GAUSS-SEIDEL

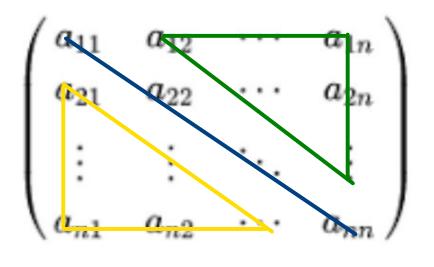
$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

$$A=egin{pmatrix} a_{11}&a_{12}&\cdots&a_{1n}\ a_{21}&a_{22}&\cdots&a_{2n}\ dots&dots&\ddots&dots\ a_{n1}&a_{n2}&\cdots&a_{nn} \end{pmatrix}, \qquad x=egin{pmatrix} x_1\ x_2\ dots\ x_n \end{pmatrix}, \qquad b=egin{pmatrix} b_1\ b_2\ dots\ b_n \end{pmatrix}.$$



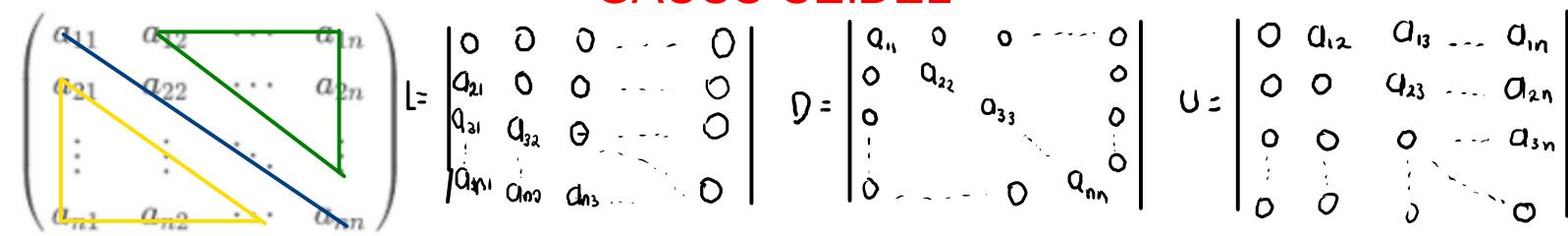
$$Ax=b$$

$$A=L+D+U$$

$$(L+D+U)x=b$$

$$Lx+Dx+Ux=b$$

$$L = \begin{cases} 0 & 0 & 0 & 0 \\ 0_{21} & 0 & 0 & 0 \\ 0_{31} & 0_{32} & 0 & 0 \\ 0_{401} & 0_{002} & 0_{03} & 0 \end{cases}$$



