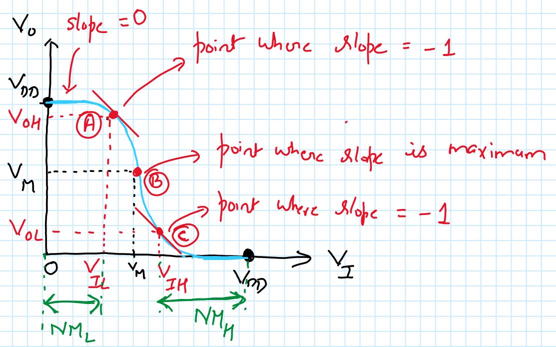
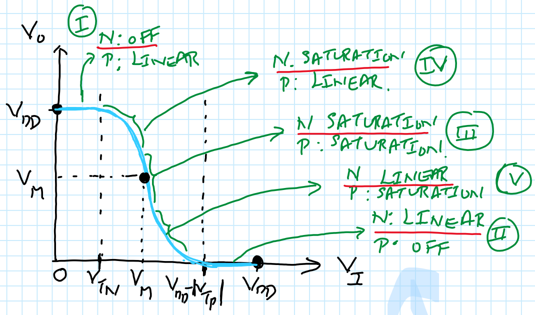
Additional Notes on Noise Margin. (Not covered in the class

* We try to detail more on the Noise Margin mathematically. The notes is self explanatory & will not be covered in class.





* We refer to the Lecture-5 notes for the Inverter transfer characteristics
* Based on our discussion, At point (A), slope = −1,

Also, in region (A), NMOS is in saturation & PMOS is in Linear IV

Equate $I_{D,N} = I_{D,P}$ for solutions

* To find V_o ($I_{D,N} = I_{D,P}$)

$$\frac{1}{2}K_N\left[V_I - V_{TN}\right]^2 = K_P\left[(V_{DD} - V_I - |V_{TP}|)(V_{DD} - V_o) - \frac{(V_{DD}-V_o)^2}{2}\right]$$

* Substitute $V_x = V_{DD} - V_o$

$$\frac{1}{\gamma^2}\left[V_I - V_{TN}\right]^2 = \left[2(V_{DD} - V_I - |V_{TP}|)V_x - V_x^2\right]$$

$$\therefore V_x^2 - 2V_x(V_{DD} - V_I - |V_{TP}|) + \frac{(V_I - V_{TN})^2}{\gamma^2} = 0$$

$$\therefore V_x = \left[2(V_{DD} - V_I - |V_{TP}|) \pm \sqrt{4(V_{DD} - V_I - |V_{TP}|)^2 - \frac{4(V_I - V_{TN})^2}{\gamma^2}}\right] \times \frac{1}{2}$$

$$= (V_{DD} - V_I - |V_{TP}|) \pm \sqrt{(V_{DD} - V_I - |V_{TP}|)^2 - (V_I - V_{TN})^2/\gamma^2}$$

$$\therefore V_o = V_I + |V_{TP}| \pm \sqrt{(V_{DD} - V_I - |V_{TP}|)^2 - (V_I - V_{TN})^2/\gamma^2}$$

− → Not a solution.

$$\therefore \boxed{V_o = V_I + |V_{TP}| + \sqrt{(V_{DD} - V_I - |V_{TP}|)^2 - (V_I - V_{TN})^2/\gamma^2}}$$

Rewriting $V_o = V_I + |V_{TP}| + \sqrt{\left[(V_{DD} - V_I - |V_{TP}|) + (V_I - V_{TN})/\gamma\right]\left[(V_{DD} - V_I - |V_{TP}|) - (V_I - V_{TN})/\gamma\right]}$ — (1)

To obtain V_{IL}, $\frac{\partial V_o}{\partial V_I} = -1$ (i.e. slope = −1).

