MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$Ax = b$

$|A| \neq 0$

$$\begin{aligned}
a_{11}x_1 &+ a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\
a_{21}x_1 &+ a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\
&\vdots \\
a_{n1}x_1 &+ a_{n2}x_2 + \cdots + a_{nn}x_n = b_n
\end{aligned}$$

$$A = \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}}_{\text{coef.}}, \quad x = \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}}_{\text{var}}, \quad b = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}}_{\text{t. ind.}}.$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$Ax=b$

$A=L+D+U$

$(L+D+U)x=b$

$$Lx+Dx+Ux=b$$

$$L=\begin{vmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & 0 \end{vmatrix} \quad D=\begin{vmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & & & 0 \\ 0 & & a_{33} & & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & & a_{nn} \end{vmatrix} \quad U=\begin{vmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & & 0 \end{vmatrix}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \quad L=\begin{vmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & 0 \end{vmatrix} \quad D=\begin{vmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & & & 0 \\ 0 & & a_{33} & & 0 \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & 0 & & a_{nn} \end{vmatrix} \quad U=\begin{vmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & & 0 \end{vmatrix}$$

$Ax=b$

$A=L+D+U$

$(L+D+U)x=b$

$$Lx^{k+1}+Dx^{k+1}+Ux^{k}=b$$

$$Dx^{k+1}=b-Lx^{k+1}-Ux^{k}$$

$$\boxed{x^{k+1}=D^{-1}[b-Lx^{k+1}-Ux^{k}]}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$x^{k+1} = D^{-1}\left[b - Lx^{k+1} - Ux^{k}\right]$$

$$\begin{vmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & 0 \end{vmatrix} \quad D = \begin{vmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & & & 0 \\ 0 & & a_{33} & & 0 \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & & 0 & a_{nn} \end{vmatrix} \quad U = \begin{vmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & & 0 \end{vmatrix}$$

$$\begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{vmatrix}^{k+1} = \begin{vmatrix} 1/a_{11} & 0 & 0 & \cdots & 0 \\ 0 & 1/a_{22} & 0 & \cdots & 0 \\ 0 & 0 & 1/a_{33} & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & & 1/a_{nn} \end{vmatrix} \left[\begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & a_{43} & & \\ a_{n1} & a_{n2} & a_{n3} & a_{n,n-1} & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{vmatrix}^{k+1} - \begin{vmatrix} 0 & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & a_{24} & \cdots & a_{2n} \\ 0 & 0 & 0 & a_{34} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & & a_{n-1,n} \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{vmatrix}^{k} \right.$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}^{k+1}
=
\begin{bmatrix}
1/a_{11} & 0 & 0 & \cdots & 0 \\
0 & 1/a_{22} & 0 & \cdots & 0 \\
0 & 0 & 1/a_{33} & & \vdots \\
\vdots & \vdots & & \ddots & 0 \\
0 & 0 & \cdots & & 1/a_{nn}
\end{bmatrix}
\left[
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}
-
\begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
a_{21} & 0 & 0 & \cdots & 0 \\
a_{31} & a_{32} & 0 & \cdots & 0 \\
\vdots & \vdots & a_{43} & & \\
a_{n1} & a_{n2} & a_{n3} & a_{n,n-1} & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}^{k+1}
-
\begin{bmatrix}
0 & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\
0 & 0 & a_{23} & a_{24} & \cdots & a_{2n} \\
0 & 0 & 0 & a_{34} & \cdots & a_{3n} \\
\vdots & \vdots & \vdots & \vdots & & a_{n-1,n} \\
0 & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}^{k}
\right]
$$

$$x_1^{k+1} = \frac{1}{a_{11}}\left(b_1 - (0x_1^{k+1} + 0x_2^{k+1} + 0x_3^{k+1} \cdots 0x_n^{k+1}) - (0x_1^k + a_{12}x_2^k + a_{13}x_3^k + a_{14}x_4^k + \cdots a_{1n}x_n^k)\right)$$

$$x_1^{k+1} = \frac{b_1 - a_{12}x_2^k - a_{13}x_3^k - a_{14}x_4^k - \cdots - a_{1n}x_n^k}{a_{11}}$$

$$x_2^{k+1} = \frac{1}{a_{22}}\left(b_2 - (a_{21}x_1^{k+1} + 0x_2^{k+1} + 0x_3^{k+1} + \cdots + 0x_n^{k+1}) - (0x_1^k + 0x_2^k + a_{23}x_3^k + a_{24}x_4^k + \cdots + a_{2n}x_n^k)\right)$$

