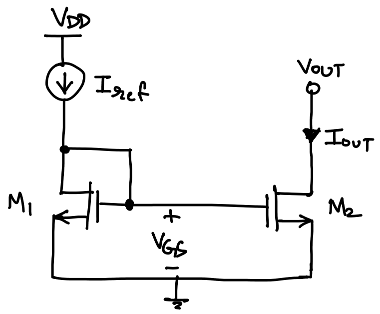
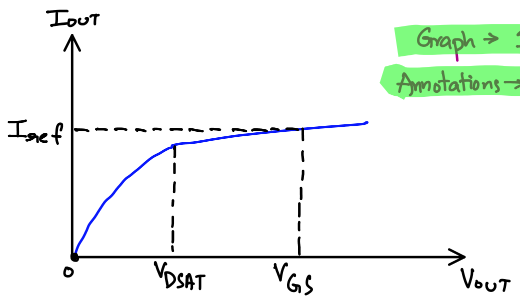
EE618 Quiz 1 Solutions

Q1.) Given : M_1 & M_2 are identical



A.)



Graph → 1 mark

Annotations → 1 mark

B) given : $V_{OUT} = V_{DSAT} = V_{GS} - V_{TN}$

from M_1 : $I_1 = I_{ref} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [V_{GS} - V_{TN}]^2 (1 + \lambda V_{GS})$ — ①

from M_2 : $I_2 = I_{OUT} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [V_{GS} - V_{TN}]^2 (1 + \lambda V_{DSAT})$ — ②

→ 1 mark

② − ① $\Rightarrow$ $I_{OUT} - I_{ref} = \left(\frac{\mu_n C_{ox}}{2} \frac{W}{L} [V_{GS} - V_{TN}]^2\right) \lambda (V_{DSAT} - V_{GS})$

$$= \left(\left(\frac{I_D}{1 + \lambda V_{DS}}\right)\right) \lambda (V_{DSAT} - V_{GS})$$

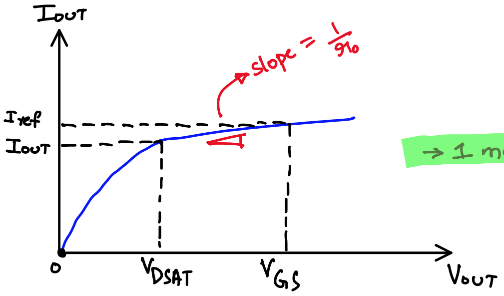
$$= \left(\frac{1}{r_o}\right)(V_{DSAT} - V_{GS})$$

$$\Rightarrow \boxed{\Delta I_{OUT} = I_{OUT} - I_{ref} = \frac{V_{DSAT} - V_{GS}}{r_o}}$$

→ 1 mark

ALTERNATE SOLUTION:

from the graph



→ 1 mark

wkt slope $= g_o = \frac{1}{r_o}$

$$\Rightarrow \quad \frac{1}{r_o} = \frac{\Delta y}{\Delta x} = \frac{I_{ref} - I_{OUT}}{V_{GS} - V_{DSAT}}$$ → 1 mark

$$\Rightarrow \quad \boxed{\Delta I_{OUT} = I_{OUT} - I_{ref} = \frac{V_{DSAT} - V_{GS}}{r_o}}$$

C.) given:

