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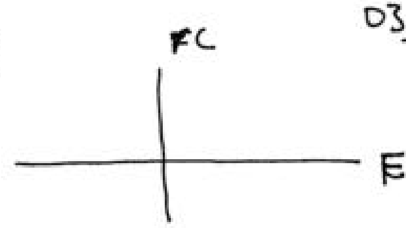
(Prob 1.1)

| | | $F^\circ$ | $C^\circ$ |
|---|---|---|---|
| ice melts | : | 32 | 0 |
| $H_2O$ Boils | : | 212 | 100 |

pts

(32, 0)

(212, 100)




(a) Assuming a linear relationship between the two

$$F - 32 = \frac{(212-32)}{100-0}(C-0) = \frac{9}{5}C$$

$$F = \frac{9}{5}C + 32 \qquad \text{Check} \quad F(100) = 212 \ \checkmark.$$

(b) $F(-273.15) = -459.67\ ^\circ F$.

(Prob 1.2)

$$R(F) = F - q \qquad R(-459.67) = -459.67 - q = 0$$

$$= q = +459.67$$

$$R(F) = F + 459.67 \qquad \underline{\underline{R = F + 459.67}}$$

$$R = \frac{9}{5}C + 32 + 459.67 \qquad C = k - 273.15$$

$$R = \frac{9}{5}(k - 273.15) + 32 + 459.67$$

$$= \frac{9}{5}k \qquad \underline{R = \frac{9}{5}k}$$

Room temp $\approx 20°C =$ ~~273K~~ $\approx 300°K. = 540°R$

Prob 1.3

(a) $\approx 100°F \Rightarrow \frac{5}{9}(100-32) \approx 35°C. \cong 305°K.$

(b) $100°C \Rightarrow 373.15\ K.$

(c) $50°F \Rightarrow 10°C \Rightarrow 293.15\ K$

(d) $-196°C \Rightarrow 77\ K$

F

(e) $327°C \Rightarrow \approx 600\ K$

Prob 1.4 Meaning would be twice as great of a temperature.

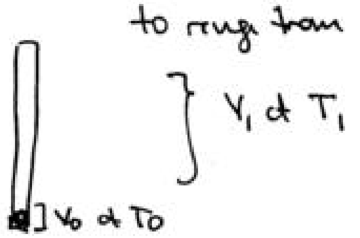
Prob 1.5 $\approx$ 2 min.

Prob 1.6 Physiological exp of ~~interta~~ intertwined hot & cold teps. See an into psycology book

Prob 1.7 (a) Size of bulb $\approx$ 3 mm $\approx$ maybe ½ cm would be a better estimate. $\Rightarrow V = \frac{4}{3}\pi(1.5\,mm)^3 = 14.13\ mm^3$

diameter tube $\approx$ 1mm. (estimate)

to range from $0°C$ to $100°C \Rightarrow \Delta T = 100°C = 100°K$



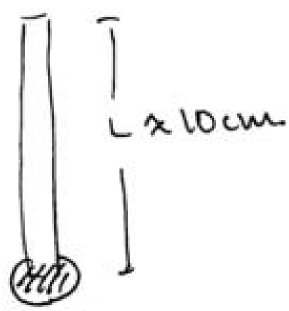
$V$ = size of initial bulb of Hg.

$$\beta = \frac{\Delta V / V}{\Delta T}$$

$$\Delta V = \frac{V\beta}{\Delta T} = \cancel{\frac{(14.13)\ mm^3 (1.81 \cdot 10^{-4}\ K^{-1})}{100\ K}}$$

$$= \beta V \Delta T = (1.81 \cdot 10^{-4}\ K^{-1})(14.13\ mm^3)(100\ K)$$

$$= 2.53 \cdot 10^{-1}\ mm^3$$



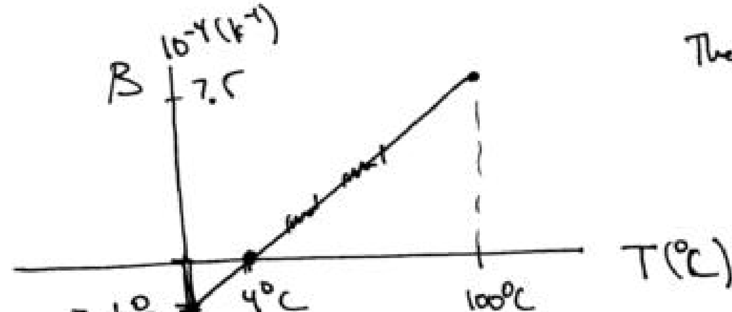

$$= L\pi r^2 \cong 31.4\ cm\ r^2$$

$$= 314\ mm\ r^2$$

$$r = 2.83 \cdot 10^{-2}\ mm$$ seems too small.

$\approx$ 3/100th of mm!!

(b)




Thermal expansion coeff of $H_2O$

$\beta > 0 \Rightarrow$

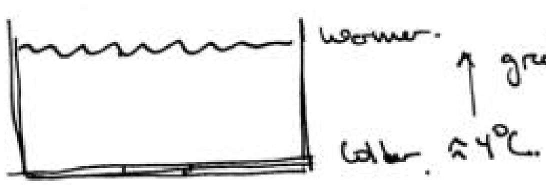
$$\beta \equiv \frac{\Delta V / V}{\Delta T}$$

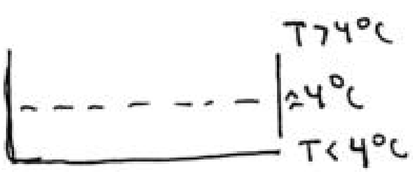
$\Delta V = \beta \Delta T V > 0$

if $\Delta T > 0$

or $\Delta V < 0$ if $\Delta T <$

$\beta < 0 \Rightarrow \Delta V > 0$ if



Think the answer is that if the thermal expansion was always positive ⇒ that lakes would freeze from the bottom up.

Wouldn't we eventually have mixing?

Prob 1.8 $\quad \alpha \equiv \dfrac{\Delta L/L}{\Delta T}$

(a) $L = 1\,km \qquad 100^\circ F = 310\,K$

$\Delta T = 16.8\,K \qquad 20^\circ C = 293.15\,K$

$\qquad\qquad\qquad\qquad 0^\circ C = 273.15\,K$

$$\Delta L_{Hot} = \alpha L \Delta T = (1.1\cdot 10^{-5}\,K^{-1})(1\,km)(16.8\,K) = 1.85\cdot 10^{-4}\,km$$

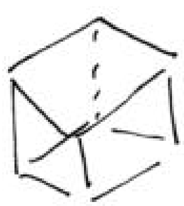
$$\Delta L_{cold} = \alpha L \Delta T = (1.1\cdot 10^{-5}\,K^{-1})(1\,km)(-20\,K) \approx -1.85\cdot 10^{-4}\,km$$

$$=$$

$$\Delta L_{total} = 2(.18\,m) = .36\,m$$

(b) Each metal has a different coefficient of thermal expansion. when sandwiched together as the temp changes the unit Bends

(c) $\quad \beta \equiv \dfrac{\Delta V/V}{\Delta T} \qquad V = L_x L_y L_z$



$\Delta V = \qquad \Delta V = \Delta L_x L_y L_z + L_x \Delta L_y L_z + L_x L_y \Delta L_z$

Thus $\dfrac{\Delta V}{V} = \dfrac{\Delta L_x}{L_x} + \dfrac{\Delta L_y}{L_y} + \dfrac{\Delta L_z}{L_z} = \Delta T \overset{\alpha_x}{\cancel{\beta_x}} + \Delta T \overset{\alpha_y}{\cancel{\beta_y}} + \Delta T \overset{\alpha_z}{\cancel{\beta_z}}$

$\Rightarrow \quad \alpha_x = \alpha_y = \alpha_z$

$$3\alpha = \frac{\Delta V / V}{\Delta T} \equiv \beta$$

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$$nR = \frac{N}{N_A} R = N\left(\frac{R}{N_A}\right) = Nk$$

$$N = nN_A \Rightarrow n = \frac{N}{N_A}$$

Problem 1.9 $n=1$, $T = 20°C = 300\,k$, $P = 1\,atm = 1.01 \cdot 10^5\,Pa$

$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{(1\,mol)(8.314\,J/mol \cdot k)(300\,k)}{1.01 \cdot 10^5\,Pa} = 2.46 \cdot 10^{-2}\,J/Pa$$

$$= 2.46 \cdot 10^{-2}\,\frac{N \cdot m}{N/m^2} = 2.46 \cdot 10^{-2}\,m^3$$

Prob 1.10 $PV = kNT$ $P = 1\,atm = 1.01 \cdot 10^5\,Pa$ $T = 300\,k$

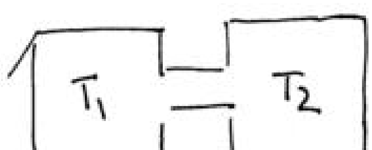
$k = 1.381 \cdot 10^{-23}\,J/k$

$V = (20\,m)^3 = 8000\,m^3$

$$N = \frac{(1.01 \cdot 10^5\,Pa)(8000\,m^3)}{(1.381 \cdot 10^{-23}\,J/k)(300\,k)}$$

$$= 1.95 \cdot 10^{29} \;!! = \cancel{300,000}\ 300,000 \text{ moles} \;!!$$

Prob 1.11




$T_2 > T_1$ Both of volume V.

$P_1V_1 = kN_1T_1$ since $P_1 = P_2$ + $V_1 = V_2$

$P_2V_2 = kN_2T_2$

$N_1T_1 = N_2T_2$ since $T_2 > T_1$ $N_2 < N_1$

Prob 1.12 $PV = NkT$ $P = 1\,atm = 1.01 \cdot 10^5$ Pa

$$P\left(\frac{V}{N}\right) = kT$$

$$\frac{V}{N} = \bar{v} = \frac{kT}{P} = \frac{(1.38 \cdot 10^{-23}\,J/_K)(300\,K)}{1.01 \cdot 10^5\,Pa} = 4.099 \cdot 10^{-26}\,m$$

$$\bar{\ell} = \sqrt[3]{\bar{v}} = 3.44 \cdot 10^{-9}\,m = 3.44\,nm$$

size of $N_2 \sim 10^{-10}$ so yes.

Prob 1.13 $N_A \hat{=} N_{protons/gram}$

$m_N \hat{=} M_p$

$m_e \ll m_N$

atomic mass $\approx$ # protons + neutrons.

mass in grams of one mole of a substance

To approx required atomic mass $\approx$ total # of protons + neutrons

$$H_2O = 2(2)\cancel{+}\ 1(8) = 12\ {}^{grams}/_{mole}.$$

$$2(1.) + 1(16) = 18\ {}^{gm}/_{mole}$$

$$N_2 = 2(14) = 28\ {}^{gm}/_{mole}$$

$$\text{lead} = Pb = 207\ {}^{gm}/_{mole}$$

$$SiO_2 = 28.08 + 2(16) = 60\ {}^{gm}/_{mole}$$

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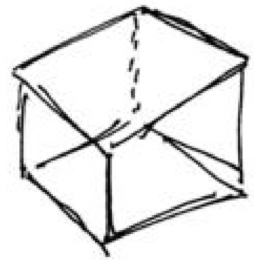
Prob 1.14 $P = 1\,atm$



$nR = Nk$

$M_{Air} = \cancel{\ldots}$

$= N_{O_2} m_{O_2} + N_{N_2} m_{N_2} + N_{Ag} m_{Ag}$ consider 1 mole of Air

$PV = 1RT$

$$V = \frac{RT}{P} = \frac{N_A kT}{P}$$ What are $N_{O_2}, N_{N_2}, N_{Ag}$ ?

$$N_A = N_{O_2} + N_{N_2} + N_{Ag}$$

Daltons law of partial pressure

$$P = P_1 + P_2 + P_3 = \frac{N_{N_2} kT}{V} + N$$

$$\| $$

$$\frac{N_A kT}{V}$$

Assuming that volume percentage is the same as molecule percentage:
How do I show this?

$N_{N_2} = .78(N_A) =$

$N_{O_2} = .21 N_A$

$N_{Ag} = .01 N_A$

$\rightarrow M_{Air} = (.78 m_{N_2} + .21 m_{O_2} + .01 m_{Ag}) N_A$

$\cancel{\overline{m}_{Air} = (.78(28) + .21(16) + .01(39.9))(6.02\cdot 10^{23})}$

$m_{\square}$ = mass per molecule = $\overline{m}_{\square}$

$$\overline{m} = \# \frac{\text{grams}}{\text{mole}} \cdot \frac{1 \text{ mole}}{N_A \text{ atoms}} = \frac{1}{N_A} \frac{\# \text{ grams}}{\text{atom.}} = \frac{1}{N_A} m_{\square}$$

Thus $$m_{Air} = \left(.78\frac{m_{N_2}}{N_A} + .21\frac{m_{O_2}}{N_A} + .01\frac{m_{Ag}}{N_A}\right)N_A$$

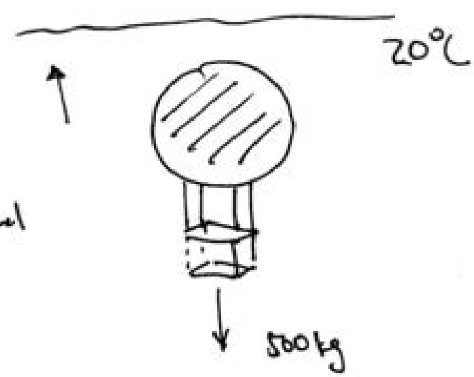
$$= (.78\, m_{N_2} + .21\, m_{O_2} + .01\, m_{Ag}) =$$

$$= .78(28) + .21(16) + .01(39.9) = 25.59 \text{ grams/mole.}$$

Prob 1.15

Boyancy force $= gV\rho_{Air \text{ displaced}}$

$\rho_{Air}$ = mass density of air around ballon. or normal air. [grams/$m^3$]




$PV = nRT$ $\qquad n = \frac{M}{m}$ $\frac{\text{total mass}}{\text{mass per unit mole}}$

$$= \frac{M}{m}RT$$

$$\rho^{-1} = \frac{V}{M} = \frac{RT}{mP} \qquad \rho^{-1}_{Air} = \frac{RT}{mP} \cong \frac{(8.314 \text{ J/mol}\cdot\text{k})(300°\text{k})}{(25.59\cdot 10^{-3} \text{ kg/mol})(1.01\cdot 10^{+5} \text{ Pa})}$$

$$= \qquad = 9.65\cdot 10^{-1} \frac{\text{J}}{\text{kg}\cdot\text{Pa}}$$

$$\frac{J}{kg\cdot Pa} = \frac{N\cdot m}{kg\cdot \frac{N}{m^2}} = \frac{m^3}{kg} \rightarrow$$

$$\Rightarrow \rho = 1.01 \ kg/m^3$$

Thus so that Boyancy force exactly balances weight

$$\cancel{g} V \rho_{Air} = (500 kg)\cancel{g}$$

$$V = \frac{500 \ kg}{1.01 \ kg/m^3} = 495 \ m^3$$ is the volumn of the air inside the ballon.

