ECE 606, Fall 2021, Assignment 10
Due: Tuesday, November 23, 11:59pm

Submission: submit your written solutions to crowdmark. There are no **[python3]** problems in this assignment.

1. Recall that we define a decision problem as a function whose codomain is $\{\mathsf{true}, \mathsf{false}\}$. Let $f\colon \{0,1\}^* \to \{\mathsf{true}, \mathsf{false}\}$ be the following function:

   for all $x \in \{0,1\}^*$, $f(x) = \mathsf{false}$

   That is, f maps every input bit string to false. Prove that the decision problem f is not **NP**-hard.

2. Let InArray be the problem: given inputs (i) an array $A[1, \ldots, n]$ of integers where n is a positive integer, and, (ii) an integer i, is $i \in A[1, \ldots, n]$?

   Let LongSimplePath be the problem: given inputs (i) connected undirected $G = \langle V, E \rangle$, (ii) two distinct $a, b \in V$, and, (iii) a positive integer k between 1 and $|V| - 1$, does there exist a simple path $a \rightsquigarrow b$ of $\geq k$ edges?

   Prove that InArray $\leq_k$ LongSimplePath.

   (*Hint*: InArray $\in$ **P**.)

3. Consider the following two problems:

   - HamPathDecision: given input an undirected graph, is there a simple path of all the vertices?
   - HamPathConstruction: given input an undirected graph, output a simple path of all the vertices if one exists, and the string "none" otherwise.

   Prove that if HamPathDecision $\in$ **P** then there exists a polynomial-time algorithm for HamPathConstruction.

4. Prove that if problem s is **NP**-hard and $s \leq_k t$, then t is **NP**-hard.