$$Ax=b$$
 $A=L+D+U$
 $(L+D+U)x=b$

$$Lx^{k+1} + Dx^{k+1} + Ux^{k} = b$$

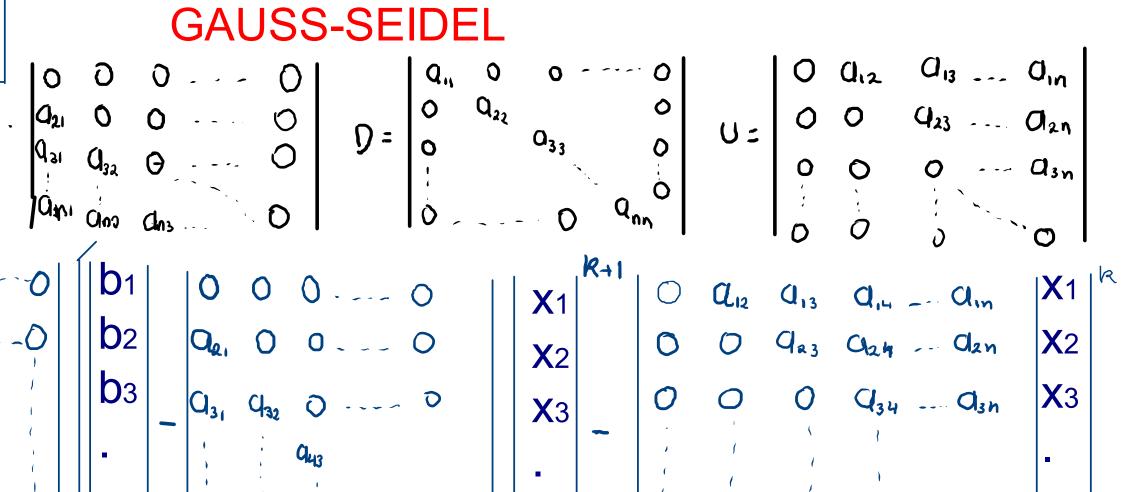
$$Dx^{k+1} = b - Lx^{k+1} - Ux^{k}$$

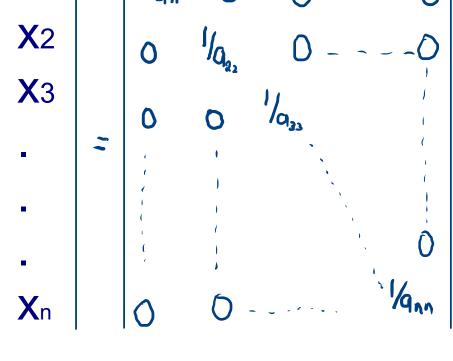
$$x^{k+i} = D^{-i}[b-Lx^{k+i}-Ux^{k}]$$

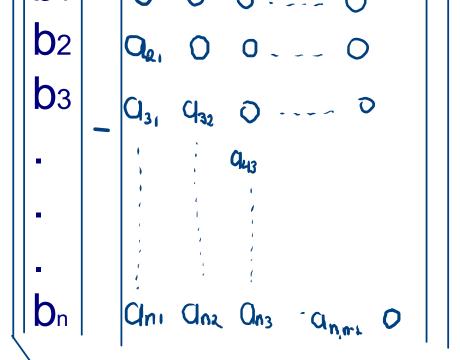
$x^{k+1} = D^{-1}[b-Lx^{k+1}-Ux^{k}]$

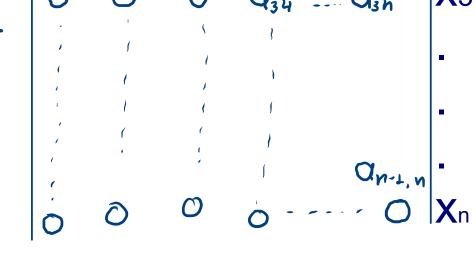
KtI

X1



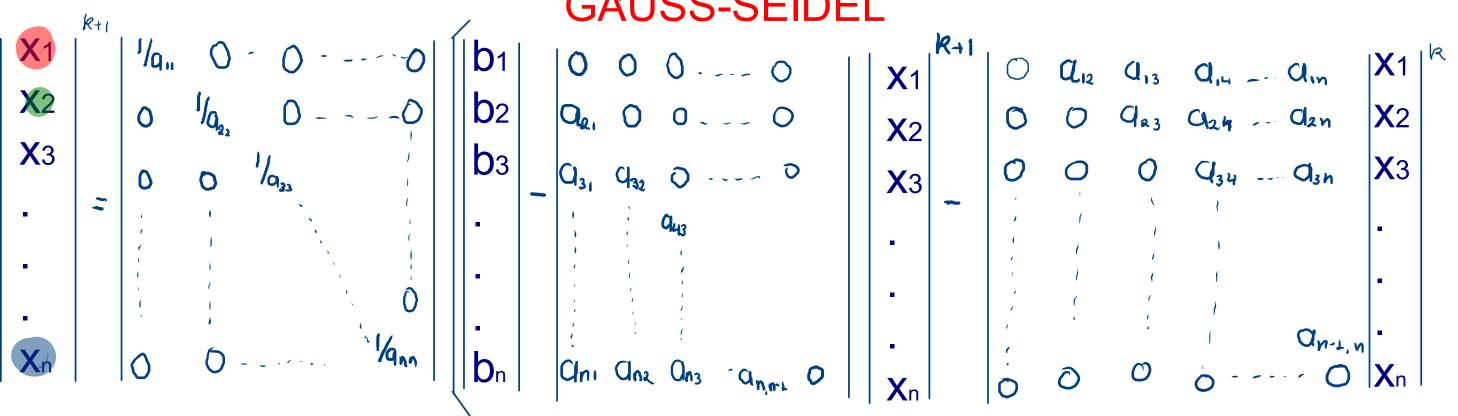






$$\begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ d_{21} & 0 & 0 & 0 & 0 \\ d_{31} & d_{32} & 0 & 0 & 0 \\ d_{31} & d_{32} & 0 & 0 & 0 \\ X_1 & X_2 & X_3 & 0 & X_4 \\ X_2 & X_3 & X_4 &$$

METODOS INDIRECTOS O ITERACT



$$X_{3}^{k+1} = b_{3} - a_{3} X_{1}^{k+1} - a_{3} X_{2}^{k+1} - a_{34} X_{4}^{k} - a_{34} X_{n}^{k}$$

$$x_{n}^{k+1} = \frac{b_{n} - \alpha_{n1} x_{1}^{k+1} - \alpha_{n2} x_{2}^{k+1} - \alpha_{n3} x_{3}^{k+1} - \cdots - \alpha_{nn-1} x_{n-1}^{k+1}}{C C_{nn}}$$

$$\chi_{i}^{k+1} = \frac{b_{i} - \sum_{j=1}^{i-1} a_{ij} \chi_{j}^{k+1} - \sum_{j=i+1}^{m} a_{ij} \chi_{j}^{k}}{a_{ii}}$$