~~Notes~~ ~~[illegible]~~

I was trying to get an estimate of the volume of the ~~[illegible]~~ ballon & hoping to then get an estimate of the temperature.

Guessing the temp in the ballon is $30°C = 310 k$.

$$PV = nRT \Rightarrow n_{Air} = \frac{(1.01\cdot 10^{+5} Pa)(495 \ m^3)}{(8.314 \ J/mol\cdot k)(310 \ k)}$$

$$= \cancel{[illegible]} \ 19397 \ \text{moles}$$

$$\text{Mass} = n_{Air}(25.59 \ \text{grams}/\text{mol}) = \underline{496 \ kg} \ \text{Air.}$$

from prob 1.14

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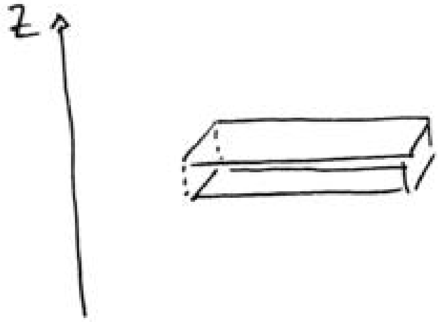
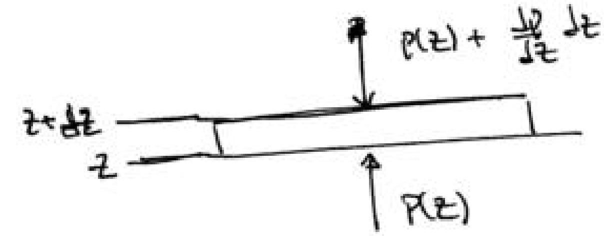
Prob 1.16






$\therefore$ ~~$p(z) + \frac{dp}{dz} dz - p(z) = \text{force}$~~

$$A\left(p(z) - p(z) - \frac{dp}{dz} dz\right) = \text{Force up} = \rho g A dz$$

(a) $$\frac{dp}{dz} = -\rho g \cancel{A}$$ $\rho$ = mass density.

$$PV = nRT = \frac{M}{m} RT \Rightarrow \frac{M}{V} = \frac{Pm}{RT} \Rightarrow \rho = \frac{Pm}{RT}$$

$$\Rightarrow \frac{dp}{dz} = -g\left(\frac{PM}{RT}\right) = -\frac{mg}{RT} P$$ same result obtained in RCZ.

m = mass per unit mole.

$$n = \frac{M}{m}$$

$$nR = k_B N$$

$$\frac{M}{m} R = kN \Rightarrow$$ ~~[scribbled out]~~

03-10-02 c

$$R = \frac{mkN}{M} = mk\left(\frac{N}{M}\right) = \frac{mk}{m_0}$$ $m_0$ = mass per molecule

— — —

~~PV = RNT~~ m = Avg mass of ~~air~~ air molecules. per mole.

— — — —

(c) Then $P(z) = P_0 e^{-\frac{mg}{RT}z}$ $PV = \frac{M}{m}RT$

$\therefore \rho = \frac{mP_0}{RT} e^{-\frac{mg}{RT}z}$. $\rho = \frac{M}{V} = \frac{mP}{RT}$

m = 25 g/mol

(d) $$P(z) = (1\,atm)\exp\left\{-\frac{(25\ g/mol)(9.8\ m/s^2)}{\underbrace{(8.314\ J/mol\cdot K)(300\ K)}}z\right\}$$

$$\text{coefficient} = \frac{(25\cdot 10^{-3})(9.8)}{8.314\cdot 300}\ \frac{kg\ m/s^2}{J} = \frac{kg\ m/s^2}{kg\cdot m^2/s^2} = \frac{1}{m}$$

$$= 2.9\cancel{4}\cdot 10^{-2}\ m^{-1} = 9.822\cdot 10^{-5}\ m^{-1}$$

$$\therefore P(z) = 1\,atm\ \exp\left\{-\frac{z}{33.9\,m}\right\}$$

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Prob 1.17

(a) $\frac{V}{n} = \frac{(8.314\ J/(mol\ K))\ T}{1.01 \cdot 10^5\ Pa}$

see ~~rama~~ file.

$1\,cm^3 = 1\,(10^{-2}m)^3$

$= 10^{-6}\,m$

The ~~???~~ 2nd virial coeff is

$$\frac{B(T)}{\left(\frac{R}{P}T\right)} = \cdots$$

All terms are smaller than 1/100.

(b) $B(T) < 0$ $T$ small $\Rightarrow$ $PV$ smaller than predicted by Ideal gas law.

$\Rightarrow$ ~~atom~~ atoms may attract?

~~(c)~~ $B(T) > 0$ $T$ large atoms may repel?

(c) $\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$

$$PV\left(1 + \frac{an^2}{PV^2}\right)\left(1 - \frac{nb}{V}\right) = nRT$$

$$PV = \frac{nRT}{\left(1+\frac{an^2}{PV^2}\right)\left(1 - \frac{nb}{V}\right)} = \frac{nRT}{\left(1 - \frac{a}{P\bar{V}^2}\right)\left(1 - \frac{b}{\bar{V}}\right)}$$

$$= \frac{nRT}{\left(1 - \frac{a}{P}x^2\right)(1 - bx)} = nRT\left(1 + \frac{b}{\bar{V}} + \left(b^2 + \frac{a}{P}\right)\frac{1}{\bar{V}^2} + \cdots\right)$$

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... incorrect given

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

$$P(V - nb) + \frac{an^2}{V^2}(V - nb) = nRT$$

$$PV\left(1 - \frac{nb}{V}\right) = nRT + -\frac{a}{\bar{V}^2}(V - nb)$$

$$PV = \frac{nRT - \frac{a}{\bar{V}^2}(\bar{V} - b)n}{\left(1 - \frac{nb}{V}\right)}$$

$$PV = nRT\,\frac{\left(1 - \frac{a}{\bar{V}^2}(\bar{V} - b)\frac{1}{RT}\right)}{\left(1 - \frac{b}{\bar{V}}\right)}$$

$$\therefore PV \cong nRT\left(1 - \frac{a}{RT}\left(\frac{1}{\bar{V}} - \frac{b}{\bar{V}^2}\right)\right)\left(1 + \frac{b}{\bar{V}} + O\left(\frac{1}{\bar{V}^2}\right)\right)$$

$$= nRT\left(1 - \frac{a}{RT}\frac{1}{\bar{V}} + \frac{a}{RT}\frac{b}{\bar{V}^2} + \frac{b}{\bar{V}} - \frac{ab}{RT}\left(\frac{1}{\bar{V}}\right)^2 + \frac{ab^2}{RT}\left(\frac{1}{\bar{V}}\right)^3 + \cdots\right)$$

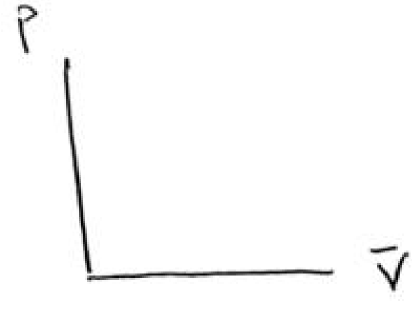
$$PV = nRT\left(1 - \left(\frac{a}{RT} - b\right)\frac{1}{\bar{V}} + \left(\frac{ab}{RT} - \frac{ab}{RT}\right)\frac{1}{\bar{V}^2} + O\left(\frac{1}{\bar{V}^3}\right)\right)$$

$$\therefore \quad B(T) = b - \frac{a}{RT} \qquad C(T) = 0$$

(d) w/

$$B(T) = b - \frac{a}{RT}$$

Calculate b + a from data given, then plot.



$$\left(P + \frac{a}{\bar{V}^2}\right)(\bar{V} - b) = RT$$

i.e. Solve for P given $\bar{V}$

$$\left(P(v) + \frac{a}{\bar{v}^2}\right)$$

$$\boxed{P(v) = \frac{RT}{\bar{v} - b} - \frac{a}{\bar{v}^2}}$$

From the data given in the text we get that

$$B(T) \hat{=} .0000642 - \frac{.02192}{T} \overset{set}{=} b - \frac{a}{RT}$$

$\Rightarrow b = 6.42 \cdot 10^{-5}$ $\quad a = .02192(8.314) = .18207$

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$$\bar{P} = \frac{\bar{F}_x}{A} = -\frac{\bar{F}_{x,m}}{A} = -\frac{m\frac{\overline{\Delta v_x}}{\Delta t}}{A}$$

$$\Delta t = \frac{2L}{v_x}$$

$$\Delta v_x = v_{x,final} - v_{x,initial} = -v_x - v_x = -2v_x$$

$$\bar{P} = \frac{-m\frac{(-2v_x)}{\frac{2L}{v_x}}}{A} = \frac{\frac{mv_x^2}{L}}{A} = \frac{mv_x^2}{LA} = \frac{mv_x^2}{V} \quad \text{eq 1.12}$$

$$\cancel{PV} = PV = mv_{1x}^2 + mv_{2x}^2 + \cdots$$

$$= Nm\overline{v_x^2}$$

$$\|$$

$$NkT$$

$$kT = mv_x^2$$

$$\overline{k_{trans}} = \frac{3}{2}kT \qquad [k] = J/K$$

$$T_{room} \cong 300°K$$

$$kT = 300°K\,(1.38\cdot 10^{-23}\,J/K) = 4.14\cdot 10^{-21}\,J$$

$$1eV = 1.6\cdot 10^{-19}\,J$$

Then

$$k_B = 1.38 \cdot 10^{-23}\, J/K = 1.38 \cdot 10^{-23} \left(1/1.6 \cdot 10^{+19}\right) eV/K$$

$$= .862 \cdot 10^{-4}\, eV/K$$

$$= 8.62 \cdot 10^{-5}\, eV/K$$

$$\therefore \quad k_B(T)\Big|_{room} = (8.62 \cdot 10^{-5}\, eV/K)(300\, K)$$

$$= 0.26 \cdot 10^{-1}\, eV \approx \frac{1}{40} eV$$

$$\overline{v^2} = \frac{3}{m} kT \qquad v_{rms} = \sqrt{\frac{3}{m} kT}$$

$$v_{rms} \gtrsim \bar{v}.$$

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Prob 1.8 $m_{N_2} = ?$

$1 \text{ mol} = 6.022 \cdot 10^{23}$

1 mole of N weighs 14.0067 g

$\therefore M_{N_2} = 2(14.00) \, g/mol$

$\Rightarrow m = 2(14.00) \frac{g}{mol} \cdot \frac{1}{6.022 \cdot 10^{23}} mol = 4.64 \cdot 10^{-23} \text{ grams}$

$V_{rms} = \sqrt{\frac{3kT}{m}} = \left(\frac{3(4.14 \cdot 10^{-21} J)}{4.64 \cdot 10^{-23} \text{ grams}}\right)^{1/2}$ $\quad J/kg = \frac{kg \cdot m^2/s^2}{kg}$

$= 516.8 \, m/s.$ $\quad \{1 m = 6.2 \cdot 10^{-4} \, mi\}$

$= 3.2 \cdot 10^{-1} \, mi/s \cdot \frac{3600 s}{hr}$

$= 1153. \, mi/hr.$

$1 \frac{m}{s} = 6.2 \cdot 10^{-4} \cdot 3600 \, mi/hr = 2.2 \, mi/hr.$

Prob 1.19

$H_2 \Rightarrow m = 2(1.007)\ g/mol \cdot \frac{1}{6.022 \cdot 10^{23}} =$

$O_2 \Rightarrow m = 2(16)\ g/mol \cdot \frac{1}{6.022 \cdot 10^{23}} =$

$$v_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{3k_BT}{m}}$$

$$\therefore v_{rms_{H_2}} = \sqrt{\frac{3k_BT}{m_{H_2}}}$$

$$v_{rms_{O_2}} = \sqrt{\frac{3k_BT}{m_{O_2}}}$$

intuitively, the smaller the mass the faster they should be travelling

$$\rightarrow \frac{v_{rms_{H_2}}}{v_{rms_{O_2}}} = \sqrt{\frac{m_{O_2}}{m_{H_2}}} = \sqrt{\frac{2(16)}{2(1.007)}}$$

$$\Rightarrow \frac{v_{rms_{H_2}}}{v_{rms_{O_2}}} = \sqrt{\frac{16}{1.007}} =$$

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Prob 1.20

$$m_{U^{238}F_6} = \frac{(6(19)+238)}{6.022 \cdot 10^{23}} \ g/\text{molecule} = 5.845 \cdot 10^{-22} \ g$$

$$m_{U^{235}F_6} = \frac{(6(19)+235)}{6.022 \cdot 10^{23}} \ g/\text{molecule} = 5.795 \cdot 10^{-22} \ g$$

$$V_{rms\,U^{238}F_6} = \sqrt{\frac{3k_BT}{m_{U^{238}F_6}}} = \sqrt{\frac{3(4.14 \cdot 10^{-21} \cdot 10^{3} \ g\,m^2/s^2)}{5.845 \cdot 10^{-22} \ g}}$$

$$= 145.7 \ m/s$$

$$V_{rms\,U^{235}F_6} = \sqrt{\frac{3(4.14 \cdot 10^{-21} \cdot 10^{3} \ g\,m^2/s^2)}{5.795 \cdot 10^{-22} \ g}} = 146.3 \ m/s$$

About 1 m/s difference in speed

Prob 1.21

$m = 2g \quad V = 15\, m/s$

$f = 30\ 1/s$


$\bar{P}_{ressure} = \dfrac{\bar{F}}{A} =$

$\bar{F} = \dfrac{\overline{\Delta p_x}}{\Delta t}$ w/ $\Delta p$ = change in momentum.
$\Delta t$ = interval of time over which this change happens

$= \dfrac{p_{x,final} - p_{x,initial}}{\Delta t} = \dfrac{(-V_0 \cos 45° - V_0 \cos 45°)m}{\Delta t}$

$= \dfrac{(-2V_0 \cos 45°)m}{\Delta t} = \dfrac{-2V_0}{\sqrt{2}} \dfrac{m}{\Delta t}$

Now $\Delta t$ = Avg time between collisions $= \dfrac{1}{f} = 30^{-1} s$

$\therefore \quad \bar{F} = -\dfrac{2(15\, m/s)}{\sqrt{2}} \dfrac{(2g)}{(30^{-1} s)} = 1272.7\ g\, m/s^2$

$= 1.27\ kg\, m/s^2 = 1.27\ N$

$\therefore\ \bar{P} = \dfrac{1.27}{.5}\ Pa = 2.54\ Pa$

~~[illegible] atm.~~ $1\,atm = 1.013 \cdot 10^5\ N/m^2 \Rightarrow \bar{P} = 2.5 \cdot 10^{-5}$ atms.

