

Prove that $\text{InArray} \leq_k \text{LongSimplePath}$. Use proof by construction to prove this. So firstly, we need to adopt a function m that maps the input of InArray to the input of LongSimplePath , which is from $\langle A, i \rangle$ to $\langle G, a, b, k \rangle$. Also, notice that the problem $\text{InArray} \in \mathbf{P}$. So for LongSimplePath , we will prepare two instances for it, i_t being the true instance, and i_f being the false instance. And for i_t , it could be $G = \langle V, E \rangle$ where $V = \{a, b\}$ and $E = \{ \langle a, b \rangle \}$, the vertices a, b , and $k = 1$. For i_f , it could be $G = \langle V, E \rangle$ where $V = \{a, b\}$ and $E = \{ \langle a, b \rangle \}$, the vertices a, b , and $k = 2$. Now, by construction, we propose the following mapping function m , given an instance $\langle A, i \rangle$, introduce a graph $G = \langle V, E \rangle$ and k . Since $\text{InArray} \in \mathbf{P}$, it can find whether i is in A in polynomial time. If i in A then $k = 1$, else $k = 2$. Finally, call $\text{LongSimplePath}(G, a, b, k)$, and return whatever it returns.

For the only if direction, suppose for InArray , i is in A , then it's true and $k = 1$, which for LongSimplePath , it would map to i_t . For the if direction, suppose for InArray , i is not in A , and for the purpose of contradiction, LongSimplePath would output true. And this means it has $k = 1$, and this is only the case when i is in A , which is a contradiction.