

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$Ax = b$$

$$|A| \neq 0$$

$$\rightarrow \begin{array}{ccccccc} & \downarrow & & \downarrow & & & \\ a_{11}x_1 & + a_{12}x_2 & + \dots & + a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + a_{22}x_2 & + \dots & + a_{2n}x_n & = & b_2 \\ & \vdots & & & & \vdots \\ a_{n1}x_1 & + a_{n2}x_2 & + \dots & + a_{nn}x_n & = & b_n \end{array}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

coef. Var t. ind.

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

$$Ax=b$$

$$A=L+D+U$$

$$(L+D+U)x=b$$

$$Lx+Dx+Ux=b$$

$$L = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ a_{21} & 0 & 0 & \dots & 0 \\ a_{31} & a_{32} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & 0 \end{pmatrix} \quad D = \begin{pmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & a_{22} & 0 & \dots & 0 \\ 0 & 0 & a_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{pmatrix} \quad U = \begin{pmatrix} 0 & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & 0 & a_{23} & \dots & a_{2n} \\ 0 & 0 & 0 & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} = L + D + U$$

$$L = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ a_{21} & 0 & 0 & \dots & 0 \\ a_{31} & a_{32} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & 0 \end{pmatrix} \quad D = \begin{pmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & a_{22} & 0 & \dots & 0 \\ 0 & 0 & a_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{pmatrix} \quad U = \begin{pmatrix} 0 & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & 0 & a_{23} & \dots & a_{2n} \\ 0 & 0 & 0 & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

$$Ax=b$$

$$A=L+D+U$$

$$(L+D+U)x=b$$

$$Lx^{k+1} + Dx^{k+1} + Ux^k = b$$

$$Dx^{k+1} = b - Lx^{k+1} - Ux^k$$

$$x^{k+1} = D^{-1} [b - Lx^{k+1} - Ux^k]$$

MÉTODOS INDIRECTOS O INTERACTIVOS

$$x^{k+1} = D^{-1} [b - Lx^{k+1} - Ux^k]$$

GAUSS-SEIDEL

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{pmatrix} \quad D = \begin{pmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & a_{22} & 0 & \dots & 0 \\ 0 & 0 & a_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{pmatrix} \quad U = \begin{pmatrix} 0 & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & 0 & a_{23} & \dots & a_{2n} \\ 0 & 0 & 0 & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

$$\left[\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{array} \right]^{k+1} = \left[\begin{array}{cccccc} 1/a_{11} & 0 & 0 & \dots & 0 \\ 0 & 1/a_{22} & 0 & \dots & 0 \\ 0 & 0 & 1/a_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & 1/a_{nn} \end{array} \right] \left[\begin{array}{c} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{array} \right] - \left[\begin{array}{cccccc} 0 & 0 & 0 & \dots & 0 \\ a_{21} & 0 & 0 & \dots & 0 \\ a_{31} & a_{32} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{n,n-1} & 0 \end{array} \right] \left[\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{array} \right]^{k+1}$$

GAUSS-SEIDEL

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$k+1$

x_1	$\frac{1}{a_{11}}$	0	0	...	0
x_2	0	$\frac{1}{a_{22}}$	0	...	0
x_3	0	0	$\frac{1}{a_{33}}$...	0
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
x_n	0	0	...	0	$\frac{1}{a_{nn}}$

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b_1	0	0	0	...	0
b_2	a_{21}	0	0	...	0
b_3	a_{31}	a_{32}	0	...	0
\vdots	\vdots	\vdots	a_{43}	\ddots	\vdots
b_n	a_{n1}	a_{n2}	a_{n3}	...	a_{nn}

$k+1$

x_1	0	a_{12}	a_{13}	a_{14}	...	a_{1n}
x_2	0	0	a_{23}	a_{24}	...	a_{2n}
x_3	0	0	0	a_{34}	...	a_{3n}
\vdots	\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
x_n	0	0	0	0	...	$a_{n-1,n}$

k

x_1	0	0	0	...	0
x_2	0	0	0	...	0
x_3	0	0	0	...	0
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots
x_n	0	0	0	...	0

$$x_1^{k+1} = \frac{1}{a_{11}} \left(b_1 - (0x_1^k + 0x_2^k + 0x_3^k + \dots + 0x_n^k) - (0x_1^k + a_{12}x_2^k + a_{13}x_3^k + a_{14}x_4^k + \dots + a_{1n}x_n^k) \right)$$

$$x_1^{R+1} = \frac{b_1 - a_{12}x_2^k - a_{13}x_3^k - a_{14}x_4^k - \dots - a_{1n}x_n^k}{a_{11}}$$

$$x_2^{k+1} = \frac{1}{a_{21}} (b_2 - (a_{21}x_1^{k+1} + 0x_2^{k+1} + 0x_3^{k+1} + \dots + 0x_n^{k+1}) - (0x_1^k + 0x_2^k + a_{23}x_3^k + a_{24}x_4^k + \dots + a_{2n}x_n^k))$$

