$$\frac{\text{Notes, 1(b)}}{\text{ECE 606}}$$

## Propositional logic and a bit beyond

- 1. Proposition: a statement with which we can associate true or false.
  - (a) E.g., "The Earth is flat."
  - (b) E.g., "This statement is true."
  - (c) Not propositions:
    - i. "Hey, you!" c2: If this is true, then it is false. But if it's false, then it's true,
    - ii. "This statement is false." so we have a contradiction.
    - iii. "The value in the variable x is positive." c3: The truth value of this statement depends on the value that x takes, so it's not a proposition.
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  2. We will sometimes refer to propositions abstractly. E.g., "Let p and q be propositions..."

  This means p and q either takes the value ture, or false, but not both.
- 3. Atomic proposition: cannot be broken down further in to propositions.
- 4. Compound proposition: constructed from other propositions. Given propositions p, q, we will say that the following as also propositions:
  - $(p), \neg p, p \land q, p \lor q, p \implies q, p \Longleftarrow q, p \rightleftarrows q, p \oplus q, \dots$
- 5. Important point with logic (and other "languages"): syntax vs. semantics
- 6. The above list of items specify syntax only. We have not yet said, for example, what the truth-value of (p) is, if p is true.
  - (a) I am hinting at a truth-table based semantics.
  - (b) There are other ways to specify semantics: beyond the scope of this course.

Truth-table based semantics:

- 1. What is the truth-value of a proposition given the truth-values of its constituent propositions?
- $\begin{array}{c|cccc} p & (p) \\ \hline 2. & E.g., & true & true \\ \hline \text{false} & \text{false} \end{array}$
- 4. A particularly interesting one: implication.

p	q	$p \implies q$
true	true	true
true	false	false
false	true	true
false	false	true

(a) E.g., "if the Earth is flat, then an emu can fly."

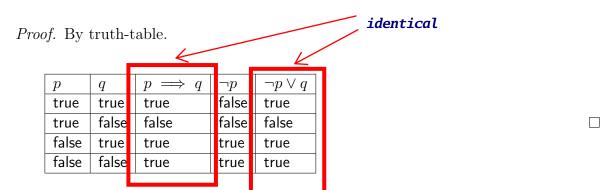
E.g., "if 1=2, then I am the pope."

Both the above statements are true.

5. Note: no gray areas. A proposition is either true or false.

1. Suppose we use " $\equiv$ " to mean "equivalent"; that is,  $a \equiv b$  for propositions a, b means that both a and b always have the same truth-value.

Claim 1. 
$$p \implies q \equiv \neg p \lor q$$



- 2. Turns out we need (),  $\neg$ ,  $\wedge$ ,  $\vee$  only. The others, e.g.,  $\implies$ ,  $\oplus$ , ... can be specified in terms of those.
- 3. E.g., ⊕:

p	q	$p \oplus q$
true	true	false
false	true	true
true	false	true
false	false	false

$$\begin{array}{l} p \oplus q \equiv (p \vee q) \wedge \neg (p \wedge q) \\ \equiv (p \vee q) \wedge (\neg p \vee \neg q) & \because \text{De Morgan's} \\ \equiv (p \wedge (\neg p \vee \neg q)) \vee (q \wedge (\neg p \vee \neg q)) & \because \wedge \text{distributes over} \vee \\ \equiv ((p \wedge \neg p) \vee (p \wedge \neg q)) \vee ((q \wedge \neg p) \vee (q \wedge \neg q)) & \because \vee \text{ distributes over} \wedge \\ \equiv (p \wedge \neg q) \vee (q \wedge \neg p) & \because a \wedge \neg a \equiv \text{ false, false} \vee a \equiv a \end{array}$$

4. Sets and propositions are related closely.

$$\cup \approx \vee, \cap \approx \wedge, \emptyset \approx \mathsf{false}, \mathbb{U} \approx \mathsf{true}, \dots$$

We need to go a bit beyond propositional logic: two quantifiers.

- 1. " $\exists$ ": existential quantifier
- 2. " $\forall$ ": universal quantifier

E.g., For **Q** the set of rationals, "
$$\sqrt{2} \notin \mathbf{Q}$$
"  $\equiv$  " $\forall y \in \mathbf{Q}, y^2 \neq 2$ ".

Rules for negation: suppose p(x) is a statement about the variable x.

- 1.  $\neg(\exists x, p(x)) \equiv \forall x, \neg p(x)$
- 2.  $\neg(\forall x, p(x)) \equiv \exists x, \neg p(x)$

E.g., "no mammal lays eggs."

Suppose M is the set of mammals. This is  $\equiv$  to, " $\forall x \in M, x$  does not lay eggs".

It turns out that the above statement is false. So its negation is true. So what is true is:

$$\neg(\forall x \in M, x \text{ does not lay eggs}) \equiv \exists x \in M, \neg(x \text{ does not lay eggs})$$
$$\equiv \exists x \in M, x \text{ lays eggs}$$