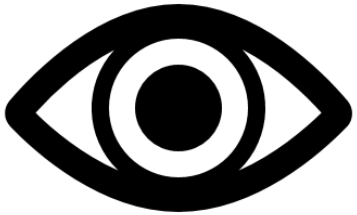


Advanced Image Processing

Image Filtering

SYDE 671

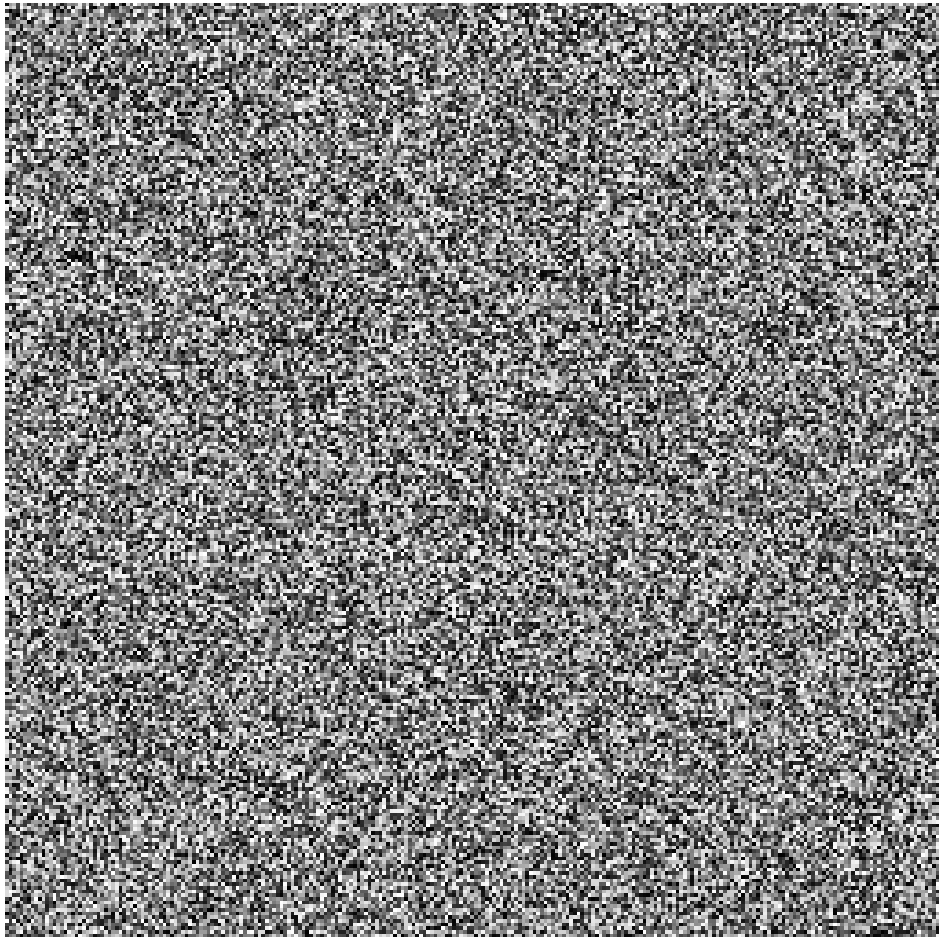
What Is an Image?

```
>>> from numpy import random as r
>>> I = r.rand(256,256);
```

- What is this? What does it look like?
- Which values does it take?
- How many values can it take?

- Is it an image?

```
>>> from matplotlib import pyplot as p
>>> I = r.rand(256,256);
>>> p.imshow(I);
>>> p.show();
```

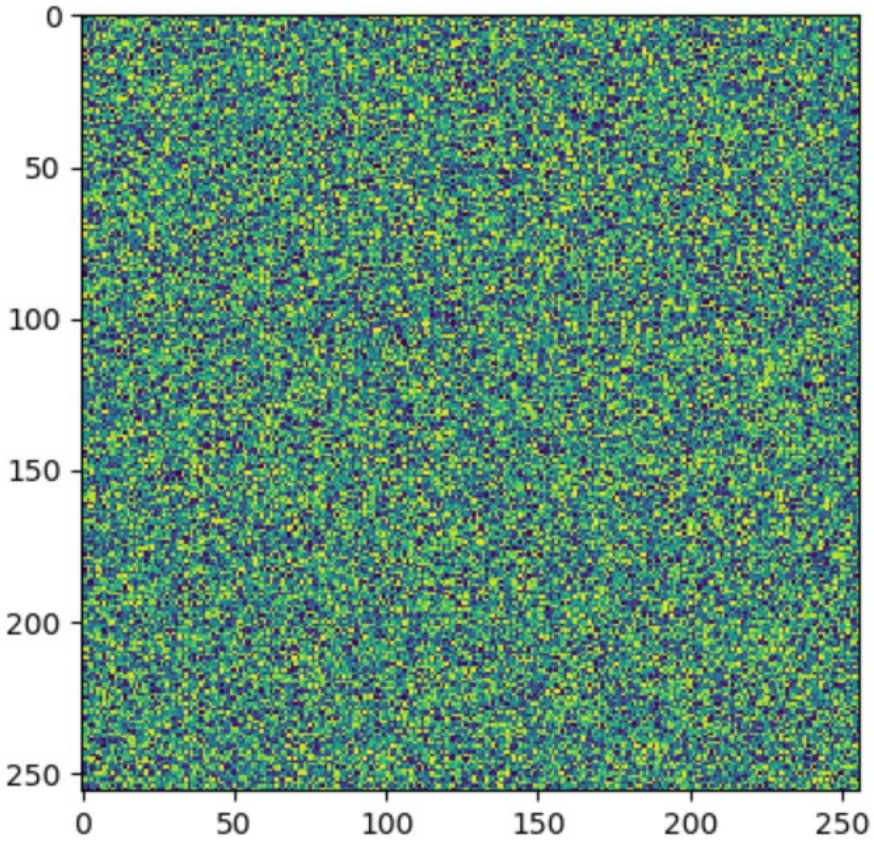


Note: matplotlib and colour maps

```
>>> from matplotlib import pyplot as p
>>> from numpy import random as r
>>> I = r.rand(256,256);
>>> p.imshow(I);
>>> p.show();
```

This code will actually produce an image like this:

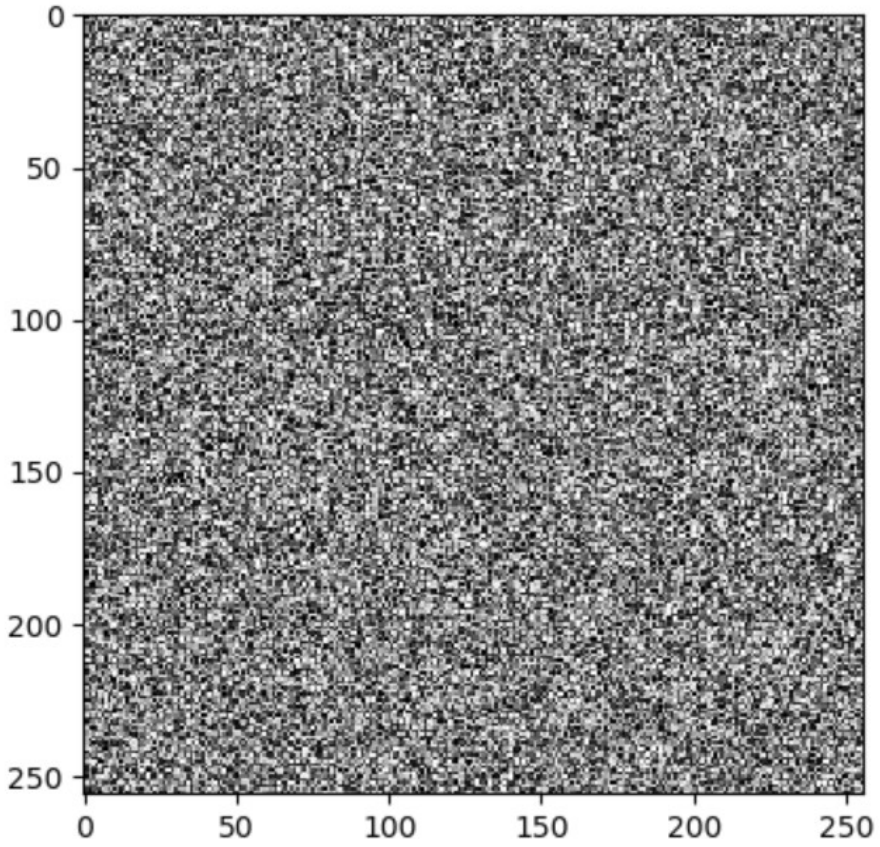
matplotlib assumes the data is ‘scientific’ and applies a colourmap.



Note: matplotlib and colour maps

```
>>> from matplotlib import pyplot as p
>>> from numpy import random as r
>>> I = r.rand(256,256);
>>> p.imshow(I,cmap='gray');
>>> p.show();
```

Better.



Dimensionality of an Image

- @ 8bit = 256 values ^ 65,536
  - Computer says ‘Inf’ combinations.
- Some depiction of all possible scenes would fit into this memory.

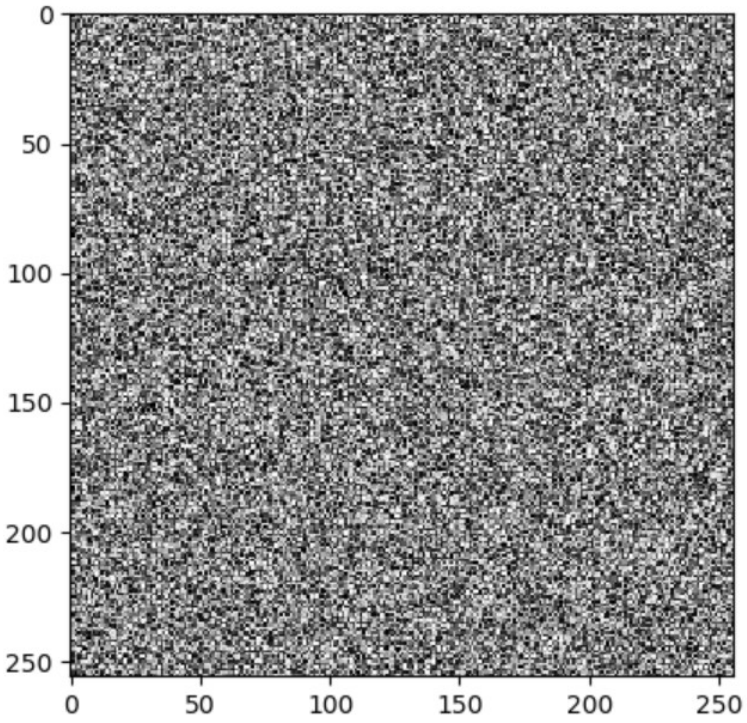


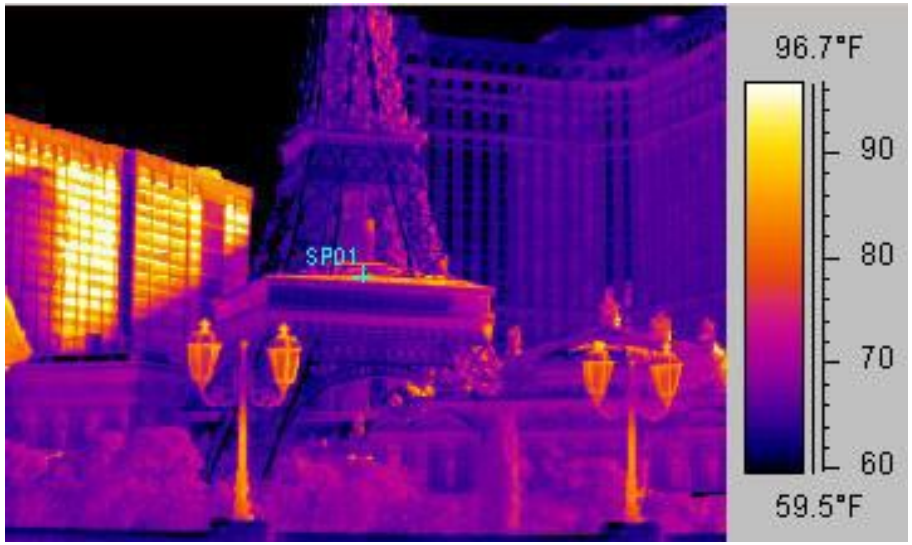
Image as a 2D sampling of signal

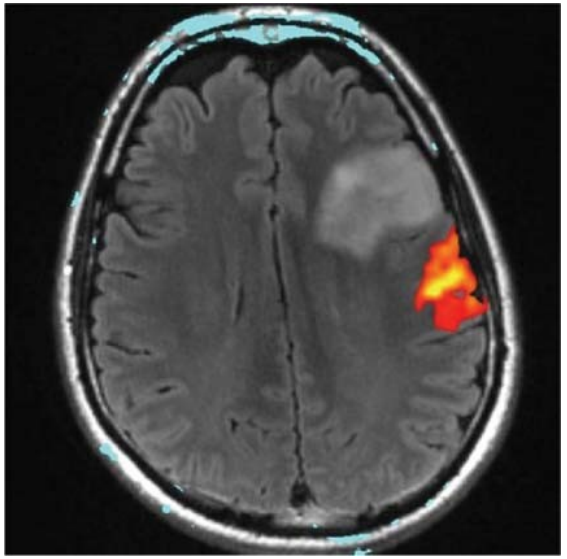
- Signal: function depending on some variable with physical meaning.

- Image: sampling of that function.
  - 2 variables: xy coordinates
  - 3 variables: xy + time (video)
  - ‘Brightness’ is the value of the function for visible light

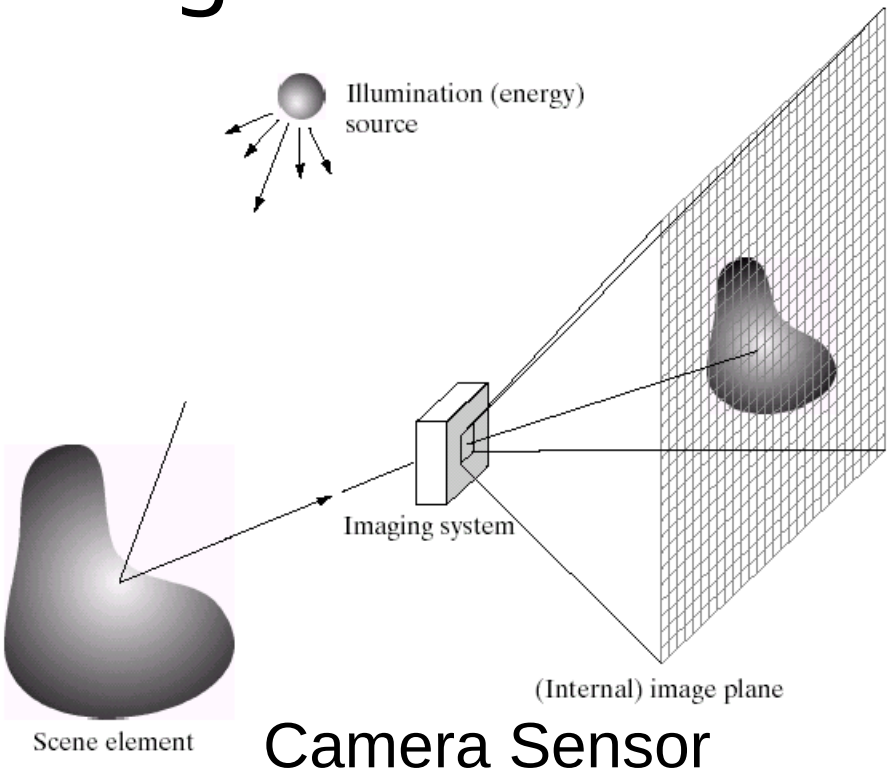
- Can be other physical values too: temperature, pressure, depth …

Example 2D Images

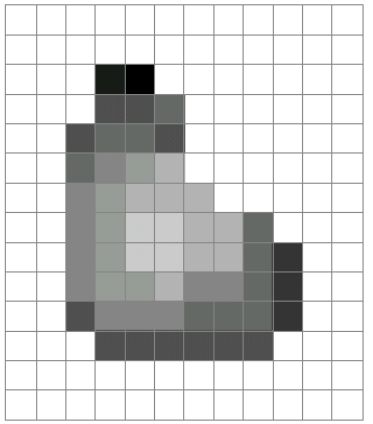




Integrating light over a range of angles.



