2.

a)

$$Me_x = u \to \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

$$Me_y = v \to \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} \to M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

b)

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (u \ v) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (e_x \ e_y) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$M = (f(e_x) \ f(e_y)) = \left(f\begin{pmatrix} 1 \\ 0 \end{pmatrix} \ f\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = (u \quad v) =$$
$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

c)

$$N = \frac{1}{ad-cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = M^{-1}$$

$$f \circ g(x) = f(g(x)) = f(Nx) = M(Nx) = (MN)x = (MM^{-1})x = I_2 x = x$$

alltså:

$$f \circ g(e_x) = e_x \qquad f \circ g(e_y) = e_y$$

d)

$N = M^{-1}$: N är inversen av M

e)

$$R \cdot M = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \Rightarrow R \cdot \cancel{M \cdot N} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \cdot \frac{1}{ad-cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$R = \begin{pmatrix} \frac{ed-fc}{ad-cb} & \frac{af-eb}{ad-cb} \\ \frac{gd-hc}{ad-cb} & \frac{ah-gb}{ad-cb} \end{pmatrix} = \frac{1}{ad-cb}\begin{pmatrix} ed-fc & af-eb \\ gd-hc & ah-gb \end{pmatrix}$$