$$x_2^{k+1} = \frac{b_2 - a_{21}x_1^{k+1} - a_{23}x_3^k - a_{24}x_4^k + \dots + a_{2n}x_n^k}{a_{22}}$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$\begin{array}{c|c|c|c|c|c}
 \begin{array}{cccccc}
 0 & 0 & 0 & \dots & 0 \\
 a_{21} & 0 & 0 & \dots & 0 \\
 a_{31} & a_{32} & 0 & \dots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn}
 \end{array} &
 \begin{array}{c}
 x_1^{(k+1)} \\
 x_2^{(k+1)} \\
 x_3^{(k+1)} \\
 \vdots \\
 x_n^{(k+1)}
 \end{array} &
 \begin{array}{c}
 a_{11} \\
 0 \\
 0 \\
 \vdots \\
 0
 \end{array} &
 \begin{array}{c}
 0 \\
 a_{22} \\
 a_{33} \\
 \vdots \\
 a_{nn}
 \end{array} &
 \begin{array}{c}
 x_1^{(k+1)} \\
 x_2^{(k+1)} \\
 x_3^{(k+1)} \\
 \vdots \\
 x_n^{(k+1)}
 \end{array} &
 \begin{array}{c}
 0 \\
 a_{12} \\
 a_{23} \\
 \vdots \\
 0
 \end{array} &
 \begin{array}{c}
 a_{13} \\
 \vdots \\
 a_{2n} \\
 \vdots \\
 a_{3n} \\
 \vdots \\
 a_{nn}
 \end{array} &
 \begin{array}{c}
 x_1^{(k)} \\
 x_2^{(k)} \\
 x_3^{(k)} \\
 \vdots \\
 x_n^{(k)}
 \end{array} &
 \begin{array}{c}
 b_1 \\
 b_2 \\
 b_3 \\
 \vdots \\
 b_n
 \end{array}
 \end{array}$$

$$(0x_1 + 0x_2 + 0x_3 + \dots + 0x_n) + (a_{11}x_1 + 0x_2 + 0x_3 + \dots + 0x_n) + (0x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n) = b_1$$

$$a_{11}x_1 + (a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n) = b_1 \Rightarrow x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^k - a_{13}x_3^k - \dots - a_{1n}x_n^k}{a_{11}}$$

$$(a_{21}x_1 + 0x_2 + 0x_3 + \dots + 0x_n) + (0x_1 + a_{22}x_2 + 0x_3 + \dots + 0x_n) + (0x_1 + 0x_2 + a_{23}x_3 + \dots + a_{2n}x_n) = b_2$$

$$a_{21}x_1 + (a_{22}x_2) + (a_{23}x_3 + \dots + a_{2n}x_n) = b_2 \Rightarrow x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^k - \dots - a_{2n}x_n^k}{a_{22}}$$

$k=1$

$k=2$

$$x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)} - \dots - a_{3n}x_n^k}{a_{33}}$$

$$x_n^{(k+1)} = \frac{b_n - a_{n1}x_1^{(k+1)} - a_{n2}x_2^{(k+1)} - a_{n3}x_3^{(k+1)} - \dots - a_{nn-1}x_{n-1}^{(k+1)}}{a_{nn}}$$

GAUSS-SEIDEL

The diagram illustrates the elimination of X_1 from the system of equations. It shows three stages of the process:

- Initial System:** A system of n equations with n variables X_1, X_2, \dots, X_n . The coefficient matrix is lower triangular with 1s on the diagonal. The right-hand side is b_1, b_2, \dots, b_n .
- Elimination Step:** The first equation is used to eliminate X_1 from the remaining equations. This results in a new system where the first equation remains unchanged, and the subsequent equations have their first column entries set to zero. The right-hand side values are updated accordingly.
- Final System:** The system is now in a form where X_1 only appears in the first equation. The coefficient matrix is block upper triangular, with the first row containing the coefficients for X_1 and the rest of the matrix being lower triangular with 1s on the diagonal. The right-hand side is $b_1, 0, \dots, 0$.

$$x_3^{k+1} = \frac{b_3 - a_{31}x_1^{k+1} - a_{32}x_2^{k+1} - a_{34}x_4^k - \dots - a_{3n}x_n^k}{a_{33}}$$

$$\underline{x}_n^{k+1} = \frac{b_n - a_{n1} \underline{x}_1^{k+1} - a_{n2} \underline{x}_2^{k+1} - a_{n3} \underline{x}_3^{k+1} - \dots - a_{nn-1} \underline{x}_{n-1}^{k+1}}{a_{nn}}$$

