Problem Set 6

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1 Analytical Exercises

1. Suppose that $Y_t = (Y_{1,t}, Y_{2,t})'$ and assume a VAR(1):

$$Y_t = \mu + \Phi Y_{t-1} + \epsilon_t, \ \epsilon_t \sim (0, \Omega_\epsilon)$$

with $\Phi = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $\mu = (\mu_1, \mu_2)'$, $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t})'$.

a) Write the VAR in the form $\Delta Y_t = \Pi Y_{t-1} + \epsilon_t$

b) Establish the order of integration of the components of Y_t and show that the cointegrating rank of the system is 1.

c) Explain why $(1, -1)$ is a cointegrating vector

d) Write down the error-correction representation of the system.

2. The following regressions have been estimated using n observations on real consumers' expenditure on nondurable goods and services (C_t) and real personal disposable income (Y_t):

$$\underset{}{C_t} = \underset{(0.41)}{0.88} + \underset{(0.002)}{0.005}t + \underset{(0.04)}{0.91}C_{t-1} - \underset{(0.07)}{0.12}\Delta C_{t-1} + e_{1t}, \quad DW = 2.19, \quad n = 131$$
$$\underset{}{Y_t} = \underset{(0.49)}{1.30} + \underset{(0.003)}{0.008}t + \underset{(0.05)}{0.87}Y_{t-1} - \underset{(0.09)}{0.18}\Delta C_{t-1} + e_{2t}, \quad DW = 2.09, \quad n = 131$$
$$\underset{}{C_t} = \underset{(0.15)}{1.21} + \underset{(0.01)}{0.87}Y_t + e_{3t}, \quad DW = 1.56, \quad n = 131$$

$$e_{3t} = \underset{(0.10)}{0.19} e_{3,t-1} + e_{4t}, \quad DW = 1.93, \quad n = 130$$

Variables are in natural logarithms, figures in parentheses are estimated standard errors, t is a time trend, n is the sample size, $\Delta X_t = X_t - X_{t-1}$, DW is the Durbin-Watson statistic, and e_{it} $(i = 1, ..., 4)$ are OLS residuals.

a) Examine whether the necessary condition for cointegration between C_t and Y_t is satisfied.

b) Formulate and test the hypothesis of cointegration between C_t and Y_t.

c) Examine whether it is advisable to estimate a model for C_t and Y_t formulated in first differences of variables rather than in levels, with or without an error-correction term.

2 R Exercises

3. Download the series Y and conduct the Engle & Granger cointegration test manually. Obtain estimates for the error-correction representation of the system.