Camera Sensor

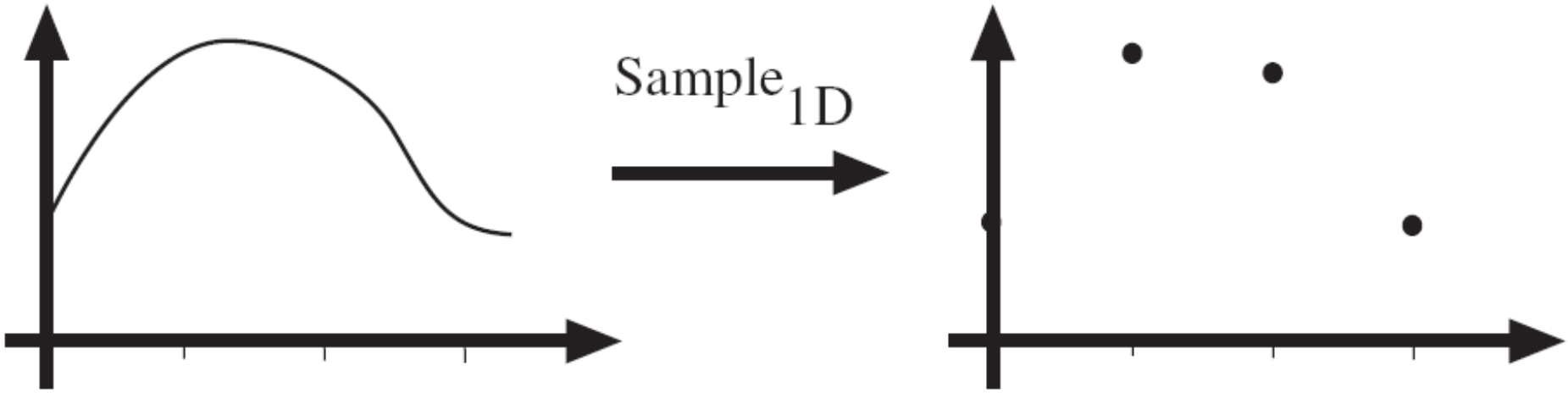


Output Image

Two things taking place:

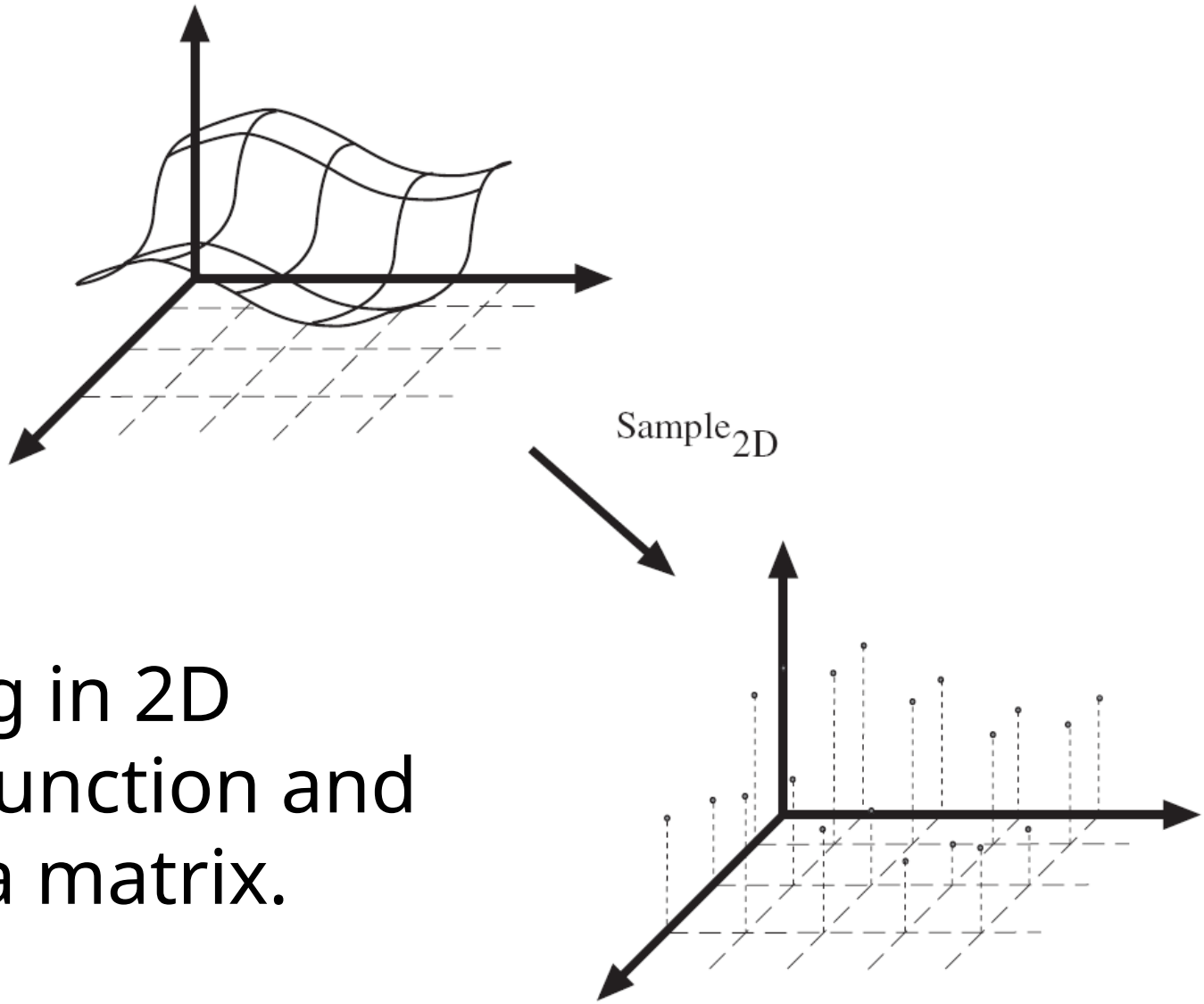
- Sampling
- Quantization

Sampling in 1D



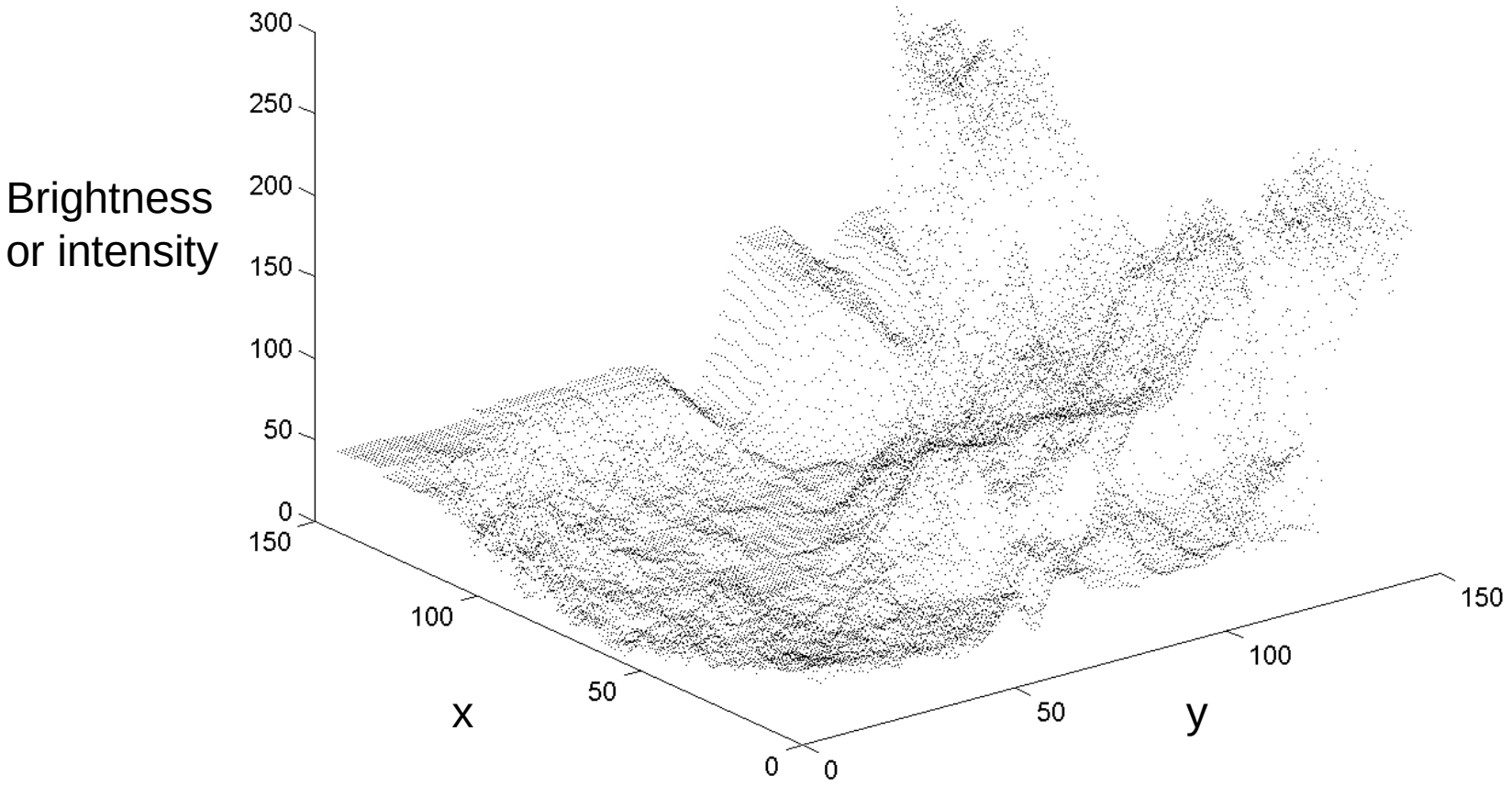
- Sampling in 1D takes a function, and returns a vector whose elements are values of that function at the sample points.

Sampling in 2D

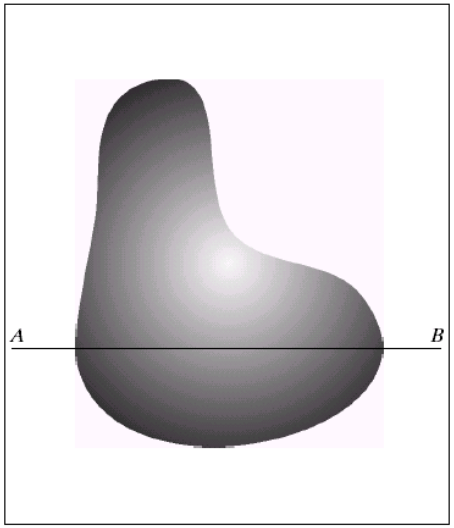


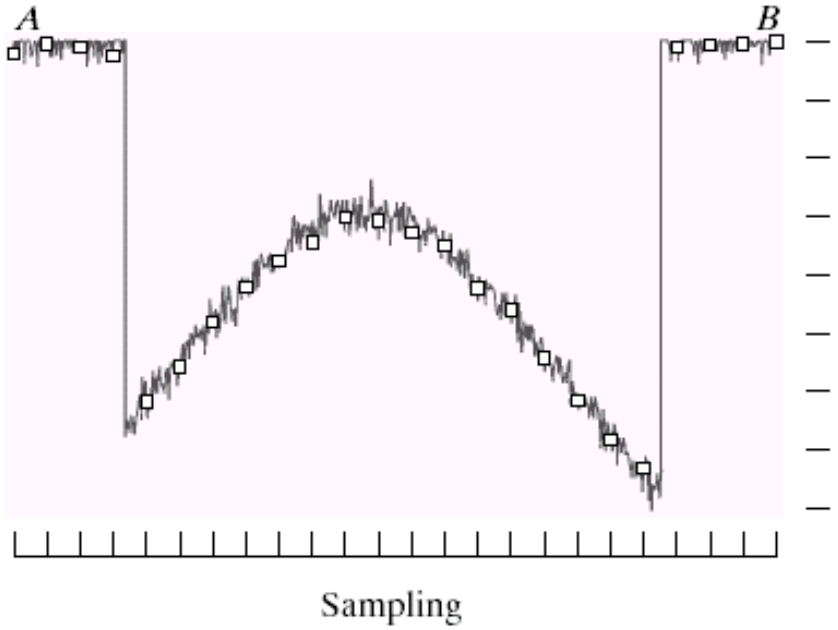
- Sampling in 2D takes a function and returns a matrix.

Grayscale Digital Image



Sampling



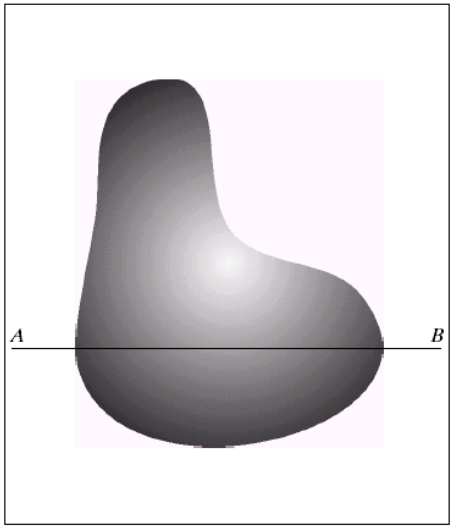


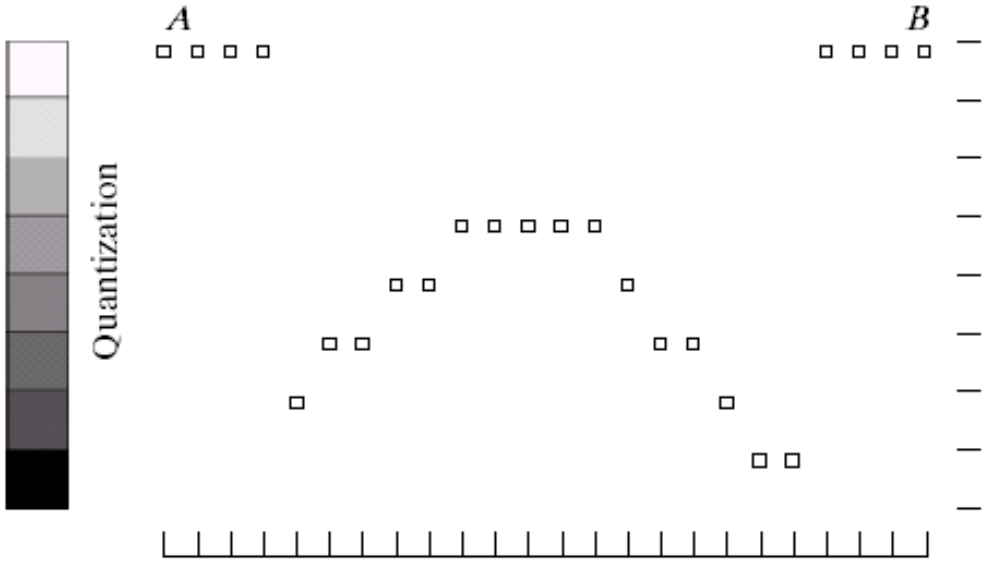
Sampling – spatial or geometric resolution

Both images are ~500x500 pixels

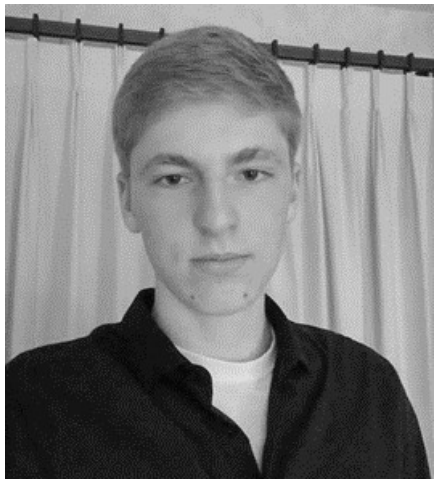
Each pixel area maps to different areas dimensions at the surface of the earth

