Notes, 2(b)

ECE 606

Data Structures – Overview, Graphs and Trees

Data structure = a way to organize and retrieve data.

Distinguish from (basic) data type: integer, character.

Examples: set, array, linked list, queue, stack, graph, tree

Every distinct data structure has some unique characteristics. Examples:

- set: unordered, unique items, dynamic size.
- array: random access, static size.
- queue: FIFO
- stack: LIFO

Algorithms that come with each data structure reflect this uniqueness. Examples:

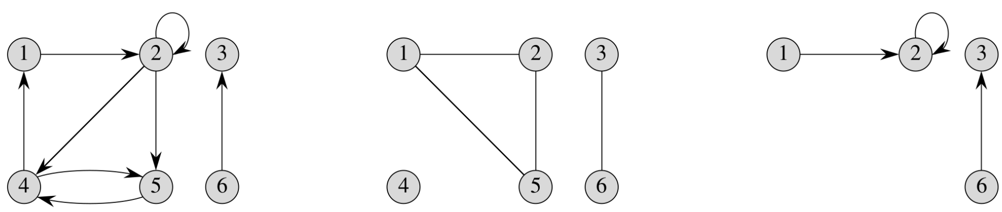
- set: S.addItem(i) – presumably S remains the same if i already in S.
- array: A.writeItem(i, j) – write item i at index j.

The array would not change only if i is already in Array A at index j.

Graph

A pair of sets, $\langle V, E\rangle$, where:

- V is a set of things or names. Called a set of *vertices* or *nodes*. E.g., $V = \{1, 2, 3\}$.
- E, set of *edges* is a relation on V^2
  - Directed graph: $E \subseteq V^2$. E.g., $E = \{\langle 2, 3\rangle, \langle 1, 2\rangle, \langle 1, 1\rangle\}$.
  - Undirected graph: $E \subseteq \{\{u, v\} \subseteq V \mid u \neq v\}$. E.g., $E = \{\{2, 3\}, \{1, 2\}\}$.
    Alternately, can perceive an undirected graph as a special-case of a directed graph.
    1. $\forall u \in V, \langle u, u\rangle \notin E$ Self loop is not allowed
    2. $\forall u, v \in V, \langle u, v\rangle \in E \iff \langle v, u\rangle \in E$. A directed edge <u,v> is in the set of edges if and only if the directed edge <v,u> is also in it.
  - E.g., $E = \{\{2, 3\}, \{1, 2\}\}$ would be written $E = \{\langle 2, 3\rangle, \langle 1, 2\rangle\}$.



(a) (b) (c)

Notions related to graphs:

- A subgraph of a graph. A subgraph of a graph induced by a subset of the vertices.
  - E.g., Graph (c) is subgraph of (a) induced by $\{1, 2, 3, 6\}$.
- Directed graph: in- and out-degree of a vertex. Undirected graph: degree of a vertex.
- Path of length k is a sequence of vertices $\langle u_0, u_1, \ldots, u_k\rangle$ such that...
- Simple path.
- Simple cycle.
  - In an undirected graph, cycle has at least three distinct vertices.
- Undirected graph is connected if... Connected components of an undirected graph.
  - Directed graph: strongly connected, strongly connected components.

More notions related to graphs:

- A graph is said to be acyclic if it contains no cycles.
- Two graphs G, G' are said to be isomorphic if...
- An undirected graph is said to be complete if...
- A bipartite graph is...

<u>Tree</u>

A (free) tree is a connected, acyclic undirected graph.

Theorem B.2 (Properties of free trees)
Let $G = (V, E)$ be an undirected graph. The following statements are equivalent.

1. G is a free tree.
2. Any two vertices in G are connected by a unique simple path.
3. G is connected, but if any edge is removed from E, the resulting graph is disconnected.
4. G is connected, and $|E| = |V| - 1$.
5. G is acyclic, and $|E| = |V| - 1$.
6. G is acyclic, but if any edge is added to E, the resulting graph contains a cycle.

A rooted tree is a free tree in which one of the nodes is distinguished from the others.

- Parent, child, ancestor, descendant, siblings, ...
- Subtree rooted at a node
- Leaf, internal node
- Height, depth
- Ordered tree
- Positional tree, special case: binary tree

Claim 1. *Number of nodes in a complete binary tree:*

1. *total, where the tree has height h, is $2^{h+1} - 1$, and,*
2. *at depth d is 2^d.*