$$x_2^{k+1} = \frac{b_2 - a_{21}x_1^{k+1} - a_{23}x_3^k - a_{24}x_4^k + \cdots + a_{2n}x_n^k}{a_{22}}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$\begin{vmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots a_{n\,n-1} & 0 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_{n-1} \\ X_n \end{vmatrix}^{k+1} \underset{(3)}{\overset{(2)\ act}{+}} \begin{vmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & & 0 \\ 0 & & a_{33} & & 0 \\ \vdots & & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & a_{nn} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{vmatrix}^{k+1} \underset{(3)}{\overset{(2)\ act}{+}} \begin{vmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & & 0 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{vmatrix}^{k} \underset{(2)}{\overset{(1)\ ant}{=}} \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{vmatrix}$$

$$(0X_1 + 0X_2 + 0X_3 \cdots 0X_n) + (a_{11}X_1 + 0X_2 + 0X_3 + \cdots 0X_n) + (0X_1 + a_{12}X_2 + a_{13}X_3 + \cdots a_{1n}X_n) = b_1$$

$$a_{11}X_1 + (a_{12}X_2 + a_{13}X_3 + \cdots a_{1n}X_n) = b_1 \Rightarrow X_1^{k+1} = \frac{b_1 - a_{12}X_2^k - a_{13}X_3^k - \cdots - a_{1n}X_n^k}{a_{11}}$$

$$(a_{21}X_1^{k+1} + 0X_2^{k+1} + 0X_3^{k+1} + \cdots 0X_n) + (0X_1^{k+1} + a_{22}X_2^{k+1} + 0X_3^{k+1} + \cdots 0X_n^{k+1}) + (0X_1 + 0X_2^k + a_{23}X_3^k + \cdots a_{2n}X_n^k) = b_2$$

$$a_{21}X_1 + (a_{22}X_2) + (a_{23}X_3 + \cdots a_{2n}X_n) = b_2 \Rightarrow X_2 = \frac{b_2 - a_{21}X_1^{k+1} - a_{23}X_3^k - \cdots - a_{2n}X_n^k}{a_{22}}$$

$k = 1$

$k = 2$

$$X_3^{k+1} = \frac{b_3 - a_{31}X_1^{k+1} - a_{32}X_2^{k+1} - \cdots a_{3n}X_n^k}{a_{33}}$$

$$X_n^{k+1} = \frac{b_n - a_{n1}X_1^{k+1} - a_{n2}X_2^{k+1} - a_{n3}X_3^{k+1} - \cdots a_{n\,n-1}X_{n-1}^{k+1}}{a_{nn}}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}^{k+1} = \begin{bmatrix} 1/a_{11} & 0 & 0 & \cdots & 0 \\ 0 & 1/a_{22} & 0 & \cdots & 0 \\ 0 & 0 & 1/a_{33} & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & & 1/a_{nn} \end{bmatrix} \left[\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & a_{43} & & \\ a_{n1} & a_{n2} & a_{n3} & a_{n,n-1} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}^{k+1} - \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & a_{24} & \cdots & a_{2n} \\ 0 & 0 & 0 & a_{34} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & & a_{n-1,n} \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}^{k} \right]$$

$$x_3^{k+1} = \frac{b_3 - a_{31}x_1^{k+1} - a_{32}x_2^{k+1} - a_{34}x_4^{k} \cdots - a_{3n}x_n^{k}}{a_{33}}$$

$$x_n^{k+1} = \frac{b_n - a_{n1}x_1^{k+1} - a_{n2}x_2^{k+1} - a_{n3}x_3^{k+1} - \cdots - a_{nn-1}x_{n-1}^{k+1}}{a_{nn}}$$

$$\boxed{x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^{k}}{a_{ii}}}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

Ax=b
A=L+D+U
(L+D+U)x=b

$$Lx^{k+1} + \underbrace{Dx^{k+1}} + Ux^{k} = b$$

$$Dx^{k+1} = b - Lx^{k+1} - Ux^{k}$$

$$D^{-1} Dx^{k+1} = D^{-1}[b - Lx^{k+1} - Ux^{k}]$$

$$\boxed{X^{k+1} = D^{-1}[b - Lx^{k+1} - Ux^{k}]}$$

$$\boxed{x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k}}{a_{ii}}}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$X^{k+1} = D^{-1}[\, b - Lx^{k+1} - Ux^{k} \,]$$

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k}}{a_{ii}}$$

1. Valores iniciales [Xo]
2. Calcular [X] (k+1)
3. Criterio de paro número de iteraciones o un error máximo
4. [Er]=abs([V(k+1)]-[V(k)]./[V(k+1)])
5. Si no cumple regresa al paso 2

****** Para garantizar convergencia la matriz de coeficientes debe ser de diagonal estrictamete dominante ******

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$\boxed{X^{k+1} = D^{-1}[\,b - Lx^{k+1} - Ux^{k}\,]}$$

Ejemplo: Resolver el siguiente sistema de ecuaciones.

