD

N	R_{LSM}	R_{LAD}	χ^2
8566	1.17×10^{-5}	2.11×10^{-6}	8.6
10563	1.24×10^{-4}	1.04×10^{-4}	14.7
37655	3.40×10^{-3}	2.37×10^{-4}	6.1
54509	1.43×10^{-4}	1.30×10^{-4}	61.7
85770	4.48×10^{-3}	1.78×10^{-3}	10.9
137120	3.04×10^{-6}	4.78×10^{-5}	4.3
138175	3.42×10^{-5}	2.38×10^{-5}	10.8
138971	1.90×10^{-5}	1.39×10^{-5}	17.2
139359	6.05×10^{-5}	4.08×10^{-4}	21.3
141432	4.41×10^{-5}	6.75×10^{-5}	3.9
141614	3.69×10^{-4}	1.12×10^{-4}	15.0
154276	6.13×10^{-6}	4.32×10^{-6}	21.9
162000	1.85×10^{-4}	1.36×10^{-4}	11.7
163373	4.34×10^{-4}	2.27×10^{-4}	4.4
184266	3.64×10^{-5}	3.61×10^{-5}	23.5
185851	3.22×10^{-5}	2.91×10^{-5}	10.7

If we assume in (3) that $E_1 = E_2 = E$, then $\Phi(E_1, E_2) = F(E)$. Let $E_2 = E^0$, where E^0 is the initial vector of the orbital parameters. Then the function

$$\Phi\left(E_1, E^0\right) = \sum_{i=1}^{N} w_i \frac{(O_i - C_i(E_1))^2}{\left|O_i - C_i\left(E^0\right)\right|} = \sum_{i=1}^{N} w_i^{'} (O_i - C_i(E_1))^2, \quad (4)$$

becomes equal to the target function of the least squares method with the modified weights $w_i^{'} = \dfrac{w_i}{\left|O_i - C_i\left(E^0\right)\right|}$. Minimizing this function by the standard LSM, we get the vector $E_1 = E^1$. Then we substitute the vector E^0 for E^1 in (4) and can again minimize function (4), etc. Eventually, we get a sequence $E^0, E^1, \ldots, E^k$, which converges to E^*, which minimizes function (4). Using this technique, we calculated the orbits of 16 selected asteroids. Moreover, the orbits were determined by the accepted improvement technique using the LSM.

The accuracy of the orbits determined by the two methods (LSM and LAD) was checked as follows. The asteroid orbits were calculated from observations at the first opposition. Then the asteroid's geocentric coordinates at the moment of the first observation at the next opposition were calculated by integrating motion equations with the initial data obtained by the LSM and LAD. The two positions were compared with the position in a more accurate orbit calculated from observations at all the oppositions. When calculating the orbit from all the oppositions, we used the accepted LSM-based improvement technique. The calculated data are presented in Table 2. The second and third columns of this table give the distances in AU between the position in the accurate orbit and those determined by the LSM and LAD from observations during one opposition, respectively. The cases, when $R_{\mathrm{LSM}} < R_{\mathrm{LAD}}$, are marked in the table by gray. The fourth column presents the relative values of Pearson's χ^2 test (Aivazyan et al., 1983), which allow one to verify the hypothesis on the normal distribution of the asteroid observation errors.

In order to calculate this test, the entire range of the relative values of the difference $O - C$ (between the observed and calculated positions determined from the nominal orbit and divided by the mean error of unit of weight σ_0 of the asteroid) at the first oppositions was divided into eight intervals $(-\infty, -3)$, $[-3, -2)$, $[-2, -1)$, ..., $[2, 3)$, $[3, \infty)$ to calculate the number of $O - C$ in each interval m_i $i = \overline{1,8}$ and the frequencies $p_i^* = m_i/N$, where N is the total number of $O - C$ at the first opposition. The nominal orbit was calculated from all the available observations by the least squares method; in the case of right ascension, the difference $O - C$ between the observed and calculated positions was multiplied by the cosine of the calculated inclination. Pearson's χ^2 statistics were calculated by the formula $\chi^2 = \sum_i^8 \dfrac{N\left(p_i^* - p_i\right)^2}{p_i}$, where p_i is the probability of a normally distributed random variable with zero expectation and unit variance. The resulting values were divided by $\chi_{7.0.01} = 18.48$, which corresponds to the 0.01 significance level for seven degrees of freedom (the number of intervals –1). Thus, if the calculated $\chi^2 > 1$, we can reject the hypothesis that the values of the difference $O - C$ are distributed according to the normal law with a probability of 0.99. The values of χ^2 in Table 2 show that, with a probability of 0.99, the distributions of the observation errors of all the asteroids under study are not normal because the calculated χ^2 are greater than one.

