

For NP, we defined it as the class of decision problems for each of which there exists a polynomial-time non-deterministic algorithm.

ProdTwoPrimes(n):

```
    if prime_checking(n) then return false
    prod ← false
    Non-deterministically pick x and y ∈ {2, ..., n-1}
    If n = x * y and prime_checking(x) and prime_checking(y) then prod ← true
    return prod
```

Let ProdTwoPrimes be the algorithm for the decision problem “given integer $n \geq 2$, is n the product of exactly two primes”, since prime_checking is in **P**, the algorithm is a polynomial-time non-deterministic algorithm, hence ProdTwoPrimes is in **NP**.

Now, by tweaking this algorithm a little bit, invert its output,

Co_ProdTwoPrimes(n):

```
    if prime_checking(n) then return false
    prod ← false
    Non-deterministically pick x and y ∈ {2, ..., n-1}
    If n = x * y and prime_checking(x) and prime_checking(y) then prod ← true
    return not prod
```

This makes the algorithm Co_ProdTwoPrimes to output exactly the opposite output of ProdTwoPrimes. And since prime_checking is in **P**, this algorithm is a polynomial-time non-deterministic algorithm, hence it's also in **NP**. Since both the algorithms are in **NP**, the problem ProdTwoPrimes is in **NP** ∩ **co-NP**.