ELECTENG 733 Digital Signal Processing

Semester 1 2020, Department of Electrical, Computer, and Software Engineering

Practice Test 1

31 March 2020

Instructions

- There are **TWENTY** questions in total, answer **ALL** of them
- This test needs to be submitted on CANVAS within 24 hours – i.e. **by 11:59 am on April 1, 2020, New Zealand Standard Time.**
- **Write your answers** on a paper using dark blue or black **PEN**, do **NOT** use pencils as these may not scan.
- The answers can be scanned **ONLY** using any of these two methods:

1. Using a good quality document scanner (that is usually available with a Printer).
2. Using Camscanner App (No other App or taking photos directly from your phone is allowed for this test.)

- Before submitting, check if you have scanned all the pages and if the images are focused correctly, so that is it readable to the marker
- You also have the option to **type your answers** and submit as well.
- If you believe you need further information than that provided, make the appropriate engineering assumption(s), state them clearly, and continue with your answer.
- Include correct units for all the quantities calculated.

What to submit:

Section	Question type	To submit
I.	Descriptive / Mathematical working	Only submit written answers with working
II.	Descriptive / Mathematical working + MATLAB	Submit written answers with working. Also, MATLAB code for Question 2 along with images used.
III.	Descriptive / Mathematical working + MATLAB	Submit written answers with working. Also, MATLAB code for Questions 14, 15.
IV.	Descriptive / Mathematical working	Only submit written answers with working
V.	Descriptive / Mathematical working	Only submit written answers with working

Upload the answer sheet as PDF file and the MATLAB files as .m files on Canvas. Canvas submission for the test supports multiple file submissions.

Questions:

I. *For this question, you need to write the mathematical steps on your answer sheet.* (5 marks)

Question 1. Given the quadratic equation $ax^2 + bx + c = 0$. If you are attempting this question at time HH:MM: SS –where HH corresponds to the hour, and MM corresponds to the minute, and SS corresponds to the second – then $a = \text{HH}, b = \text{MM}, c = \text{SS}$. For example, if you are attempting this question at 9:15:01, then $a = 9, b = 15$ *and* $c = 1$. Using these values of a, b, c, complete the squares to find the roots of this equation. Show the working for completing the squares and obtaining the roots with these values of a, b, c.

II. *In this question, you will read some image signals into MATLAB, make some observations and then complete the rest of the computations on your answer paper. You have to submit the MATLAB code and the written answers*. (10 marks)

a) Read five image signals from your collection to MATLAB. (You can also use any five images from the set used for Tutorial 2). Name them as image1, image2, image3, image4, image5.

b) Convert all the images to greyscale.

c) Define a random variable X: which is the *white pixel %* in the image signal. Measure the Random variable X: *white pixel %* in each of the images – let the definition of white pixel be – *pixels with amplitude value* > 128. So, *white pixel %* will be;

$$White\ pixel\ \% = \frac{White\ pixel\ count}{Total\ number\ of\ pixels\ in\ the\ image} \times 100$$

Question 2: Include a table with each image name and its white pixel count. Your table should look like this:

Image name	**White pixel count**	**Total pixel count**	**White pixel %**
image1			
image2			
image3			
image4			
image5			

Include the MATLAB code for the feature extraction along with your answers. Include the 5 images you used as well for submission.

d) **Question 3:** For the *white pixel %* of image1, image2, image3 and image4 estimate a probability distribution for the 4 points using Maximum Likelihood Estimation. Assume that the distribution is a Normal distribution. Show the steps of the estimation method, clearly stating any assumptions made. Include a table with the parameters estimated for the distribution like the ones shown below:

Parameter	**Estimated value**
…	…
…	…
…. etc.	… etc.

e) Question 4: Write the Probability density function of the probability distribution estimated in Question 3.

f) Question 5: Obtain the Cumulative distribution function of random variable X from the probability distribution estimated in Question 3. Show steps of how CDF can be obtained from PDF.