Seems very small.

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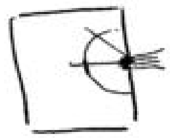
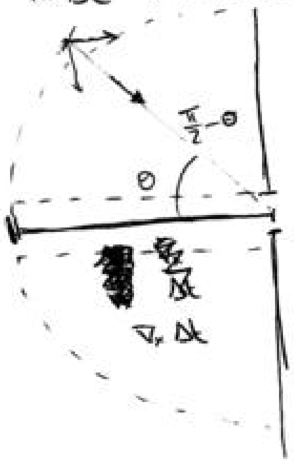
07-09-02

(Prob 1.22)



(a)

in $\Delta t$ # molecules colliding is coming from a distance $\left(\frac{\Delta t}{v_x}\right)^{-1}$ away or nearer




Assume all molecules in the ½ sphere centered at the effusion hole go through the hole

$$\frac{1}{2}\left(\frac{4}{3}\pi(\bar{v}_x \Delta t)^3\right) = V_0.$$

# of molecules in this vol. is

$PV = NkT$

- - - - - -

Assume the pressure at this point comes from only molecule inputs.

Then $\bar{P} = \frac{\bar{F}}{A} = \frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right) = \frac{1}{A}\frac{(+2\bar{v}_x)m}{\Delta t} \cdot N$

$$\Rightarrow N = \frac{AP\Delta t}{2\bar{v}_x m}$$

(b) This follows directly from 1.15 $\quad m\bar{v_x}^2 = kT$

$$\Rightarrow \bar{v_x^2} = \sqrt{\frac{kT}{m}}$$

(c) Now the difficulty is that $P = P(V) = P(N)$ & $dt$ for an infinitesimal time increment $\frac{AP\,dt}{2\bar{v}_x m} = \frac{AP}{2m}dt\sqrt{\frac{m}{kT}}$ of $N$ is lost put negative sign

$$\Rightarrow dN = -\frac{AP}{2m}\sqrt{\frac{m}{kT}}\,dt$$

$$\frac{dN}{dt} = \frac{-A\sqrt{m}}{2m}\left(\frac{kNT}{V}\right)\sqrt{kT}$$

$\left\{ P = \frac{kNT}{V} \right\}$ $\{PV = NkT\}$

$$\Rightarrow \quad \frac{dN}{dt} = \frac{-A}{2\sqrt{m}}\frac{\sqrt{kT}}{V} N$$ ✓. integrating th

$$\frac{dN}{N} = -\frac{A}{2V}\sqrt{\frac{kT}{m}}\, dt \equiv -\frac{1}{\tau}\, dt \quad \text{w/ } \tau \equiv \frac{2V}{A}\sqrt{\frac{m}{kT}}$$

$$\ln N = \frac{t}{\tau} + C$$

$$N = N_0 e^{-t/\tau}$$

$[\tau] = L$

$[kT] = J$

$\left[\frac{m}{kT}\right] = \frac{M}{ML^2/T^2} = \frac{T^2}{L^2}$

$\therefore [\tau] = L\frac{T}{L} = T$ ✓.

(d) $T = 300K$ room temp.
$V = 1$ liter $= 10^{-3}\, m^3$
$A = 1\, mm^2$

$$m_{Air} = .7 m_{N_2} + .3 m_{O_2}$$

$$m_{N_2} = 2(14)\, g/mol \cdot \frac{1}{6.022\cdot 10^{23}\ atoms/mol} = 4.649\cdot 10^{-23}\ g/atom$$

$$m_{O_2} = \frac{2(16)\ g/mol}{6.022\cdot 10^{23}\ atoms/mol} = 5.313\cdot 10^{-23}\ g/molecule$$

$$\therefore \quad m_{Air} \hat{=} (.7(4.649) + .3(5.313))\cdot 10^{-23}\ g/molecule$$
$$= 4.84\cdot 10^{-23}\ g/molecule$$

$$kT = (1.38\cdot10^{-23}\ J/K)(300K)$$
$$= 4.14\cdot10^{-21}\ J$$
$$\sqrt{\frac{m}{kT}} = \sqrt{\frac{4.84\cdot10^{-3}\cdot10^{-23}}{4.14\cdot10^{-21}}\ \frac{kg}{J}} = 3.42\cdot10^{-3}\ \left(\frac{s}{m}\right)$$

$$\frac{A}{V} = \frac{1(10^{-3})^2\ m^2}{10^{-3}\ m^3} = \frac{10^{-3}}{m}$$

$$\tau = 2\cdot10^3\,(3.42\cdot10^{-3})\ s$$
$$\therefore\ n(t) = N_0 e^{-t/6.84s}$$

(e) $e^{-1} = .367$
$e^{-2} = .135$
$e^{-3} = .05 \leftarrow$ this is about zero.

$t \doteq 1hr = 3600\ s. \qquad \tau =$

$$\Rightarrow\ -\frac{t}{\tau} \doteq -3$$
$$\Rightarrow\ t \doteq 3\tau = 3\cdot\frac{2V}{A}\sqrt{\frac{m}{kT}} = 3\cdot\frac{2V}{A}\left(3.42\cdot10^{-3}\ s/m\right)$$

$V \approx 1$ liter? How know.

$$\Rightarrow\ 3600\ s = 3\cdot\frac{2(10^{-3}\ m^3)}{A}(3.42\cdot10^{-3}\ s/m)$$

$\Rightarrow \quad A = 5.7 \cdot 10^{-9}\ m^2$

$= 5.7 \cdot 10^{-9}\,(10^{6})\ mm^2$

$= 5.7 \cdot 10^{-3}\ mm^2$ ... seems to small.

(d) know $v_{rms} \cong$ hundreds of meters / second.

It is doubtful that they could move the fork.

(Prob 1.23) $U_{thermal} = N \cdot f \cdot \frac{1}{2}kT$ $\qquad f_{diatomic} = 5.$

$$N = \frac{PV}{k_B T}$$

$$U_{thermal} = \left(\frac{PV}{\cancel{k_B T}}\right) f \left(\frac{1}{2}\cancel{k_B T}\right) = \frac{f}{2}PV.$$

$$U_{thermal} = \frac{5}{2}(1\,\text{atm})(1\,\text{liter}) = 2.5(10^5\,\text{Pa})(10^{-3}\,\text{m}^3)$$

$$= 2.5 \cdot 10^2\,\text{J}. = 250\,\text{J}$$

$1\,\text{atm} = 10^{+5}\,\text{Pa}.$

Calculation would be the same

(Prob 1.24) $U_{thermal} = N \cdot f \cdot \frac{1}{2}kT$ $\qquad f_{solids} = 3(2)$

one for potential & one for ~~thermal~~ ~~energy~~ kinetic

$N = 1\,\text{gram}$

$1\,\text{mole} = \cancel{208}\ 207.\ \text{g}$

$1\,\text{gram} = \frac{1}{207}\,\text{mole}$

$$\therefore N = \frac{6.022 \cdot 10^{23}}{207}\,\text{atoms} = 2.9 \cdot 10^{21}\ \text{atoms}$$

$$U_{th} = (2.9 \cdot 10^{21})(6)(.5)(1.38 \cdot 10^{-23}\,\text{J/K})(300\,\text{K}) = 36.1\,\text{J}$$

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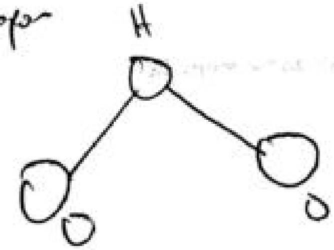
Prob 1.26 $H_2O$ vapor



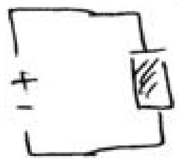

3 degress for translational.

3 degress for rotational.

2 degrees ∀ excellent bond

1 degree for ~~x~~ spring motion ~~[illegible]~~ in hing of molecule.

(Prob 1.26)



Classify it as work since it would not happen spontaneously.

& one is moving electrons along

from the resistor to the $H_2O$ is heat since this is happening spontaneously.

(Prob 1.27) Pumping a bicycle tire

melting an ice cube, or any process where

work is generated as a by product of the heat added.

(Prob 1.28) $T_i = 300°K$ $T_f = 100 + 273 = 373°K$.

600 watt microwave

$\frac{\Delta U}{\Delta t} = 600 J/s \rightarrow \Delta U = 600 J/s \; \Delta t$

Assuming the fluid ($H_2O$) does no volume work.

$U_{thermal} = Nf \frac{1}{2} kT$

$\Delta U = (Nf \frac{1}{2} k) \Delta T$ know $\Delta T = 73°K$

$(600\ J/s\ \Delta t) = (N f \frac{1}{2} k)\ \Delta T$

$f = 3.$

$$\Delta t = \frac{N f \frac{1}{2} k \Delta T}{600\ J/s}$$

$N =$

$M = 2(1) + 16 = 18\ g/mol$

1 cup $\doteq$ 50 grams. (more like 200 grams)

$$\Rightarrow N = \left(\frac{50\ gram}{18\ g/mol} 6.022 \cdot 10^{23}\right) = 1.67 \cdot 10^{24} \text{ molecule}$$

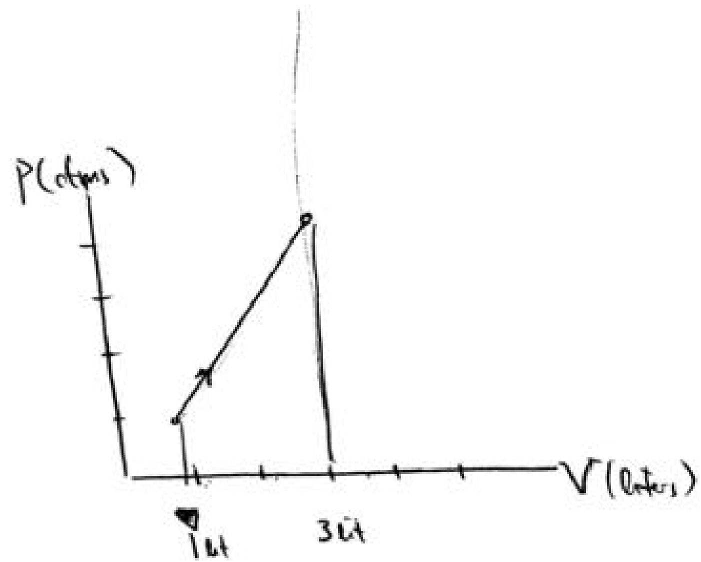
$\Delta t = 4.2\ s$ fast!!

Prob 1.29 Nothing. The increase in temperature indicates that energy has flown into the $H_2O$. But this energy could have come from 1) heat or 2) work.

Prob 1.30 ?

Prob 1.31 $P = \alpha V$

(a)




(b) $W_{\text{on gas}} = -\int_{V_i}^{V_f} P\,dV$

$= -\frac{\alpha}{2}(V_f^2 - V_i^2) = -\frac{\alpha}{2}(9-1)$ 

$= -4\alpha$

$T = \frac{PV}{kN}$

(d) $\Delta U = Q + W$

$\therefore$

$Q = \Delta U - W$

$= \frac{\alpha f}{2}(V_f^2 - V_i^2)$

$+ \frac{\alpha}{2}(V_f^2 - V_i^2)$

(c) $\Delta U = N f \frac{1}{2} k \Delta T$

$= N f \frac{1}{2} k \Delta\left(\frac{PV}{kN}\right)$

$= \frac{f}{2}\Delta(V \alpha V)$

$= \frac{\alpha f}{2}\Delta(V^2)$

$= \frac{\alpha f}{2}(V_f^2 - V_i^2) =$

$Q = \frac{\alpha}{2}(1+f)(V_f^2 - V_i^2)$

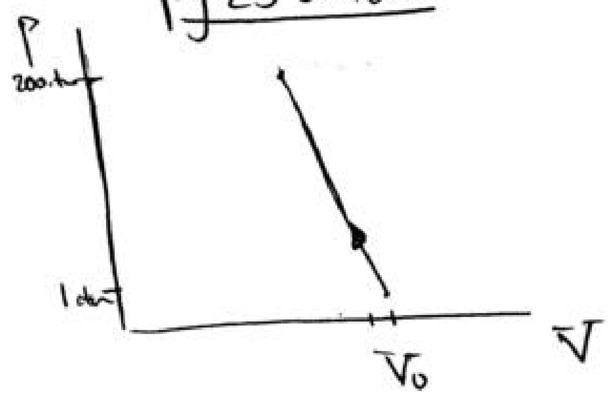
$= \frac{\alpha}{2}(4)(8) = 16\alpha$

helium $f = 3$

$= \alpha\frac{(3)}{2}(8) = 12\alpha$

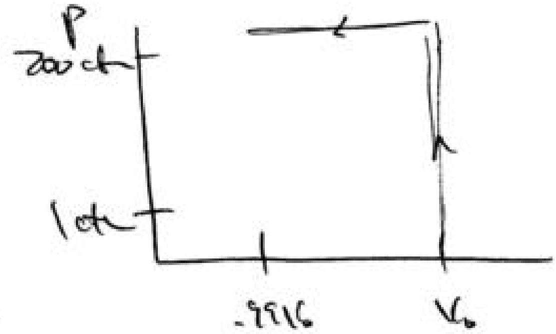
(e) Heat the container (or allow heat to flow).

(Prob 1.32)




$$W = -\int_{V_i}^{V_f} P\,dV$$

Assume constant pressure where the transition would look more like the following




$= -200\,\text{atm}(V_f - V_i)$

$= -200\,\text{atm}(.99V_i - V_i)$

$= 200(.01)\ \text{atm}\cdot\text{m}^3\,[V_i]$

$= 2\ \text{atm}\cdot\text{m}^3$ ?