12x1+5x2-x3=15
X1-6x2-4x3=9
2x1-3x2+8x3=5
Con valores iniciales x1= 1 , x2= 1 , x3= 1

$$\begin{bmatrix} 12 & 5 & -1 \\ 1 & -6 & -4 \\ 2 & -3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix}$$

$|12| > |5| + |-1| \Rightarrow 12 > 6$

$|-6| > |1| + |-4| \Rightarrow 6 > 5$

$|8| > |2| + |-3| \Rightarrow 8 > 5$

MÉTODOS INDIRECTOS O ITERACTIVOS GAUSS-SEIDEL

$$\boxed{X^{k+1} = D^{-1}[\, b - Lx^{k+1} - Ux^{k}\,]}$$

$$X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$L = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{vmatrix} \quad D = \begin{vmatrix} 12 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 8 \end{vmatrix} \quad U = \begin{vmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\begin{bmatrix} 12 & 5 & -1 \\ 1 & -6 & -4 \\ 2 & 3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix}$$

$$D^{-1} = \begin{vmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{vmatrix}$$

$k = 0$

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}^{1} = \begin{vmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{vmatrix} \left(\begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}^{1} - \begin{vmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}^{0} \right)$$

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}^{1} = \begin{vmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{vmatrix} \left(\begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}^{1} - \begin{vmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}^{0} \right) = \begin{pmatrix} 0.917 \\ -2.014 \\ 0.359 \end{pmatrix}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}^{(2)} = \begin{vmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{vmatrix} \left(\begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}^{(2)} - \begin{vmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}^{(1)} \right) = \begin{bmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{bmatrix}$$

$$e = \left| \frac{\begin{bmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{bmatrix} - \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}}{\begin{bmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{bmatrix}} \right| \times 100 = \begin{bmatrix} 9.09\ \% \\ 146.15\ \% \\ 60.0\ \% \end{bmatrix}$$

$k = 2$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

Resuelva el siguiente sistema de ecuaciones lineales por medio del método de Gauss Seidel, con una tolerancia de 1%

$$3x_1 + -0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \qquad Xo = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}^0$$

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k}}{a_{ii}}$$

$k=0$

$$x_1^1 = \frac{b_1 - \sum_{j=2}^{3} a_{ij} x_j^0}{a_{11}} = \frac{b_1 - a_{12}x_2^0 - a_{13}x_3^0}{a_{11}} = \frac{7.85 + 0.1(1) + 0.2(1)}{3} = 2.716667$$

$$x_2^1 = \frac{b_2 - \sum_{j=1}^{1} a_{2j} x_j^1 - \sum_{j=3}^{3} a_{2j} x_j^0}{a_{22}} = \frac{b_2 - a_{21}x_1^1 - a_{23}x_3^0}{a_{22}} = \frac{-19.3 - (0.1)(2.716667) - (-0.3)(1)}{7} = -2.753095$$

$$x_3^1 = \frac{b_3 - \sum_{j=1}^{2} a_{3j} x_j^1 - \cancel{\sum_{j=4}^{3} a_{3j} x_j}}{a_{33}} = \frac{b_3 - a_{31}x_1^1 - a_{32}x_2^1}{a_{33}} = \frac{71.4 - (0.3)(2.716667) - (-0.2)(-2.753095)}{10} = 7.003438$$

0

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$k=0$

$$x_1^1 = \frac{b_1 - \sum_{j=2}^{3} a_{ij} x_j^0}{a_{11}} = \frac{b_1 - a_{12}x_2^0 - a_{13}x_3^0}{a_{11}} = \frac{7.85 + 0.1(1) + 0.2(1)}{3} = 2.716667$$

$$x_2^1 = \frac{b_2 - \sum_{j=1}^{1} a_{2j} x_j^1 - \sum_{j=3}^{3} a_{2j} x_j^0}{a_{22}} = \frac{b_2 - a_{21}x_1^1 - a_{23}x_3^0}{a_{22}} = \frac{-19.3 - (0.1)(2.716667) - (-0.3)(1)}{7} = -2.753095$$