g) Question 6: Calculate the expectation of the *white pixel %* (Random variable X) for the five images. Show the steps of the calculation. Let this value be called $M = E[X]$.

h) Question 7: Consider a random variable Y, which is a function of random variable X. Let $Y = X/2$. Calculate the Expectation of Y. Show the steps of the calculation. Let us name $E[Y] = N$.

i) Question 8: Calculate the probability that the value of random variable X is less than N. Show the steps of the calculation.

j) Question 9: Calculate the first moment, second moment, first central moment and second central moment of the random variable X. Show the steps for each.

III. *In this question, you will discuss a real-life application of exponential distribution. You need to include answers to these questions, along with sufficient citations to support your answer. Also, include MATLAB code as .m file for Question 14. For all other questions, write the answers in the paper you are submitting.* (10 marks)

Question 10: Give an example of a real-life application where the random variable follows an exponential distribution. Include the link to this application along with the explanation you provide.

Question 11: Consider a random variable Z that has an exponential distribution with decay factor = M from Question 6. Plot the PDF of this random variable.

Question 12: Derive the CDF of Z from the PDF. Show the steps to do this derivation.

Question 13: Write the PDF of a Laplace distribution having the same decay factor as random variable Z.

Question 14: Using MATLAB plot the PDF of the random variable Z for all values of Z. Implement the equation of the PDF.

Question 15: Using MATLAB plot the CDF of the random variable Z for all values of Z. Implement the equation of the CDF.

Question 16: What happens to the probability distribution of Z if the value of the decay factor is increased?

Question 17: Find the area under the PDF of random variable Z. Show the required calculations.

IV. *For this question, only the working is needed in your answer sheet, along with steps and any assumptions made. Define any new parameter you are introducing to the calculations.*

(5 marks)

A speech signal is transmitted through a ternary communication channel. The symbols 0 and 1 are sent through the channel, but noise can switch 1 to 0 or vice versa with certain probabilities. Let the probability that a digit transmitted as 0 is received as 1 is $1 - \beta$, with a similar random corruption probability when the digit 1 is transmitted and 0 is received. It has been observed that across a large number of transmitted signals, 0s and 1s are transmitted in the ratio 1:2.

Question 18: Given that the sequence 001 is received, what is the probability of receiving a sequence where all bits are corrupted? Assume that transmission and reception are independent events.

V. *In this question, you will show working for implementing an image classification system*.

(10 marks)

Question 19: The aim of this question is to test an image recognition system.

Consider the Random variable with Normal distribution obtained in Section I, Question 4. Let it be called Random Variable X.

Consider another Random variable Y which is the *white pixel %* of images taken of a *particular object O* to be recognized by an image recognition system. Random variable Y follows exponential distribution with the decay factor 0.1.

If a new image falls under the probability distribution of random variable Y, then the object will be classified as *present* in the image. If a new image falls under the probability distribution of random variable X, then the object will be classified as *not present* in the image.

Consider a new image *image_x* that will be tested by the image recognition system, which has *white pixel %* = 20%.

Using Naïve Bayes classifier with Maximum A Posteriori criteria, make the decision if the image is that of the *particular object O* or not.

Your answer should contain the following steps:

(i) Describe the problem in terms of probabilities needed for a Naïve Bayes classifier. In this stage, you should also define what the random variables to the system are, and what the decisions to be made are.
(ii) Calculate the pre-requisites needed for the classifier.
(iii) Plot the likelihoods for the two classes sharing the same x-axis. Comment on how well the two classes can be differentiated based on their probability distributions.
(iv) Make the decision for *particular object O* based on the Naïve Bayes classifier.

Question 20: Derive the Expectation of the two random variables X and Y based on their PDFs. The Expectation of these random variables may be known to you already as a formula. But for this question, you need to show all mathematical calculations to prove it.

END