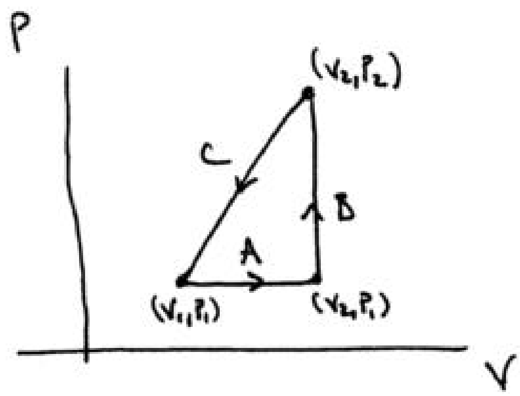
(Prob 1.33) (a) ~~A: (a) Negative work done on gas~~

~~(b) Don't think I can calculate this.~~

~~(c)~~

~~$U_{thermal} = N \cdot f\,\frac{1}{2}kT$~~

||




Prob 1.33

**A:**

(a) Work done on gas is negative

$$W = -\int_{V_1}^{V_2} P_1\,dV = -P_1(V_2-V_1) < 0$$

4 #'s $V_1, V_2$
$P_1, P_2$.

(b) $\Delta U = N \cdot f \frac{1}{2} k\,\Delta T$

$$T = \frac{PV}{Nk} \qquad \Delta T = \frac{1}{Nk}\Delta(PV)$$

$$\therefore \Delta U = \cancel{N}\cdot f\,\frac{1}{2} \times \frac{1}{\cancel{Nk}}\Delta(PV) = \frac{f}{2}\Delta(PV) = \frac{f}{2}P_1(V_2-V_1) > 0$$

(c) $\Delta U = W + Q$

$$\frac{fP_1}{2}(V_2-V_1) = -P_1(V_2-V_1) + Q \Rightarrow Q = P_1(V_2-V_1)\left\{\frac{f}{2}+1\right\} > 0$$

**B:**

(a) $W = 0$ since $dV = 0$

(b) $\Delta U = \frac{f}{2}\Delta(PV) = \frac{f}{2}V_2(P_2-P_1) > 0$

(c) $Q = \Delta U = \frac{f}{2}V_2(P_2-P_1) > 0$

**C:** Along path C $P(V) = P_1 + \alpha(V-V_1)$ w/ $\alpha = \frac{P_2-P_1}{V_2-V_1}$

(a) $\therefore W = -\int_{V_2}^{V_1} P(V)\,dV = -P_1(V_1-V_2) - \alpha\int_{V_2}^{V_1}(V-V_1)\,dV$

$$W = -P_1(V_1 - V_2) - \alpha \int_{V_2 - V_1}^{0} v\,dv$$

$$= -P_1(V_1 - V_2) - \frac{\alpha}{2}\left(0 - (V_2 - V_1)^2\right)$$

$$= -P_1(V_1 - V_2) + \frac{\alpha}{2}(V_2 - V_1)^2 = (V_2 - V_1)\left\{ +P_1 + \frac{1}{2}(P_2 - P_1) \right\}$$

$$= (V_2 - V_1)\left\{ \frac{P_2}{2} + \frac{P_1}{2} \right\} = \frac{1}{2}(V_2 - V_1)(P_1 + P_2) > 0$$

(b) $\Delta U = \frac{f}{2}\Delta(PV) = \frac{f}{2}(P_1V_1 - P_2V_2) = \frac{f}{2} \overset{?}{\gtrless} 0$

(c) The $Q = \Delta U - W$

$$= \frac{f}{2}(P_1V_1 - P_2V_2) - \frac{1}{2}(V_2 - V_1)(P_1 + P_2)$$

$$= \frac{f}{2}(P_1V_1 - P_2V_2) -$$

<u>Overall</u>:

(a) work $= \frac{1}{2}(V_2 - V_1)(P_2 - P_1)$

check: $\overset{?}{=} W_A + W_B + W_C$

$$= -P_1(V_2 - V_1) + 0 + \frac{1}{2}(V_2 - V_1)(P_1 + P_2)$$

$$= (V_2 - V_1)\left\{ -P_1 + \frac{P_1}{2} + \frac{P_2}{2} \right\} = (V_2 - V_1)\frac{1}{2}\left\{ P_2 - P_1 \right\}$$ As above ✓.

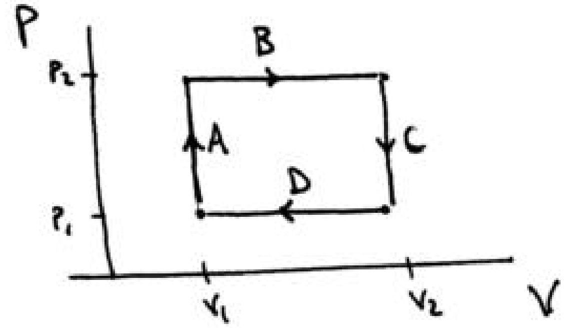
(b)

(b) since $\Delta U = 0$

$$Q = -W = -(V_2 - V_1)\frac{1}{2}(P_2 - P_1) < 0$$

(c) $\Delta U = 0$

The process is a refrigerator.




Prob 1.34

(a)

A: $W = 0$

$$\Delta U = \frac{f}{2}\Delta(PV)$$

$$= \frac{f}{2}V_1(P_2 - P_1) > 0$$

$$Q = \frac{f}{2}V_1(P_2 - P_1) > 0$$

B: $W = -P_2(V_2 - V_1) < 0$

$$\Delta U = \frac{f}{2}P_2(V_2 - V_1) > 0$$

$$Q = \Delta U - W = \left(\frac{f}{2} + 1\right)P_2(V_2 - V_1) > 0$$

$\cancel{\frac{f}{2} + 1 > 0}$

$\Rightarrow f > -2$ ✓.

C: $W = 0$

$$\Delta U = \frac{f}{2}V_2(P_1 - P_2) < 0$$

$$Q = \frac{f}{2}V_2(P_1 - P_2) < 0$$

D: $W = -P_1(V_1 - V_2) > 0$

$$\Delta U = \frac{f}{2}P_1(V_1 - V_2) < 0$$

$$Q = \frac{f}{2}P_1(V_1 - V_2) + P_1(V_1 - V_2) = \left(\frac{f}{2} + 1\right)P_1(V_1 - V_2) < 0$$

(b) A: Heat is added while the volume is kept fixed

B: Heat is still added this time allowing the volume to expand.

C: Heat is removed at constant volume.

D: I put work in while heat is still flowing out.

(c)
$$W = W_A + W_B + W_C + W_D$$
$$= 0 + -P_2(V_2-V_1) + 0 + -P_1(V_1-V_2)$$
$$= (V_2-V_1)(P_1-P_2) < 0$$
I get work out.

$$Q = \sum Q_i$$
$$= \frac{f}{2}V_1(P_2-P_1) + \left(\frac{f}{2}+1\right)P_2(V_2-V_1)$$
$$+ \frac{f}{2}V_2(P_1-P_2) + \left(\frac{f}{2}+1\right)P_1(V_1-V_2)$$

$$= \frac{f}{2}\left\{ V_1P_2 - V_1P_1 + P_2V_2 - P_2V_1 + V_2P_1 - V_2P_2 + P_1V_1 - P_1V_2 \right]$$

$$+ P_2(V_2-V_1) + P_1(V_1-V_2) = (V_2-V_1)(P_2-P_1) > 0$$

$$\cancel{= (V_2-V_1)(-P_1+P_2)} < 0$$

$$\Delta U = 0.$$

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Prob 1.35

$$VT^{f/2} = \text{const}$$

$$T = \frac{PV}{kN}$$

$$\Rightarrow V P^{f/2} V^{f/2} = \text{const}$$

$$P^{f/2} V^{\frac{2+f}{2}} = \text{con.}$$

$$P V^{\frac{2+f}{f}} = \text{const.}$$

$$\Rightarrow PV^{\gamma} = \text{const} \quad \text{w/} \quad \gamma = \frac{f+2}{f}$$

(Prob 1.36) $P_i = 1$ atm,

$P_f = 7$ atm

(a) Adiabatic curves follow $PV^\gamma = \text{const.}$ $\gamma = \frac{f+2}{f}$

For diatomic air $f = 5$ $\gamma = \frac{7}{5} = 1.4$.

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$V_f = V_i \left(\frac{P_i}{P_f}\right)^{1/\gamma}$$

$$V_f = (1 \text{ liter}) \left(\frac{1 \text{ atm}}{7 \text{ atm}}\right)^{5/7} = .249 \text{ liters.}$$

(b) $$W = -\int_{V_i}^{V_f} P\,dV \qquad P_{\text{Adiabatic}} = \frac{P_0}{V_0^\gamma} V^\gamma$$

$$= -\frac{P_0}{V_0^\gamma} \int_{V_i}^{V_f} V^\gamma dV = -\frac{P_0}{V_0^\gamma}\left(\frac{V_f^{\gamma+1}}{\gamma+1} - \frac{V_i^{\gamma+1}}{\gamma+1}\right)$$

$$= -\frac{1 \text{ atm}}{(1 \text{ liter})^\gamma} \frac{1}{(\gamma+1)} \left((.249 \text{ liter})^{\gamma+1} - (1 \text{ liter})^{\gamma+1}\right)$$

$$= \frac{1 \text{ atm}}{(2.4)} \left(1 - (.249)^{2.4}\right) \text{ liter.}$$

$$= .401 \text{ atm} \cdot \text{liter}$$

$1 \text{ atm} = 10^5$ Pa $\quad 1 \text{ liter} = 10^{-3} \text{ m}^3$

$\therefore \; W = .401 \cdot 10^{2}\,J$

$= 40.1\,J.$

(c) $T_f = ?$ know $P_f$ & $V_f$

$P_i V_i = N k T_i$

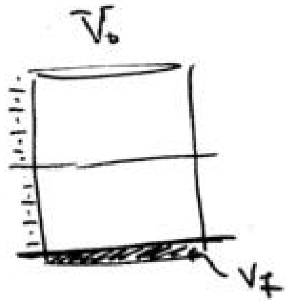
$P_f V_f = N k T_f$

$\Rightarrow$

$$\frac{P_i V_i}{P_f V_f} = \frac{T_i}{T_f} \qquad T_f = T_i \left(\frac{P_f V_f}{P_i V_i}\right)$$

$$\therefore \; T_f = 300\,k \left( \frac{7\,\text{atm} \cdot .249\ \text{liters}}{1\,\text{atm} \cdot 1\ \text{liter}} \right) = 522\,k.$$

Prob 1.37




Since compression is so fast process occurs adiabatically.

$T_i \cong 300K$ ; $V_i = V_0$ ; $P_i = 1atm$.

$T_f = ?$ $V_f = \frac{1}{20}V_0$

For an adiabatic process $VT^{f/2} = V_i T_i^{f/2}$

Since Air is mostly diatomic $\Rightarrow f = 3+2 = 5$

$$\frac{V_i}{20} T_f^{f/2} = V_i (300)^{f/2}$$

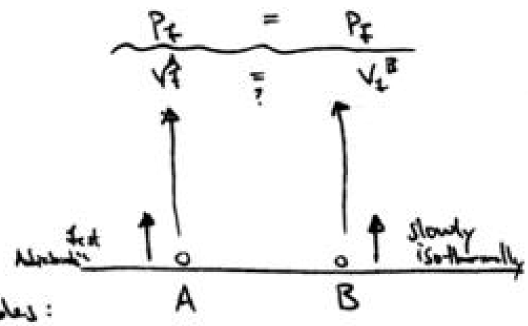
$$\Rightarrow T_f = \cancel{20} \quad 20^{2/f}(300) = 20^{2/5}(300) = 994\,K$$

this temperature must be close the flash point of ~~[illegible]~~ diesel ~~fuel~~.

Fuel.

Prob 1.38

Bubbles:




$V_i^A, T_i^A, P_i^A = V_i^B, T_i^B, P_i^B$ initially, all variables are equal.

Bubble A undergoes an adiabatic transformation

$\Rightarrow \quad P_f V_f^{\gamma} = P_i V_i^{\gamma}$ $\qquad V_{f\,\text{adiabatic}} = \left(\frac{P_i}{P_f}\right) V_i^{\gamma}$

Bubble B undergoes an isothermal transformation

$\Rightarrow \quad P_f V_f = NkT = P_i V_i$

Thus $\qquad V_{f\,\text{isothermal}} = \left(\frac{P_i}{P_f}\right) V_i$

Thus is $\quad V_{f\,\text{adiabatic}} \overset{?}{\gtrless} V_{f\,\text{isothermal}}$

$\Leftrightarrow \quad V_i^{\gamma} \gtrless V_i$

$\Leftrightarrow \quad V_i^{\gamma-1} \gtrless 1$

$\Leftrightarrow \quad \gamma - 1 \gtrless 0 \quad \Leftrightarrow \quad \gamma \gtrless 1$

Since $\gamma > 1$ $\qquad \gamma = 1.4$ for a diatomic gas

then $\quad V_{f\,\text{adiabatic}} > V_{f\,\text{isothermal}}$

Prob 1.39

$$C_s = \sqrt{\frac{B}{\rho}}$$

$$B \equiv \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)} = -V \frac{\Delta P}{\Delta V} \qquad P = \frac{kNT}{V}$$

(a) $$B_{isothermal} = -\frac{V}{\Delta V} \Delta\left(\frac{kNT}{V}\right)$$

$$= -\frac{V}{\Delta V} kNT \left(\frac{-1}{V^2}\right) \Delta V = \frac{V}{V^2} kNT$$

$$= \frac{kNT}{V} = P$$

$$B_{adiabatic} = \frac{-V}{\Delta V} \Delta(P)$$

$$P = V^{-\gamma} P_i V_i^{\gamma} \qquad \Delta P = P_i V_i^{\gamma} (-\gamma) V^{-\gamma-1} \Delta V$$

$$\therefore B_{Adiabatic} = -V P_i V_i^{\gamma} (-\gamma) V^{-\gamma-1}$$

$$= \gamma P_i V_i^{\gamma} V^{-\gamma} = \gamma P$$

(b) Adiabatic process occur very quickly & compression of gas in a sound wave is this type of process.

