ECE 606, Fall 2021, Assignment 11
Due: Tuesday, November 30, 11:59pm

Submission: submit your written solutions to crowdmark. There are no [**python3**] problems in this assignment.

1. Consider the function from instances of CnfSAT to Clique that underlies the proof for the **NP**-hardness of Clique in Claim 73, pdf page 6-7 of Lecture 11 of your textbook.

   Prove that the function is not a bijection.

2. Prove by construction that Clique $\leq_k$ CnfSAT. "By construction" means you need to devise a function from instances of Clique to instances of CnfSAT that satifies the properties specified in Definition 15, pdf page 8 of Lecture 10 of your textbook.

   (*Hint*: suppose the input instance of Clique is $\langle G = \langle V, E \rangle, k \rangle$. Think of your clique of size k as a k-tuple. For each $u \in V$, for each $i = 1, \ldots, k$, introduce a boolean variable $x_{u,i}$ with the semantics that $x_{u,i} = 1$ if and only if the vertex u occupies position i in the k-tuple that is your clique. Now express all the constraints the $x_{u,i}$'s must satisfy. Each such constraint will be a clause in your output instance of CnfSAT.)

3. Show that the following problem is **NP**-complete. Given a weighted undirected graph $G = \langle V, E, w \rangle$ and a real number $k \in \mathbb{R}$, is there a cycle in G whose sum of edge weights is exactly k? (*Hint*: to show that it is **NP**-hard, reduce from HamCycle.)

4. Let Iso be the problem: given two undirected graphs G_1, G_2, is G_1 isomorphic to G_2? Also, adopt from Chapter 11 of your textbook the problem SubgraphIso, which is the problem: given two undirected graphs G_1, G_2, is G_1 isomorphic to a subgraph of G_2?

   Show that Iso $\leq_k$ SubgraphIso.

5. Consider the decision problem ProdTwoPrimes from towards the end of Chapter 9 of your textbook: given as input integer $n \geq 2$, is n the product of exactly two primes.

   Show that ProdTwoPrimes $\in$ **NP**$\cap$**co-NP**. Assume that the problem of checking whether an integer $k \geq 2$ is prime is in **P**.