$$\frac{\partial v_O}{\partial v_I} = 1 + \frac{\left[V_{DD} - V_I - |V_{TP}| + (V_I - V_{TN})/\gamma\right]\left[-1 - 1/\gamma\right] + \left[V_{DD} - V_I - |V_{TP}| - (V_I - V_{TN})/\gamma\right]\left[-1 + 1/\gamma\right]}{2\sqrt{\left[(V_{DD} - V_I - |V_{TP}|) + (V_I - V_{TN})/\gamma\right]\left[(V_{DD} - V_I - |V_{TP}|) - (V_I - V_{TN})/\gamma\right]}} = -1$$

After simplifications,

$$4V_I^2\beta(\beta+4) + 8V_I(\beta+4)\left[V_1 - V_{TN}/\gamma^2\right] - 12V_1^2 - \frac{8V_{TN}V_1}{\gamma^2} + \frac{4V_{TN}^2}{\gamma^2}\left(\frac{1}{\gamma^2}+4\right) = 0$$

where, $V_1 = V_{DD} - |V_{TP}|$ and $\beta = \frac{(1-\gamma^2)}{\gamma^2}$

$$\therefore V_I = V_{IL} = \boxed{\frac{V_{TN}^2\left[1/\gamma^2 + 4\right]/\gamma^2 - 2V_{TN}(V_{DD} - |V_{TP}|)/\gamma^2 - 3(V_{DD} - |V_{TP}|)^2}{-(\beta+4)\left[V_{DD} - |V_{TP}| - V_{TN}/\gamma^2\right] \pm \sqrt{(\beta+4)^2\left[V_{DD} - |V_{TP}| - V_{TN}/\gamma^2\right]^2 - \beta(\beta+4)\left[\frac{V_{TN}^2(1/\gamma^2+4)}{\gamma^2} - \frac{2V_{TN}(V_{DD} - |V_{TP}|)}{\gamma^2} - 3(V_{DD} - |V_{TP}|)^2\right]}}}$$

* Under the special case, $\gamma = 1$ $\left(\beta = \frac{1-\gamma^2}{\gamma^2} = 0\right)$

$$V_{IL} = \frac{5V_{TN}^2 - 2V_{TN}(V_{DD} - |V_{TP}|) - 3(V_{DD} - |V_{TP}|)^2}{-4\left[(V_{DD} - |V_{TP}|) - V_{TN}\right] \pm 4\left[V_{DD} - |V_{TP}| - V_{TN}\right]}$$

with $\gamma = 1$, the quadratic equation collapses to a linear eq^n & there is only one solution

$$\therefore V_{IL} = \frac{3(V_{DD} - |V_{TP}|)^2 + 2V_{TN}(V_{DD} - |V_{TP}|) - 5V_{TN}^2}{8\left[(V_{DD} - |V_{TP}|) - V_{TN}\right]}$$

$$= \frac{\left[3(V_{DD} - |V_{TP}|) + 5V_{TN}\right]\cancel{\left[(V_{DD} - |V_{TP}|) - V_{TN}\right]}}{8\cancel{\left[(V_{DD} - |V_{TP}|) - V_{TN}\right]}}$$

$$\boxed{\therefore V_{IL} = \frac{3V_{DD} - 3|V_{TP}| + 5V_{TN}}{8}}$$

* Substituting this V_{IL} in (1) to get V_{OH}, we get

$$\boxed{V_{OH} = \frac{7V_{DD} + V_{TN} + |V_{TP}|}{8}} = V_{DD} - \frac{V_{DD} - (V_{TN} + |V_{TP}|)}{8}$$

* Under the condition, $\gamma = 1$ and $V_{TN} = |V_{TP}| = V_T$

$$\underline{V_{IL} = \frac{3V_{DD} + 2V_T}{8}} \quad \text{and} \quad \underline{V_{OH} = \frac{7V_{DD} + 2V_T}{8}}$$

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* Now, shift focus to V_{IH}

* Based on our discussion, At point (c), slope $= -1$,

Also, in region (c), PMOS is in saturation & NMOS is in Linear [(V) in the transfer curve]

Equating $I_D = I$ for solution

Also, in region (C), PMOS is in saturation & NMOS is in Linear [(V) in the transfer curve]

