AAG1
Domáci úkol II.

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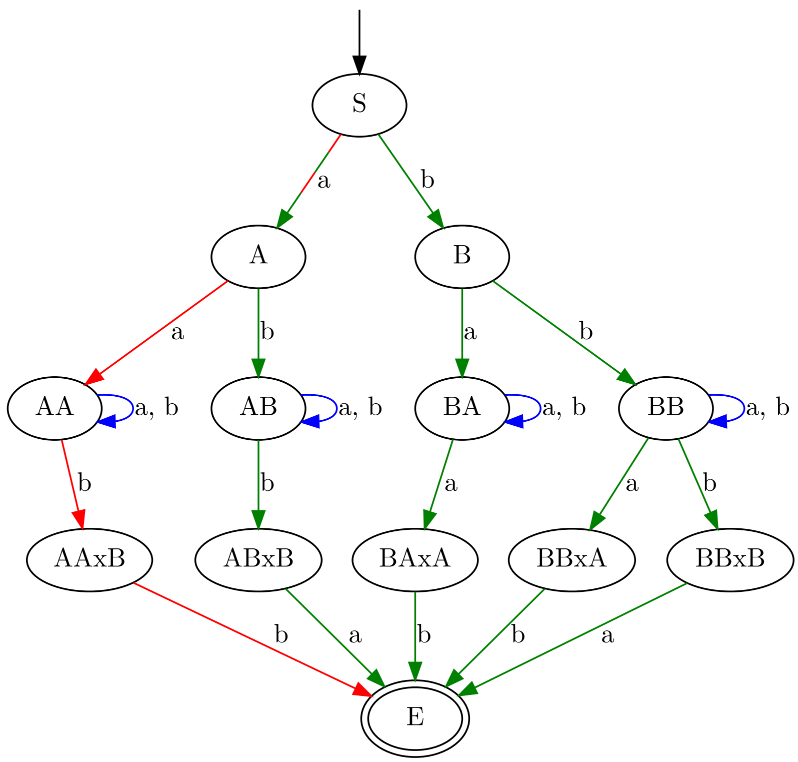
$$L = \{w : w \in \{a,b\}^* \wedge w = uvz \wedge |u| = |z| = 2 \wedge$$
$$\wedge\ u \neq z\ \wedge (z \in \{ab, ba\} \Leftrightarrow |u|_a < 2)\}$$

We can break equivalence from last condition into:

$$L =$$
$$\{w : w \in \{a,b\}^* \wedge w = uvz \wedge |u| = |z| = 2\ \wedge u \neq z \wedge$$
$$\wedge\ ((z \in \{ab, ba\} \wedge |u|_a < 2) \vee (z \notin \{ab, ba\} \wedge |u|_a \geqslant 2))\}$$
$$=$$
$$\{w : w \in \{a,b\}^* \wedge w = uvz \wedge |u| = |z| = 2\ \wedge u \neq z \wedge$$
$$\wedge\ z \in \{ab, ba\} \wedge |u|_a < 2\} \cup$$
$$\cup\ \{w : w \in \{a,b\}^* \wedge w = uvz \wedge |u| = |z| = 2\ \wedge u \neq z \wedge$$
$$\wedge\ z \notin \{ab, ba\} \wedge |u|_a \geqslant 2\}$$
$$=$$
$$\{w : w \in \{a,b\}^* \wedge w = uvz \wedge |u| = |z| = 2\ \wedge u \neq z \wedge$$
$$\wedge\ z \in \{ab, ba\} \wedge u \in \{ab, ba, bb\}\} \cup$$
$$\cup\ \{w : w \in \{a,b\}^* \wedge w = uvz \wedge |u| = |z| = 2\ \wedge u \neq z \wedge$$
$$\wedge\ z \in \{aa, bb\} \wedge u = aa\}$$
$$=$$
$$\{w : w \in \{a,b\}^* \wedge w = uvz \wedge$$
$$\wedge\ (u, z) \in \{(ba, ab), (bb, ab), (ab, ba), (bb, ba)\}\} \cup$$
$$\cup\ \{w : w \in \{a,b\}^* \wedge w = aavbb\}$$

We can create nondeterministic finite automaton from this description:



Of course we could join states $\{ABxB, BBxB\}$ and $\{AAxB, BAxA, BBxA\}$.

