

To prove the problem is NP-complete, can first prove that it's NP-hard, since all NP-hard problems are at least NP-complete. In order to prove it's NP-hard, can use Karp reduction to prove that  $\text{HAMCYCLE} \leq_k \text{Problem}$  since HAMCYCLE is NP-hard. Use prove by construction, first, for the given graph  $G$ , choose a vertex  $u$  and another vertex  $v$  that's connected to it. Attach a new vertex  $x$  to  $u$ , connect them with a new edge, and attach a new vertex  $y$  to  $v$  and connect them with a new edge, then connect vertex  $x$  and  $y$ . Call HAMCYCLE until found such vertices  $u$  and  $v$  such that it returns true, if HAMCYCLE returns False for all  $u, v \in V$ , it then means there's no simple cycle of all its vertices within the original graph  $G$ . In the case of HAMCYCLE returns true for tweaked graph, let the edge  $\langle u, x \rangle$  and  $\langle v, y \rangle$  each have weights  $-2/k$ , then add all the weights of other edges with them to see if the final weight is 0, is yes, then return true. In the case of HAMCYCLE returns false for tweaker graph for all  $u, v \in V$ , there's no simple cycle and hence return false.