Equate $I_{D,N} = I_{D,P}$ for solution

* To find V_o ($I_{D,N} = I_{D,P}$)

$$\frac{1}{2} K_N [V_I - V_{TN}]^2 = K_P \left[(V_{DD} - V_I - |V_{TP}|)(V_{DD} - V_o) - \frac{(V_{DD} - V_o)^2}{2} \right]$$

* Substitute $V_x = V_{DD} - V_o$

$$K_N \left[(V_I - V_{TN}) V_o - \frac{V_o^2}{2} \right] = \frac{1}{2} K_P \left[(V_{DD} - V_I - |V_{TP}| \right]^2.$$

$$2(V_I - V_{TN}) V_o - V_o^2 = \gamma^2 [V_{DD} - V_I - |V_{TP}|]^2.$$

$$V_o^2 - 2V_o (V_I - V_{TN}) + \gamma^2 [V_{DD} - V_I - |V_{TP}|]^2 = 0$$

$$\therefore V_o = \frac{2(V_I - V_{TN}) \pm \sqrt{4(V_I - V_{TN})^2 - 4\gamma^2 [V_{DD} - V_I - |V_{TP}|]^2}}{2}$$

$$\boxed{V_o = (V_I - V_{TN}) \pm \sqrt{(V_I - V_{TN})^2 - \gamma^2 [V_{DD} - |V_{TP}| - V_I]^2}}$$

(+ not solution)

Rewrite $V_o = (V_I - V_{TN}) - \sqrt{[V_I - V_{TN} + \gamma (V_{DD} - |V_{TP}| - V_I)][V_I - V_{TN} - \gamma (V_{DD} - |V_{TP}| - V_I)]}$ —— (2)

To obtain V_{IH}, $\frac{\partial V_o}{\partial V_I} = -1$ (i.e. slope $= -1$).

(You can solve this yourselves).

Under the special case of $\gamma = 1$

$$\boxed{V_{IH} = \frac{5V_{DD} + 3V_{TN} - 5|V_{TP}|}{8}}$$

* Substituting V_{IH} in (2) to get V_{OL}, we get

$$V_{OL} = \frac{V_{DD} - (V_{TN} + |V_{TP}|)}{8}$$

* Under the condition, $\gamma = 1$ and $V_{TN} = |V_{TP}| = V_T$

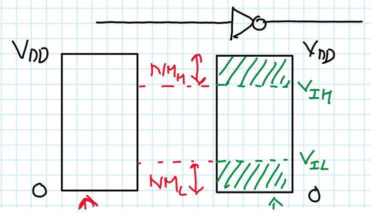
$$V_{IH} = \frac{5V_{DD} - 2V_T}{8} \quad \text{and} \quad V_{OL} = \frac{V_{DD} - 2V_T}{8}$$

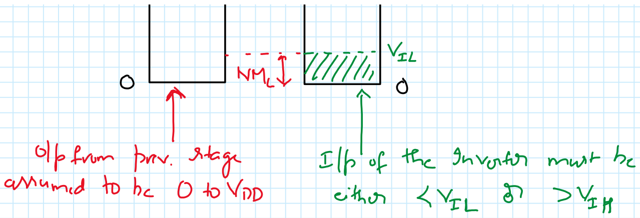
x —— x

* Re-evaluate Noise Margin

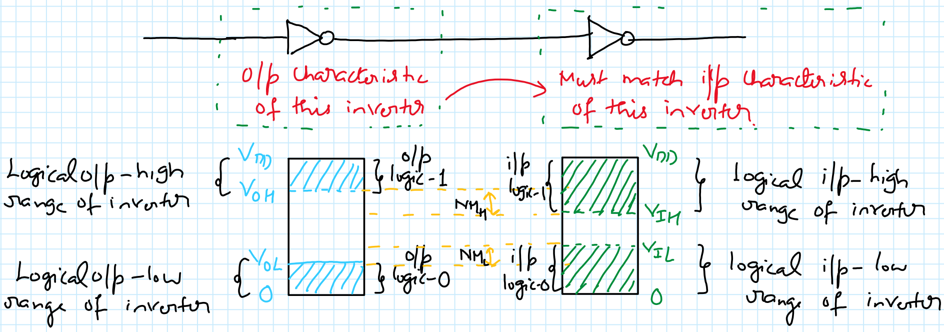
* In the class, we discussed NM_H & NM_L as

$NM_H = V_{DD} - V_{IH}$ & $NM_L = V_{IL} - 0$ considering a single inverter.