Quantization



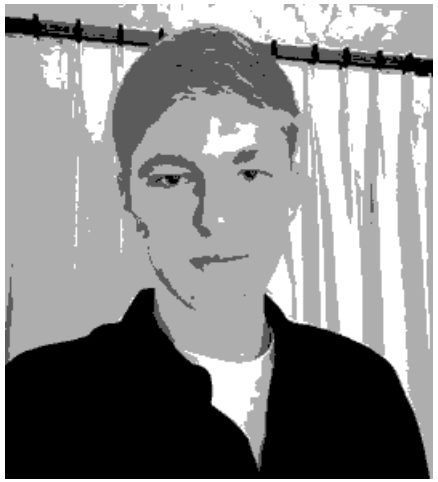


Quantization Effects – Radiometric Resolution



8 bit – 256 levels

4 bit – 16 levels



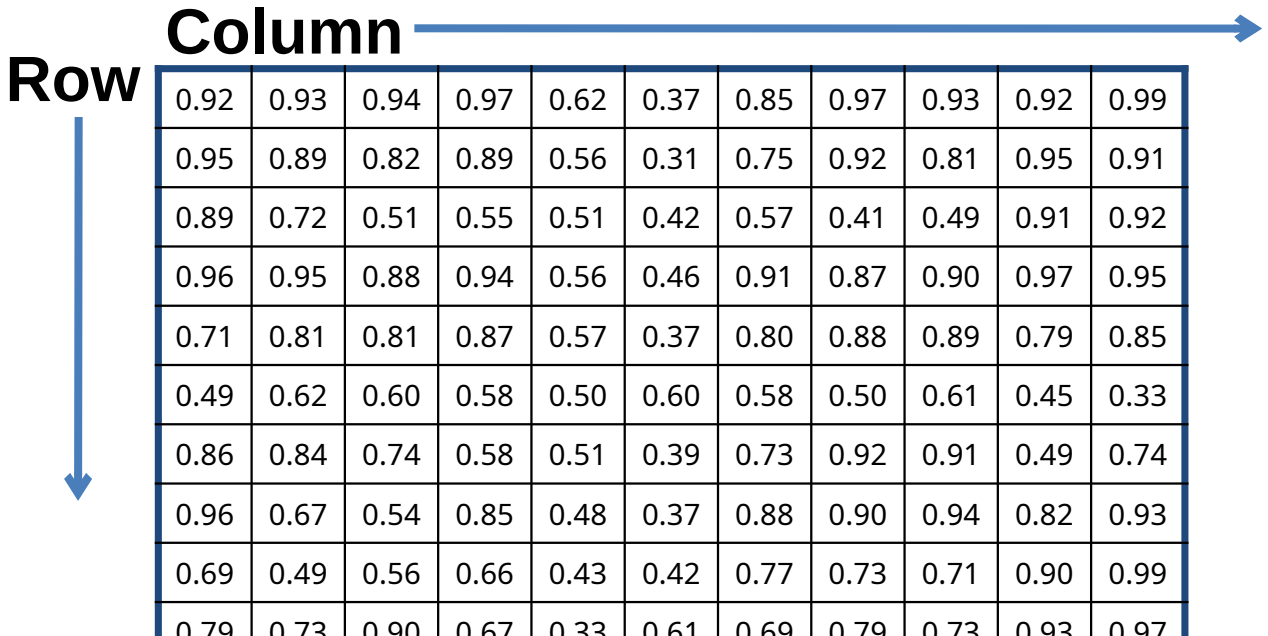
2 bit – 4 levels

1 bit – 2 levels

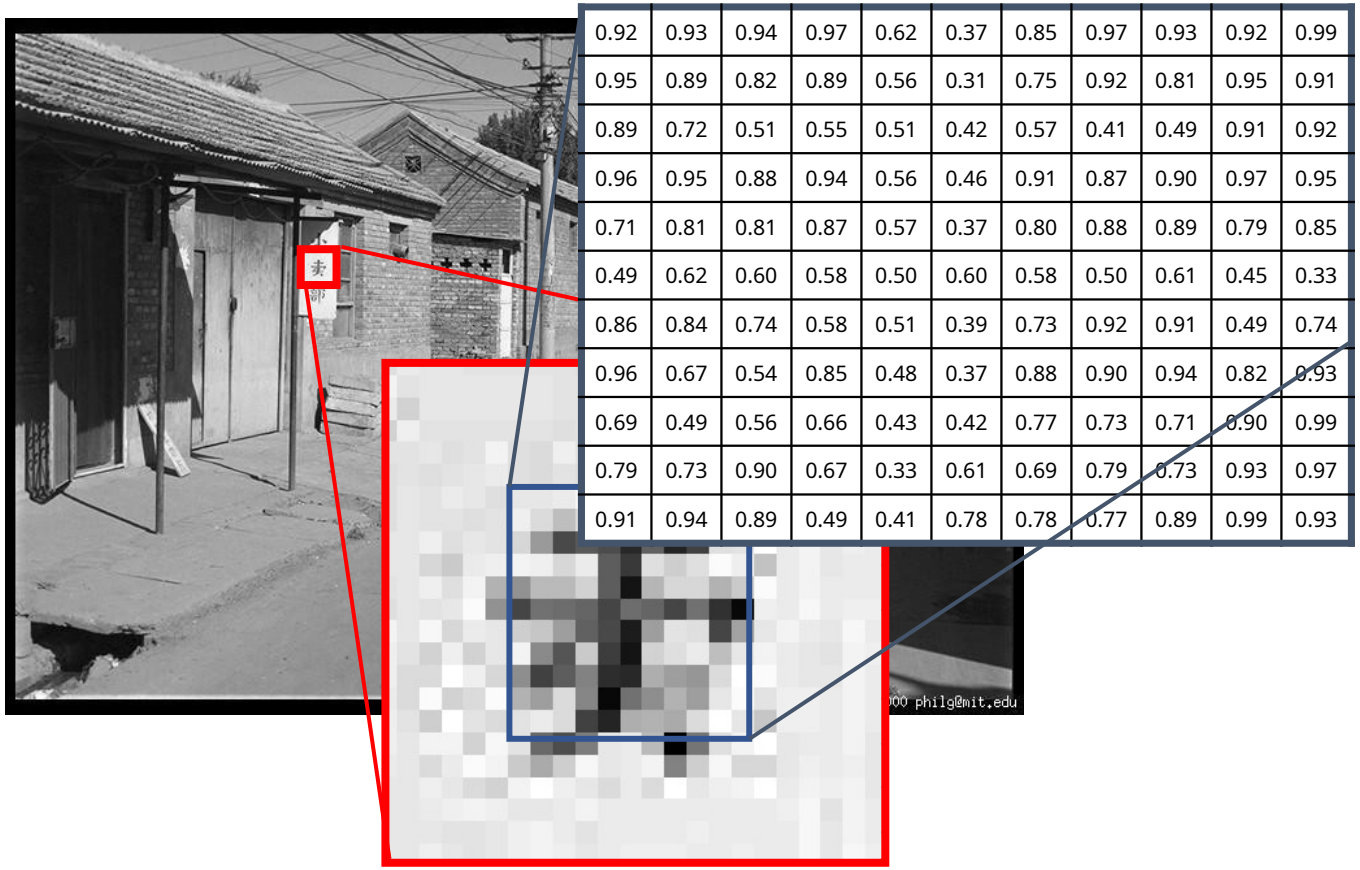
Images in Python Numpy

N x M grayscale image “im”

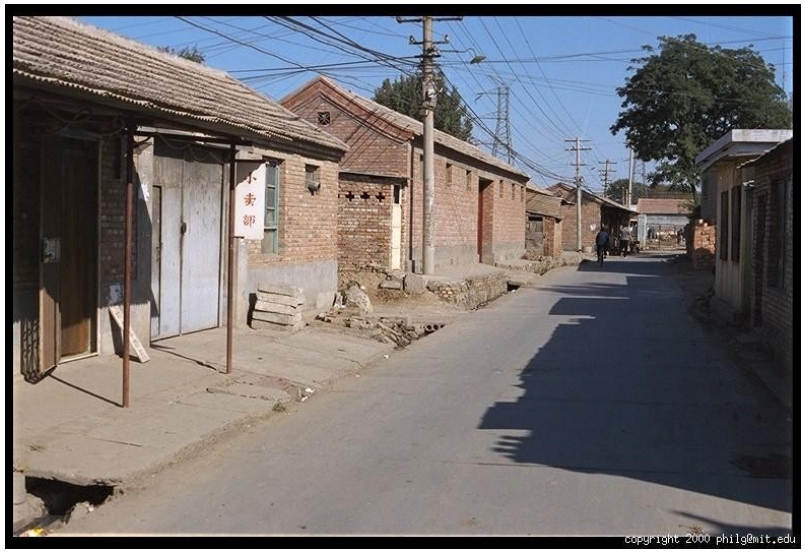
- im[0,0] = top-left pixel value
- im[y, x] = y pixels down, x pixels to right
- im[N-1, M-1] = bottom-right pixel

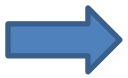


Grayscale intensity



Color


copyright 2000 philg@mit.edu




R
G
B
copyright 2000 philg@mit.edu

Images in Python Numpy

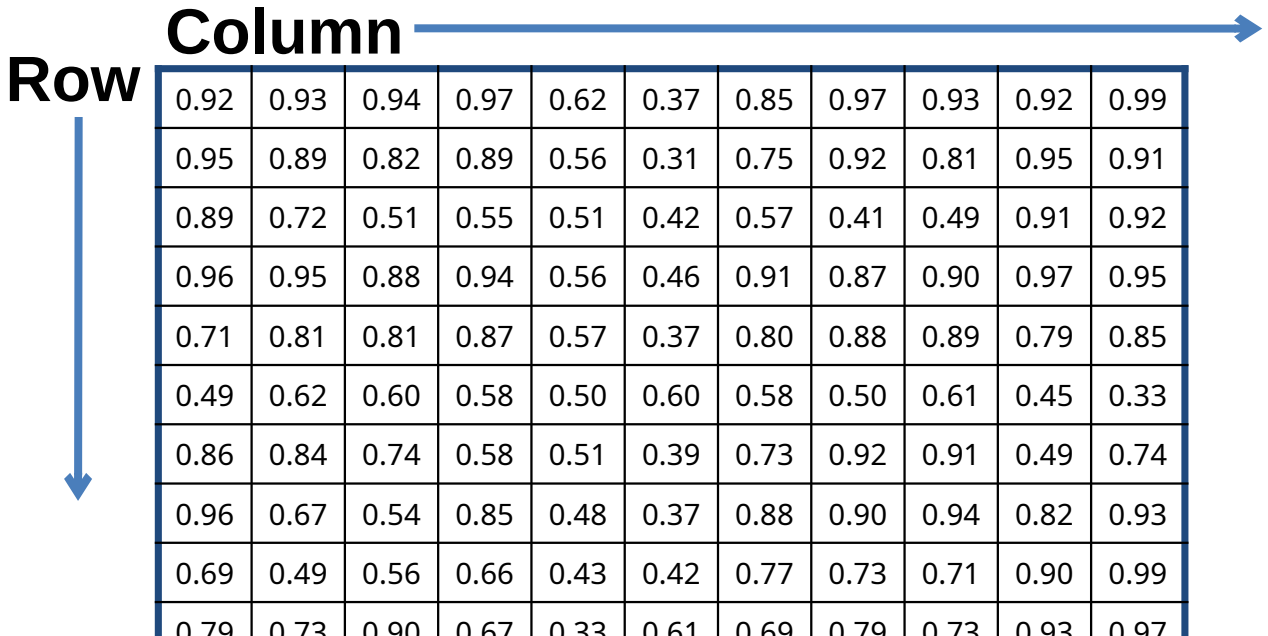
N x M RGB image “im”

- im[0,0,0] = top-left pixel value in R-channel
- Im[x, y, b] = x pixels to right, y pixels down in the bth channel
- Im[N-1, M-1, 3] = bottom-right pixel in B-channel

Images in Python Numpy

Take care between types! (io and img_as_float32 are part of skimage library)

- uint8 (values 0 to 255) – io.imread("file.jpg")
- float32 (values 0 to 255) – io.imread("file.jpg").astype(np.float32)
- float32 (values 0 to 1) – img_as_float32(io.imread("file.jpg"))



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97

Image filtering

- Image filtering:
  - Compute function of local neighborhood at each position

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Image filtering

- Image filtering:
  - Compute function of local neighborhood at each position

h=output f=filter I=image

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

2d coords=k,l 2d coords=m,n

[ ] [ ] [ ]

Example: box filter

$f[\cdot,\cdot]$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10							

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20						

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30					

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
						?			
				50					

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood

- Achieve smoothing effect (remove sharp features)

- Why does it sum to one?

$f[\cdot,\cdot]$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Smoothing with box filter

James Hays

$f[\cdot,\cdot]$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

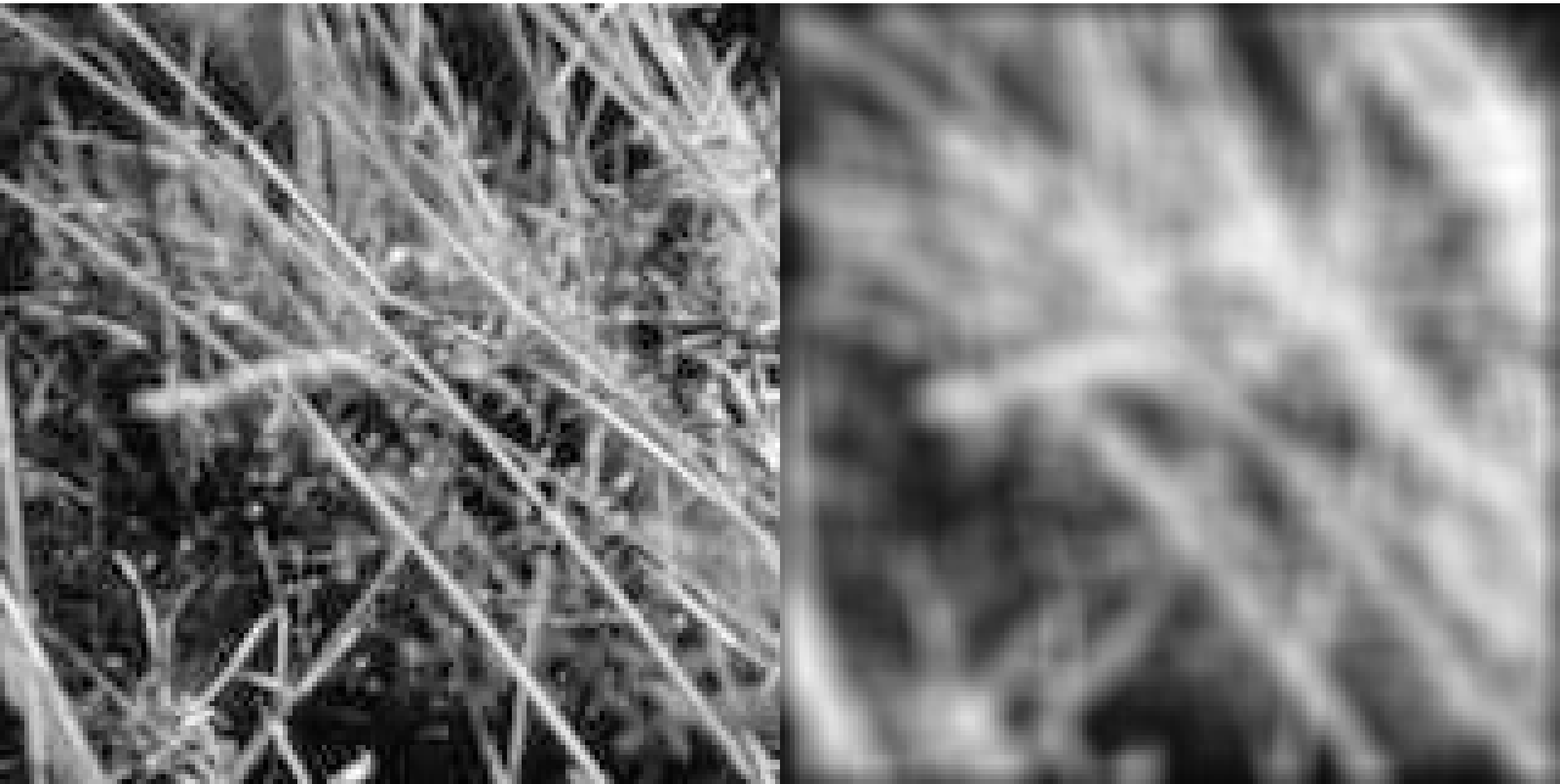


Image filtering

- Image filtering:
  - Compute function of local neighborhood at each position

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

- Really important!
  - Enhance images
    - Denoise, resize, increase contrast, etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching

1.

0	0	0
0	1	0
0	0	0

2.

0	0	0
0	0	1
0	0	0

3.

1	0	-1
2	0	-2
1	0	-1

4.

0	0	0
0	2	0
0	0	0

$-$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

1. Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

1. Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

2. Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

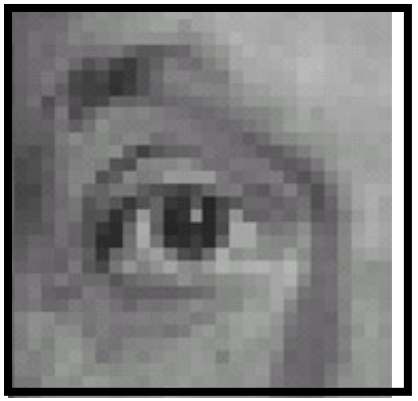
?