It follows from Table 2 that in 13 cases the distances between the position in the LAD orbit and the accurate position in the orbit calculated from all the observations proved to be smaller than those for the LSM orbits. Only in three cases, for asteroids 137120, 139359, and 141432, the LSM orbits from observations at the first opposition proved to be more accurate. Note, however, that asteroid 137120 has 252 observations at the first opposition in the month-and-a-half arc (from January 14, 1999, to February 22, 1999); hence, the least squares method should give a rather good orbit. Moreover, the distribution of the errors of these 252 observations is close to the normal law. This follows from the relatively small $\chi^2 = 4.3$

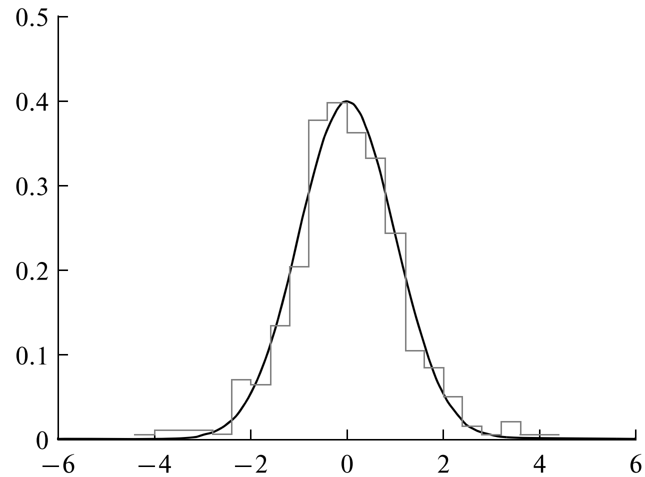

Fig. 1. The $O-C$ histogram for positional observations of asteroid 137120.

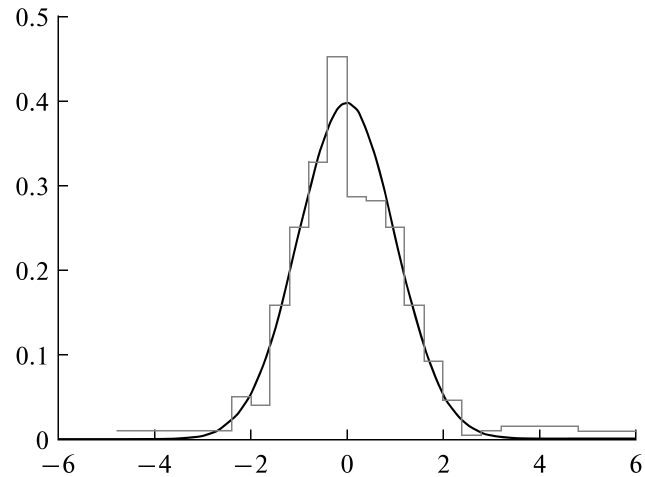

Fig. 2. The $O-C$ histogram for positional observations of asteroid 8566.

among the 16 asteroids and the $O-C$ histogram of asteroid 137120 in Fig. 1.

In this figure, the abscissa axis plots the relative values of the difference $O-C$ (between the observed and calculated positions determined from the nominal orbit and divided by the mean error of unit of weight $\sigma_0 = 0''.386$ of the asteroid), and the ordinate axis plots the values of the distribution density function. The thin line shows the $O-C$ histogram as such; the solid line shows the Gaussian curve with zero expectation and unit variance.