* But practically, each inverter drives another inverter (or other logic)



* The next stage inverter will consider any voltage below V_{IL} @ its input as logic-low & will consider any voltage above V_{IH} @ its input as logic-high.

* The previous stage can o/p any voltage between V_{DD} & V_{OH} as output-high (depending on its i/p) & it can o/p any voltage between 0 & V_{OL} as output-low (depending on its i/p)

* When the previous stage outputs V_{OH} as o/p (it is still a valid logic-high voltage), the amount of margin we have for the noise is reduced (i.e an addition of noise can make the i/p to the next inverter below V_{IH} or $(V_{OH} - \text{Noise}) < V_{IH}$, leading to wrong o/p).

$\therefore$ The worst case Noise margins are

$$NM_H = V_{OH} - V_{IH}$$

& $$NM_L = V_{IL} - V_{OL}$$

Best case:-

$$NM_H = V_{DD} - V_{IH}$$

$$NM_L = V_{IL} - 0$$

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* For the special case of $\gamma = 1$

$$NM_H = V_{OH} - V_{IH}$$

$$= \frac{7V_{DD} + |V_{TP}| + V_{TN}}{8} - \frac{(5V_{DD} - 5|V_{TP}| + 3V_{TN})}{8}$$

$$= \frac{2V_{DD} + 6|V_{TP}| - 2V_{TN}}{8}$$

$$\boxed{\therefore NM_H = \frac{V_{DD} + 3|V_{TP}| - V_{TN}}{4}}$$

$$NM_L = V_{IL} - V_{OL}$$

$$= \frac{3V_{DD} - 3|V_{TP}| + 5V_{TN}}{8} - \frac{(V_{DD} - |V_{TP}| - V_{TN})}{8}$$

$$= \frac{2V_{DD} - 2|V_{TP}| + 6V_{TN}}{8}$$

$$\therefore NM_L = V_{DD} - |V_{TP}| + 3V_{TN}$$

$$\therefore NM_L = \boxed{\frac{V_{DD} - |V_{TP}| + 3V_{TN}}{8}}$$

* Under the special case, $\gamma = 1$ & $V_{TN} = |V_{TP}| = V_T$

$$\boxed{NM_H = \frac{V_{DD} + 2V_T}{4} \quad \& \quad NM_L = \frac{V_{DD} + 2V_T}{4}}.$$

Summary of static parameters:

Parameter	$\gamma = 1$	$\gamma = 1$ and $V_{TN} = \|V_{TP}\| = V_T$.
V_M	$\frac{V_{DD} - \|V_{TP}\| + V_{TN}}{2}$	$V_{DD}/2$
V_{IL}	$\frac{3V_{DD} - 3\|V_{TP}\| + 5V_{TN}}{8}$	$(3V_{DD} + 2V_T)/8$
V_{IH}	$\frac{5V_{DD} - 5\|V_{TP}\| + 3V_{TN}}{8}$	$(5V_{DD} - 2V_T)/8$
V_{OH}	$V_{DD} - \frac{V_{DD} - \|V_{TP}\| - V_{TN}}{8}$	$(7V_{DD} + 2V_T)/8$
V_{OL}	$\frac{V_{DD} - \|V_{TP}\| - V_{TN}}{8}$	$(V_{DD} - 2V_T)/8$.
NM_H	$\frac{V_{DD} + 3\|V_{TP}\| - V_{TN}}{4}$	$(V_{DD} + 2V_T)/4$
NM_L	$\frac{V_{DD} - \|V_{TP}\| + 3V_{TN}}{4}$	$(V_{DD} + 2V_T)/4$.