Problem Set 5

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1 Analytical Exercises

1. Prove that $\sum_{i=1}^{n} k = n(n+1)/2$ and $\sum_{i=1}^{n} k^2 = n(n+1)(2n+1)/6$.
2. * Consider a series $y_t = c_t + T_t$ with deterministic trend $T_t = \mu + \delta_t$ where $t = 1, 2, \ldots$. Further assume that, $E(c_t) = 0$. Denote as $\theta = (\mu, \delta)'$ and estimate it, using OLS. Derive the asymptotic properties of $\hat{\theta}$ (**Hint**: you might want to use results from problem 1) What are the asymptotic rates?
3. Let y_t be an $(n \times 1)$ vector of $I(1)$ variables. Denote $g \equiv (n-1)$ and partition $y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}$ where y_{2t} denotes a $(q \times 1)$ vector. Now consider the OLS regression of the first variable on the g variables and a constant as follows:

$$y_{1t} = c + \beta' y_{2t} + u_t$$

Derive the OLS coefficient estimates and express them in matrix form (i.e what is $\begin{bmatrix} \hat{c}_T \\ \hat{\beta}_T \end{bmatrix}$)

2 R Exercises

4. Simulate two random walk series (say y and x)of $T = 500$ where $y_0 = x_0 = 0$ and the error terms are standard normally distributed. Discard the first 100 observations(this is typical to get rid of the effect of initial values). Run a regression of y on x. What do you observe? Next, run a unit root test on the residuals. What do you observe? Finally, run a regression of the differenced y on differenced x and conduct a t-test for $\beta = 0$.