Source: D. Lowe

2. Practice with linear filters

Original

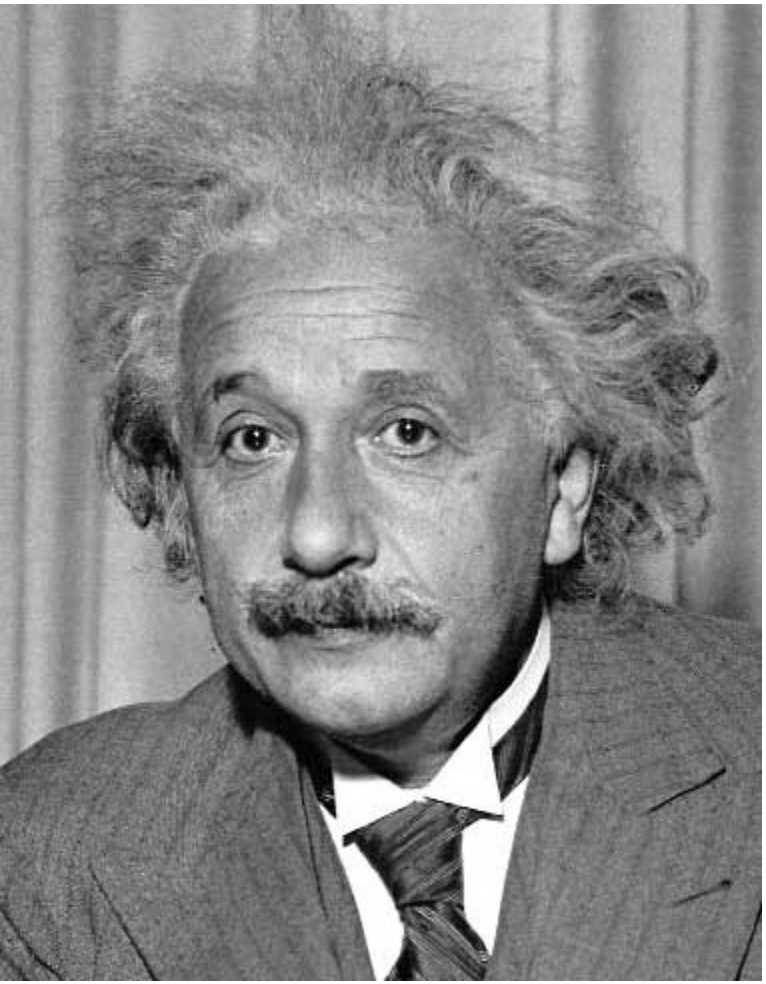
0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

Source: D. Lowe

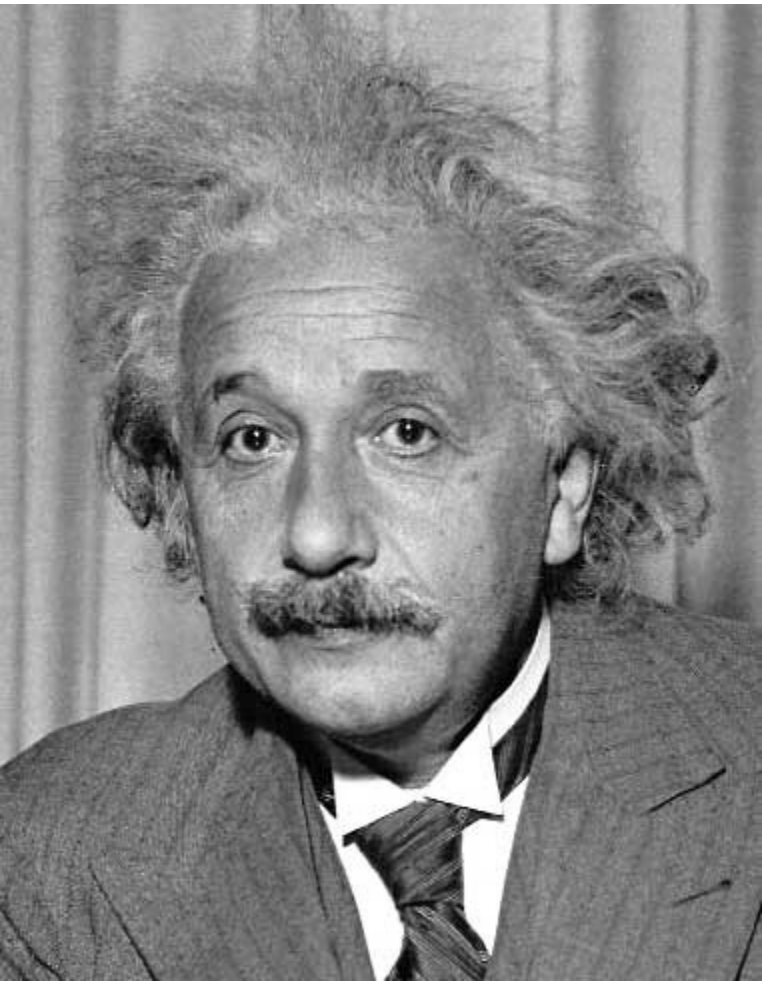
3. Practice with linear filters



1	0	-1
2	0	-2
1	0	-1

?

3. Practice with linear filters

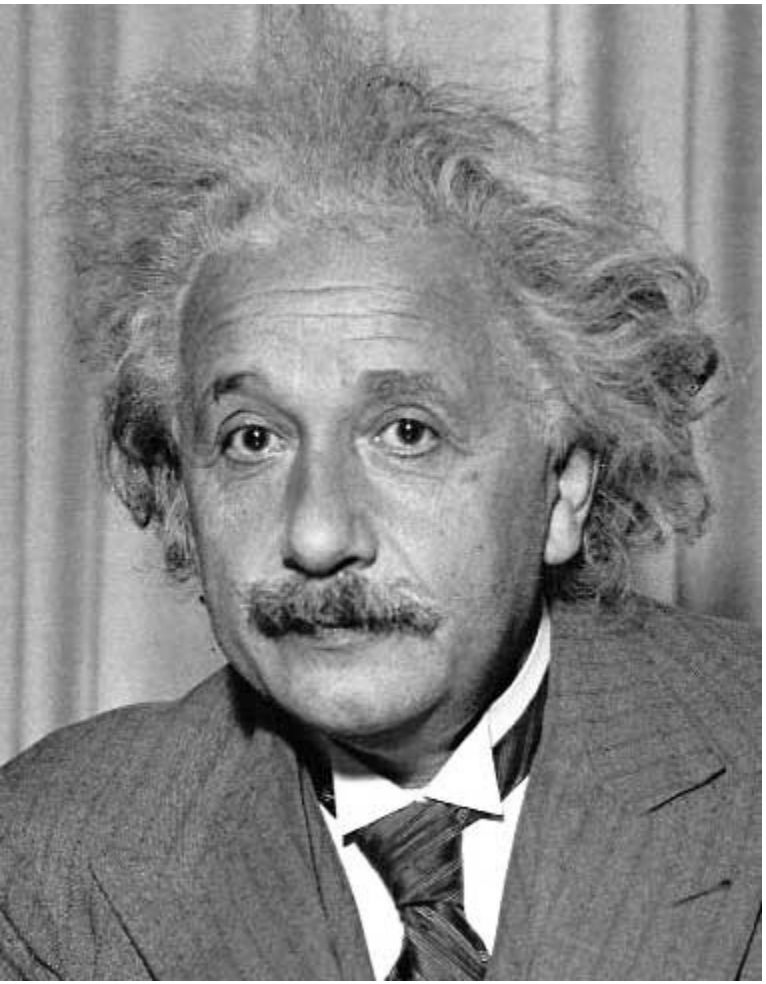


1	0	-1
2	0	-2
1	0	-1

Sobel

Vertical Edge
(absolute value)

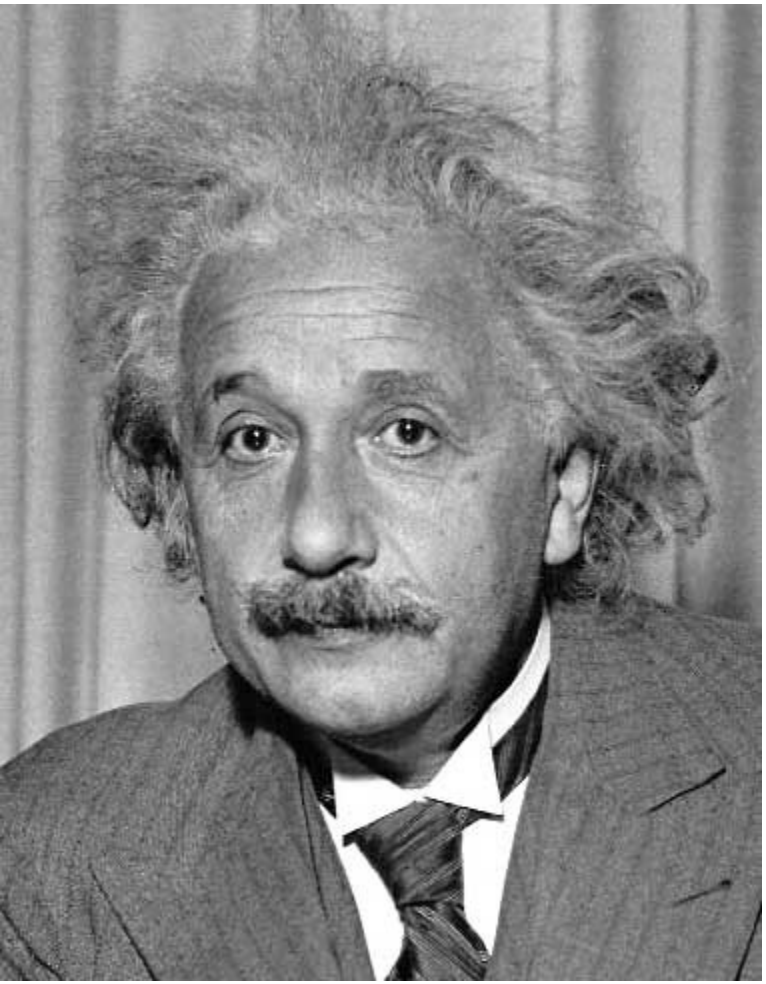
3. Practice with linear filters



1	2	1
0	0	0
-1	-2	-1

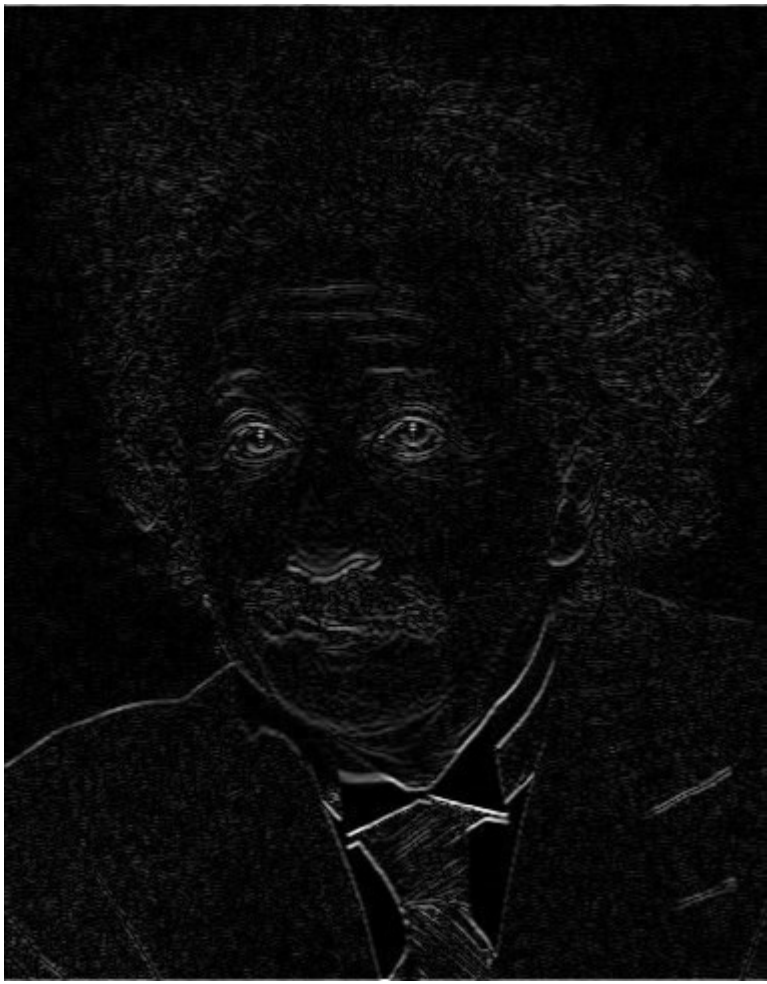
?

3. Practice with linear filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

David Lowe

4. Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

(Note that filter sums to 1)

Source: D. Lowe

4. Practice with linear filters

Original

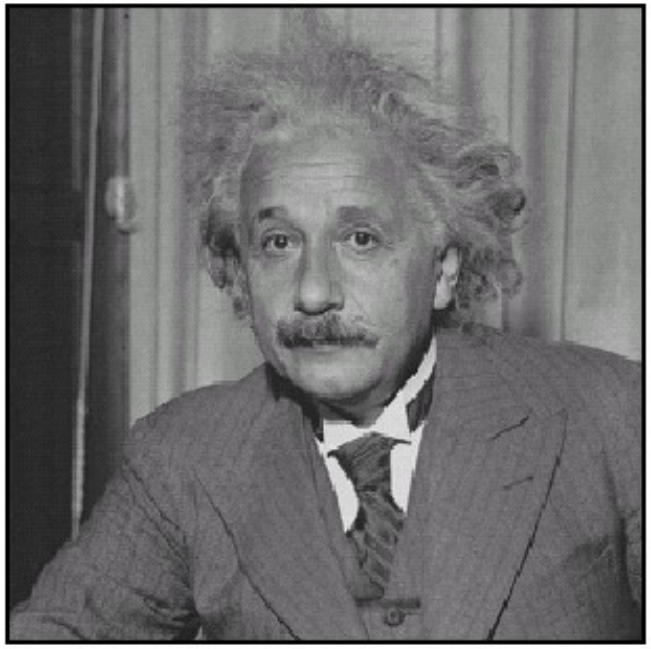
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

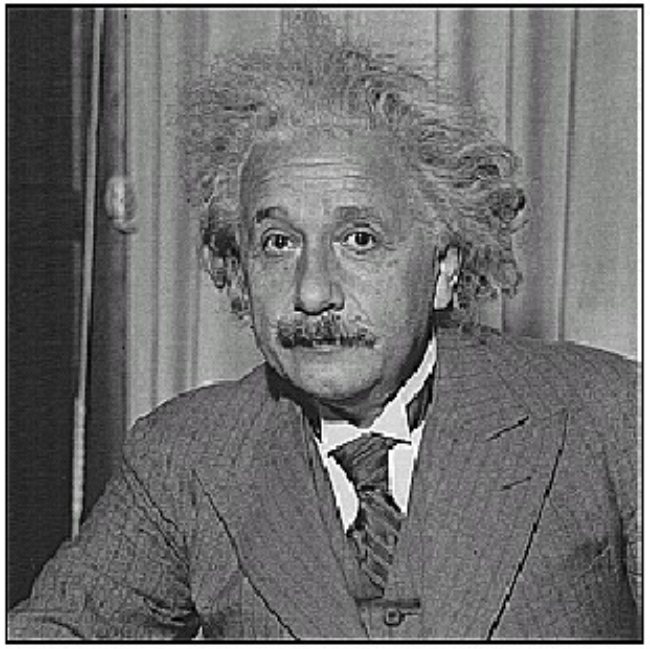
- Accentuates differences with local average

Source: D. Lowe

4. Practice with linear filters



before



after

Source: D. Lowe

Key properties of linear filters

Linearity:

$\texttt{imfilter(I, } f_1 + f_2\texttt{)} =$

$\texttt{imfilter(I,} f_1\texttt{)} + \texttt{imfilter(I,} f_2\texttt{)}$

Shift invariance:

Same behavior given intensities regardless of pixel location m,n

`imfilter(I,shift(f)) = shift(imfilter(I,f))`

Any linear, shift-invariant operator can be represented as a convolution.

Correlation and Convolution

- 2d correlation

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

```
e.g., h = scipy.signal.correlate2d(f,I)
```

Correlation and Convolution

- 2d correlation

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

```
e.g., h = scipy.signal.correlate2d(f,I)
```

- 2d convolution

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m-k, n-l]$$

```
e.g., h = scipy.signal.convolve2d(f,I)
```

Convolution is the same as correlation with a 180° rotated filter kernel.
Correlation and convolution are identical when the filter kernel is symmetric.

Convolution properties

Commutative: $a * b = b * a$

- Conceptually no difference between filter and signal
- But particular filtering implementations might break this equality, e.g., image edges

Associative: $a * (b * c) = (a * b) * c$

- Often apply several filters one after another:
- $(((a * b_1) * b_2) * b_3)$

Source: S. Lazebnik

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- This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$
- Correlation is

Source: S. Lazebnik

Convolution properties

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  - But particular filtering implementations might break this equality,

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- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$
  - Correlation is _not_ associative (rotation effect)
  - Why important?