$V_{OUT} = V_{DSAT} = 0.2\,V$ $\quad$ $I_{ref} = 0.2\,mA$

$V_{TN} = 0.45V$ $\quad$ $r_o = 45\,K\Omega$

$V_{DSAT} = V_{GS} - V_{TN}$

$\Rightarrow V_{GS} = V_{DSAT} + V_{TN} = 0.65\,V$ → 0.5 marks

$$\Delta I_{OUT} = \frac{V_{DSAT} - V_{GS}}{r_o} = \frac{-0.45}{45\,K}$$

$$= -10\,\mu A$$ → 1 mark

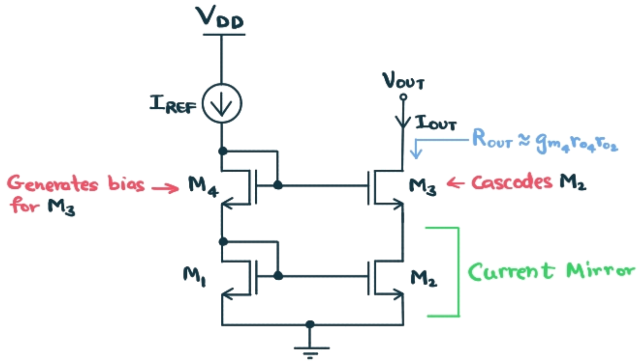
$I_{OUT} = I_{ref} + \Delta I_{OUT} = 190\,\mu A$

$= 0.19\,mA$ → 0.5 marks

D:)

Drawback: R_{OUT} of this current source is not very high.
($R_{OUT} = r_o$) → 0.5 marks

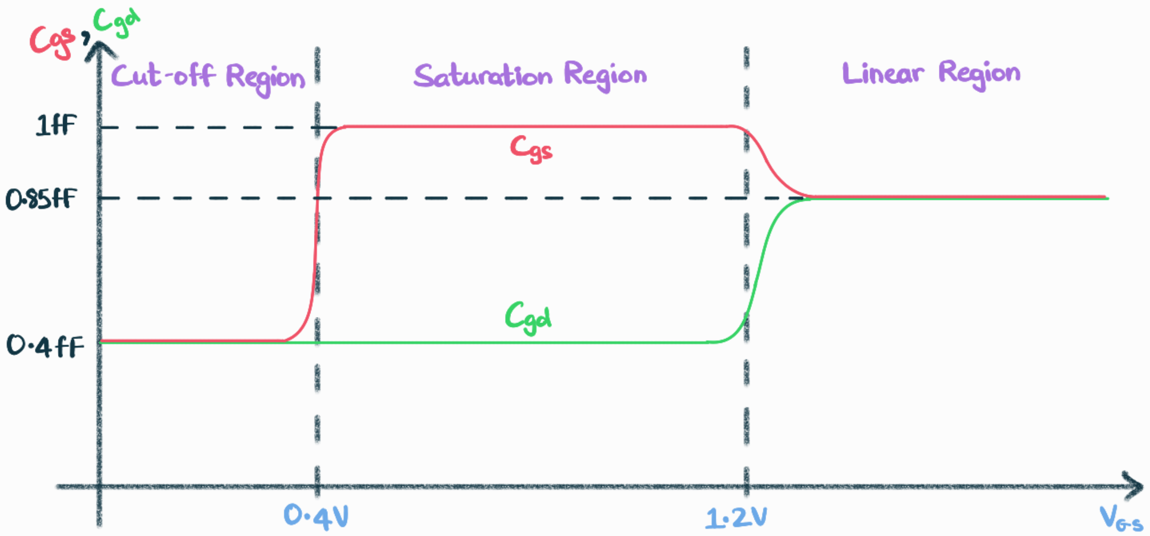
Modified schematic: (Use a cascode)



→ 1 mark

Trade-off: Voltage required at V_{OUT} to keep M_2 & M_3 in saturation is higher now. → 0.5 marks

Q2]