For comparison, Fig. 2 presents the $O-C$ histogram at the first opposition for asteroid 8566, whose LSM orbit proved to be considerably worse than the LAD orbit. This figure uses the same designations as Fig. 1. The mean error of unit of weight for this asteroid is $\sigma_0 = 0''.530$, and its $\chi^2 = 8.6$. It is seen from this pattern that the distribution of the observation errors for this asteroid at the first opposition is different from that of a normal distribution (the histogram is not symmetric with respect to zero), which explains the low accuracy of the LSM orbit. The situation with asteroid 141432 is roughly the same as with asteroid 137120. There is quite a large number of observations in the time interval (>1 month); the error distribution is close to the normal law, and Pearson's test is $\chi^2 = 3.9$ (the lowest value in the sample). Unfortunately, χ^2 is not a decisive estimate of the quality of an LSM orbit. This follows from the example of asteroid 139359, which has a large $\chi^2 = 21.3$, which indicates that the distribution of the observation errors at the first opposition for this asteroid is substantially deviated from the normal law. However, the large number of observations over a long observation interval (from May 25, 2001, to July 4, 2001) and the shape of the histogram (see Fig. 3) explain why this asteroid has a confident LSM orbit.

Figure 3 shows that the distribution of $O-C$ is different from the normal law, but the main differences are concentrated in the near-zero intervals and consist in an excessive number of accurate observations (less than $\sigma_0 = 0''.435$), which, apparently, was no handicap in obtaining a quite accurate orbit by the classical LSM.

MODIFIED DIFFERENTIAL METHOD

The second method tested in terms of the possibility of calculating a more accurate orbit from observations during one opposition for NEAs is the modified differential method (MDM). Within this method, conditional equations for observations with the apparent angular velocity greater than 0.2″ per second were written for the deviations parallel (ΔG) and perpendicular (Δg) to the asteroid's apparent path, and not for

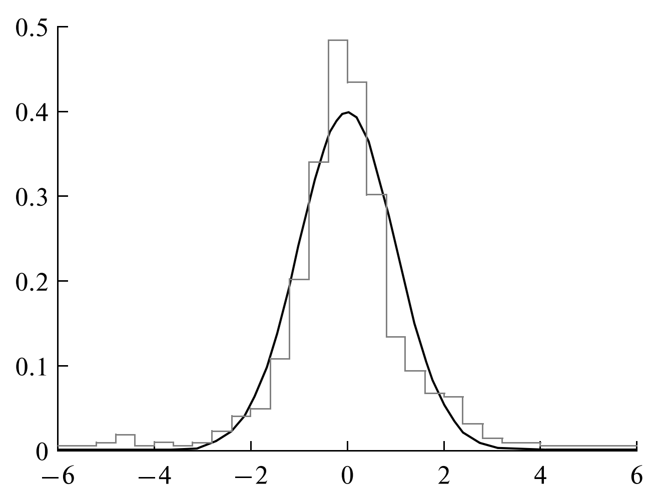

Fig. 3. The $O-C$ histogram for positional observations of asteroid 139359.

Table 3. Comparison of the more accurate orbit with those determined by the LSM and MDM

N	R_{LSM}	R_{MDM}
8566	1.16×10^{-5}	1.37×10^{-5}
10563	1.23×10^{-4}	1.20×10^{-3}
37655	3.40×10^{-3}	4.13×10^{-5}
54509	1.43×10^{-4}	3.36×10^{-4}
85770	4.48×10^{-3}	4.53×10^{-3}
137120	3.04×10^{-6}	8.72×10^{-6}
138175	3.42×10^{-5}	3.88×10^{-5}
138971	1.90×10^{-5}	6.89×10^{-6}
139359	6.05×10^{-5}	2.79×10^{-5}
141432	4.41×10^{-5}	1.00×10^{-5}
141614	3.69×10^{-4}	3.69×10^{-4}
154276	6.14×10^{-6}	4.07×10^{-6}
162000	1.85×10^{-4}	1.81×10^{-4}
163373	4.34×10^{-4}	4.48×10^{-4}
184266	3.64×10^{-5}	4.68×10^{-5}
185851	3.22×10^{-5}	3.64×10^{-5}

the deviation in the right ascension $\Delta\alpha\cos\delta$ and in the inclination $\Delta\delta$, as in the generally accepted method. This technique was first proposed by D.K. Kulikov and Yu.V. Batrakov (1960) to improve the orbits of artificial satellites when observation points have large errors. The applications of the technique found it to be highly effective in improving artificial satellite orbits, i.e., for objects with quick apparent motions; therefore, it is suggested that this technique may be helpful for NEAs.

