AAG3

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1

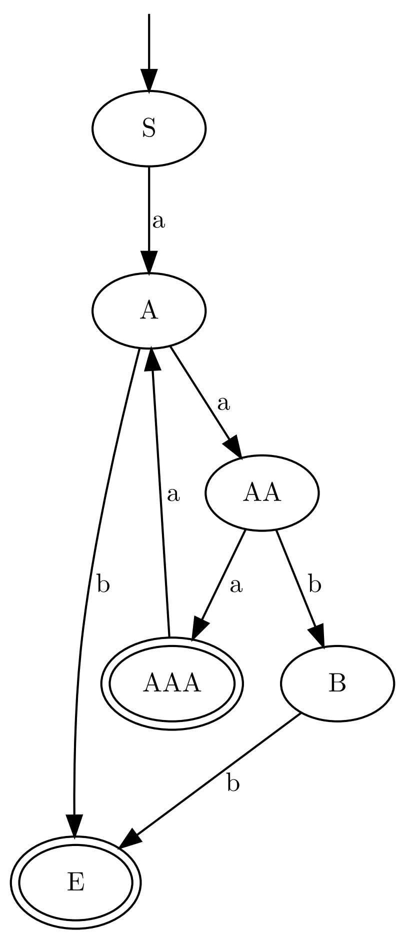
$$L_1 = \{w : w \in \{\mathtt{a}, \mathtt{b}\}^* \wedge |w|_\mathrm{a} \bmod 3 = |w|_\mathrm{b}\} \cap \left\{\mathtt{a}^m \mathtt{b}^j \mathtt{b}^i : m, j, i \in \mathbb{N}_0 \wedge j < m\right\} =$$

$$= \{\mathtt{a}^m \mathtt{b}^j \mathtt{b}^i : m, j, i \in \mathbb{N}_0 \wedge |w|_\mathrm{a} \bmod 3 = |w|_\mathrm{b} \wedge j < m\} =$$

$$= \{\mathtt{a}^m \mathtt{b}^j \mathtt{b}^i : m, j, i \in \mathbb{N}_0 \wedge m \bmod 3 = j + i \wedge m > j\} =$$

$$= \{\mathtt{a}^k \mathtt{b}^{k \bmod 3} : k \in \mathbb{N}\}$$

This language can be described by finite deterministic automaton:



Shortest word from language is $|\mathtt{ab}| = 2$, so smallest p we will consider is $p = 3$. $p = 3$ does not work because for word $\mathtt{aaa}; y \in \{\mathtt{a}, \mathtt{aa}, \mathtt{aaa}\}$ no y is suitable. Next p we can choose is $p = 4$ but again for word $\mathtt{aabb}$, all choices y do not work. For $p = 5$ smallest word we can test is $\mathtt{aaaab}$, for which choice of $x = \varepsilon, y = \mathtt{aaa}, z = \mathtt{ab}$ does satisfy pumping lemma because $(\forall k \in \mathbb{N}_0)(\mathtt{x}\mathtt{y}^k\mathtt{z} \in L_1)$. Generally for all $w = \mathtt{aaaa}^k\mathtt{b}^{(k+3) \bmod 3} = \mathtt{aaaa}^k\mathtt{b}^{k \bmod 3}; k \in \mathbb{N}$ the choice of $x = \varepsilon, y = \mathtt{aaa}, z = \mathtt{a}^k\mathtt{b}^{k \bmod 3}$ does satisfy pumping lemma.

2

$$L_2 = \{2^m 1^n 2^j : m, n, j \in \mathbb{N} \wedge m > 1 \wedge n, j \geq 1 \wedge j \neq m + n\}$$

By negating pumping lemma, we get following implication:

$$(\forall p \geq 1)(\exists w \in L)[|w| \geq p \wedge (\forall x, y, z \in \Sigma^*)((w = xyz \wedge |xy| \leq p \wedge |y| \geq 1) \Rightarrow (\exists k \geq 0) xy^k z \notin L)] \tag{1}$$
$$\implies L \textit{ is not regular language}$$

For any p we can choose $w = 2^{p+1}12^{p!+p+2}$, which is from language L_2 because $p+1 = m \geq 2 \wedge m + n = p + 1 + 1 = p + 2 < p! + p + 2 = j$ because $p! > 0$. From series of implication we can see that premise (1) holds:

$$(\forall x, y, z \in \Sigma^*)((w = xyz \wedge |xy| \leq p \wedge |y| \geq 1) \Rightarrow$$
$$\Rightarrow (x = 2^a, y = 2^b, c = 2^{p+1-a-b}12^{p!+p+2}; a, b \in \mathbb{N}_0; a + b \leq p \wedge b \geq 1) \Rightarrow$$
$$\Rightarrow (w' = xy^k z = 2^{kb+p+1-b}12^{p!+p+2}; k \in \mathbb{N}_0) \Rightarrow$$
$$\Rightarrow (m' = kb + p + 1 - b \wedge n' = 1 \wedge j' = p! + p + 2 \wedge k = 1 + \frac{p!}{b}) \Rightarrow$$
$$\Rightarrow (p! + p + 2 = (1 + \frac{p!}{b})b + p + 1 - b + 1) \Rightarrow$$
$$\Rightarrow (j' = m' + n') \Rightarrow$$
$$\Rightarrow (xy^k z = w' \notin L_2) \Rightarrow$$
$$\Rightarrow (\exists k \geq 0) xy^k z \notin L)$$

and therefore(from negation of pumping lemma), L_2 is not a regular language. $\square$