$$Ax=b$$
 $A=L+D+U$
 $(L+D+U)x=b$

$$\Gamma x_{k+1} + D x_{k+1} + \Omega x_{k} = p$$

$$Dx^{k+1} = b - Lx^{k+1} - Ux^{k}$$

$$D_{T}DX_{KY}=D[p-\Gamma X_{K+1}-\Omega X_{K}]$$

$$X_{k+1} = D_{1}[p-\Gamma X_{k+1}-\Omega X_{k}]$$

$$\chi_{i}^{k+1} = b_{i} - \sum_{j=1}^{i-1} a_{ij} \chi_{j}^{k+1} - \sum_{j=i+1}^{n} a_{ij} \chi_{j}^{k}$$

$$a_{ii}$$

$$X_{k''} = D_{1}[p-\Gamma X_{k''}-\Omega X_{k'}]$$

$$\chi_{i}^{k+1} = b_{i} - \sum_{j=1}^{i-1} a_{ij} \chi_{j}^{k+1} - \sum_{j=i+1}^{m} a_{ij} \chi_{j}^{k}$$

$$a_{ii}$$

- 1. Valores iniciales [Xo]
- 2. Calcular [X] (k+1)
- 3. Criterio de paro número de iteraciones o un error máximo
- 4. $[Er]=abs([V(k+1)]-[V(k)]_{-}/[V(k+1)])$
- 5. Si no cumple regresa al paso 2

****** Para garantizar convergencia la matriz de coeficientes debe ser de diagonal estrictamete dominante ******

$$X_{k''} = D_{1}[p-rx_{k''}-nx_{k''}]$$

Ejemplo: Resolver el siguiente sistema de ecuaciones.

$$12x1+5x2-x3=15 \\ X1-6x2-4x3=9 \\ 2x1-3x2+8x3=5 \\ Con valores iniciales x1=1 , x2=1 , x3=1$$

$$\begin{bmatrix} 12 & 5 & -1 \\ 1 & -6 & -4 \\ 2 & -3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix}$$

$$X''' = D_1^{-1}[p-\Gamma x_{k+} - \Omega x_k]$$

$$X'' = D_{R}^{-1} \begin{bmatrix} b - Lx''' - Ux'' \end{bmatrix} \qquad \chi_{6} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad L = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 12 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 8 \end{bmatrix} \qquad U = \begin{bmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 5 & -1 \\ 1 & -6 & -4 \\ 2 & 3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix}$$

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} = \begin{vmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{vmatrix} \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{vmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} - \begin{vmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{vmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0$$

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}^{\frac{1}{2}} = \begin{vmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{vmatrix} = \begin{vmatrix} 15 \\ 9 \\ 5 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 5 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0.917 \\ -2.014 \\ 0.359 \end{vmatrix}$$

MÉTODOS INDIRECTOS O ITERACTIVOS

$$\begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix}^{\frac{1}{2}} = \begin{vmatrix} 1/12 & 0 & 0 \\ 0 & -1/6 & 0 \\ 0 & 0 & 1/8 \end{vmatrix} \begin{pmatrix} 15 \\ 9 \\ 5 \end{pmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -3 & 0 \end{vmatrix} \begin{pmatrix} 0 & 5 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{vmatrix} \begin{pmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{pmatrix} = \begin{pmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{pmatrix} = \begin{pmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{pmatrix}$$

$$\begin{vmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{vmatrix} = \begin{pmatrix} 0.916666 \\ -2.166666 \\ 0.625 \end{pmatrix}$$

Resuelva el siguiente sistema de ecuaciones lineales por medio del método de Gauss

Seidel, con una tolerancia de 1%

Seidel, con una tolerancia de 1%
$$3x_1 + -0.1x_2 - 0.2x_3 = 7.85$$

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$

$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

$$3x_1 - 0.2x_2 + 10x_3 = 71.4$$

K=0

$$\chi'_{1} = \frac{b_{1} - \sum_{j=2}^{3} a_{ij} \chi'_{j}}{a_{11}} = \frac{b_{1} - a_{12} \chi'_{2} - a_{i3} \chi'_{3}}{a_{11}} = \frac{7.85 + 0.1(1) + 0.2(1)}{3} = 2.716667$$

$$\chi_{2}^{1} = \frac{b_{2} - \sum_{j=1}^{L} Q_{2j} \chi_{j}^{1} - \sum_{j=3}^{3} Q_{2j} \chi_{j}^{0}}{Q_{22}} = \frac{b_{2} - Q_{2j} \chi_{j}^{0} - Q_{23} \chi_{3}^{0}}{Q_{22}} = \frac{-19.3 - (0.1)(2.716667) - (-0.3)(1)}{7} = -2.753095$$

$$\chi_{3}^{'} = \frac{b_{3} - \sum_{j=1}^{2} Q_{3j} \chi_{j}^{'} - \sum_{j=1}^{3} Q_{3j} \chi_{j}^{'}}{Q_{33}} = \frac{b_{3} - Q_{31} \chi_{1}^{'} - Q_{32} \chi_{2}^{'}}{Q_{33}} = \frac{71.4 - (0.3)(2.716667) - (-0.2)(-2.753695)}{10} = 7.003438$$