(c) $$C_s = \sqrt{\frac{\gamma P}{\rho}} \qquad [\rho] = \frac{m}{V}.$$

$PV = NkT$ $\qquad N = \frac{m}{M} \cdot N_A.$

$[m]$ = grams (mass of gas) $\quad M$ = Any molecular ~~weight~~ mass $\quad$ mass/mol

$[M]$ = grams/mole

$$\Rightarrow \quad PV = \frac{m}{M} N_A k T$$

$Nk = nR$

$$\boxed{N_A k = R}$$

$$\rho = \left(\frac{V}{m}\right)^{-1}$$

$$\Rightarrow \quad P = \frac{\rho R T}{M}$$

why do I remember $P = \tilde{\rho} R T$ from gas dynamics? $\tilde{\rho}$ there is molar density not mass density

$$\therefore \quad c_s = \sqrt{\frac{\gamma R T}{M}}$$

if $[\tilde{\rho}] = \frac{\text{moles}}{\text{vol.}}$

$$\left[\tilde{\rho} \frac{1}{M}\right] = \frac{\text{mole}}{\text{vol}} \cdot \frac{1}{\text{grams/mole}} = \frac{\text{mol}}{\text{vol}}.$$

$$[\tilde{\rho} M] = \frac{\text{mole}}{\text{vol}} \cdot \frac{\text{g mass}}{\text{mol}} = \rho_{\text{mass}} = \rho$$

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

$$M = m N_A$$

$$\tilde{\rho} = \frac{\rho}{M}$$

$\tilde{\rho} M = \rho$

$P = \tilde{\rho} R T$

$P = \frac{\rho}{M} R T$

So

$$c_s = \sqrt{\frac{\gamma R T}{m N_A}} = \sqrt{\frac{\gamma k T}{m}}$$

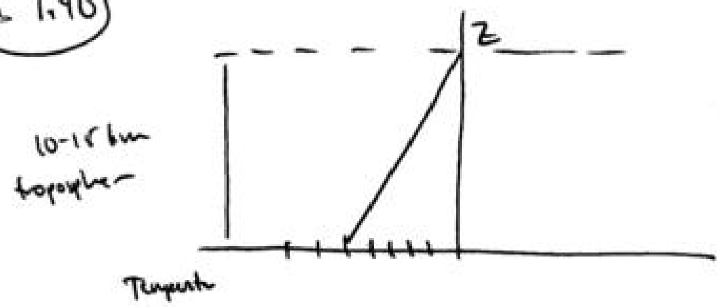
V.S. $\quad V_{rms} = \sqrt{\frac{\gamma k T}{m}}$

(c) $C_s^2 = \dfrac{1.4(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}{\cancel{.7(2)(14) + .3(2)(16)}\ (.7(2)(14) + .3(2)(16))\ \text{g/mol}}$

$$\Rightarrow C_s = \sqrt{\frac{3.49\cdot 10^3\ \text{J}}{29.10^{-3}\ \text{kg}}} = 345\ \text{m/s}.$$

(d) At A higher altitude the Avg temperature is lower & thus I would expect the sound speed to decrease pg Schroeder    07-11-02 r

Prob 1.40




(a) $Pv^{\gamma} =$ const.   &   $VT^{f/2} =$ const   $\gamma =$

$$\frac{T^{\frac{f+2}{2}}}{P} = \text{const.}$$

$$\frac{f+2}{2}\ln T - \ln P = \text{const.}$$

$$\frac{f+2}{2}\frac{1}{T}\,dT - \frac{1}{P}\,dP = 0$$

$$\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P}$$

(b) $$\frac{dP}{dz} = \frac{-mg}{kT}P$$

$$\frac{dT}{dz}\cdot\frac{dz}{dP} = \frac{2}{f+2}\frac{T}{P}$$   Assume $\frac{dT}{dz} =$ const.

$$\frac{dT}{dz}\cdot\left(\frac{-k\cancel{T}}{mg}\right)\frac{1}{\cancel{P}} = \frac{2}{f+2}\frac{\cancel{T}}{\cancel{P}}$$

$$\Rightarrow \frac{dT}{dz} = -\frac{mg}{k}\left(\frac{2}{f+2}\right)$$

$$m_{Air} = .7m_{N_2} + .3m_{O_2} = .7\,\frac{2(14)}{6.022\cdot10^{23}} + \frac{.3\;2(16)}{6.022\cdot10^{23}} = 4.84\cdot10^{-23}\ \text{g/molecule}$$

$$\frac{dT}{dz} = \cancel{(4.84 \cdot 10^{-26})(9.}$$

$$\frac{dT}{dz} = \cancel{-(4.84 \cdot 10^{-23}\ J/\text{molecule})}\left(\cancel{980\ g\,cm/s^2}\right)$$

$$\frac{dT}{dz} = -\frac{(4.84 \cdot 10^{-26}\ kg/\text{molecule})(9.8\ kg\,m/s^2)}{(1.38 \cdot 10^{-23}\ J/K)} \; \frac{2}{(1.4+2)}$$

$$= 2.0 \cdot 10^{-2}\ \frac{K}{m}$$

$$= 2.0 \cdot 10^{-2}\ \frac{K}{10^{-3}\ km} = 20\ \frac{K}{km}.$$

... mistake somewhere.

Prob 1.41

(a) After significant time metal is at 100°C.

$\Delta U_{\text{heat content of metal}} = C\Delta T = C_{metal}(T_f - T_i)$

$\Delta U_{metal} + \Delta U_{H_2O} = 0$

$C_p(H_2O) = 75.29\ J/K$ per mole of substance.

$C_{metal}(24-100) + \left(\frac{250g}{18g}\right)(75.29\ J/K)(24-20) = 0$

1 mole $H_2O$ = 18g.

∴ 75.29 J/K per 18 grams

⇒

(a) Heat gained by $H_2O$

$Q = 13.8(75.29\ J/K)(4) = 4.18 \cdot 10^3\ J. = 4183\ J.$

(b) Heat lost by metal = $-4183\ J$

(c) $C_{metal} = \frac{-4183\ J}{-76\ k} = 55.03\ J/K$

(d) $c = \frac{C_{metal}}{100g} = .55\ J/g \cdot k.$

Prob 1.42 $c = 1.8\ J/g\,C°$

don't know the final temp of $H_2O$ + pasta mix. Might be lower than 100°C

$Q_{\text{Absorbed by Pasta}} = cm\Delta T = 1.8\ J/g\,C°(340g)(\cancel{100}°C - 25°C)$

$= 45900\ J$

This amount of heat will be pulled from the $H_2O$

$$\underset{\parallel}{\Delta Q_{pasta}} + \Delta Q_{H_2O} = 0$$

$$c m \Delta T + c_{H_2O} m_{H_2O} \Delta T = 0$$

$c_{H_2O} = 75.29\ J/mol \cdot k$

$c_{H_2O} = \frac{75.29}{18}\ J/g \cdot k$

$$(1.8\ J/g°c)(340\,g)(T_f - 25°c)$$

$$+ \left(\frac{75.29}{18}\frac{J}{gk}\right)(10^3 \cdot 1.5\,g)(T_f - 100°c) = 0$$

1.5 liter =

1 liter $= 10^{-3}\ m^3$.

$1\,cm^3 = 1\,g$ for $H_2O$.

$1m = 100cm$

$\therefore$ 1 liter $= 10^{-3} \cdot (10^2)^3\ cm^3$

$= 10^{-3} \cdot 10^6\ g$

$= 10^3\ g$

$$\Rightarrow\ 612(T_f - 25) + 6.27 \cdot 10^3 (T_f - 100) = 0$$

$$6.88\ 10^3\ T_f = 6.42 \cdot 10^5$$

$$T_f = 93.27\ °C \quad \text{drops}.$$

Prob 1.43 Eq $U_{thermal} = f \cdot N \cdot \frac{1}{2} k_B T$

$$C = \frac{U_{thermal}}{\Delta T}$$

From 1.46 $C_V = \frac{\partial U}{\partial T}\Big|_V = \frac{N \cdot f k}{2}$

Then $\frac{C_V}{N} = \frac{f k}{2}$

we know for liquid $H_2O$ ~~$C_p(l\ H_2O)$~~ $C_p(H_2O:l) = \frac{75.29}{18}\ J/g\cdot$

$C_p(H_2O{:}l) = 75.29\ J/mol\cdot K$

$$\therefore \frac{C_p}{N} = \frac{75.29\ J/K}{6.022\cdot 10^{23}} = \frac{1}{2} f\,(1.38\cdot 10^{-23}\ J/K)$$

$$\Rightarrow f = 18.11 \quad !!$$

Prob 1.44 $\quad C_p = C_v + Nk = N\frac{f}{2}k + Nk$

$$= \left(\frac{f+2}{2}\right)N\cdot k.$$

For monatomic gasses $f=3$ ($\frac{5}{2}=2.5$), diatomic gasses $f=5$ ($\frac{7}{2}=3.5$), solids $f=6$ ($\frac{8}{2}=4$)

$N\cdot k = 6.022\cdot 10^{23}\cdot(1.38\cdot 10^{-23}) = 8.31$

$\therefore$

| $C_p$ | $C_p$ | |
|---|---|---|
| monatomic gasses | 20.77 | match well |
| diatomic gasses | 29.0 | match well |
| solids | 33.24 | don't seem to match well |

Prob 1.45 $\quad w = xy \qquad x = yz$

(a) $w(x,z) = \frac{x^2}{z} \qquad w(y,z) = y^2 z$

(b) $\left.\frac{\partial w}{\partial x}\right|_y = y \qquad \left.\frac{\partial w}{\partial x}\right|_z = \frac{2x}{z} = 2y$ Not equal !!

(c) $\left.\frac{\partial w}{\partial y}\right|_x = x$ $\qquad$ $\left.\frac{\partial w}{\partial y}\right|_z = 2yz = 2x$

$\left.\frac{\partial w}{\partial z}\right|_x = -\frac{x^2}{z^2} = -y^2$ $\qquad$ $\left.\frac{\partial w}{\partial z}\right|_y = y^2$

None!! of the derivatives are ~~both~~ ~~found~~. equivalent!!

Prob 1.46

(a) $\beta = \frac{1}{V_1}\left.\frac{\partial V}{\partial T}\right|_P$

$\beta = \frac{1}{V_1}\left.\frac{\partial V_1}{\partial T}\right|_P \Rightarrow \partial V_1 = V_1 \beta\, \partial T$

(b) $k_T = -\frac{1}{V_2}\left.\frac{\partial V}{\partial P}\right|_T$

$\partial V_2 = -V_2 k_T\, \partial P$

(c) expands due to temp increase & we compress as change

$\Rightarrow \partial V_1 = -\partial V_2$

$V_1 \beta\, \partial T = V_2 k_T\, \partial P$ $\qquad$ Not actually true that $V_1 = V_2$ but to a good approximation.

$$\left.\frac{\partial P}{\partial T}\right|_V = \frac{\beta}{k_T} = \frac{\frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P}{-\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_T} = -\frac{(\partial V/\partial T)_P}{(\partial V/\partial P)_T}$$

(d) $\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_p$ $V = \frac{Nk_BT}{p}$

$pV = Nk_BT$

$= \frac{1}{V}\frac{Nk_B}{p} = \frac{1}{V}\frac{V}{T} = \frac{1}{T}$ ideal gas

$k_T = -\frac{1}{V}\frac{\partial V}{\partial p}\Big|_T = -\frac{1}{V}\left(-\frac{Nk_BT}{p^2}\right) = \frac{Nk_BT}{p}\frac{1}{Vp} = \frac{1}{p}.$ ideal gas

$\cancel{\frac{\partial B}{\partial V}} \cdot \frac{\partial P}{\partial T}\Big|_V = \frac{\beta}{k_T} = \frac{1/T}{1/p} = \frac{P}{T} = \frac{Nk}{V}.$

Check $\frac{Nk}{V} \stackrel{?}{=} \left(\frac{-(Nk/p)}{-NkT/p^2}\right) = \frac{P}{T}$ ✓

(e) $\beta = 2.57 \cdot 10^{-4}\,K^{-1}$ $k_T = 1.81 \cdot 10^{-4}\,K^{-1}$

$\frac{\Delta P}{\Delta T}\Big|_V = +\frac{\beta}{k_T}$

$\Delta P = \frac{\beta}{k_T}\Delta T = \left(\frac{2.57 \cdot 10^{-4}\,K^{-1}}{1.81 \cdot 10^{-4}\,\cancel{K}}\right)(10\,K) = 14.19\,\cancel{K}\ Pa$

=

$\Delta P_{mercury} = \left(\frac{1.81 \cdot 10^{-4}\,K^{-1}}{4.04 \cdot 10^{-11}\,1/Pa}\right)(10\,K) = 4.48 \cdot 10^{7}\,Pa$

$= .045\ GPa.$

Prob 1.47

$$\Delta Q_{H_2O} + \Delta Q_{ICE} = 0$$

$$\parallel$$

$$cm\Delta T + c_{Ice} m_{Ice} \Delta T_{ice} + L m_{Ice} + c_{H_2O} m_{H_2O} \Delta T_{H_2O} = 0$$

to bring $H_2O$ at 100°C to 65°C. — to go down ice at -15°C to ice at 0°C. — to transform ice at 0°C to water at 0°C. — to transform

$$c_p(H_2O:l) = \frac{75.29 \text{ J/K mol}}{18 \text{ g/mol}} = 4.18 \text{ J/K·g}$$

$$m(H_2O) = 200 \text{ g}$$

$$\Delta T = 65 - 100 = -35 \text{ K}$$

$$c_{Ice}\ c_p(H_2O:s) = .5 \text{ cal/g°C}$$

$$= 2.093 \text{ J/gK}$$

~~1 cal = 4.18 J~~

1 cal = 4.186 J

$$m_{Ice} = ?$$

$$\Delta T_{ice} = (0 - (-15)) = 15 \text{ K}$$

$$L(\text{ice} \to H_2O) = 333 \text{ J/g}$$

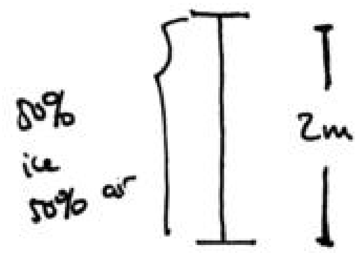
$$\Rightarrow (4.18)(200)(-35) + (2.093)m(15) + 333m + (4.18)m(65-0) = 0$$

$$-2.927\cdot 10^4 + 31.39\,m + 333\,m + 271.7\,m = 0$$

$$636.09\,m = 2.927\cdot 10^4$$

$$m = 46.02\,g$$

Prob 1.48




Sunlight ≈ $1000\ W/m^2$.

