

For this Claim, saying the change-making possesses the greedy choice means there exists a locally optimal solution that could lead to a globally optimal solution. And in this case, the locally optimal solution is choosing the coins of the highest denomination first. For change-making problems, it fails to possess the greedy choice because the number A that got chosen is the multiple of a denomination that's not the largest amongst the coins in D , and in this way, change-making does not possess the greedy choice as choosing the coin with largest denomination first would result in more total number of coins being chosen as to choosing the coin with the certain denomination for the multiple of times. For Claim 44, use proof by contradiction, and suppose that for $D = \langle 1, 5, 10, 25 \rangle$, change-making does not possess the greedy choice, which means that there exists a number A such that it's a multiple of one of the first three numbers, hence if handing out most coins of the highest denomination first would give the wrong answer.

For A being a multiple of 1, any number below 5 would give us a same result that greedy strategy would give us, and for any number above 5, greedy strategy would let us choose 5, or 10, or 25 first, which would give us the optimal solution compared with choosing all 1's.

For A being a multiple of 5, 5 is just choosing itself, 10 is the denomination of the third coin, 15 can be achieved by choosing three coins with denomination of 5, while the greedy strategy would let us choose 10 and 5, which requires only 2 coins in total. And for any number above 15, the greedy strategy would require us to choose much less coins than choosing all 5's.

For A being a multiple of 10, 10 is just choosing, 20 is less than 25, making 10 the largest denomination to choose, which is the greedy strategy. And for 30, while choosing three coins with denomination of 10, the greedy strategy would let us choose 25 and 5, which is only 2 coins being chosen. And for any number above 30, the greedy strategy would choose 25 first and will result is less coins than choosing all 10's.

Hence, there's no such number that would result in having less total number of coins without choosing the highest denomination first for $D = \langle 1, 5, 10, 25 \rangle$. So, by contradiction, for $D = \langle 1, 5, 10, 25 \rangle$, change-making possess the greedy choice of handing out most coins of the highest denomination first, then second-highest and so on.