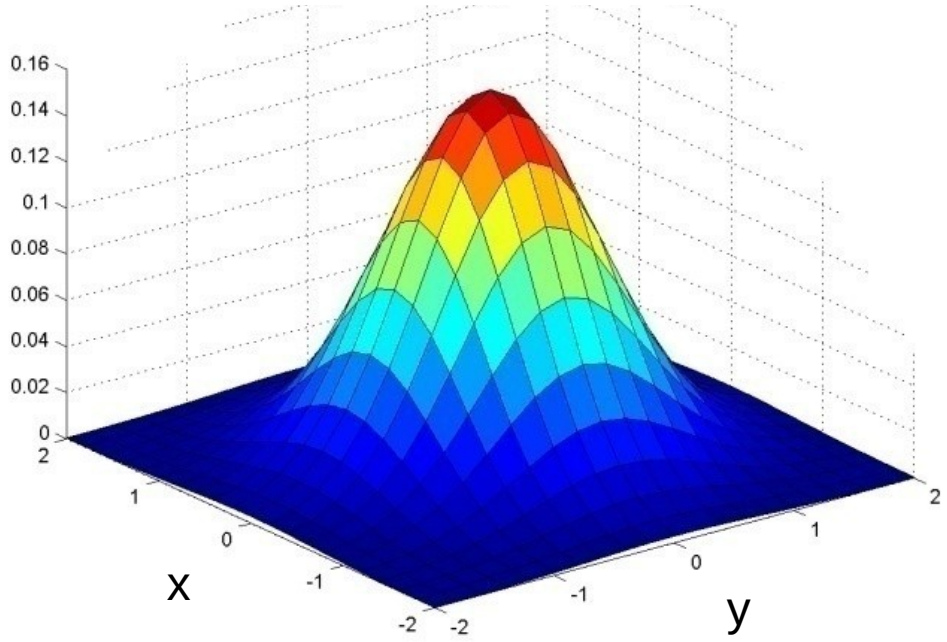
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$

- Scalars factor out: $ka * b = a * kb = k (a * b)$

Source: S. Lazebnik

Important filter: Gaussian

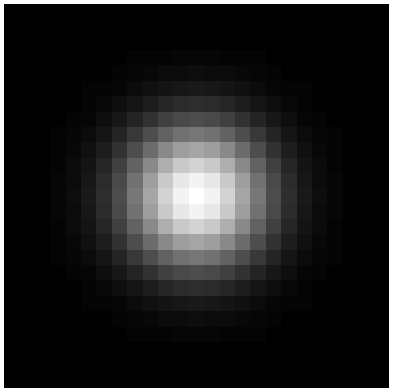
Weight contributions of neighboring pixels by nearness



x

y					
	0.003	0.013	0.022	0.013	0.003
	0.013	0.059	0.097	0.059	0.013
	0.022	0.097	0.159	0.097	0.022
	0.013	0.059	0.097	0.059	0.013
	0.003	0.013	0.022	0.013	0.003

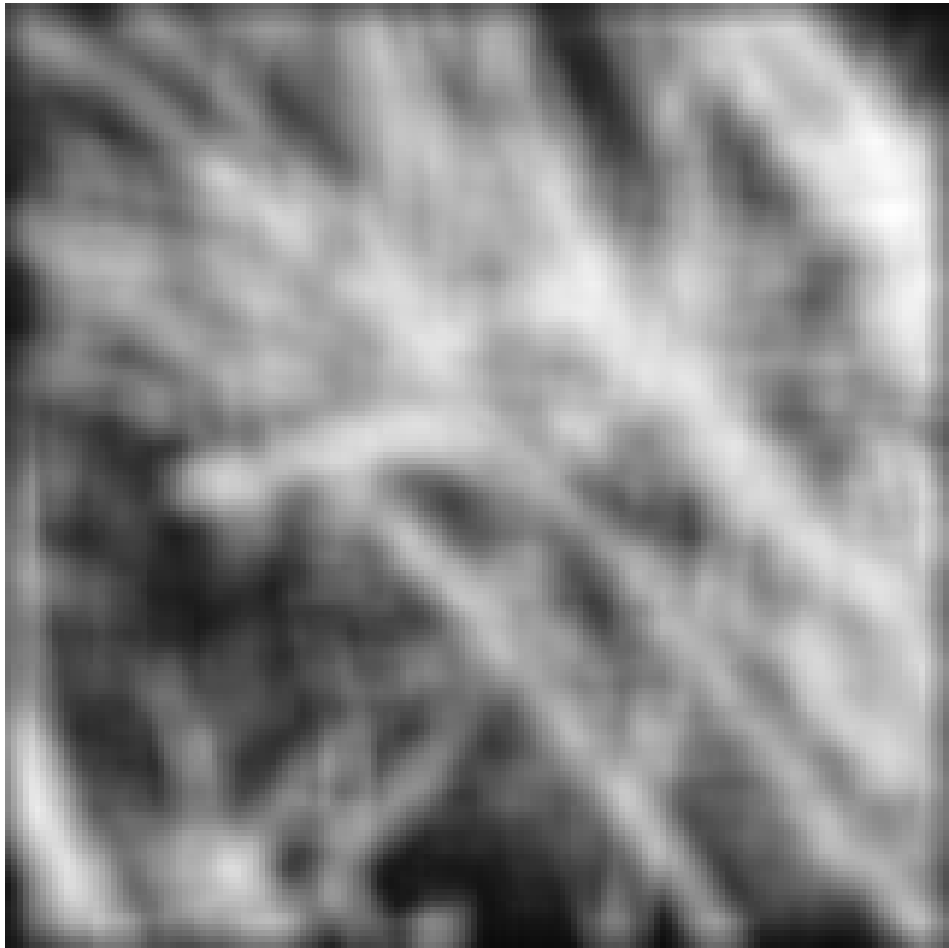
5 x 5, σ= 1



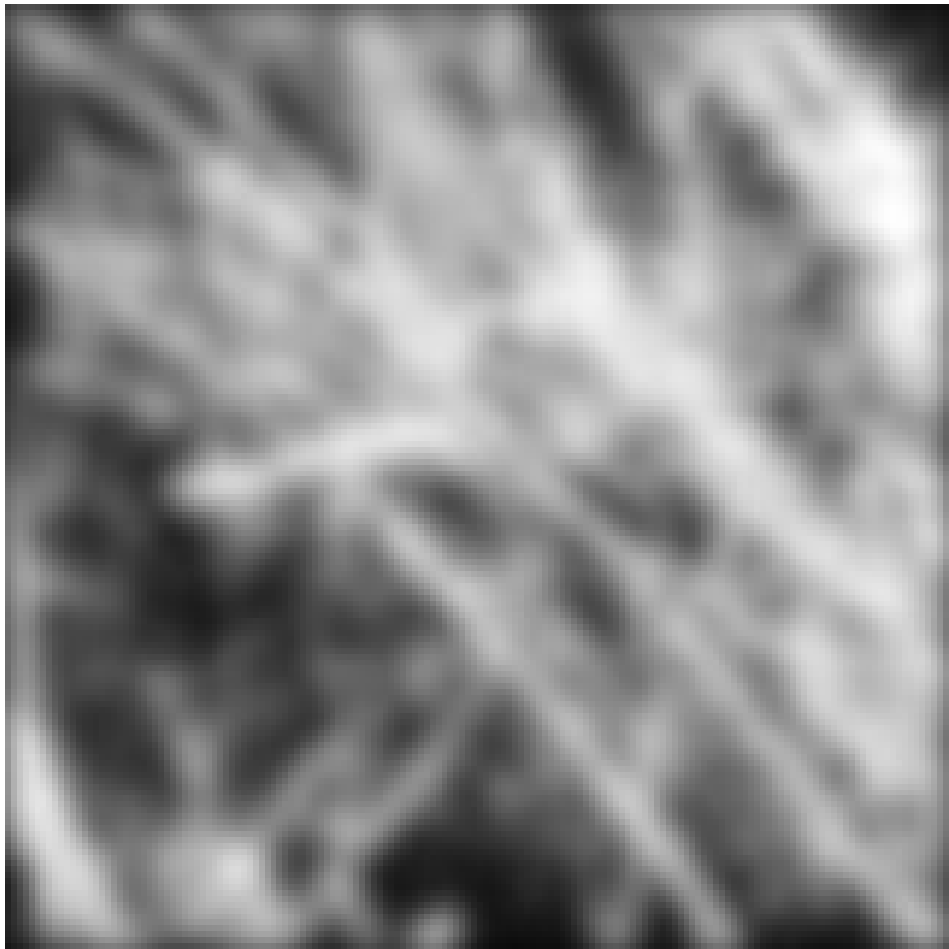
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Slide credit: Christopher Rasmussen

Smoothing with box filter



Smoothing with Gaussian filter



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
  - Images become more smooth
- Gaussian convolved with Gaussian…
  …is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving twice with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

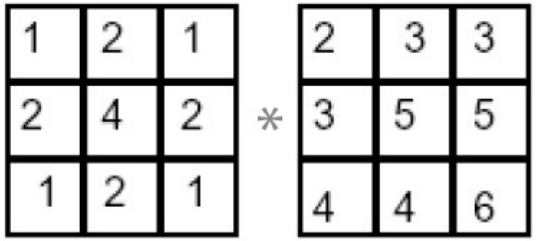
The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

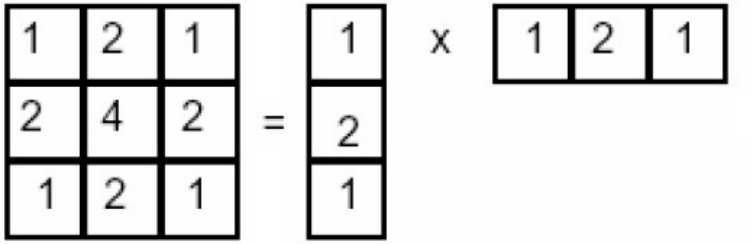
Source: D. Lowe

Separability example

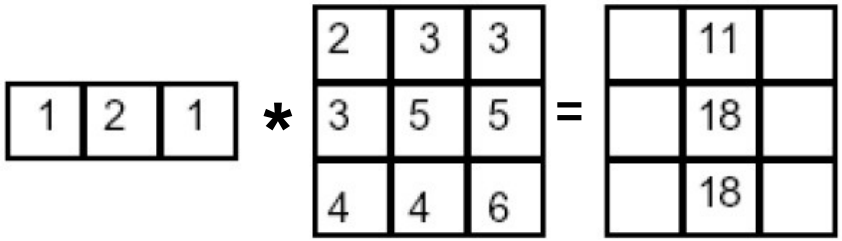
2D convolution
(center location only)



The filter factors
into a product of 1D
filters:



Perform convolution
along rows:



Followed by convolution
along the remaining column:

Source: K. Grauman

Separability

Why is separability useful in practice?

MxN image, PxQ filter

- 2D convolution: ~MNPQ multiply-adds
- Separable 2D: ~MN(P+Q) multiply-adds

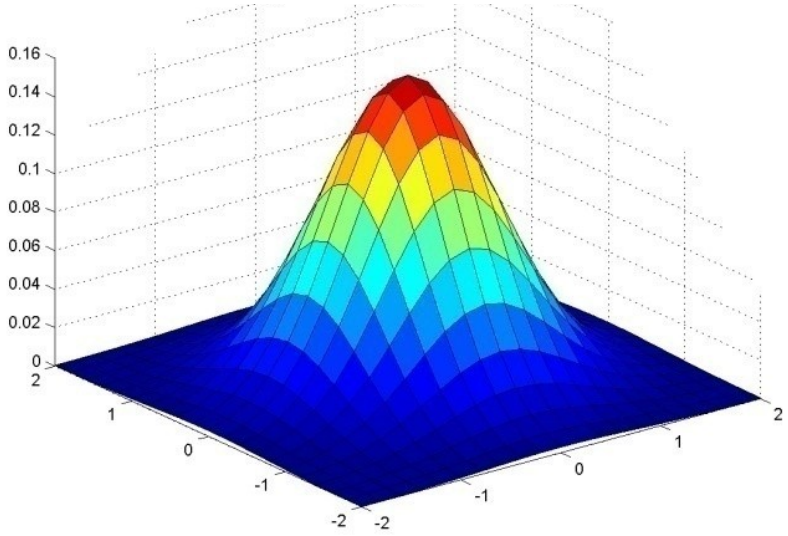
Speed up = PQ/(P+Q)

9x9 filter = ~4.5x faster

Practical matters: Parameter Selection

How big should the filter be?

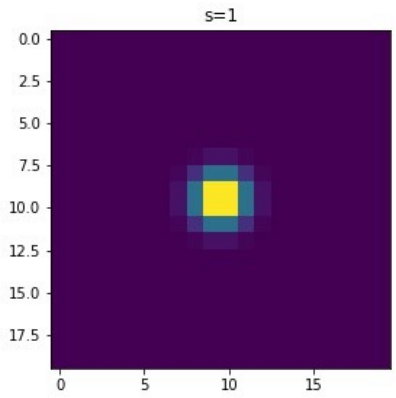
- Values at edges should be near zero
- Gaussians have infinite extent…
- Rule of thumb for Gaussian: set filter half-width to about 3 σ

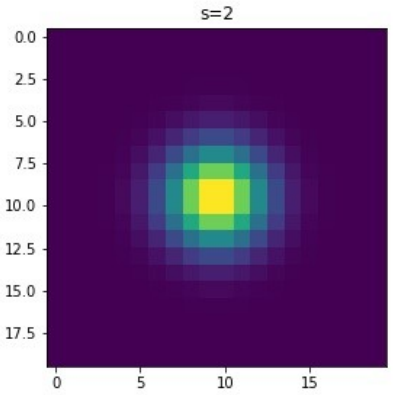


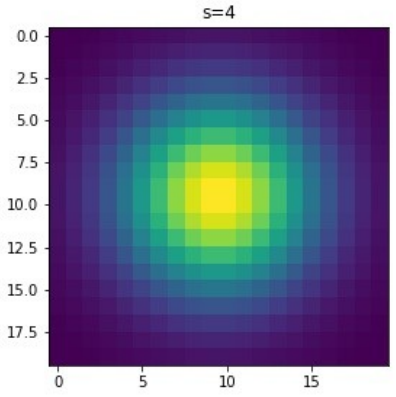
James Hays

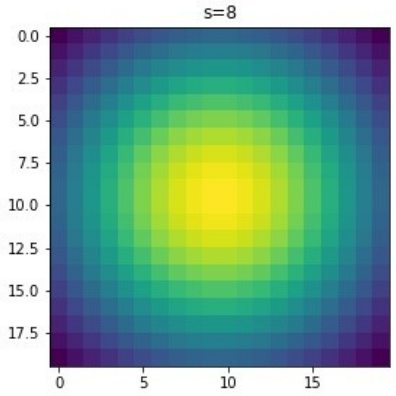
Practical matters: Parameter Selection

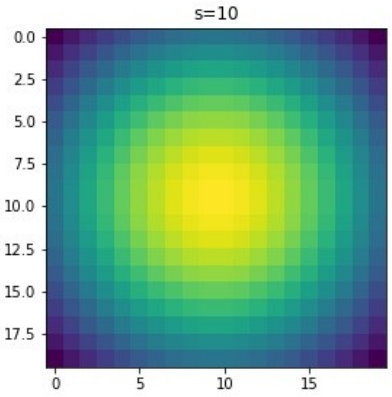
Window = 21x21

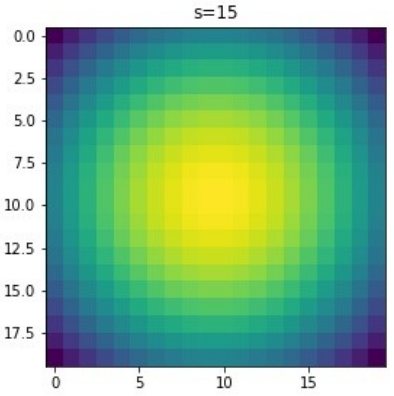












Practical matters: Image boundaries

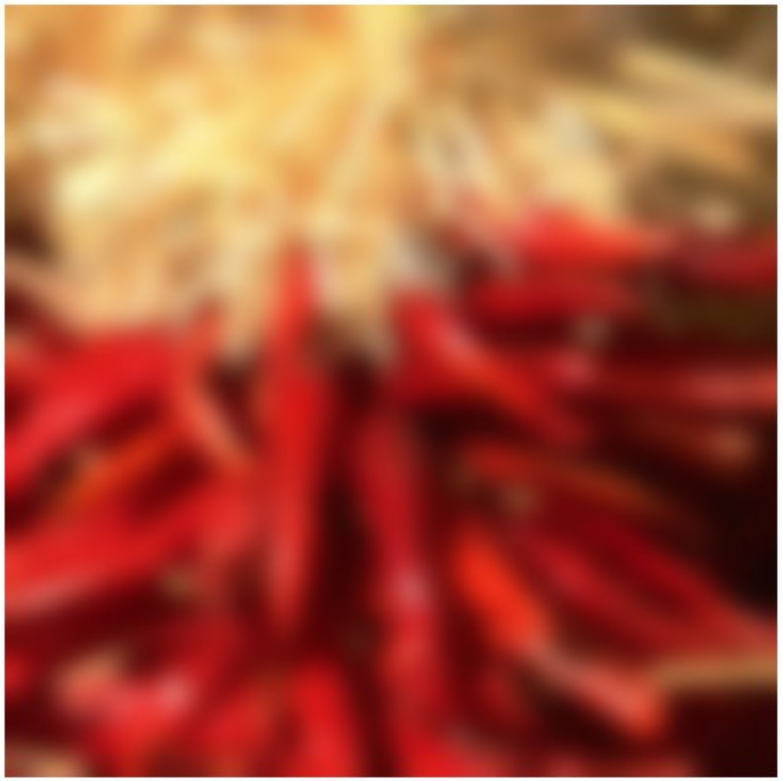
What about near the edge?

Source: S. Marschner

Practical matters: Image boundaries

What about near the edge?

- The filter window falls off the edge of the image



Source: S. Marschner

Practical matters: Image boundaries

What about near the edge?

- The filter window falls off the edge of the image
- Need to extrapolate
- methods:
  - clip filter (black)
  - wrap around
  - copy edge
  - reflect across edge

Source: S. Marschner

Practical matters: Sobel negative values !

- Throwback: Sobel Filter

```
>> I = img_to_float32( io.imread( ‘luke.jpg’ ) );
>> h = convolve2d( I, sobelKernel );
```

1	0	-1
2	0	-2
1	0	-1

Sobel (Vertical edge detector)

`plt.imshow(h);`

`plt.imshow(h+0.5);`

Practical matters: Sobel negative values !

`h(:,:,1) < 0`

`h(:,:,1) > 0`

Practical matters: Sobel Negative values !