Assume if ½ is ice, then that 1 meter thick of ice is present that would need to be melted.

perimeter of ice

req enough heat to melt & turn it to $H_2O$

$$Q = L_{ice \to H_2O} \cdot m$$

$$= (333\ J/g)\,m$$

~~m(1m)~~

$$\rho_{ICE} = 0.917\ gm/cm^3$$
$$= .917\ g/(10^{-2})^3 m^3$$
$$= .917\cdot 10^6\ g/m^3$$
$$= 917\cdot 10^3\ g/m^3$$

$$\therefore\ m_{ICE} = (1m)(917\cdot 10^3\ g/m^3) = 917\cdot 10^3\ g/m^2.$$

Thus $W\Delta t = (333\ J/g)(917\cdot 10^3\ g/m^2)$

$$= 3.05\cdot 10^8\ J/m^2$$

$$\Delta t = 3.53\ \text{days}.$$

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$PV = 3100\,J$

$\Delta H = 40{,}660\,J$

$\frac{PV}{\Delta H} = .076 \approx 8\%$

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Prob 1.49

$1H_2 + \frac{1}{2}O_2 \rightarrow H_2O.$

$$U_{thermal} = \frac{5}{2}(6.022\cdot 10^{23})(1.38\cdot 10^{-23})(300) \quad \left\{ U_{thermal} = \frac{f}{2}NkT \right.$$

$$U_{thermal} = \frac{5}{2}(6.022\cdot 10^{23}\ \text{atoms/mole})(1\,\text{mole})(1.38\cdot 10^{-23}\ \text{J/K})(300\,\text{K}) \quad = \frac{f}{2}nRT$$

$$+ \frac{5}{2}(6.022$$

$$= \frac{f}{2}1RT + \frac{f}{2}\frac{1}{2}RT = \frac{f}{2}\cdot\frac{3}{2}\cdot RT.$$

$$= \frac{5}{2}\cdot\frac{3}{2}\cdot(8.314\ \text{J/mol}\cdot\text{K})(300\,\text{K}) = 9.35\cdot 10^3\ \text{J}$$

The PV work done by the atmosphere upon contraction, would be:

$$W = -P\Delta V = 1\,\text{atm}\,V_0 = n1RT = (8.314\ \text{J/mol}\cdot\text{K})(300\,\text{K})$$

$$+ \frac{1}{2}(8.314\ \text{J/mol}\cdot\text{K})(300\,\text{K})$$

$$= \frac{3}{2}(2.494\cdot 10^3) = 3.741\cdot 10^3\ \text{J}.$$

Then

$$\Delta U = \underset{\text{lost}}{-\Delta U_{thermal}} + \underset{\text{work done by atmosphere}}{W}$$

$$= -9.35\cdot 10^3\ \text{J} + 3.741\cdot 10^3\ \text{J} = -5.6\cdot 10^3\ \text{J}\ ?$$

What about thermal energy stored in $H_2O$?

$U_{thermal}$

for liquids $f \approx 3$ ...

$$U_{thermal\ liquid} = \frac{3}{2} NkT$$

$$= \frac{3}{2}(6.02 \cdot 10^{23}\ \text{molecules/mole})(1.38 \cdot 10^{-23}\ \text{J/K})(300\,\text{K})$$

$$= 3.73 \cdot 10^3\ \text{J}.$$

$\therefore$ $U_{Before} = \underset{Gas}{U_{thermal}} + \underset{by\ atmosphere}{\text{work done}}$ $\doteq$

$U_{After} = \underset{liquid}{U_{thermal}} + \underset{\parallel}{Q_{reaction}}$

$U_{Before} = U_{After} + Q$

$$Q_{reaction} = \underset{gas}{U_{thermal}} + \text{work by atmosphere} - \underset{liquid}{U_{thermal}}$$

$$= (9.35 + 3.741 - 3.73)\ \text{kJ}$$

$$= -3.7\ \text{kJ}.$$

? Not correct.

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Prob 1.50

(a) Reactant: $CH_4$ $\longrightarrow$ product: $C + 2H_2$

$\Delta_f H(C) + 2\cdot\Delta_f H(H_2) - \Delta_f H(CH_4)$

$= 0 + 2\cdot 0 - (-74.81)$ kJ/mole $= 74.81$ kJ/mole

(b) reactant: $C + 2H_2 + 2O_2$ $\rightarrow$ product: $CO_2 + 2H_2O$

$\Delta_f H(CO_2) + 2\Delta_f H(H_2O) - \Delta_f H(C) - 2\Delta_f H(H_2) - 2\Delta_f H(O_2)$

$-393.51 + 2(-241.82) - 0 - 2\cdot 0 - 2\cdot 0$

$= -877.15$ kJ/reaction

(c) $CH_4 + 2O_2 \xrightarrow{(1)} C + 2H_2 + 2O_2 \xrightarrow{(2)} CO_2 + 2H_2O$

$\Delta H$(2st reaction) $-$ $\Delta H$(1nd reaction)

$= (-877.15 - 74.81)$ kJ/reaction $= -951.9$ kJ/reaction

Check $\Delta H_f$(reac products) $- \Delta H_f$(products)

$= \Delta H_f(CO_2) + 2\Delta H_f(H_2O) - \Delta H_f(CH_4) - 2\Delta H_f(O_2)$

$$= -393.51 + -2(241.82)$$
$$-(-74.81) - 2(0) = -802 \text{ kJ}.$$ Not !! the same??

(d) $\Delta H = \cancel{\Delta U}\ Q + W_{other}$ (constant P) eq 1.57.

$\therefore Q_{\substack{given \\ off}} = -802.$ kJ.

(e) $\Delta H = \Delta U + P\Delta V$
$\quad \|$
$$-802 \text{ kJ} = \Delta U + (1\text{atm})(V_f - V_0) \qquad V = \frac{nRT}{P}$$
$$= \Delta U + \underbrace{(1\text{atm})(3\text{mole} - 3\text{moles})}_{=0.}$$

This would be different in the following way (assuming vol. of liquid is insignificant)

$$-802 \text{ kJ} = \Delta U + (1\text{atm})\left(\frac{(1\text{mole})(R)T}{P} - \frac{(3\text{ mole})RT}{P}\right)$$

$\Rightarrow \Delta U = \ldots$

(f) Assuming the sun was made of entirly methan $CH_4$.

$m = \qquad M(\text{molar } \cancel{\phantom{xx}} \text{mass}) = (12+4)\ \text{g/mole}.$

$$n = \frac{2 \cdot 10^{30}\text{ kg}}{(16 \cdot 10^{-3}\text{ kg/mole})} = 1.25 \cdot 10^{32}$$

Each mole of methane $CH_4$ burned produces $-841$ (say) kJ of heat.

$\therefore$ total energy produced

$$(1.25 \cdot 10^{32})(841\ \text{kJ}) = (3.9 \cdot 10^{26}\ \text{W})\,\Delta t$$

$$\Rightarrow \Delta t = 2.69 \cdot 10^{11}\ \text{s} = 8.54 \cdot 10^{3}\ \text{yr}.$$

Prob 1.51

$$\Delta H = 6(-393.51) + 6(-241.82)$$
$$-1(-1273) - 0$$
$$= 2.5 \cdot 10^3 \text{ kJ}$$

Prob 1.52 $\Delta H_{comb}$ = 31,000 kcal per gallon gasoline

$\Delta H_{comb}$ = 100 kcal per 28g of corn flakes

1 gallon of gas ≅ \$2 (US)

1 Box of corn flakes 14 oz box cost ≈ \$2.52

∴ ~~$\Delta H$~~ cost $\left(\frac{2}{31 \cdot 10^6 \text{ cal}}\right) = 6.45 \cdot 10^{-8}$ \$/cal

So for gas

for cornflakes $\dfrac{2.52}{100 \cdot 10^3 \left(\frac{14}{28}\right)} = 5.04 \cdot 10^{-5}$ \$/cal

So per ~~cal~~ calory gas is cheaper

Prob 1.53 $\Delta H_f = 217.97$ kJ/mole $\frac{1}{2}H_2 \longrightarrow H$

~~#~~ of molecules is $\frac{1}{2}(6.022 \cdot 10^{23})$

in this reaction

$$\therefore \quad \Delta H_f = \frac{217.97 \text{ kJ/~~mol~~ reaction}}{\frac{1}{2}(6.022 \cdot 10^{23}) \text{ molecules/reaction}}$$

$$= 7.23 \cdot 10^{-19} \text{ kJ/molec } H_2$$

$1 J = ?\ eV$

$1 eV = 1.602 \cdot 10^{-19} J \qquad 1 J = 6.2 \cdot 10^{18} eV$

$\therefore \quad \Delta H_f = 4513 \text{ eV/molecule } H_2.$

Prob 1.54 $\quad 1 kg = 2.2 \text{ lbs.}$

(a) $E_{\text{at top (potential)}} = mgy.$

$= (60 kg)(9.8 m/s^2)(1500 m) = 8.82 \cdot 10^5 J$

$B \cdot (100 \text{ kcal}) \cdot \text{fact.}$

$B = 8.42$ bowls.

1 kcal =

1 cal = 4.186 J.

(b) $Q = 2.6 \cdot 10^6 J$. of energy would have to be dissipated.

$Q = C_v \Delta T \qquad$ What is $C_v$ for a human?

(c) If $Q = 2.6 \cdot 10^6$ J $= m\ell_{H_2O(l) \to H_2O(g)}$

$\ell = 580$ cal/g ...

Prob 1.55

(a)

Potential Energy $= -\dfrac{Gm^2}{r^2}$

total KE $= \frac{1}{2}m(\omega \frac{r}{2})^2 + \frac{1}{2}m(\omega \frac{r}{2})^2 = \dfrac{m\omega^2 r^2}{4}$.

Are $\omega$ + $r$ independent? No. Force to keep mass in rotation must be provided by gravitation

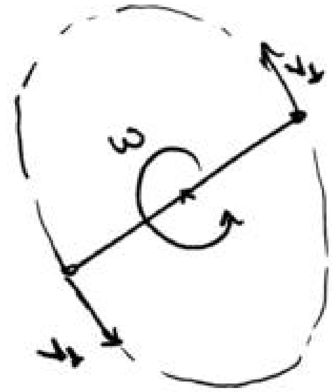
$$F = +\frac{Gm^2}{r} = \text{centripetal acceleration} = m\frac{v^2}{r} = \frac{m(\omega r/2)^2}{(r/2)}$$

$$= m\omega^2 \frac{r}{2}$$

$$\therefore \frac{m\omega^2}{2} = +\frac{Gm^2}{r^2} = \text{Potential Energy}$$

$$\frac{\omega^2}{2} = \frac{Gm}{r^2}$$

Prob 1.55




let $l$ = distance between two masses.

$$U_{potential} = -\frac{Gm^2}{l} \quad + \quad KE_{total} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2$$

$$= mv_1^2 = m(\omega^2 l/2)^2$$

$$= \frac{m\omega^2 l^2}{4}$$

Now a force Balance on one mass due to the other would give:

$$F_{gravitational} = \frac{Gm^2}{l^2} = \text{"} m\frac{v_1^2}{r} \text{"} = \text{centrifugal force}$$

$$= \frac{m(\omega\, l/2)^2}{l/2} = m\omega^2 \frac{l}{2}$$

Thus $\dfrac{Gm^2}{l} = \dfrac{m\omega^2 l^2}{2}$

$$\parallel$$

$$-U_{potential} = 2KE(total).$$

$$\Rightarrow U_{potential} = -2U_{kinetic}.$$

(b) If one adds simply energy into a system one would expect at the equilibrate the distribution at

$$\frac{U_{potential}}{U_{kinetic}} = -2.$$

$U_{potential} + 2U_k = 0$

$U^0_{total} = U^0_{potential} + U^0_{kin}$ and $U_p = -2U_k$

$U_{total} + \Delta U = -2U_{kin} + U_k = -U_{kin}.$

Not sure...

(c) $U_{total} = U_{potential} + U_{kin} \underset{\uparrow}{=} -U_{kin} =$

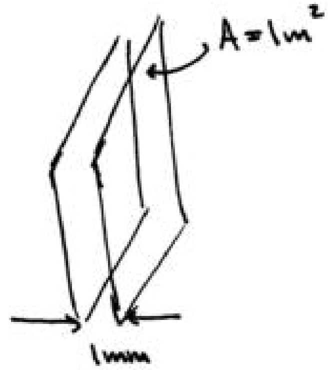
by virial thm

?

(d) ?

(e) ?

Prob 1.56




$$\frac{Q}{\Delta t} \propto \frac{A \Delta T}{\Delta x} \qquad Q \propto \frac{A\Delta T}{\Delta x} \cdot \Delta t$$

$$\frac{Q}{\Delta t} = -k_t \frac{A \Delta T}{\Delta x}$$

$$\therefore \frac{Q}{\Delta t} = \frac{-(.026 \, W/mK)(1m^2)(20K)}{(1mm)}$$

$$= \frac{-(.026 \, W/mK)(1m^2)(20K)}{10^{-3} m} = 5.2 \cdot 10^2 \, W.$$

Prob 1.57 $R = \frac{\Delta x}{k_t}$

(a) $R_{glass} = \frac{(3.2 \cdot 10^{-3} m)}{.8\ W/mk} = 4 \cdot 10^{-3}\ \frac{m^2 k}{W}$

$R_{Air} = \frac{1 \cdot 10^{-3} m}{.026\ W/mk} = 3.84 \cdot 10^{-2}\ \frac{m^2 k}{W}.$

(b) $\frac{°F \cdot ft^2 \cdot hr}{Btu} =$

$T_C = \frac{5}{9}(T_F - 32)$

$1°F = \frac{9}{5}°C = \frac{9}{5}k$

$\therefore$

$1\ (Btu) = (4.53 \cdot 10^{-1} kg)\, c_{H_2O} \left(\frac{9}{5}\right)$

$1 lb = 4.53 \cdot 10^{-1} kg$

$c_{H_2O} = 75.29\ \frac{J}{k} \left(\frac{1 mole}{18 \cdot 10^{-3} kg}\right) \frac{1}{mole} = 4.18 \cdot 10^{6}\ J/kg$

$\therefore\ 1 Btu = 3.41 \cdot 10^{6} J$

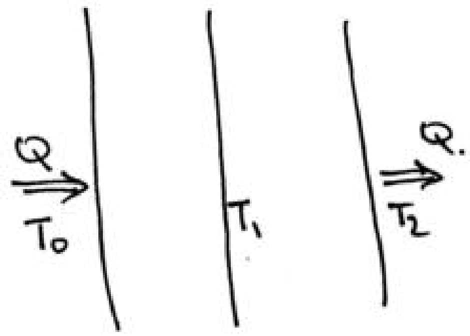
$\therefore\ 1\ \frac{°F \cdot ft^2\, hr}{Btu} = \frac{(\frac{9}{5}k)(.3 m)^2 (3600 s)}{3.41 \cdot 10^{6} J} = 1.7 \cdot 10^{-4}\ \frac{m^2 k}{W}.$

$\therefore\ 4 \cdot 10^{-3}\ \frac{m^2 k}{W} = 23.5\ \frac{F \cdot ft^2\, hr}{Btu}$

