Solutions

Practice 1

$P(A \cup B \cup C)$

$$P(A \cup B \cup C) = P(A \cup B) + P(C) - P(A \cup B \cap C)$$

$$P(A \cup B \cup C) = P(A) + P(B) - P(A \cap B) + P(C) - P(A \cup B \cap C)$$

$$P(A \cup B \cup C) = P(A) + P(B) - P(A \cap B) + P(C) - P(A \cap C) - P(A \cap B) + P(A \cap B \cap C)$$

Solution 2

X = "ducks killed" ; Ω = {0,1,2,3}

1. (1,1,0) (1,0,1) (0,1,1); $P(X = 2) = 3 * p^2 * (1 - p)$

Event (0,0,1); X = "number of failure before first success"

2. (0,0,1) ; $P(X = 2) = (1 - p)^2 p$

3. Here it is:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$
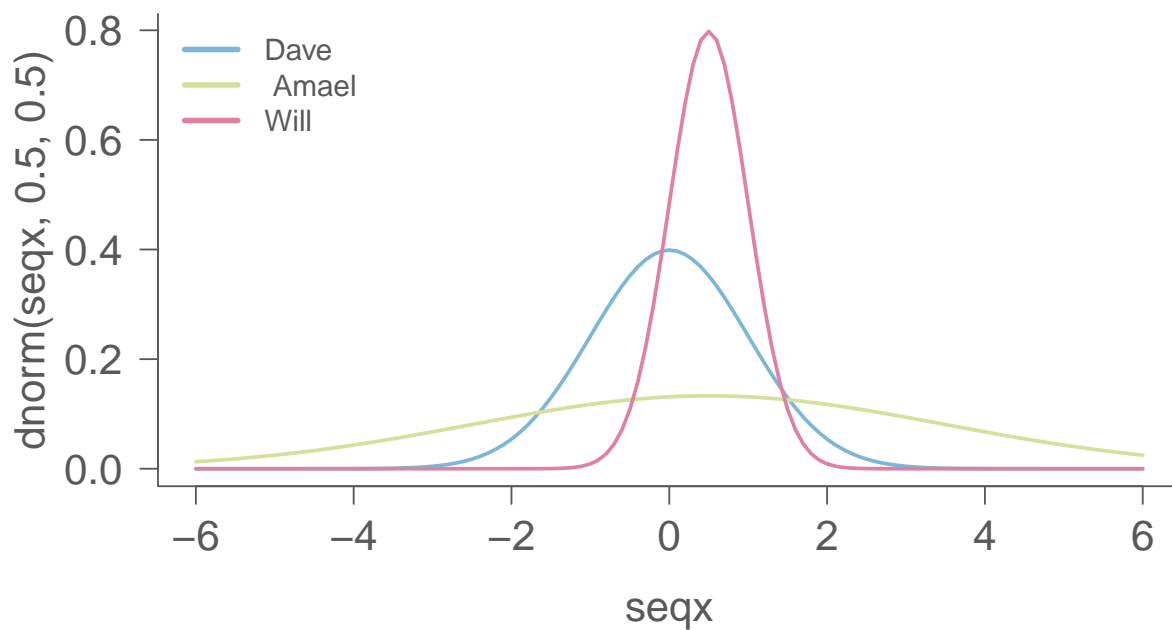
4. Here it is:

$$P(X = k) = p(1 - p)^{n-1}$$

Practice 3

Model

1- Dave: N(0,1)

2- Amael: N(.5,3)

3- Will: N(.5,.5)

Starting on time

$$P("all there before 0") = P(X_1 < 0)P(X_2 < 0)P(X_3 < 0)$$

Start delayed by at least half an hour

$$P("at least one 30 min late") = 1 - P("all there before")$$

$$P("at least one 30 min late") = 1 - P(X_1 < .5)P(X_2 < .5)P(X_3 < .5)$$

Starting the meeting

ontime / avec .5 min de retard / à quelle heure commencera

when all three are present

"start time x" = "all there at x" = "the last arrive at x the two other where there"

Z =

$$P(X_1 < x)P(X_2 < x)f_3(x) + P(X_1 < x)P(X_3 < x)f_2(x) + P(X_2 < x)P(X_3 < x)f_1(x)=$$

Solution 4

60$ = 20*3$

$$E(X) < 20$$

$$\frac{1-p}{p} < 20$$

$$p < \frac{1}{21}$$

Solution 5

2 random variables:

- X = 1 ("sick"); X = 0 ("sane")

- T = 1 ("test positive"); T = 0 ("test negative")

We know $P(X = 1)$ (external study "prior", we'll use it as the parameter of a binomial variable)

In 1. we need to determine $P(X = 1 | T = 1)$

$$P(X = 1 | T = 1) = \frac{P(X = 1 \cap T = 1)}{P(T = 1)}$$

$$P(X = 1 | T = 1) = \frac{P(T = 1 | X = 1)P(X = 1)}{P(T = 1 | X = 1)P(X = 1) + P(T = 1 | X = 0)P(X = 0)}$$

$$P(X = 1 | T = 1) = \frac{1}{1 + \frac{P(T=1|X=0)P(X=0)}{P(T=1|X=1)P(X=1)}}$$

$$P(X = 1 | T = 1) = \frac{1}{1 + \frac{\alpha(1-\pi)}{(1-\alpha)\pi}}$$

For 2. determine $P(X = 1 | T = 0)$

$$P(X = 1 | T = 0) = \frac{P(X = 1 \cap T = 0)}{P(T = 0)}$$

$$P(X = 1 | T = 0) = \frac{1}{1 + \frac{P(T=0|X=0)P(X=0)}{P(T=0|X=1)P(X=1)}}$$

$$P(X = 1 | T = 0) = \frac{1}{1 + \frac{(1-\beta)(1-\pi)}{\beta\pi}}$$

Practice 6