Solution for display purposes we can:

- Add the mean of the result before displaying it.
  - Shifts the image towards the positive domain, but the contrast would remain and may suffer from saturation.
- Use the absolute values of the result (stretched between 0 and 255).
  - This makes very negative and very positive gradients appear brighter
- Other solution coming when we see edge detection

Exercise

* = Convolution operator

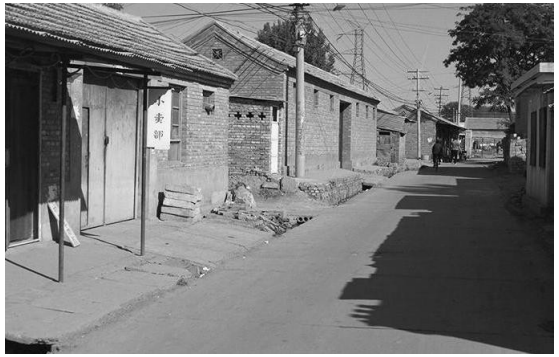
a) _ = D * B

b) F = D * _

c) _ = D * D

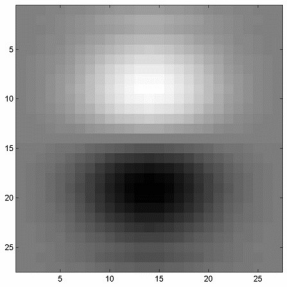
d) A = _ * _

D

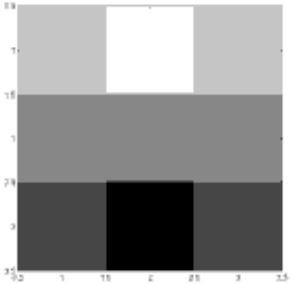


H

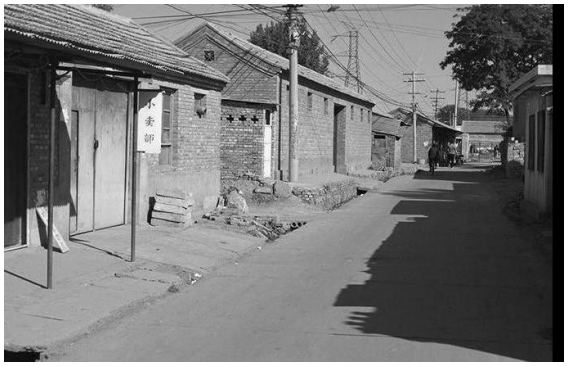
A



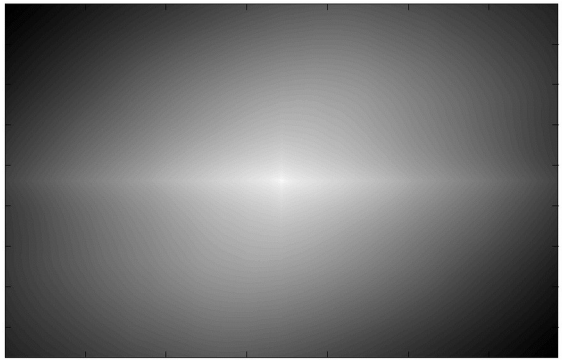
B



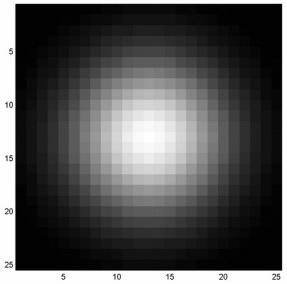
F



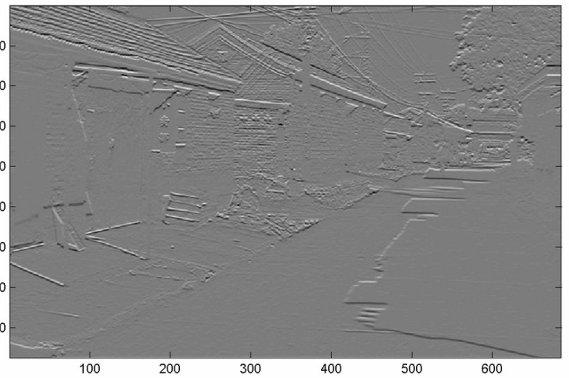
I



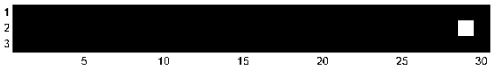
C



G

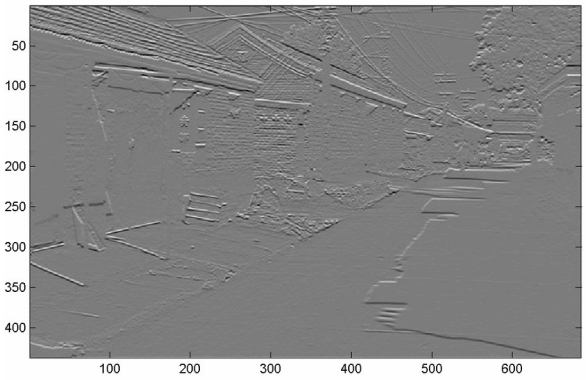


E



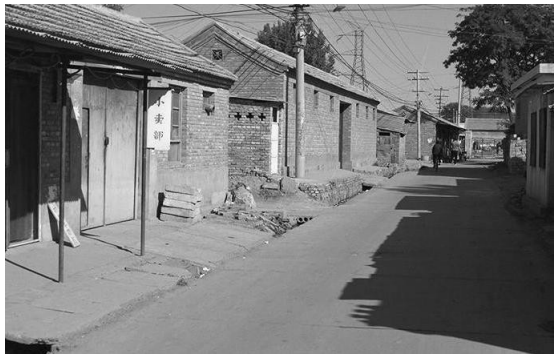
Exercise

- a) `G = D * B` (some sort of horizontal detection)



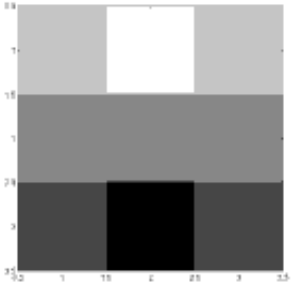
G

=



D

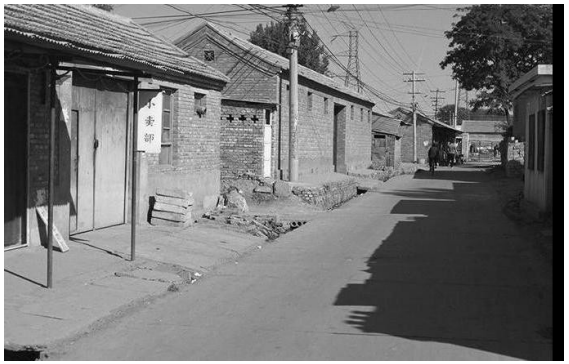
*



B

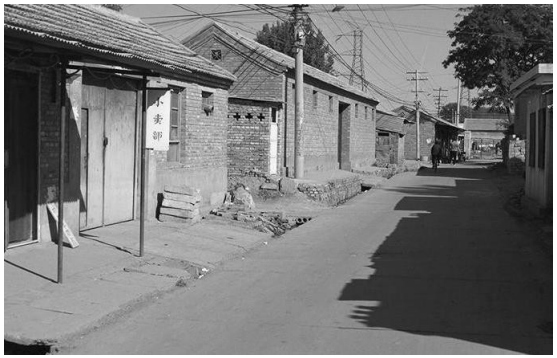
Exercise

b) `F = D * E` - Shift to the left.



F

=



D

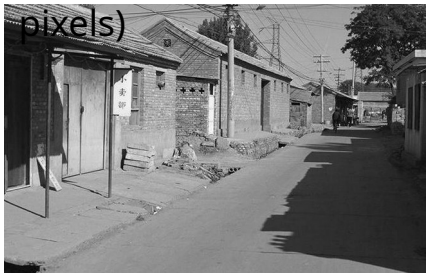
*

E

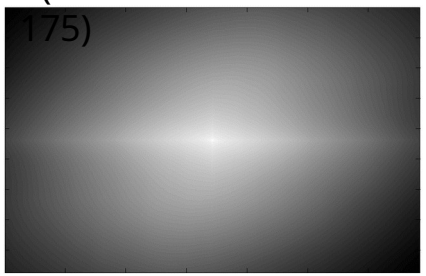
Exercise

c) I = D * D

D (275 x 175 pixels)



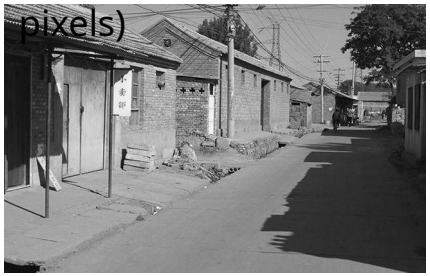
I (from slide – 275 x 175)



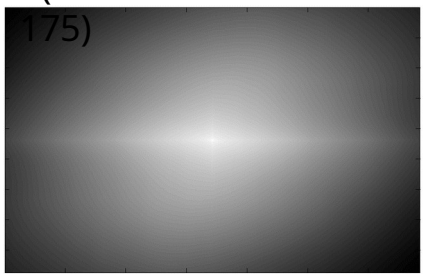
“…something to do with lack of content (black) at edges…”

I = D * D

D (275 x 175 pixels)



I (from slide – 275 x 175)



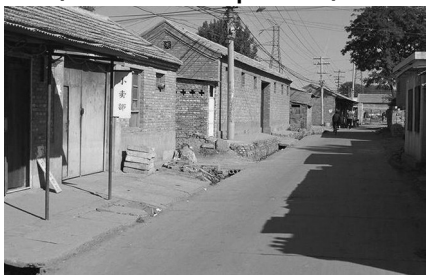
```
>> D = img_as_float32( io.imread( ‘convexample.png’ ) )
>> I = convolve2d( D, D )
>> np.max(I)
1.1021e+04
```

I_norm

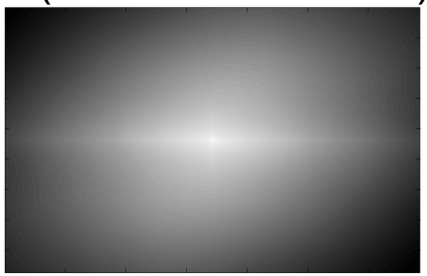
```
# Normalize for visualization
>> I_norm = (I – np.min(I)) / (np.max(I) – np.min(I))
>> plt.imshow( I_norm )
```

I = D * D

D (275 x 175 pixels)



I (from slide – 275 x 175)

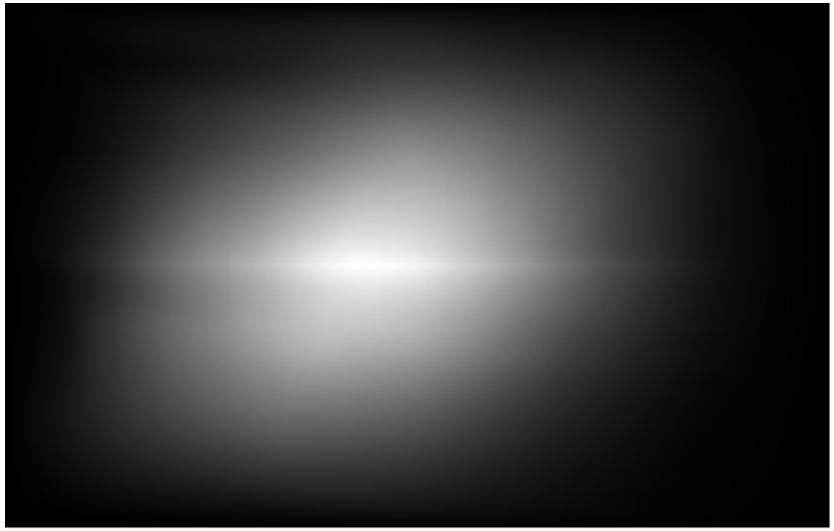


(275-1)/2

(275-1)/2

For x: 275 + (275-1)/2 + (275-1)/2 = 549

I_norm (549 x 349 pixels)



Sklearn Convolve2d parameters

mode : *str {'full', 'valid', 'same'}, optional*

A string indicating the size of the output:

`full`

The output is the full discrete linear convolution of the inputs. (Default)

`valid`

The output consists only of those elements that do not rely on the zero-padding. In 'valid' mode, either *in1* or *in2* must be at least as large as the other in every dimension.

`same`

The output is the same size as *in1*, centered with respect to the 'full' output.

boundary : *str {'fill', 'wrap', 'symm'}, optional*

A flag indicating how to handle boundaries:

`fill`

pad input arrays with fillvalue. (default)

`wrap`

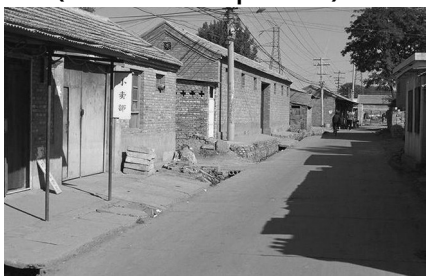
circular boundary conditions.

`symm`

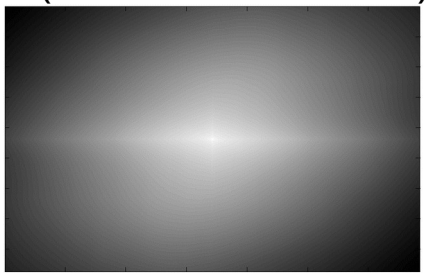
symmetrical boundary conditions.

I = D * D

D (275 x 175 pixels)

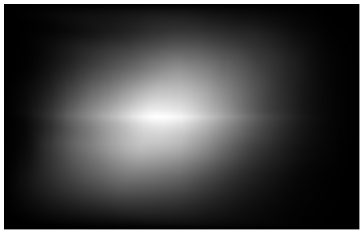


I (from slide – 275 x 175)



>> I = convolve2d(D, D, mode='full')

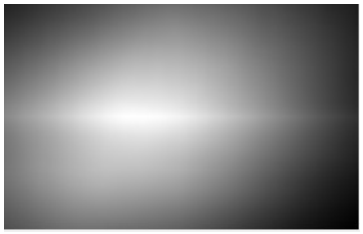
(Default; pad with zeros)



549 x 349

>> I = convolve2d(D, D, mode='same')

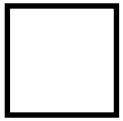
(Return same size as D)



275 x 175

>> I = convolve2d(D, D, mode='valid')

(No padding)

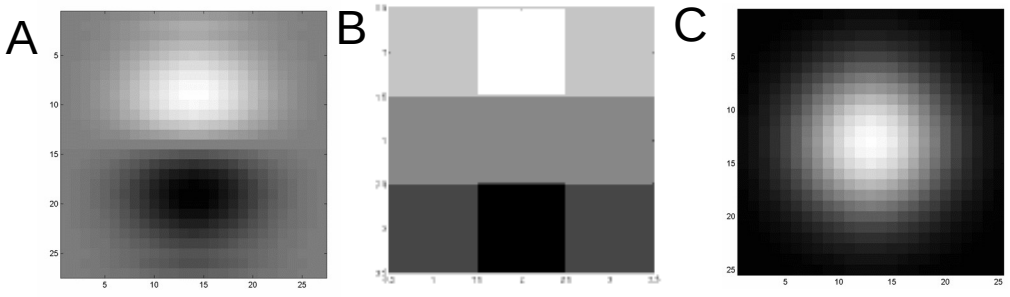


Value = 10528.3

1x1

Exercise

d) `A = B * C` - “because it kind of looks like it.”



C is a Gaussian filter
(or something close to it it),
and we know that it ‘blurs’.

Difference between Convolution and Correlation:

- **Convolution** is the response of one function when subjected to another function.
- **Correlation**: A measure of similarity between two functions. For example: ‘template matching’

For symmetric filters: use either convolution or correlation.

For nonsymmetric filters: correlation is template matching.

Filtering: Correlation and Convolution

- 2d correlation

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

```
e.g., h = scipy.signal.correlate2d(f,I)
```

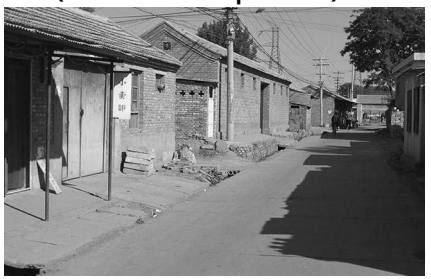
- 2d convolution

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m-k, n-l]$$

```
e.g., h = scipy.signal.convolve2d(f,I)
```

Convolution is the same as correlation with a 180° rotated filter kernel.
Correlation and convolution are identical when the filter kernel is symmetric.

D (275 x 175 pixels)



OK, so let's test this idea. Let's see if we can use correlation to 'find' the parts of the image that look like a template.

>> f = D[57:117, 107:167]

Expect response 'peak' in middle of I

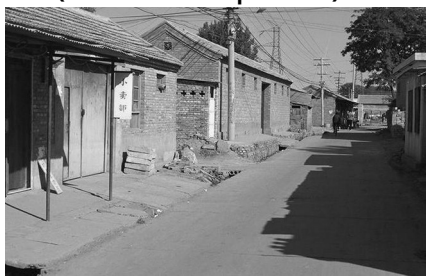
f
61 x 61

>> I = correlate2d(D, f, 'same')

[Thanks to Robert Collins @ Penn State]

OK, so let's test this idea. Let's see if we can use correlation to 'find' the parts of the image that look like a template.

D (275 x 175 pixels)



>> f = D[57:117, 107:167]

Expect response 'peak' in middle of I

f
61 x 61

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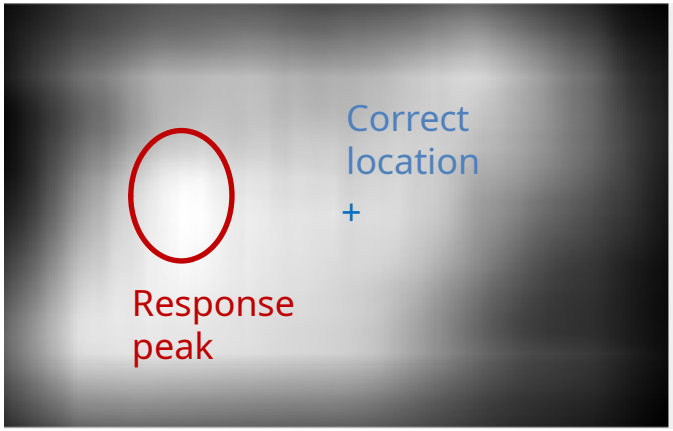
I



Hmm…
That didn't work – why not?

[Thanks to Robert Collins @ Penn State]

Correlation

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

```
e.g., h = scipy.signal.correlate2d(f,I)
```

As brightness in *I* increases, the response in *h* will increase, as long as *f* is positive.