$$x_3^1 = \frac{b_3 - \sum_{j=1}^{2} a_{3j} x_j^1 - \underbrace{\cancel{\sum_{j=4}^{3} a_{3j} x_j^1}}_{0}}{a_{33}} = \frac{b_3 - a_{31}x_1^1 - a_{32}x_2^1}{a_{33}} = \frac{71.4 - (0.3)(2.716667) - (-0.2)(-2.753095)}{10} = 7.003438$$

$$e_{a\%} = \left| \frac{\begin{vmatrix} 2.716667 \\ -2.753095 \\ 7.003438 \end{vmatrix} - \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}}{\begin{vmatrix} 2.716667 \\ -2.753095 \\ 7.003438 \end{vmatrix}} \right| \times 100\% = \begin{vmatrix} 63.19 \\ 136.32 \\ 85.72 \end{vmatrix}$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$k=1$

$$x_1^2 = \frac{b_1 - \sum_{j=2}^{3} a_{ij} x_j^1}{a_{11}} = \frac{b_1 - a_{12}x_2^1 - a_{13}x_3^1}{a_{11}} = \frac{7.85 + 0.1(-2.753095) + (0.2)(7.003438)}{3} = 2.991793$$

$$x_2^2 = \frac{b_2 - \sum_{j=1}^{1} a_{2j} x_j^2 - \sum_{j=3}^{3} a_{2j} x_j^1}{a_{22}} = \frac{b_2 - a_{21}x_1^2 - a_{23}x_3^1}{a_{22}} = \frac{-19.3 - (0.1)(2.991793) - (-0.3)(7.003438)}{7} = -2.499735$$

$$x_3^2 = \frac{b_3 - \sum_{j=1}^{2} a_{3j} x_j^2 - \cancelto{0}{\sum_{j=4}^{3} a_{3j} x_j^1}}{a_{33}} = \frac{b_3 - a_{31}x_1^2 - a_{32}x_2^2}{a_{33}} = \frac{71.4 - (0.3)(2.991793) - (-0.2)(-2.499735)}{10} = 7.000252$$

$$e_{r\%} = \left| \frac{\begin{vmatrix} 2.991793 \\ -2.499735 \\ 7.000252 \end{vmatrix} - \begin{vmatrix} 2.716667 \\ -2.753095 \\ 7.003438 \end{vmatrix}}{\begin{vmatrix} 2.991793 \\ -2.499735 \\ 7.000252 \end{vmatrix}} \right| * 100 = \begin{vmatrix} 9.14 \\ 10.13 \\ 0.045 \end{vmatrix} \%$$

MÉTODOS INDIRECTOS O ITERACTIVOS
GAUSS-SEIDEL

$k=2$

$$x_1^3 = \frac{b_1 - \sum_{j=2}^{3} a_{ij} x_j^2}{a_{11}} = \frac{b_1 - a_{12}x_2^2 - a_{13}x_3^2}{a_{11}} = \frac{7.85 + 0.1(-2.499735) + (0.2)(7.000252)}{3} = 3.000026$$

$$x_2^3 = \frac{b_2 - \sum_{j=1}^{1} a_{2j} x_j^3 - \sum_{j=3}^{3} a_{2j} x_j^2}{a_{22}} = \frac{b_2 - a_{21}x_1^3 - a_{23}x_3^2}{a_{22}} = \frac{-19.3 - (0.1)(3.000026) - (-0.3)(7.000252)}{7} = -2.49999$$

$$x_3^3 = \frac{b_3 - \sum_{j=1}^{2} a_{3j} x_j^3 - \cancel{\sum_{j=4}^{3} a_{3j} x_j^2}^{\,0}}{a_{33}} = \frac{b_3 - a_{31}x_1^3 - a_{32}x_2^3}{a_{33}} = \frac{71.4 - (0.3)(3.000026) - (-0.2)(-2.49999)}{10} = 6.99999$$

$$e_{R\%} = \left| \frac{\begin{vmatrix} 3.000026 \\ -2.49999 \\ 6.99999 \end{vmatrix} - \begin{vmatrix} 2.991793 \\ -2.499735 \\ 7.000252 \end{vmatrix}}{\begin{vmatrix} 3.000026 \\ -2.49999 \\ 6.99999 \end{vmatrix}} \right| * 100 = \left| \quad \right| \%$$

$$x^0 = \begin{bmatrix} b_1/a_{11} \\ b_2/a_{22} \\ b_3/a_{33} \\ \vdots \\ b_n/a_{nn} \end{bmatrix}$$