$+\ 3.84 \cdot 10^{-2}\ \frac{m^2 k}{W} = 2.25 \cdot 10^{2}\ \frac{F \cdot ft^2\, hr}{Btu}.$

Prob 1.57

(c)




$$Q \propto \frac{\Delta t A \Delta T}{\Delta x} \Rightarrow \frac{Q}{\Delta t} = -k \frac{A \Delta T}{\Delta x}.$$

For entire object:

$$\frac{Q}{\Delta t} = -k_{total} \frac{A(T_2 - T_0)}{(\Delta x_1 + \Delta x_2)}$$

$$\therefore \frac{Q}{A\Delta t} = \frac{-k_{total}(T_2 - T_0)}{\Delta x_1 + \Delta x_2}$$

For object #1 material #1

$$\frac{Q}{\Delta t} = -k_1 \frac{A(T_1 - T_0)}{\Delta x_1}$$

For object #2 material #2

$$\frac{Q}{\Delta t} = -k_2 \frac{A(T_2 - T_1)}{\Delta x_2}$$

$$\frac{Q}{A\Delta t} = \frac{-k_1(T_1 - T_0)}{\Delta x_1} \; ; \; \frac{Q}{A\Delta t} = -k_2 \frac{(T_2 - T_1)}{\Delta x_2}$$

$$\Downarrow$$

$$T_1 - T_0 = -\frac{\Delta x_1}{k_1} \frac{Q}{A\Delta t} \; ; \; T_2 - T_1 = -\frac{\Delta x_2}{k_2} \frac{Q}{A\Delta t}$$

$$\Rightarrow T_1 - T_0 = -R_1 \frac{Q}{A\Delta t} \; ; \; T_2 - T_1 = -R_2 \frac{Q}{A\Delta t}$$

Adding:

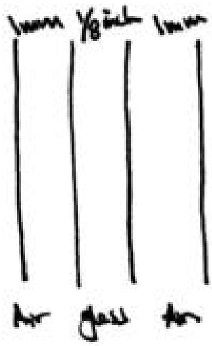
$$T_2 - T_0 = -(R_1 + R_2)\frac{Q}{A\Delta t}$$

$$\therefore \quad Q = -\frac{A\Delta t(T_2 - T_0)}{R_1 + R_2}$$

comparing to $\quad Q = -\frac{A\Delta t(T_2 - T_0)}{R_{total}}$

$\Rightarrow R_{total} = R_1 + R_2$ ✓.

(d)




$$R_{effective} = 2R_{Air} + R_{glass}$$
$$= 2\left(3.84 \cdot 10^{-2}\,\frac{m^2 k}{W}\right)$$
$$+ 4 \cdot 10^{-3}\,\frac{m^2 k}{W}$$
$$= 8.08 \cdot 10^{-2}\,\frac{m^2 k}{W}.$$

$$\frac{Q}{\Delta t} = -\frac{1}{R_{eff}}(20k)(1m^2)$$

$$= 247.\ W.$$

(Prob 1.58) From problem 1.57 $R_{\substack{Air\\ still}} = 2.25 \cdot 10^2 \ \frac{{}^{\circ}F \cdot ft^2 hr}{Btu}$

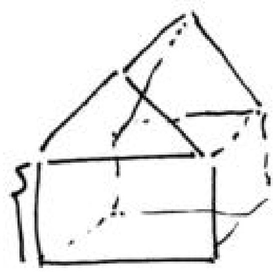
per mm

$$R_{\substack{Air\\ still\\ 3.5\ inch}} = \left(3.5 \text{ inch} \cdot \frac{? \text{ mm}}{1 \text{ inch}}\right)\left(2.25 \cdot 10^2 \ --\right)$$

$$= (3.5 \cdot 25.4)(\cdots) = 20{,}000. \ !! \text{ seems to great?}$$

If Book gets 1.

(Prob 1.59) Est. wood house:



Say 1000 sq ft living space

w/ 10 ft ceilings. $\Rightarrow$ $10^4 \ ft^3$ of volume

$\approx (10^4)^{2/3}$ area. Thermal losses due to conduction would be:
" A

$$\frac{\cancel{Q}}{\cancel{\Delta t}} \quad Q = -\frac{\Delta t \, A \, k_{wood} \, (\Delta T)}{\Delta x}$$

estimate $\Delta t = 1$ month

$A = (10^4)^{2/3} \ ft^2$

$k_{wood} = .08 \ W/m \cdot k$

$\Delta T = 30 k$

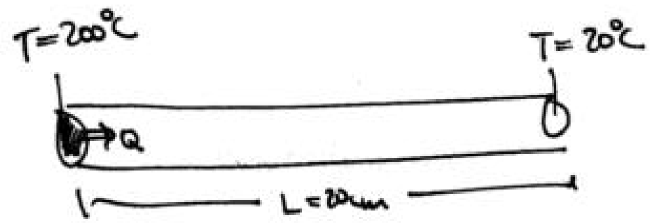
$\Delta x = 3.5$ inch.

$\Rightarrow Q = \cdots$ 

Amt heat generated by electricity

Amt heat generated by ~~fed~~ gas.

Prob 1.60




Heat conduction is governed by

$$\frac{Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x} = -kA\frac{dT}{dx}.$$

$\parallel$

constant.

if no work is done.

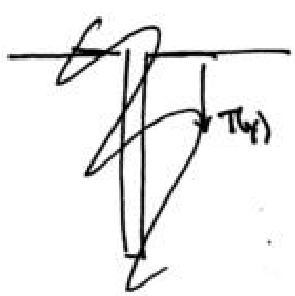
$Q = mC_V\Delta T$

$= mC_V\Delta T$

$$\therefore \quad \frac{mC_V\Delta T}{\Delta t} = -kA\frac{dT}{dx}.$$

??

Prob 1.61



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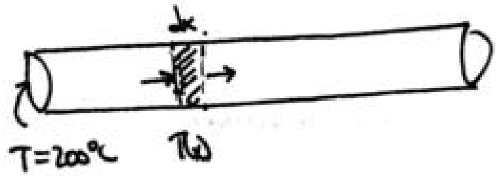
Probl. 60




Heat conduction is governed by $\frac{Q}{\Delta t} = -kA\frac{dT}{dx}$

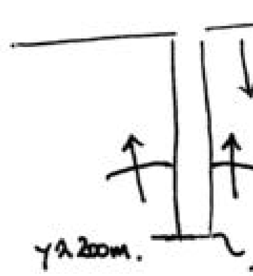
w/ $Q = mC\Delta T$

$\Rightarrow$

$mC\,T_t = -kA\,T_x$

$mC\,T_t + kA\,T_x = 0$ ??

Prob 1.61




$y$, $T(y) = T_0 + 20y$ $[y] = \text{km}$.

$k_t = 2.5 \frac{W}{m \cdot K}$

$T(200\text{m}) \cong T_0 + 20(200) = 4 \cdot 10^3 + T_0$.

$$\frac{Q}{\Delta t} \propto \frac{A \Delta T}{\Delta x}$$

At 200m ≈ 30 millionths of the fraction of the earth.

$$\frac{Q}{\Delta t} = -k_t A \frac{\Delta T}{\Delta x}$$

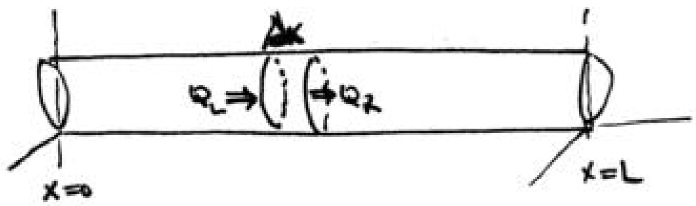
$\frac{Q}{A \Delta t}$ = rate of heat conduction per $m^2$. $= -\left(2.5 \frac{W}{m \cdot K}\right)\left(\frac{4 \cdot 10^3 K}{200\text{ m}}\right)$

$$= -20 \cdot 2.5 \frac{W}{m^2}.$$

$$A_{earth} = 4\pi (6400\text{ km})^2$$
$$= 5.14 \cdot 10^{14} m^2$$

$Q_{Earth} = 2.57 \cdot 10^{16}$ W. seems too large?

Prob 1.62




$$\frac{Q}{\Delta t} \propto \frac{A \Delta T}{\Delta x} \qquad \frac{Q}{\Delta t} = -k_t A \frac{\Delta T}{\Delta x}$$ for instantaneous heat flow

for discrete heat flow

But $\quad Q = E = C_v T$

w/ no work (PV work)

$E = Q + W$.

At steady st

$Q_R - Q_L =$ Heat absorbed into the material + stored as thermal energy.

Here $Q_L$ + $Q_R$ are the heat flowing at a given time

$$\therefore \quad \frac{Q_R}{\Delta t} - \frac{Q_L}{\Delta t} = c \rho (A \Delta x) \frac{\Delta T}{\Delta t}$$

By Fourier's heat flow, how heat flow affects temperature

Specific heat capacity

$$-k_t A \frac{dT}{dx}\bigg|_{x=x_R} + k_t A \frac{dT}{dx}\bigg|_{x=x_L} = c \rho A \Delta x \frac{dT}{dt}$$

$$\Rightarrow \quad -\frac{d^2T}{dx^2} = \left(\frac{c\rho}{k_t}\right) T_t$$

$$\underbrace{\phantom{xx}}_{\frac{1}{k}}$$

$T_t = k\,T_{xx}$ what chat minus sign?

**Prob 1.63**

$$\ell \approx \frac{1}{4\pi r^2}\left(\frac{V}{N}\right)$$

$PV = kNT$

$$\frac{V}{N} = \frac{kT}{P}$$

$$\ell \approx \frac{1}{4\pi r^2}\left(\frac{kT}{P}\right)$$

$$\ell \approx 10\,\text{cm} = \frac{1}{4\pi(1.5\cdot 10^{-10}\,\text{m})^2}\left(\frac{1.38\cdot 10^{-23}\,\text{J/K}\,(300\,\text{K})}{P}\right)$$

$$\Rightarrow \quad P = 7.41\cdot 10^{-41}\ \text{Pa}.$$

**Prob 1.64** He. is monatomic gas.

$$k_t = \frac{1}{2}\left(\frac{C_V}{V}\right)\ell\bar{v}$$

$$= \frac{1}{2}\left(\frac{\frac{f}{2}Nk}{V}\right)\ell\bar{v} = \frac{1}{2}\left(\frac{\frac{f}{2}P}{T}\right)\ell\bar{v} \cong \frac{1}{2}\left(\frac{\frac{f}{2}P}{T}\right)\ell\, v_{rms}$$

For a monatomic gas $f = 3$

(Prob 1.64)

$$k_t = \frac{1}{2}\left(\frac{C_V}{V}\right) \ell \bar{v}$$

$$PV = kNT$$
$$\frac{V}{N} = \frac{kT}{P}$$

$$\frac{C_V}{V} = \frac{\frac{f}{2}Nk}{V} = \frac{f}{2}\frac{P}{T}$$

$$\ell = \frac{1}{4\pi r^2}\frac{V}{N}$$
$$= \frac{1}{4\pi r^2}\left(\frac{kT}{P}\right)$$

$$\bar{v} \hat{=} v_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\therefore \quad k_t = \frac{1}{2}\frac{f}{2}\cdot\frac{\cancel{P}}{\cancel{T}}\cdot\frac{1}{4\pi r^2}\left(\frac{k\cancel{T}}{\cancel{P}}\right)\cdot\sqrt{\frac{3kT}{m}}$$

$$= \frac{1}{4} f \frac{1}{4\pi r^2} k \sqrt{\frac{3kT}{m}}.$$

For Helium ( ~~...~~ monatomic gas $f = 3$)

$r \hat{=} 1 \text{Å} = 10^{-10}$ m. $\quad m = ? = \frac{4.00 \text{ g/mole}}{6.022 \cdot 10^{23} \text{ Atoms/mole}} = 6.64 \cdot 10^{-24}$ g/Atom

$$\therefore k_t = \frac{1}{4}(3)\frac{1}{4\pi(10^{-10}\text{ m})^2}(1.38 \cdot 10^{-23} \text{J/k})\sqrt{\frac{3(1.38\cdot 10^{-23}\text{J/k})(300\text{ k})}{(6.64\cdot 10^{-24}\text{ g/Atom})}}$$

$$k_t = 1.12 \cdot 10^{-1} \frac{W}{m \cdot k} = .01 \frac{W}{m \cdot k}.$$

It differs 1) Because the gas is monatomic rather than diatomic

2) The mass are ~~so~~ diff

$$\frac{m_{Air}}{m_{He}} = \frac{.7(m_{N_2}) + .3(m_{O_2})}{m_{He}} = \frac{.7(28) + .3(32)}{4} = 7.3!!$$

For some reason this result is not correct w/ Fig 1.19. Perhaps $r \hat{=} 10^{-10}$ m is too small?

Prob 1.65

$$k_t = \frac{1}{4} \cdot 7 \cdot \ldots$$

I don't see how to do this w/o & having to know $\ell$, or $\bar{v}$, which I don't think would be credible

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Prob 1.66


As w/ heat conduction

$$\frac{Q}{\Delta t} = -k_D A \frac{dv}{dx}$$

Here Q is PV shearing force work $\Rightarrow Q = F_x \cdot \ell$.

$\therefore \quad F_x \cdot \ell$

... Not sure how to do this ...

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$$\frac{N}{A\,\Delta t} = \frac{D\,(N/V)}{\Delta x}$$

$$\frac{\cancel{N}}{\cancel{A\Delta x}\cdot\Delta t} = \frac{D(\cancel{N}/V)}{\Delta x^2}$$

$$\frac{1}{\Delta t} = \frac{D}{\Delta x^2} = \frac{10^{-9}\ m^2/s}{(.1m)^2} = \frac{10^{-9}\ m^2/s}{10^{-2}\ m^2} = 10^{-7}\ \cancel{m}/s$$

$$\Delta t = 10^7\ s$$

Prob 1.67

$$\frac{N}{A\,\Delta t} = D\,\frac{(N/V)}{\Delta x}$$

$$\Rightarrow \quad \frac{1}{A\Delta t} = \frac{D}{V\Delta x} \qquad \text{let } V \simeq A\Delta x$$

$\Delta t = 1$ min

$D = 10^{-9}\ m^2/s$

$$\Downarrow$$

$$\frac{1}{\Delta t} \cong \frac{D}{\Delta x^2}$$

$$\therefore \quad \Delta x^2 = \Delta t \cdot D \rightarrow \Delta x \cong \sqrt{D\Delta t} = \sqrt{10^{-9}\ m^2/s \cdot 60\,s} =$$

$$= 2.45 \cdot 10^{-4}\,m$$

$$= .24\ mm.$$

Prob 1.68 Room ~ $30 \times 20\ ft^2$.

