a) In order for C to be a valid vertex cover, it has to contain every vertex with degree ≥ 1. Which means for each x ∈ C, x needs to have at least one edge connecting to it in Graph G. And every edges that connect to every vertex x in C would sum up to E, which means that let e_i be the edge connecting to vertex i, and $\sum_{i \in C}^{C} e_i = E$. In this case, V \ C will take away all the vertices that have edges connecting to them along with all the edges connecting to them, and V \ C would only leave behind vertices and no edges since every edge connect to every vertex x in C would sum up to E. Hence, the set V \ C are vertices such that they are not connected to each other or any other vertices, so they would form an independent set of the undirected graph G.

b) No, it's not true. For the algorithm ApproxIndSetSize, consider it in three cases. Firstly, suppose that c* is the minimum size for a vertex cover of an undirected graph G.

Case 1: The returned vertex cover C in line 1 is of size c*.

Case 2: The returned vertex cover C in line 1 is of size 2c*.

Case 3: The returned vertex cover C in line 1 is of size between c* and 2c*.

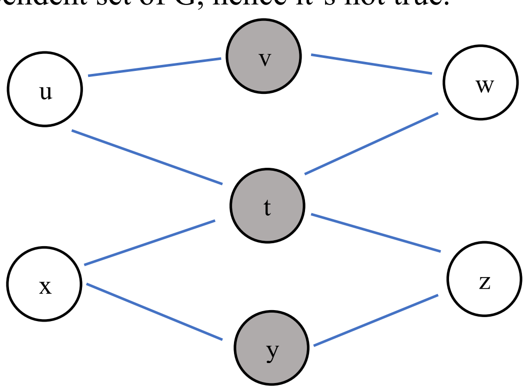
Suppose that i* is the minimum size for an independent set of an undirected graph G.

Doesn't matter for which case, if the if statement in line 2 evaluates to True, then Alice's claim is guaranteed to be true, since it's obvious that (|V| / 2) ≥ $\frac{1}{2}$ i* since i* ≤ |V|. So, we only have to consider line 3.

So, in order to get the maximum-sized independent set of G, we need to get the minimum-sized vertex cover c*, and i* = V \ c*.

For case 1, C= c*, and in line 3, |V \ C| = |V|-|c*| = i* is returned, which is the maximum-sized independent set.

For case 2, C = 2c*, but for the following graph, the returned value is less than half of a maximum-sized independent set of G, hence it's not true.



For this graph, the vertex cover could be {u,w,x,z} or {v,t,y}. But {v,t,y} is the minimum-sized vertex cover, and its size c* = 3. And since V = {u,v,w,t,x,y,z}, |V| = 7. Hence, the independent set = {u,w,x,z}, and i* = 4. In this case, $|V \setminus C| = |V|\text{-}2c^* = 7 - 3 - 3 = 1$. But $1 \leq \frac{1}{2}i^* = \frac{1}{2} \cdot 4 = 2$. Which contradicts with Alice's claim. Hence, it's not true.