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^n a_{ij} x_j^k}{a_{ii}}$$

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GAUSS-SEIDEL

$$Ax=b$$

$$A=L+D+U$$

$$(L+D+U)x=b$$

$$Lx^{k+1} + \underbrace{Dx^{k+1}} + Ux^k = b$$

$$Dx^{k+1} = b - Lx^{k+1} - Ux^k$$

$$D^{-1} Dx^{k+1} = D^{-1} [b - Lx^{k+1} - Ux^k]$$

$$X^{k+1} = D^{-1} [b - Lx^{k+1} - Ux^k]$$

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^n a_{ij} x_j^k}{a_{ii}}$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$X^{k+1} = D^{-1} [b - Lx^{k+1} - Ux^k]$$

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^n a_{ij} x_j^k}{a_{ii}}$$

1. Valores iniciales $[X_0]$
2. Calcular $[X]$ (k+1)
3. Criterio de paro número de iteraciones o un error máximo
4. $[Er] = \text{abs}([V(k+1)] - [V(k)] ./ [V(k+1)])$
5. Si no cumple regresa al paso 2

***** Para garantizar convergencia la matriz de coeficientes debe ser de diagonal estrictamente dominante *****

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$X^{k+1} = D^{-1} [b - Lx^{k+1} - Ux^k]$$

Ejemplo: Resolver el siguiente sistema de ecuaciones.

$$12x_1 + 5x_2 - x_3 = 15$$

$$x_1 - 6x_2 - 4x_3 = 9$$

$$2x_1 - 3x_2 + 8x_3 = 5$$

Con valores iniciales $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

$$\begin{bmatrix} 12 & 5 & -1 \\ 1 & -6 & -4 \\ 2 & -3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix}$$

$$|12| > |5| + |-1| \Rightarrow 12 > 6$$

$$|-6| > |1| + |-4| \Rightarrow 6 > 5$$

$$|8| > |2| + |-3| \Rightarrow 8 > 5$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$X^{k+1} = D^{-1} [b - Lx^{k+1} - Ux^k]$$

$$x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 12 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 5 & -1 \\ 1 & -6 & -4 \\ 2 & 3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix}$$

$k=0$

$$D^{-1} = \begin{bmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^1 = \begin{bmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{bmatrix} \left(\begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^1 - \begin{bmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^0 \right)$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^1 = \begin{bmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{bmatrix} \left(\begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 0.917 \\ -2.014 \\ 0.359 \end{bmatrix}$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$$\begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix}^{(2)} = \begin{bmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{bmatrix} \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{bmatrix} \begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix}^{(2)} - \begin{bmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 1 \\ 1 \end{matrix}^{(2)} = \begin{bmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{bmatrix}$$

$$e = \frac{\begin{bmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{bmatrix} - \begin{bmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{bmatrix}}{\begin{bmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{bmatrix}}$$

$$\times 100 = \begin{bmatrix} 9.09 \% \\ 146.15 \% \\ 60.0 \% \end{bmatrix}$$

$$k = 2$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

Resuelva el siguiente sistema de ecuaciones lineales por medio del método de Gauss Seidel, con una tolerancia de 1%

$$\begin{aligned} 3x_1 + -0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned} \quad \begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \quad X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^0$$

$$x_i^{k+1} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^n a_{ij} x_j^k}{a_{ii}}$$

$k=0$

$$x_1' = \frac{b_1 - \sum_{j=2}^3 a_{1j} x_j^0}{a_{11}} = \frac{b_1 - a_{12} x_2^0 - a_{13} x_3^0}{a_{11}} = \frac{7.85 + 0.1(1) + 0.2(1)}{3} = 2.716667$$

$$x_2' = \frac{b_2 - \sum_{j=1}^1 a_{2j} x_j' - \sum_{j=3}^3 a_{2j} x_j^0}{a_{22}} = \frac{b_2 - a_{21} x_1' - a_{23} x_3^0}{a_{22}} = \frac{-19.3 - (0.1)(2.716667) - (-0.3)(1)}{7} = -2.753095$$

$$x_3' = \frac{b_3 - \sum_{j=1}^2 a_{3j} x_j' - \sum_{j=4}^3 a_{3j} x_j^0}{a_{33}} = \frac{b_3 - a_{31} x_1' - a_{32} x_2'}{a_{33}} = \frac{71.4 - (0.3)(2.716667) - (-0.2)(-2.753095)}{10} = 7.003438$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$k=0$

$$x_1' = \frac{b_1 - \sum_{j=2}^3 a_{1j} x_j^0}{a_{11}} = \frac{b_1 - a_{12} x_2^0 - a_{13} x_3^0}{a_{11}} = \frac{7.85 + 0.1(1) + 0.2(1)}{3} = 2.716667$$

