Recurrence example: Merge sort

Running times of recursive programs are solved using recurrence equations:

$$MergeSort(A,p,r)$$

// Let $n=r-p+1$
if $p < r$ then $q \leftarrow (p+r)/2$
 $MergeSort(A,p,q)$
 $MergeSort(A,q+1,r)$
 $Merge(A,p,q,r)$

- \bullet Let T(n) be the time taken by MergeSort to sort n = r - p + 1 elements.
- MergeSort recurrence: T(n) = 2T(n/2) + cn, T(1) = 1.

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Algorithm Design and Analysis

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Merging two sorted subarrays

Algorithm to merge sorted subarrays A[p..q] and A[q+1..r]. Its running time is O(n), where n=r-p+1.

$$Merge(A,p,q,r)$$

$$\mathsf{Copy}\ A[p..q]\ \mathsf{into}\ L[1..q-p+1]$$

$$\mathsf{Copy}\ A[q+1..r]\ \mathsf{into}\ R[1..r-q]$$

$$\mathsf{Set}\ \mathsf{sentinels}\ L[q-p+2]=R[r-q+1]=\infty$$

$$i\leftarrow 1;\quad j\leftarrow 1$$

$$for\ k\leftarrow p\ \mathsf{to}\ r\ \mathsf{do}$$

$$\mathsf{if}\ L[i]\leq R[j]\ \mathsf{then}$$

$$A[k]\leftarrow L[i];\quad i\leftarrow i+1$$

$$\mathsf{else}$$

$$A[k]\leftarrow R[j];\quad j\leftarrow j+1$$