In order to prove there exists a polynomial-time algorithm for HamPathConstruction, we need to prove that HamPathConstruction $\leq_k$ HamPathDecision. And for this one, use proof by construction.

```
HamPathConstruction(G<V, E>)
1   if not HamPathDecision(G<V, E>) then return none
2   Path = []
3   vert = none # This would be the starting vertex for Ham
4   for i in V
5       Let V’ ← V ∪ {x}, where x ∉ V
6       Let E’ ← E ∪ {<x, i>}
7       if HamPathDecision(G<V’, E’>) then vert = i
8   Let V’ ← V \ {vert}
9   Path = Path ∪ {vert}
10  While V’ is not empty
11      for i in V’
12          Let VV ← V’ \ {i}
13          Let E’ ← E \ {edge connecting to i}
14          if HamPathDecision(G<VV, E’>)
15              Path = Path ∪ {i}
16              V’ ← V’ \ {i}
```

For this algorithm, it takes polynomial-time to execute. Of the first for loop in line 4, it would take linear time in the size of V, which is $\theta(|V|)$. For the while loop of line 10, it would take at most time $O(|V|^2)$. Hence, for this algorithm, it takes only polynomial-time to execute.