

For this problem, there's also a possibility that $j = k$. In this case, according to the recurrence, $\pi_{ij}^{(k)} = \pi_{kj}^{(k-1)}$ if $d_{ij}^{(k-1)} \geq d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$. But when $j = k$, $\pi_{ij}^{(k)} = \pi_{jj}^{(k-1)}$ if $d_{ij}^{(k-1)} \geq d_{ij}^{(k-1)} + d_{jj}^{(k-1)} = d_{ij}^{(k-1)}$, which means that $\pi_{ij}^{(k)} = \pi_{jj}^{(k-1)}$, but $\pi_{jj}^{(k-1)} = NIL$ and $\pi_{ij}^{(k)} \neq NIL$ in the following graph. Hence, the recurrence is not correct.

