

So for this problem, will use proof by contradiction. In order for the DFA to accept, the input must be in the language, which means the input has to satisfy the property that input length  $\geq 2$ , and the first bit has to be the same with the second-from-last bit. Now suppose that a DFA of 7 or fewer states exists for this problem. And by the pigeonhole principle, if we were to choose 8 distinct strings over the alphabet, then there exists at least 2 distinct strings,  $x$  and  $y$ , that would end up in the same state  $q_n$ . For any string  $z$ , if  $x$  and  $y$  ends up in the same state, then  $xz$  and  $yz$  would end up in another same state  $q_i$ . So, for this state  $q_i$ , if it's both accepting and non-accepting to  $xz$  and  $yz$ , then there's a contradiction. Now choose one of  $x$  and  $y$  to start with 0 and another to start with 1. And just assume that  $x$  starts with 0 and  $y$  starts with 1. So by choosing  $z = 01$  and append it to each of  $x$  and  $y$ , both  $xz$  and  $yz$  would end up with 01. Since  $y$  starts with 1, and end with 01, it can't be in the language, but  $xz$  is in the language. However, the computation of the DFA would end up in the same state, and in this case, that exact state is both accepting and non-accepting, which is a contradiction and hence, DFA of 7 or fewer states does not exist for this problem.