Because the images of NEAs (or stars) are stretched along the apparent orbit, large errors could be expected in the direction of the apparent motion. Therefore, the respective conditional equations were assigned smaller weights. Several variants of weight-assigning schemes were considered. Here we give the results for the variant whereby the weight p for the conditional equations ΔG was assigned by the formula $p = \sqrt{0.2/\vartheta}$, where ϑ is the apparent angular velocity of the asteroid. Thus, in the case when an asteroid had apparent angular velocity greater than 0.2″ per second, the respective conditional equations for the deviations along the apparent path were assigned a smaller weight $p \le 1$. As in the case of the LAD effectiveness testing, we calculated two orbits. The first orbit was calculated by the generally accepted differential method (referred to as the LSM orbit, as in the previous section); the second orbit was determined by the modified differential method (MDM orbit). Both techniques used observations at the first opposition only. The accuracy of the orbits was estimated by comparing the calculated positions with that at the next opposition in a more accurate orbit calculated from all the observations. The calculated results are given in Table 3.

Like Table 2, this table presents, in its second and third columns, the distances in AU between the more accurate position in the orbit calculated from all the observations and those in the orbits calculated by the accepted and modified methods, respectively, at the moment of the first observation at the second opposition. Here, like in Table 2, the cases when $R_{LSM} < R_{MDM}$ are marked with gray.

Analysis of Table 3 shows that the use of MDM does not lead to any perceptible increase in accuracy. The accuracy of MDM orbits is close to that of LSM orbits. However, for asteroids 139359 and 141432, MDM yields more accurate orbits than LAD and LSM.

CONCLUSIONS

This paper considers issues related to the improvement of the orbits of newly discovered NEAs. It compares three orbit improvement techniques. The first is a classical technique based on the LSM principle; the second is based on the principle of LAD; the third technique transforms the differences $O-C$ to fit into a new coordinate system whereby the axes are parallel and perpendicular to the asteroid's apparent path. The effectiveness of the techniques is tested on a sample of 16 asteroids with two or more observed oppositions, ≥250 observations. All the asteroids in the sample also have observations at the first opposition with an apparent angular velocity of 0.2″ per second or greater. The observations at the first opposition of each asteroid are used to calculate their orbits by three methods: (1) the classical differential method using the LSM principle for conditional equations for the deviations in the right ascension $\Delta\alpha\cos\delta$ and in the inclination $\Delta\delta$; (2) the differential method using the LAD principle for $\Delta\alpha\cos\delta$ and $\Delta\delta$; and (3) the differential method using the LSM principle for deviations in the two mutually perpendicular directions parallel and perpendicular to the apparent path. Then, at the moment of the first observation at the next observed opposition, the heliocentric distances for each asteroid in these three orbits are calculated. Moreover, the orbits of all the 16 asteroids are calculated by the accepted differential method from all the available observations. The accuracy of the orbits calculated from the observations at the first opposition is estimated by comparing the position of the asteroid in the relevant orbit with its position at the next opposition in a more accurate orbit determined from all the observations.

The comparisons show that in 13 cases out of 16 the LAD orbits are more accurate than LSM orbits. The improvement of the orbits by the modified differential method, which includes the $O - C$ transformation, does not produce any perceptible increase in the accuracy when compared to the orbits calculated by the classical method. However, for two asteroids whose LAD orbits proved to be worse than their LSM orbits, the modified method yields more accurate results.

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REFERENCES

Aivazyan, S.A., Enyukov, I.S., and Meshalkin, L.D., *Prikladnaya statistika. Osnovy modelirovaniya i pervichnaya obrabotka dannykh* (Applied Statistics. Foundations of Simulation and Data Preprocessing), Moscow: Finansy i statistica, 1983.

Kulikov, D.K. and Batrakov, Yu.V., The Way to Improve the Artificial Earth Satellites Orbits by Using Observations with Approximated Moments, *Byull. ITA,* 1960, vol. 7, no. 90, pp. 554–569.

Linnik, Yu.V., *Metod naimen'shikh kvadratov i osnovy teorii obrabotki nablyudenii* (Least-Squares Method and Foundations of the Data Processing Theory), Moscow: Fizmatlit, 1962.

Makarova, E.N., Observations Processing of Selected Minor Planets by Means of Least Modules Method, *Trudy IPA RAN,* 2000, issue 5, pp. 190–196.

Mudrov, V.I. and Kushko, V.L., *Metod naimen'shikh modulei* (Least Modules Method), Moscow: Znanie, 1971, issue 7.