

- a) One thing that's better with algorithm `RANDFINDINDEX1` is that it has better space-efficiency than `RANDFINDINDEX2` as it doesn't need to allocate space for `doneSet`.

One thing that's better with algorithm `RANDFINDINDEX2` is that because it has `doneSet`, and for every while iteration, it would randomly pick a value that has not been chosen before, so this algorithm is guaranteed to halt.

- b) Because each trial in Line(3) is a Bernoulli trial, with a success probability of  $1/n$ . Under the assumption that  $n$  is a finite natural number, this algorithm terminates in expectation. As each trial in Line(3) is a Bernoulli trial, we expect to succeed after  $n$  trials. Thus, we expect  $n$  iterations of the while loop before we return in Line(6) with the correct value for the index of  $i$ .
- c) For `RANDFINDINDEX2`, because it has `doneSet`, so for each iteration, the trial at Line(14) would only pick  $j \in \{1, \dots, n\} \setminus \text{doneSet}$ . This means the while loop would have at most  $n$  iterations before it terminates, and  $n$  here is the upper bound for the number of iterations. In the best case, the trial at Line(14) would pick  $j \in \{1, \dots, n\} \setminus \text{doneSet}$  such that  $i = A[j]$  at the first time, and it would return in Line(15), and 1 here is the lower bound for the number of iterations. Now, let  $l$  be the lower-bound, and  $u$  be the upper-bound, and  $X$  be the random variable that's the number of iterations of the while loop, then:

$$\begin{aligned}
 E[X] &= \sum_{i=l}^u i \times \Pr\{X = i\} \\
 &= \sum_{i=1}^n i \times \Pr\{X = i\} \\
 &= 1 \times \frac{1}{n} + 2 \times \left( \frac{n-1}{n} \times \frac{1}{n-1} \right) + 3 \times \left( \frac{n-1}{n} \times \frac{n-2}{n-1} \times \frac{1}{n-2} \right) + \dots \\
 &\quad + n \times \left( \frac{n-1}{n} \times \frac{n-2}{n-1} \times \dots \times \frac{n-(n-2)}{n-(n-3)} \times \frac{1}{2} \times \frac{1}{1} \right) \\
 &= \frac{n+1}{2}
 \end{aligned}$$

Hence, in the expected-case, the number of iterations of the while loop is  $(n+1) / 2$ .