MÉTODOS INDIRECTOS O ITERACTIVOS

$$\chi'_{1} = \frac{b_{1} - \sum_{j=2}^{3} a_{1j} \chi'_{j}}{a_{11}} = \frac{b_{1} - a_{12} \chi'_{2} - a_{13} \chi'_{3}}{a_{11}} = \frac{7.85 + 0.1(1) + 0.2(1)}{3} = 2.716667$$

$$\chi_{2}^{1} = \frac{b_{2} - \sum_{j=1}^{L} Q_{2j} \chi_{j}^{1} - \sum_{j=3}^{3} Q_{2j} \chi_{j}^{0}}{Q_{22}} = \frac{b_{2} - Q_{2j} \chi_{j}^{1} - Q_{23} \chi_{3}^{0}}{Q_{22}} = \frac{-19.3 - (0.1)(2.716667) - (-0.3)(1)}{7} = -2.753095$$

$$\chi_{3}^{1} = \frac{b_{3} - \sum_{j=1}^{2} Q_{3j} \chi_{j}^{1} - \sum_{j=1}^{3} Q_{3j} \chi_{j}^{1}}{Q_{33}} = \frac{b_{3} - Q_{31} \chi_{1}^{1} - Q_{32} \chi_{2}^{1}}{Q_{33}} = \frac{71.4 - (0.3)(2.716667) - (-0.2)(-2.753695)}{10} = 7.003438$$

METODOS INDIRECTOS O ITERACTIVOS

$$\chi_{1}^{2} = \frac{b_{1} - \sum_{j=2}^{3} a_{1j} \chi_{j}^{2}}{a_{11}} = \frac{b_{1} - a_{12} \chi_{2}^{2} - a_{13} \chi_{3}^{2}}{a_{11}} = \frac{7.85 + 0.1(-2.753095) + (0.2)(7.003438)}{3} = 2.991793$$

$$\chi_{2}^{2} = \frac{b_{2} - \sum_{j=1}^{L} Q_{2j} \chi_{j}^{2} - \sum_{j=3}^{3} Q_{2j} \chi_{j}^{2}}{Q_{22}} = \frac{b_{2} - Q_{21} \chi_{1}^{2} - Q_{23} \chi_{3}^{2}}{Q_{22}} = \frac{-19.3 - (0.1)(2.991793) - (-0.3)(7.003438)}{7} = -2.4997435$$

$$\chi_{3}^{2} = \frac{b_{3} - \sum_{j=1}^{2} Q_{3j} \chi_{j}^{2} - \sum_{j=4}^{3} Q_{3j} \chi_{j}^{2}}{Q_{33}} = \frac{b_{3} - Q_{31} \chi_{1}^{2} - Q_{32} \chi_{2}^{2}}{Q_{33}} = \frac{71.4 - (0.3)(2.551733) - (-0.2)(-2.455735)}{10} = \frac{7.000252}{10}$$

MÉTODOS INDIRECTOS O ITERACTIVOS

$$\chi_{1}^{3} = \frac{b_{1} - \sum_{j=2}^{3} a_{1j} \chi_{j}^{2}}{a_{11}} = \frac{b_{1} - a_{12} \chi_{2}^{2} - a_{13} \chi_{3}^{2}}{a_{11}} = \frac{3.0000252}{3} = \frac{3.0000252}{3}$$

$$\chi_{2}^{3} = \frac{b_{2} - \sum_{j=1}^{L} Q_{2j} \chi_{j}^{3} - \sum_{j=3}^{3} Q_{2j} \chi_{j}^{2}}{Q_{22}} = \frac{b_{2} - Q_{21} \chi_{1}^{3} - Q_{23} \chi_{3}^{2}}{Q_{22}} = \frac{-19.3 - (0.1)(3.000026) - (-0.3)(7.0000252)}{7} = -2.49999$$

$$\chi_{3}^{2} = \frac{b_{3} - \sum_{j=1}^{2} Q_{3j} \chi_{j}^{3} - \sum_{j=1}^{3} Q_{3j} \chi_{j}^{2}}{Q_{33}} = \frac{b_{3} - Q_{31} \chi_{1}^{3} - Q_{32} \chi_{2}^{3}}{Q_{33}} = \frac{71.4 - (0.3)(3.000026) - (-0.2)(-2.49999)}{10} = 6.99999$$

