Notes, 1(c)

ECE 606

Dis/proof strategies

All our strategies are based on logical deduction.

Strategies:

- Case analysis

  For $x, y, z \in \mathbf{N}$ with $x + y = z$, if any two of x, y, z is divisible by 3, then so is the third.

- Contradiction: focus on only row of truth-table for which $p \implies q$ is false.

  $x = \sqrt{2} \implies x \notin \mathbf{Q}$

- Contrapositive: $p \implies q \equiv \neg q \implies \neg p$.

  Given $x, y \in \mathbf{Z}^+$, $\left(\sum_{i=1}^{x} i = \sum_{i=1}^{y} i\right) \implies (x = y)$

  - Note: contrapositive is <u>not</u> the same as converse. The converse of $p \implies q$ is $q \implies p$.

- Construction: for statements of the form $\exists x \ldots$

  Given $x, y \in \mathbf{R}$ with $x < y$, $\exists z \in \mathbf{R}, x < z < y$.

- Induction

  - $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$
  - Suppose $n \in \mathbf{N}$ whose digits are $n_{k-1}n_{k-2}\ldots n_0$. If $\sum_{i=0}^{k-1} n_i$ is divisible by 3, then n is divisible by 3.

To disprove a statement S, we prove it's negation, $\neg S$.

- $\neg(\forall n \in \mathbf{N}, n^2 - n + 11 \text{ is prime}) \equiv \exists n \in \mathbf{N}, n^2 - n + 11 \text{ is not prime}$.

  Can prove this by construction, i.e., a "counterexample."

- Suppose $\mathbf{P}$ is the set of all primes, i.e., $\mathbf{P} = \{2, 3, 5, 7, \ldots\}$.

  $\neg(\exists p, q \in \mathbf{P}, p - q = 513) \equiv \forall p, q \in \mathbf{P}, p - q \neq 513$.