Overall brighter regions will give higher correlation response -> not useful!

OK, so let’s subtract the mean

>> f = D[57:117, 107:167]

>> f2 = f – np.mean(f)

>> D2 = D – np.mean(D)

Now zero centered.
Score is higher only when dark parts match and when light parts match.

>> I2 = correlate2d(D2, f2, ‘same’)

D2 (275 x 175

f2
61 x 61

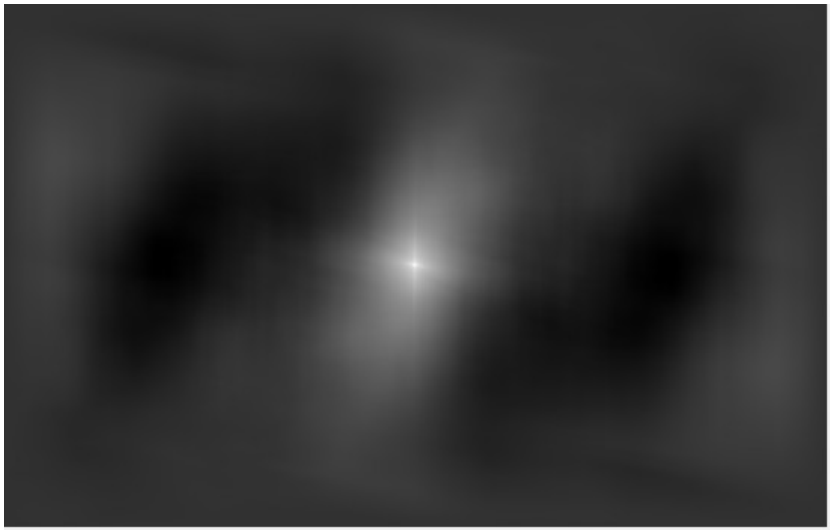
I2

D2 (275 x 175

Or even

>> I3 = correlate2d(D2, D2, ‘full’)

I3



What happens with convolution?

```
>> f = D[ 57:117, 107:167 ]
>> f2 = f – np.mean(f)
>> D2 = D – np.mean(D)

>> I2 = convolve2d( D2, f2, ‘same’ )
```

D2 (275 x 175

f2
61 x 61

What happens with convolution?

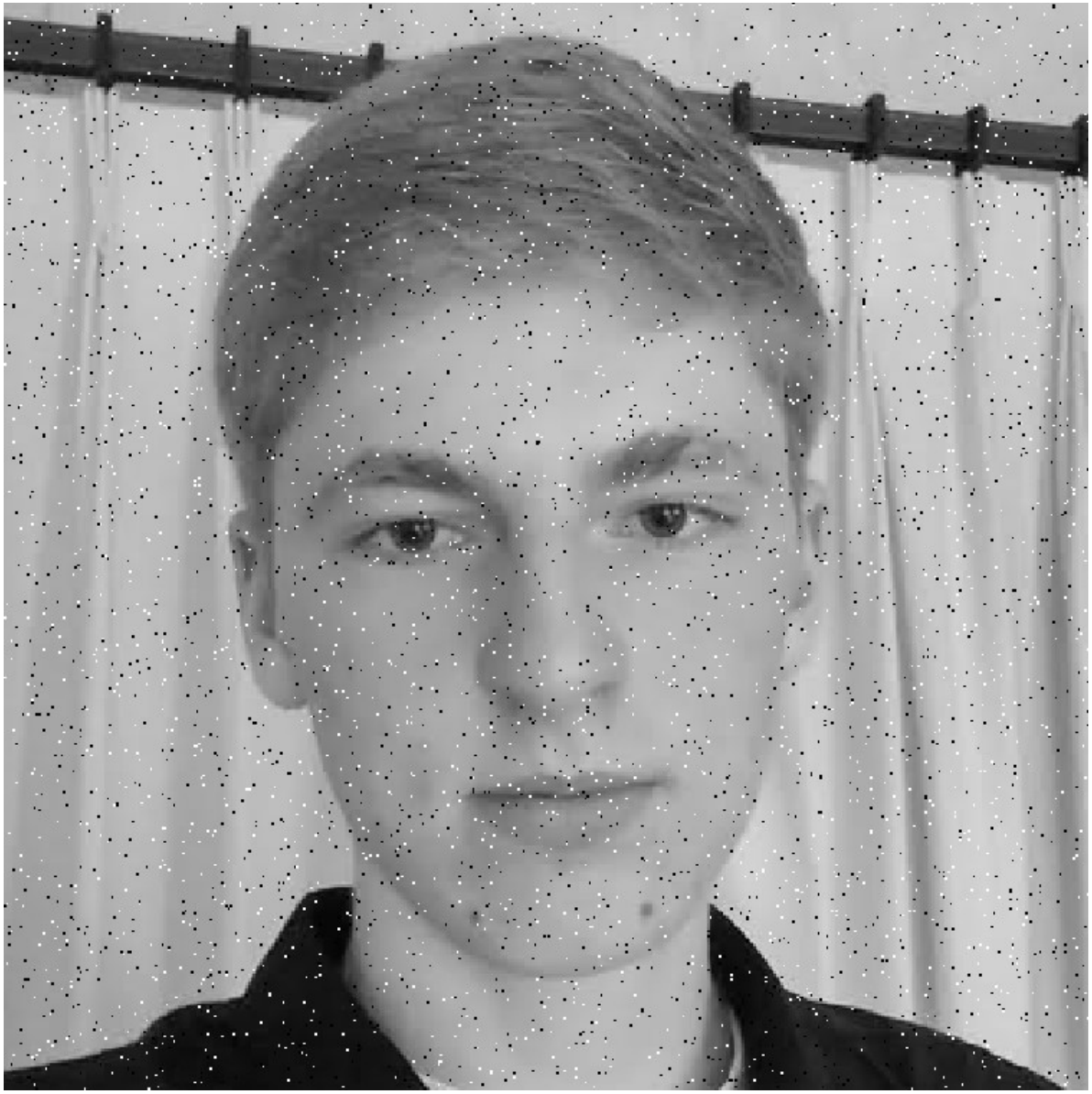
```
>> f = D[ 57:117, 107:167 ]
>> f2 = f – np.mean(f)
>> D2 = D – np.mean(D)

>> I2 = convolve2d( D2, f2, ‘same’ )
```

D2 (275 x 175

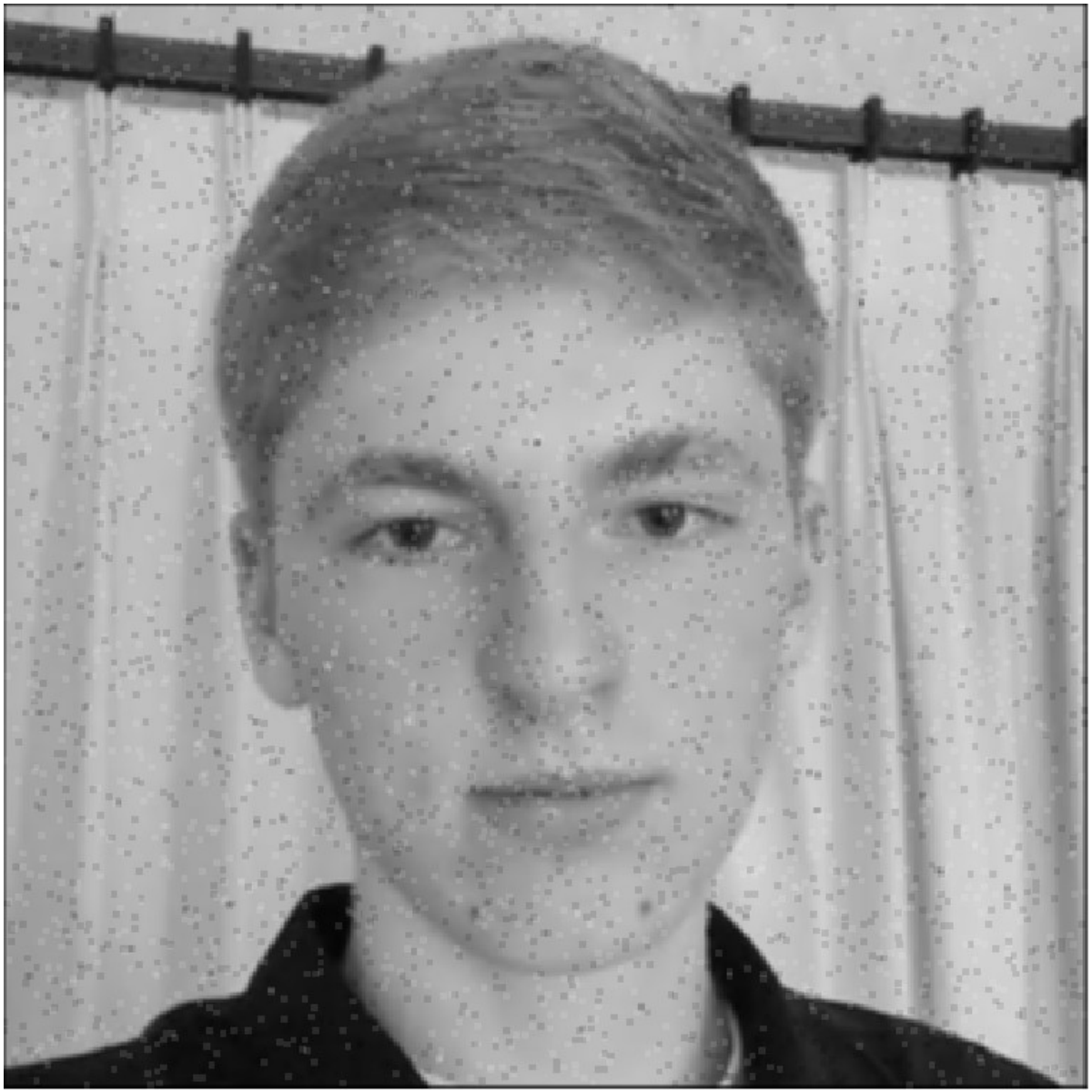
f2
61 x 61

I2

Noise – Salt and Pepper Jack



Mean Jack – 3 x 3 filter



Very Mean Jack – 11 x 11 filter

NON-LINEAR FILTERS

Median filters

- Operates over a window by selecting the median intensity in the window.
- Part of the ‘Rank’ filter family, based on ordering of gray levels
  - E.G., min, max, range filters

Image filtering - mean

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering - mean

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

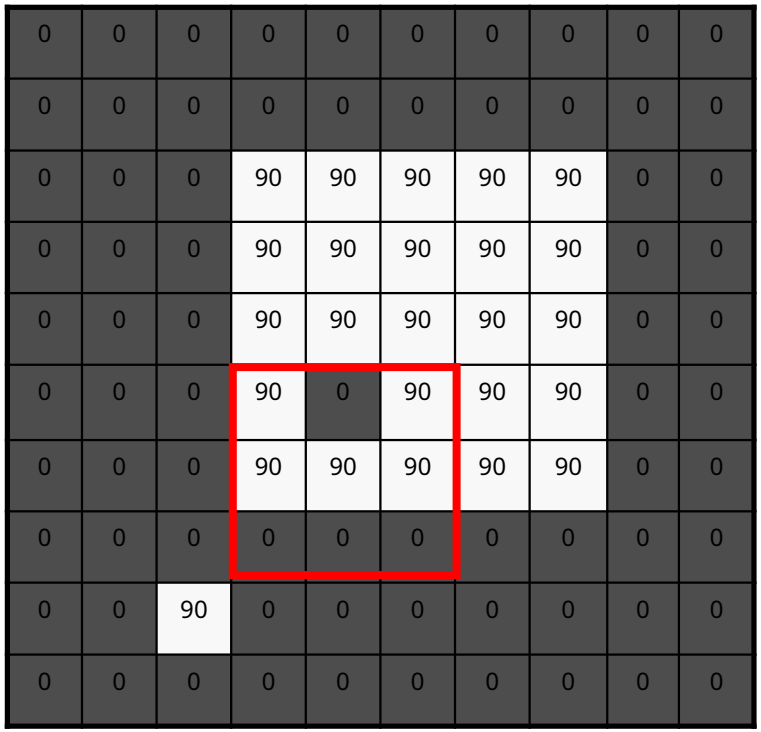
	0	10	20	30	30				
				50					

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

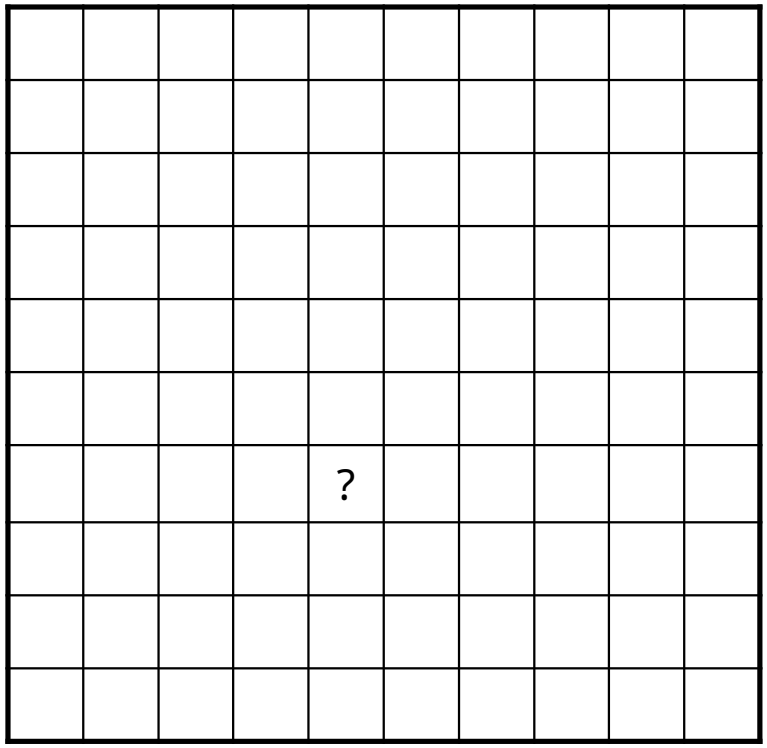
Credit: S. Seitz

Median filter?