Formal definition of this automaton:

$$
\begin{gathered}
M = (\{S, A, B, AA, BB, AB, BA, BB, AAx, ABx, BAx, BBx, E\},\\
\{a, b\},\\
\{\delta(S, a) = \{A\},\\
\delta(S, b) = \{B\},\\
\delta(A, a) = \{AA\},\\
\delta(A, b) = \{AB\},\\
\delta(B, a) = \{BA\},\\
\delta(B, b) = \{BB\},\\
\delta(AA, a) = \{AA\},\\
\delta(AA, b) = \{AA, AAxB\},\\
\delta(AB, a) = \{AB\},\\
\delta(AB, b) = \{AB, ABxB\},\\
\delta(BA, a) = \{BA, BAxA\},\\
\delta(BA, b) = \{BA\},\\
\delta(BB, a) = \{BB, BBxA\},\\
\delta(BB, b) = \{BB, BBxB\},\\
\delta(AAxB, b) = \{E\},\\
\delta(ABxB, a) = \{E\},\\
\delta(BAxA, b) = \{E\},\\
\delta(BBxA, b) = \{E\},\\
\delta(BBxB, a) = \{E\}\},\\
S,\\
\{E\})
\end{gathered}
$$

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We can create deterministic finite automaton with states that keep track of original states. Each state of new automaton keeps information about subset of original automaton. For example deterministic state $\{A, B, C\}$ would mean that we are in states A, B, C of nondeterministic automaton at the same time. We start from states $\{A\}$, $\{B\}$. From state $\{A\}$ we can get to states $\{A, C, F\}$ in single a-transition ($\delta(\{A\}, a) = \{A, B, F\}$). This way we can create transition table:

Q	a	b	ε
$\leftrightarrow \{A\}$	$\{A, C, F\}$	$\{B\}$	$\{C\}$
$\{B\}$	$\{B, D\}$	$\emptyset$	$\emptyset$
$\leftarrow \{C\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\rightarrow \{D\}$	$\{A, F\}$	$\{C, D\}$	$\{A\}$
$\{B, D\}$	$\{A, B, D, F\}$	$\{C, D\}$	$\{A\}$
$\leftarrow \{C, D\}$	$\{A, F\}$	$\{C, D\}$	$\{A\}$
$\leftarrow \{A, F\}$	$\{A, C, F\}$	$\{B\}$	$\{C\}$
$\leftarrow \{A, C, F\}$	$\{A, C, F\}$	$\{B\}$	$\{C\}$
$\leftarrow \{A, B, D, F\}$	$\{A, B, C, D, F\}$	$\{B, C, D\}$	$\{A, C\}$
$\leftarrow \{A, C\}$	$\{A, C, F\}$	$\{B\}$	$\{C\}$
$\leftarrow \{B, C, D\}$	$\{A, B, D, F\}$	$\{C, D\}$	$\{A\}$
$\leftarrow \{A, B, C, D, F\}$	$\{A, B, C, D, F\}$	$\{B, C, D\}$	$\{A, C\}$

New states of automaton:

$$\begin{gathered} Q' = \{\{A\}, \{B\}, \{C\}, \{D\}, \\ \{A, C\}, \{A, F\}, \{B, D\}, \{C, D\}, \\ \{A, C, F\}, \{B, C, D\} \\ \{A, B, D, F\}, \{A, B, C, D, F\}\} \end{gathered}$$

Finite automaton accept states that contain at least one accept state of original automaton:

$$\begin{gathered} F' = \{\{A\}, \{C\}, \{A, C\}, \{A, F\}, \{C, D\}, \\ \{A, C, F\}, \{B, C, D\} \\ \{A, B, D, F\}, \{A, B, C, D, F\}\} \end{gathered}$$

Final deterministic automaton:

$$M' = (Q', \{a, b\}, \delta', \{\{A\}, \{D\}\}, F')$$

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We can easily transform automaton M into nondeterministic automaton with ε-transitions with single start-state. We can introduce new start-state S with ε-transitions into original start-states A, D.

Q	a	b	ε
$\rightarrow S$	$\emptyset$	$\emptyset$	$\{A, D\}$
$\leftarrow A$	$\{A, C, F\}$	$\{B\}$	$\{C\}$
B	$\{B, D\}$	$\emptyset$	$\emptyset$
$\leftarrow C$	$\emptyset$	$\emptyset$	$\emptyset$
D	$\{A, F\}$	$\{C, D\}$	$\{A\}$
E	$\{B, C\}$	$\emptyset$	$\{A, B, C, D, E, F\}$
F	$\emptyset$	$\emptyset$	$\emptyset$