Cut-off region

$C_{gd} = C_{ov}W = 0.4fF$ — 0.5 mrks

$C_{gs} = C_{ov}W = 0.4fF$ — 0.5 mrks

Saturation region

$C_{gd} = C_{ov}W = 0.4fF$ — 0.5 mrks

$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov}W = 1fF$ — 0.5 mrks

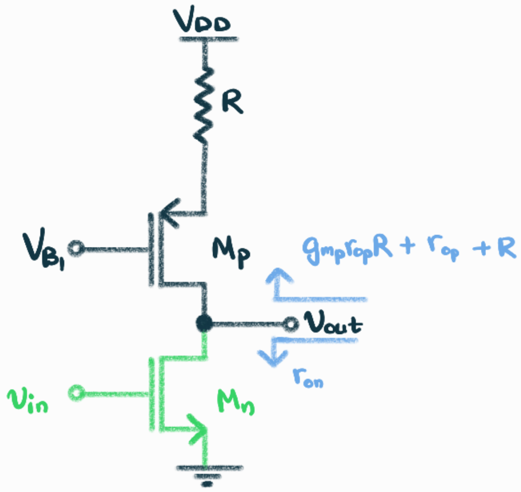
Linear / Triode / Non-sat region

$C_{gd} = \frac{1}{2}WLC_{ox} + C_{ov}W = 0.85fF$ — 0.5 mrks

$C_{gs} = \frac{1}{2}WLC_{ox} + C_{ov}W = 0.85fF$ — 0.5 mrks

Q3] a) Note: Devices shown in green are providing effective G_m

ⓐ

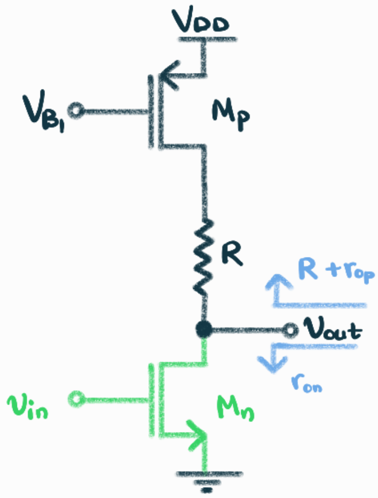


$G_m = g_{mn}$ — 0.5 mrks

$R_{out} = r_{on} \,||\, (g_{mp} r_{op} R + r_{op} + R)$ — 0.5 mrks

$A_v = -g_{mn}[r_{on} \,||\, (g_{mp} r_{op} R + r_{op} + R)]$ — 0.5 mrks

ⓑ

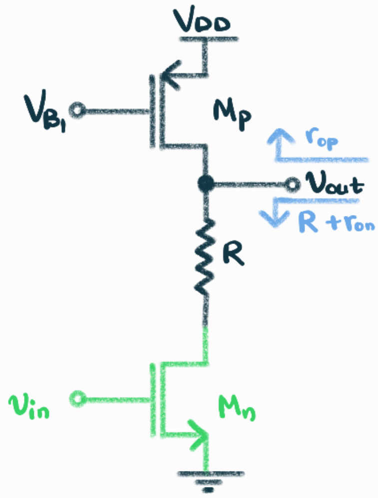


$G_m = g_{mn}$ — 0.5 mrks

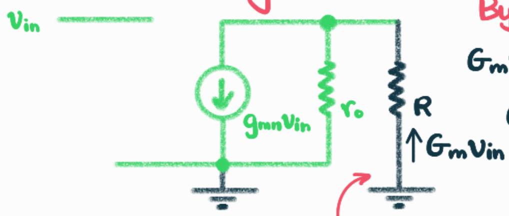
$R_{out} = r_{on} \,||\, (R + r_{op})$ — 0.5 mrks

$A_v = -g_{mn}[r_{on} \,||\, (R + r_{op})]$ — 0.5 mrks

ⓒ



Calculating G_m



O/P shorted to GND

By Current Division

$G_m V_{in} = g_{mn} V_{in} \left(\frac{r_{on}}{r_{on} + R} \right)$

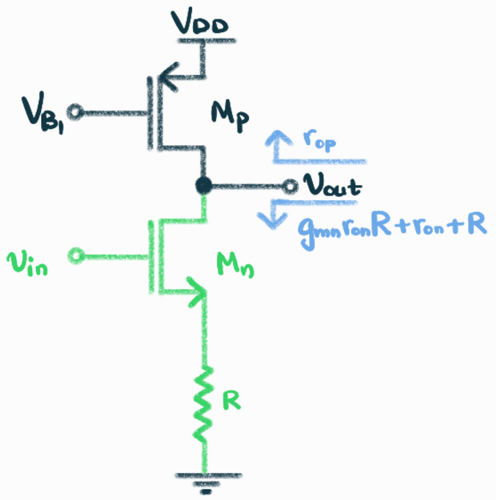
$G_m = g_{mn} \left(\frac{r_{on}}{r_{on} + R} \right)$