$$x_2' = \frac{b_2 - \sum_{j=1}^1 a_{2j} x_j' - \sum_{j=3}^3 a_{2j} x_j^0}{a_{22}} = \frac{b_2 - a_{21} x_1' - a_{23} x_3^0}{a_{22}} = \frac{-19.3 - (0.1)(2.716667) - (-0.3)(1)}{7} = -2.753095$$

$$x_3' = \frac{b_3 - \sum_{j=1}^2 a_{3j} x_j' - \sum_{j=4}^3 a_{3j} x_j^0}{a_{33}} = \frac{b_3 - a_{31} x_1' - a_{32} x_2'}{a_{33}} = \frac{71.4 - (0.3)(2.716667) - (-0.2)(-2.753095)}{10} = 7.003438$$

$$e_{R\%} = \frac{\begin{vmatrix} 2.716667 \\ -2.753095 \\ 7.003438 \end{vmatrix} - \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}}{\begin{vmatrix} 2.716667 \\ -2.753095 \\ 7.003438 \end{vmatrix}} \times 100\% = \begin{vmatrix} 63.19 \\ 136.32 \\ 85.72 \end{vmatrix}$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$k=1$

$$x_1^2 = \frac{b_1 - \sum_{j=2}^3 a_{1j} x_j^1}{a_{11}} = \frac{b_1 - a_{12} x_2^1 - a_{13} x_3^1}{a_{11}} = \frac{7.85 + 0.1(-2.753095) + (0.2)(7.003438)}{3} = 2.991793$$

$$x_2^2 = \frac{b_2 - \sum_{j=1}^1 a_{2j} x_j^2 - \sum_{j=3}^3 a_{2j} x_j^1}{a_{22}} = \frac{b_2 - a_{21} x_1^2 - a_{23} x_3^1}{a_{22}} = \frac{-19.3 - (0.1)(2.991793) - (-0.3)(7.003438)}{7} = -2.499735$$

$$x_3^2 = \frac{b_3 - \sum_{j=1}^2 a_{3j} x_j^2 - \sum_{j=4}^3 a_{3j} x_j^1}{a_{33}} = \frac{b_3 - a_{31} x_1^2 - a_{32} x_2^2}{a_{33}} = \frac{71.4 - (0.3)(2.991793) - (-0.2)(-2.499735)}{10} = 7.000252$$

$$e_{R\%} = \frac{\begin{vmatrix} 2.991793 & 2.716667 \\ -2.499735 & -2.753095 \\ 7.000252 & 7.003438 \end{vmatrix} - \begin{vmatrix} 2.991793 & 2.991793 \\ -2.499735 & -2.499735 \\ 7.000252 & 7.000252 \end{vmatrix}}{\begin{vmatrix} 2.991793 & 2.716667 \\ -2.499735 & -2.753095 \\ 7.000252 & 7.003438 \end{vmatrix}} * 100 = \begin{vmatrix} 9.19 \\ 10.13 \\ 0.045 \end{vmatrix} \%$$

MÉTODOS INDIRECTOS O ITERATIVOS

GAUSS-SEIDEL

$k=2$

$$x_1^3 = \frac{b_1 - \sum_{j=2}^3 a_{1j} x_j^2}{a_{11}} = \frac{b_1 - a_{12} x_2^2 - a_{13} x_3^2}{a_{11}} = \frac{7.85 + 0.1(-2.499735) + (0.2)(7.000252)}{3} = 3.000026$$

$$x_2^3 = \frac{b_2 - \sum_{j=1}^1 a_{2j} x_j^3 - \sum_{j=3}^3 a_{2j} x_j^2}{a_{22}} = \frac{b_2 - a_{21} x_1^3 - a_{23} x_3^2}{a_{22}} = \frac{-19.3 - (0.1)(3.000026) - (-0.3)(7.000252)}{7} = -2.49999$$

$$x_3^3 = \frac{b_3 - \sum_{j=1}^2 a_{3j} x_j^3 - \sum_{j=4}^3 a_{3j} x_j^2}{a_{33}} = \frac{b_3 - a_{31} x_1^3 - a_{32} x_2^3}{a_{33}} = \frac{71.4 - (0.3)(3.000026) - (-0.2)(-2.49999)}{10} = 6.99999$$

$$e_{R\%} = \frac{\begin{vmatrix} 3.000026 \\ -2.49999 \\ 6.99999 \end{vmatrix} - \begin{vmatrix} 2.991793 \\ -2.499735 \\ 7.000252 \end{vmatrix}}{\begin{vmatrix} 3.000026 \\ -2.49999 \\ 6.99999 \end{vmatrix}} * 100 = \begin{vmatrix} \\ \\ \end{vmatrix} \begin{vmatrix} \\ \\ \end{vmatrix} \%$$

$$x^0 = \begin{bmatrix} b_1/a_{11} \\ b_2/a_{22} \\ b_3/a_{33} \\ \vdots \\ b_n/a_{nn} \end{bmatrix}$$