$I[.,.]$



$h[.,.]$

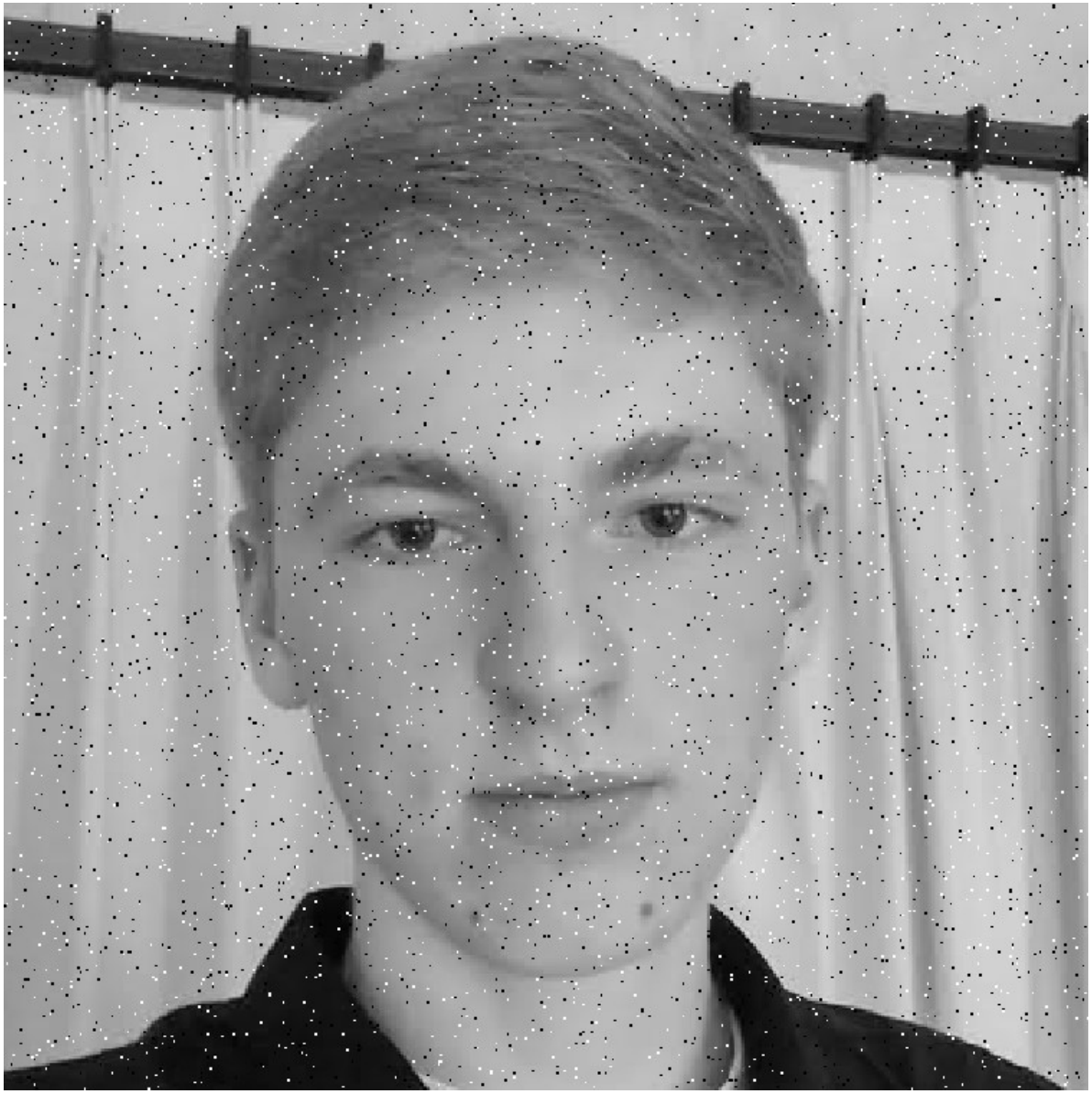


Credit: S. Seitz

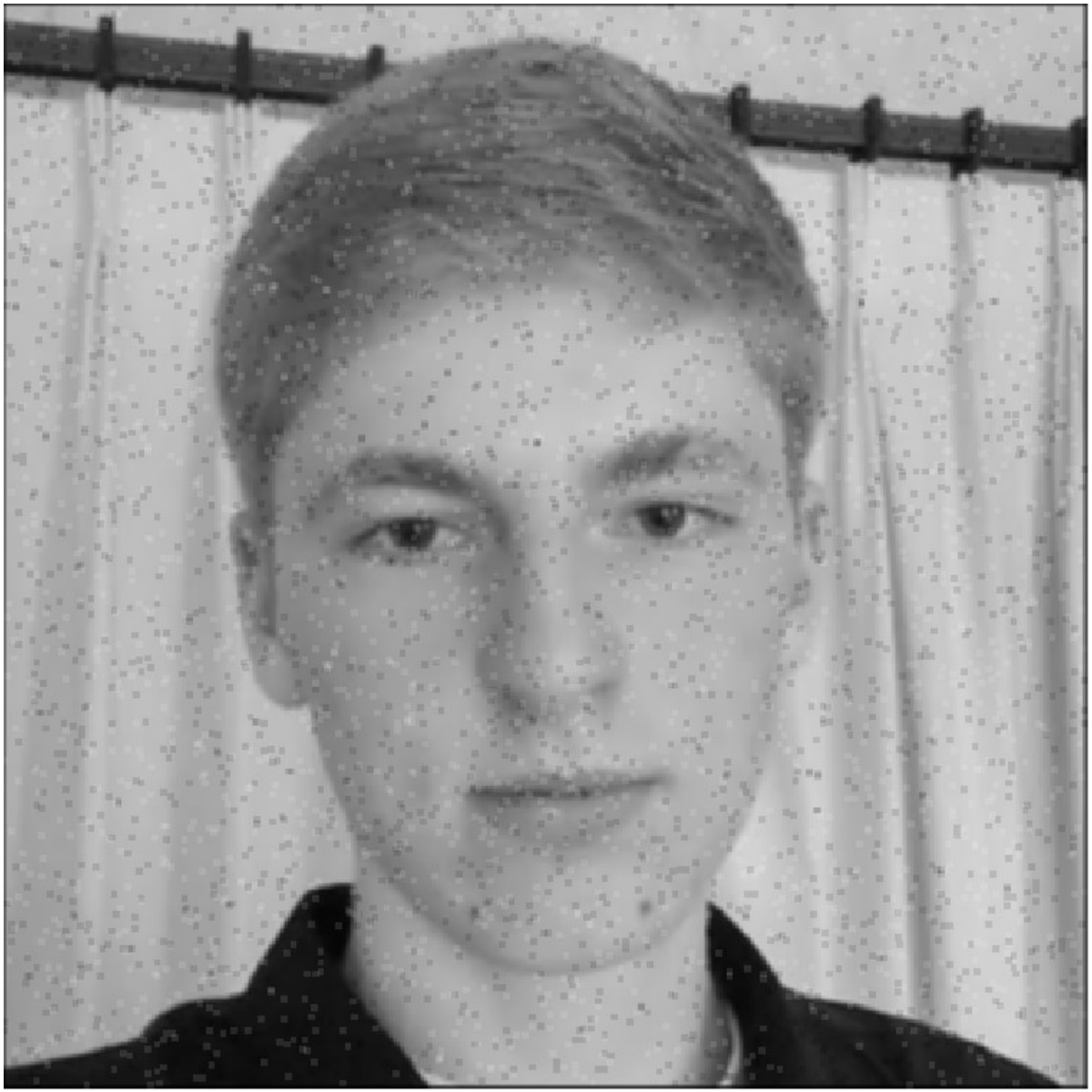
Median filters

- Operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?

Noise – Salt and Pepper Jack

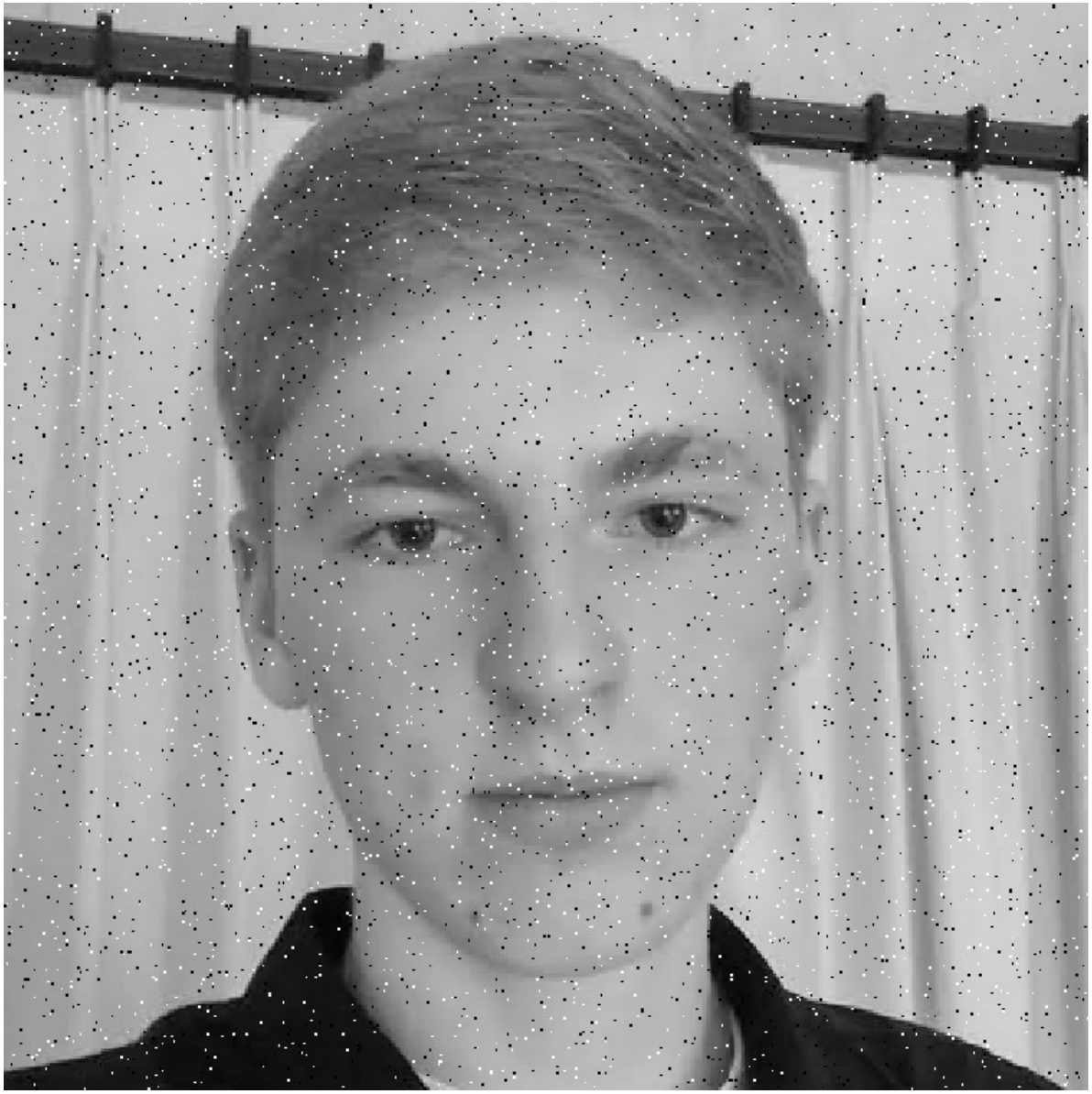


Mean Jack – 3 x 3 filter

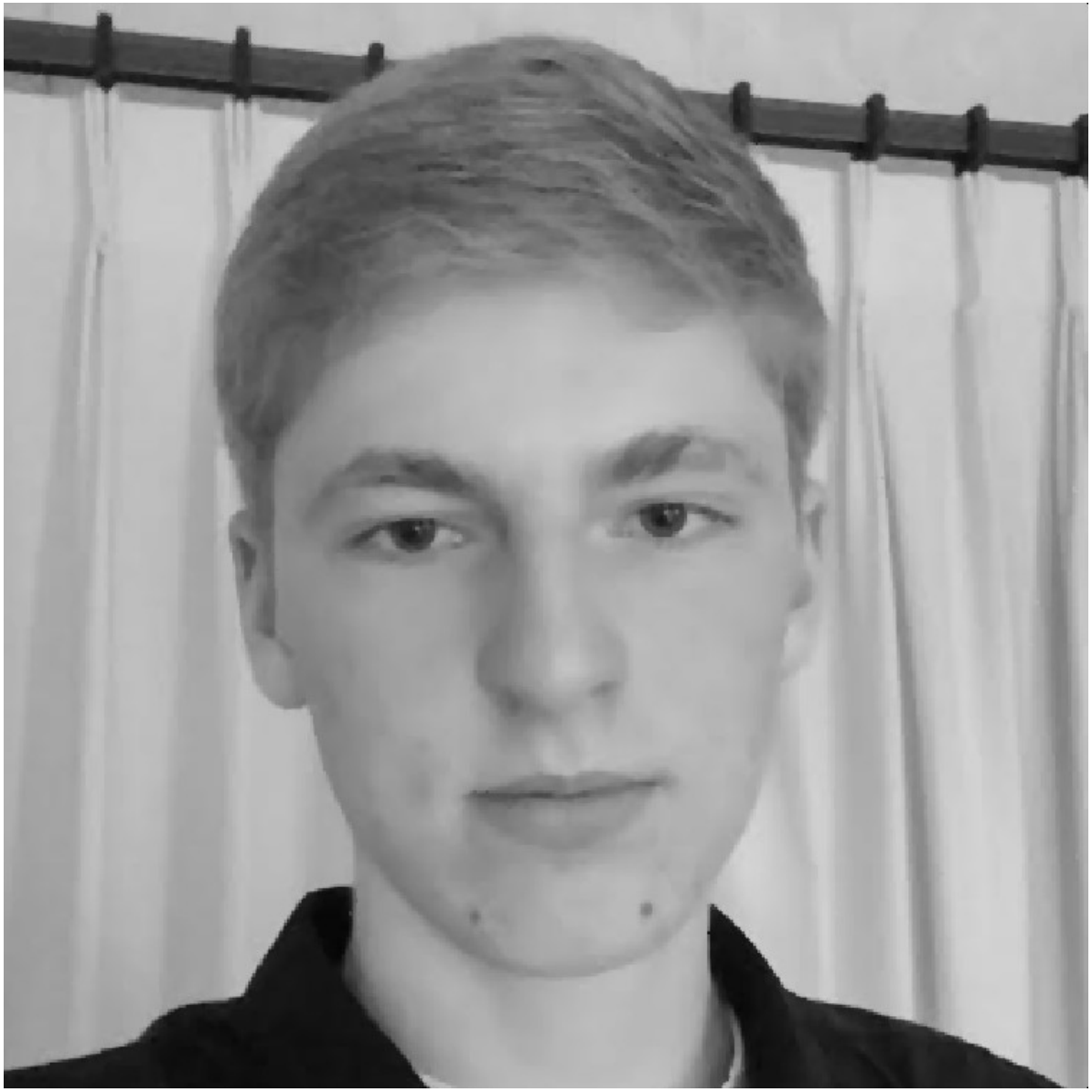


Very Mean Jack – 11 x 11 filter

Noisy Jack – Salt and Pepper



Median Jack – 3 x 3



Very Median Jack – 11 x 11

Median filters

- Operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

Median filters

- Operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

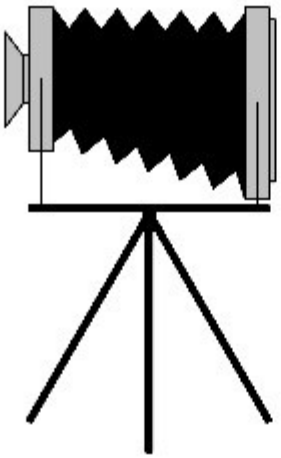
Interpretation: Median filtering is sorting.


Ben Thomas

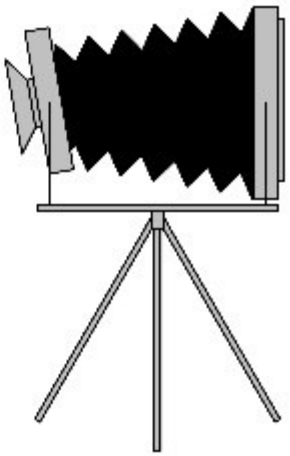
Tilt-shift photography

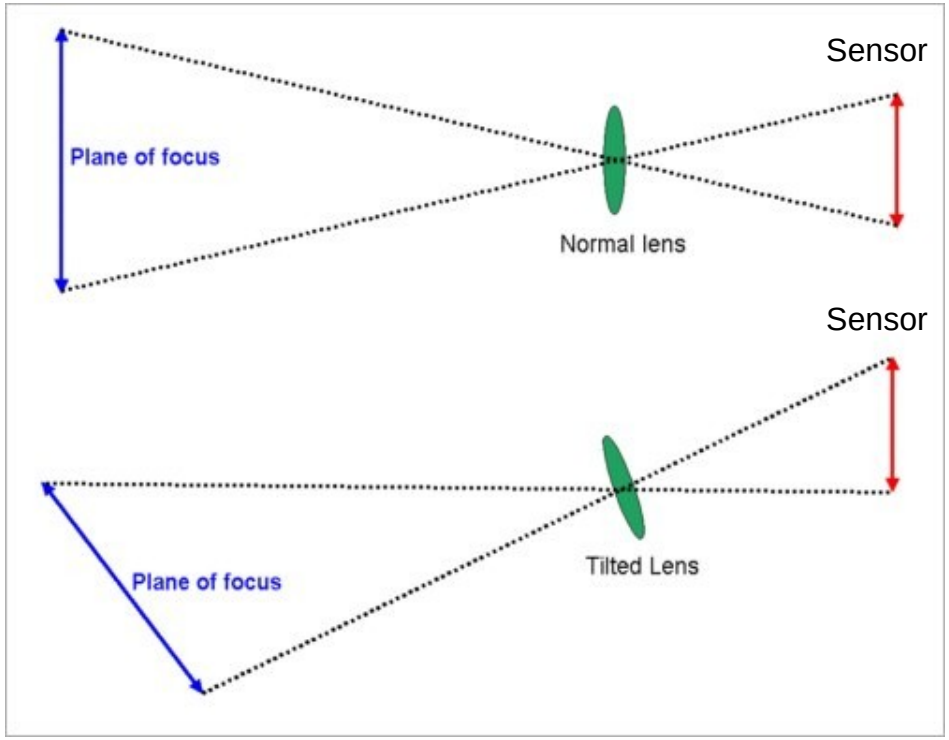
Tilt shift camera

Shift

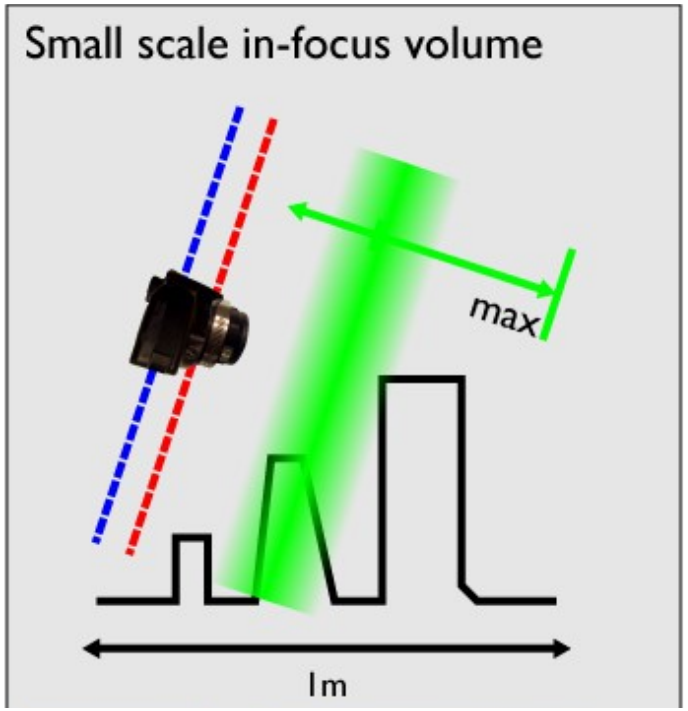


Tilt





Macro photography



Can we fake tilt shift?

- We need to blur the image
  - OK, now we know how to do that.

Can we fake tilt shift?

- We need to blur the image
  - OK, now we know how to do that.

- We need to blur progressively more away from our ‘fake’ focal point

But can I make it look more like a toy?

- Boost saturation – toys are very colorful
- We'll learn how to do this when we discuss color
- For now: transform to Hue, Saturation, Value instead of RGB

Next class: Thinking in Frequency