Introduction and Decomposition

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1 Derivations

1) (i) The data are generated as $Y_i = X_i'\beta_0 + \epsilon_i, i = 1, \ldots, n$, $\beta_0 \in \mathbb{R}^k$
   (ii) X is a nonstochastic and finite $n \times k$ matrix, $n \geq k$
   (iii) $X'X$ is nonsingular
   (iv) $E(\epsilon) = 0$
   (v) $\epsilon \sim N(0, \sigma_0^2 I)$, with $\sigma_0^2 < \infty$

   a) Prove that $\hat{\beta}_n$ exists and is unique. What assumptions are needed?
   b) Prove the unbiasedness i.e $E[\hat{\beta}_n] = \beta_0$. What assumptions are needed?
   c) Prove the normality i.e $\hat{\beta}_n \sim N(\beta_0, \sigma_0^2(X'X)^{-1})$

2) In exercise 1, consider what happens when assumption (ii) fails i.e X are now stochastic. Show that the $\hat{\beta}_n$ is unbiased. Are there any extra assumptions we need, to ensure that $\hat{\beta}_n$ is unbiased?

2 R exercise

1) Use library "astsa" and plot the "Quarterly Earnings per Share of Johnson and Johnson," the "Global Temperature Deviations" and the "speech" datasets. (For

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that you will need to check the documentation of the library atsa) Plot together the "Southern Oscillation Index" and the "Recruitment".

Next generate a time-series of length 365, that follows a standard normal distribution and plot it. Use the acf function to plot the autocorrelations(we will see more on that in our next lecture). What do you observe?

2) Sometimes we want to remove the trend (detrend the series) and seasonally adjust it (remove the seasonality effect). Seasonally adjust the log liquor sales data using a linear trend plus seasonal dummy model. **Hint**: You may need to use library(forecast)