$G_m = g_{mn} \left(\frac{r_{on}}{r_{on} + R} \right) \approx g_{mn}$ — 0.5 mrks

$R_{out} = r_{op} \,||\, (R + r_{on})$ — 0.5 mrks

$A_v \approx -g_{mn}[r_{op} \,||\, (R + r_{on})]$ — 0.5 mrks

(d)



(Source Degeneration)

$$G_m = \frac{g_{mn} r_{on}}{r_{on} + R + g_{mn} r_{on} R} \approx \frac{g_{mn}}{1 + g_{mn} R}$$ — 0.5 mrks

$$R_{out} = r_{op} \| (g_{mn} r_{on} R + r_{on} + R)$$ — 0.5 mrks

$$A_v \approx -\frac{g_{mn}}{1 + g_{mn} R}\left[r_{op} \| (g_{mn} r_{on} R + r_{on} + R)\right]$$ — 0.5 mrks

Q3] b) For $r_{op} = r_{on} = r_o$, $g_{mp} = g_{mn} = g_m$, $r_o >> R$

$|A_a| \approx g_m r_o$

$|A_b| \approx g_m r_o / 2$

$|A_c| \approx g_m r_o / 2$

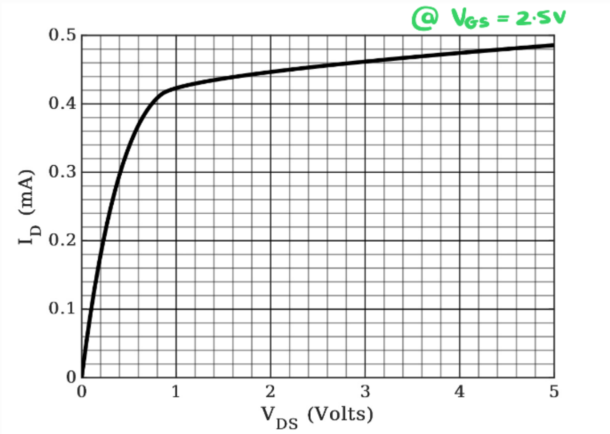
$|A_d| \approx r_o / R$

Descending Order

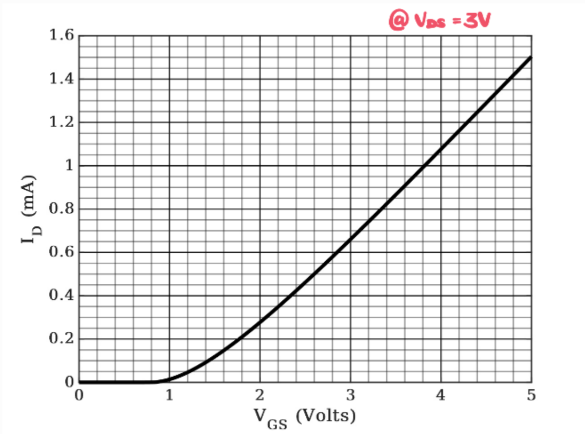
$|A_a| > |A_b| = |A_c| > |A_d|$ — 2 mrks
Only if order fully correct

Q4]

(a) Output Characteristics



Transfer Characteristics

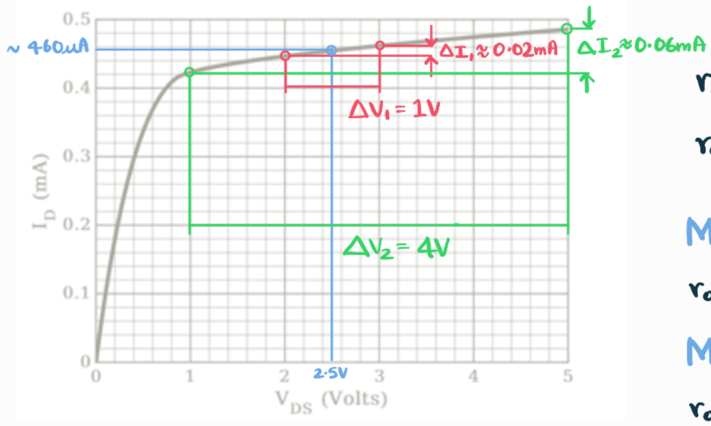


Clearly from the above plots, $I_D > 0$ for $V_{GS} > 0$ and $V_{DS} > 0$

$\therefore$ 'G' & 'D' at higher potential than 'S' $\Rightarrow$ NMOS — 1 mrks

(b) For $V_{DS} = 3V$, $V_{GS} = 2.5V$:

I_D & r_o calculation :



$I_D = 0.46mA$

r_o calculation: Find 2 suitable points around $V_{DS} = 0.3V$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\text{slope}} \quad \text{(in saturation)}$$

Method 1: Choose points $V_{DS} = 2V, 3V$

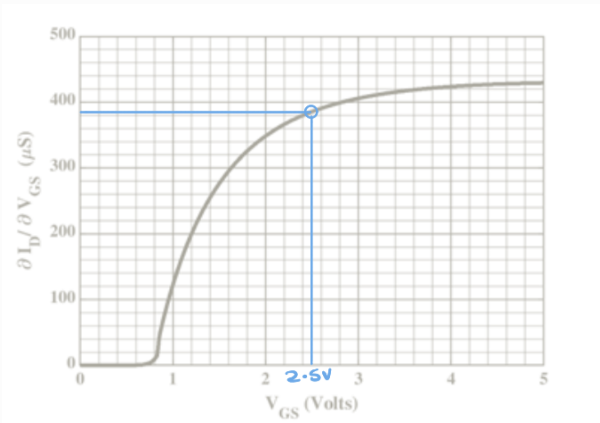
$$r_o = \frac{3V - 2V}{0.46mA - 0.44mA} = 50K\Omega$$

Method 2: Choose points $V_{DS} = 5V, 1V$

$$r_o = \frac{5V - 1V}{0.48mA - 0.42mA} = 66.67K\Omega$$

$r_o = 50K\Omega$ to $70K\Omega$ — 1 mrks

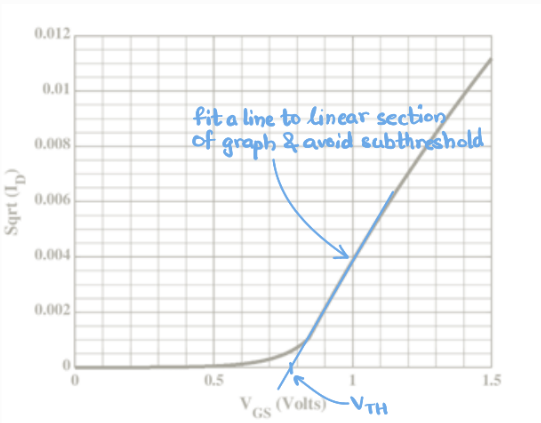
g_m calculation :



$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS} = 2.5V}$$

$g_m = 380\mu A/V$ to $400\mu A/V$ — 1 mrks

V_{TH} calculation :



V_{TH} = x-intercept of fitted line

$V_{TH} = 0.75V$ to $0.85V$ — 1 mrks

(c) $r_o = \frac{1}{\lambda I_D} \Rightarrow \lambda = \frac{1}{r_o I_D}$ $I_D = 0.46\,mA$ @ $V_{GS} = 2.5V$, $V_{DS} = 3V$

$\lambda = 0.03\,V^{-1}$ to $0.045\,V^{-1}$ — 1 mrks

$$\mu_n C_{ox} = \frac{I_D}{\frac{W}{2L}(V_{GS} - V_{TH})^2(1+\lambda V_{DS})} \quad \text{or} \quad \mu_n C_{ox} = \frac{g_m}{\frac{W}{L}(V_{GS} - V_{TH})(1+\lambda V_{DS})}$$

$\mu_n C_{ox} = 40\,\mu A/V^2$ to $60\,\mu A/V^2$ — 1 mrks

The End