Roughly $D \cong \frac{\Delta x^2}{\Delta t} \rightarrow \Delta t = \frac{\Delta x^2}{D.} = \frac{\left(30\,ft \cdot \frac{.3m}{1ft}\right)^2}{10^{-9}\ m^2/s}$

$\Delta t = 8.1 \cdot 10^{10}\ s. = 2.5 \cdot 10^{3}$ years.

principle transport mechanism must be convection.

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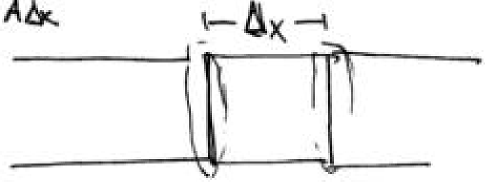
Prob 1.69

$$C(x) = \frac{n(x)}{V} = \frac{n(x)}{A\Delta x} \Rightarrow$$




Now Fick's 1st law is

$$J_x = -D\frac{dn}{dx} \quad \text{flux of particles (i.e. \# particles per unit time)}$$

Now concentration in small piece of length $\Delta x$ is given by

$$\frac{n(x)}{\cancel{\text{[illegible]}}} = J_x\Big|_{x_R} - J_x\Big|_{x_L} = -D\frac{dn}{dx}\Big|_{x_R} + D\frac{dn}{dx}\Big|_{x_L}$$

$$= -D\Delta x \frac{d^2n}{dx^2}$$

$$\frac{\Delta n(x)}{\Delta t} = J_x\Big|_{x_L} - J_x\Big|_{x_R} = -\left(J_x\Big|_{x_R} - J_x\Big|_{x_L}\right)$$

$$\frac{\Delta n(x)}{\Delta t} = -\left(-D\frac{dn}{dx}\Big|_{x_R} + D\frac{dn}{dx}\Big|_{x_L}\right) = D\left(\frac{dn}{dx}\Big|_{x_R} - \frac{dn}{dx}\Big|_{x_L}\right)$$

$$n_t = D\Delta x\frac{d^2n}{dx^2} \qquad n(x) \equiv A\Delta x \cdot C(x)$$

$$c_t = D\underbrace{\Delta x}\frac{d^2c}{dx^2}$$

How do I get rid of this term?

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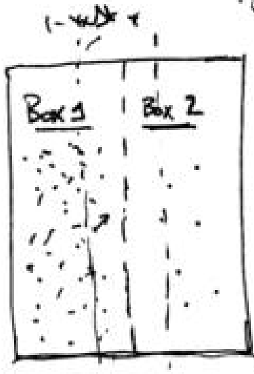
Prob 1.70




# particles crossing from Box #1 into Box #2 in time $\Delta t$

$\Delta N =$ Avg # of particles in Box 1 w/ $v_x$ positive & closer than $\bar{v}_x \Delta t$ away − Avg # of particles in Box 2 w/ $v_x$ negative & closer than $\bar{v}_x \Delta t$ away from interface

$$= \frac{1}{2}(A\bar{v}_x\Delta t)\left(\frac{P}{kT}\right) - \frac{1}{2}(A\bar{v}_x \Delta t)\left(\frac{P}{kT}\right)$$

$\uparrow$

$\frac{1}{2}$ of all particles have + $v_x$ velocities say.

$$\left\{ N = \frac{PV}{kT} \right.$$

$$\Rightarrow \frac{\Delta N}{A\Delta t} = \frac{1}{2}\left(\frac{1}{3}\bar{v}\right)\frac{P}{kT} - \frac{1}{2}\left(\frac{1}{3}\bar{v}\right)\frac{P}{kT}$$

Assuming the temperature is the same on both ½'s

$\bar{v} \simeq \bar{v}_{rms} = \sqrt{\frac{3kT}{m}}$ Then $\Rightarrow J = \frac{1}{6}\frac{\bar{v}}{kT}\frac{dP}{dx}\cdot \ell$

$\Rightarrow$ $\cancel{\quad}$ =

$$\Rightarrow \quad D = \frac{1}{6}\frac{\bar{v}}{kT}\ell \cong \frac{1}{6}\frac{v_{rms}}{kT}\ell$$

$$D = \frac{1}{6}\sqrt{\frac{3kT}{m}}\,\frac{1}{kT}\,(150\ \text{nm})$$

$$= \frac{1}{6}\sqrt{\frac{3(1.38\cdot 10^{-23}\ J/K)(300K)}{\left(\frac{(.7(28)+.3(32))}{6.022\cdot 10^{23}}\cdot 10^{-3}\right)}}\ \frac{150\cdot 10^{-9}m}{(1.38\cdot 10^{-23}\ J/K)\cdot 300}$$

$$= 3.05\cdot 10^{15}\ !$$

Maybe this is $D^{-1}$? Not very close

Prob 2.1

(a)

| | C1 | C2 | C3 | C4 | |
|---|---|---|---|---|---|
| | H | H | H | H | All Heads |
| | T | H | H | H | one tail 3 heads |
| | H | T | H | H | |
| | H | H | T | H | |
| | H | H | H | T | |
| $\binom{4}{2} = \frac{4!}{2!2!} = \frac{4\cdot 3}{2} = 6$ ✓. | T | T | H | H | two heads. |
| | T | H | T | H | |
| | T | H | H | T | |
| | H | T | T | H | |
| | H | T | H | T | |
| | H | H | T | T | |
| | T | T | T | H | one head |
| | T | T | H | T | |
| | T | H | T | T | |
| | H | T | T | T | |
| | T | T | T | T | No heads |

total # of microstates $= 16 = 2^4$

(b) Macrostates would be

4 Heads $P = 1/16$

3 heads $P = 4/16 = 1/4$

2 heads $P = 6/16 = 3/8$

1 head $P = 4/16 = 1/4$

0 heads $P = 1/16$

$$\sum = \frac{1+4+6+4+1}{16} = 1 \checkmark$$

(c) $\Omega(4,0) = \binom{4}{0} = 1$ ✓

$\Omega(4,1) = \binom{4}{1} = 4$ ✓

$\Omega(4,2) = \binom{4}{2} = 6$ ✓

$\Omega(4,3) = \binom{4}{3} = 4$ ✓

$\Omega(4,4) = 1$ ✓

Prob 2.2

(a) 20 fair coins.

$2^{20}$ microstates

(b) $\frac{1}{2^{20}} = .95 \cdot 10^{-7}$.

(c) $\Omega(20,12) = \frac{\binom{20}{12}}{2^{20}} = \frac{\frac{20!}{8!\,12!}}{2^{20}} - \frac{\frac{20 \cdot 19 \cdot \ldots 13}{8!}}{2^{20}} = \ldots$

(d)

Prob 2.3

(a) $2^{50}$

(b) $\Omega(50,25) = \binom{50}{25}$

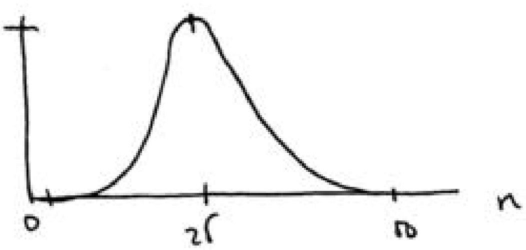
(c) $\frac{\binom{50}{25}}{2^{50}}$

(d) $\frac{\binom{50}{30}}{2^{50}}$

(e) $\frac{\binom{50}{40}}{2^{50}}$

(f) $\frac{1}{2^{50}}$

(g) $\frac{\binom{50}{n}}{2^{50}}$



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Prob 2.4

# hands of cards $\binom{52}{5}$

Prob of getting a royal flush is $\frac{4}{\binom{52}{5}}$.

A,K,Q,J,10 in each suit

$$\Omega(3,0) = \binom{3-1}{0} = 1$$

$$\Omega(N,q) = \binom{q+N-1}{q}$$

$$\Omega(3,1) = \binom{4-1}{1} = \binom{3}{1} = 3$$

$$\Omega(3,2) = \binom{4}{2} = \frac{4\cdot 3}{2} = 6$$

$$\Omega(3,3) = \binom{5}{3} = \frac{5\cdot 4\cdot \cancel{3!}}{\cancel{3!}\, 2!} = 10$$

**Prob 2.25**

(a) $N = 3 \quad q = 4$

$$\Omega(N,q) = \binom{N+q-1}{q}$$

$$= \binom{6}{4} = \frac{6 \cdot 5}{2} = 15.$$

| Oscillator #1 | #2 | #3 |
|---|---|---|
| 4 | 0 | 0 |
| 0 | 4 | 0 |
| 0 | 0 | 4 |
| 3 | 1 | 0 |
| 3 | 0 | 1 |
| 1 | 3 | 0 |
| 0 | 3 | 1 |
| 1 | 0 | 3 |
| 0 | 1 | 3 |
| 2 | 2 | 0 |
| 2 | 0 | 2 |
| 2 | 1 | 1 |
| ~~2~~ | ~~2~~ | ~~0~~ |
| 0 | 2 | 2 |
| 1 | 2 | 1 |
| ~~2~~ | ~~0~~ | ~~2~~ |
| ~~0~~ | ~~2~~ | ~~2~~ |
| 1 | 1 | 2 |

[illegible]?
15 total ✓

(b) $N = 3 \quad q = 5$

$\Omega(N,q) = \binom{N+q-1}{q}$

$\Omega(3,5) = \binom{7}{5} = \frac{7 \cdot 6}{2} = 7 \cdot 3 = 21$

| oscillator | #1 | #2 | #3 |
|---|---|---|---|
| | 5 | 0 | 0 |
| | 0 | 5 | 0 |
| | 0 | 0 | 5 |
| | 4 | 1 | 0 |
| | 4 | 0 | 1 |
| | 1 | 4 | 0 |
| | 0 | 4 | 1 |
| | 1 | 0 | 4 |
| | 0 | 1 | 4 |
| | 3 | 2 | 0 |
| | 3 | 0 | 2 |
| | 3 | 1 | 1 |
| | 2 | 3 | 0 |
| | 0 | 3 | 2 |
| | 1 | 3 | 1 |
| | 2 | 0 | 3 |
| | 0 | 2 | 3 |
| | 1 | 1 | 3 |
| | 2 | 2 | 1 |
| | 2 | 1 | 2 |
| | 1 | 2 | 2 |

21 total ✓

(c) $N = 3 \quad q = 6$

$$\Omega(N,q) = \binom{N+q-1}{q}$$

$$\Omega(3,6) = \binom{8}{6} = \frac{8 \cdot 7}{2} = 28$$

| oscillator: #1 | #2 | #3 |
|---|---|---|
| 6 | 0 | 0. |
| 0 | 6 | 0. |
| 0 | 0 | 6. |
| 5 | 1 | 0. |
| 5 | 0 | 1. |
| 1 | 5 | 0. |
| 0 | 5 | 1. |
| 1 | 0 | 5. |
| 0 | 1 | 5. |
| 4 | 2 | 0. |
| 4 | 0 | 2. |
| 4 | 1 | 1. |
| 2 | 4 | 0. |
| 0 | 4 | 2. |
| 1 | 4 | 1. |
| 2 | 0 | 4. |
| 0 | 2 | 4. |
| 1 | 1 | 4. |
| 3 | 3 | 0. |
| 3 | 0 | 3. |
| 3 | 2 | 1. |
| 3 | 1 | 2. |
| 3 | 3 | 0. |
| 0 | 3 | 3. |
| 2 | 3 | 1. |
| 1 | 3 | 2. |
| ~~3~~ | ~~0~~ | ~~3~~ x |
| ~~0~~ | ~~3~~ | ~~3~~ x |
| 2 | 1 | 3. |
| 1 | 2 | 3. |

28th

(d)

$N=4 \quad q=2$

$\Omega(N,q) = \binom{N+q-1}{q}$

$\Omega(4,2) = \binom{5}{2} = \frac{5\cdot 4}{2} = 10$

| oscillator #1 | #2 | #3 | #4 |
|---|---|---|---|
| 2 | 0 | 0 | 0 |
| 0 | 2 | 0 | 0 |
| 0 | 0 | 2 | 0 |
| 0 | 0 | 0 | 2 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 |

/10 ✓.

(e) $N=4 \quad q=3$

$\Omega(4,3) = \binom{6}{3} = \frac{6\cdot 5\cdot 4}{3\cdot 2} = 20$

| Oscillator #1 | #2 | #3 | #4 |
|---|---|---|---|
| 3 | 0 | 0 | 0 |
| 0 | 3 | 0 | 0 |
| 0 | 0 | 3 | 0 |
| 0 | 0 | 0 | 3 |
| 2 | 1 | 0 | 0 |
| 2 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 |
| 1 | 2 | 0 | 0 |
| 0 | 2 | 1 | 0 |
| 0 | 2 | 0 | 1 |
| 1 | 0 | 2 | 0 |
| 0 | 1 | 2 | 0 |
| 0 | 0 | 2 | 1 |

| #1 | #2 | #3 | #4 |
|---|---|---|---|
| 1 | 0 | 0 | 2 |
| 0 | 1 | 0 | 2 |
| 0 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |

/20 ✓.

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(f) $N=1$ $q=$ anyth

Oscillator #1
q.

$$\Omega(N,q) = \binom{1+q-1}{q} = \binom{q}{q} = 1$$

(g) $N=$ anything, $q=1$

$$\Omega(N,q) = \binom{N+1-1}{1} = N$$

Prob 2.6

$$\Omega(30,30) = \binom{30+30-1}{30} = \binom{59}{30} = \frac{59!}{30!\,29!}$$

$$= 59 \cdot 58 \cdot \dots$$

Prob 2.7 $N=4$ $q=2$

$$\Omega(N,q) = \binom{5}{2} = \frac{5 \cdot 4}{2} = 10$$

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<table>
<tr><th>#1</th><th>#2</th><th>#3</th><th>#4</th><th>dot + line representation</th></tr>
<tr><td>2</td><td>0</td><td>0</td><td>0</td><td>··|||</td></tr>
<tr><td>0</td><td>2</td><td>0</td><td>0</td><td>|··||</td></tr>
<tr><td>0</td><td>0</td><td>2</td><td>0</td><td>||··|</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>2</td><td>|||··</td></tr>
<tr><td>1</td><td>1</td><td>0</td><td>0</td><td>·|·||</td></tr>
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<tr><td>1</td><td>0</td><td>0</td><td>1</td><td>·|||·</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>1</td><td>|·||·</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>1</td><td>||·|·</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>0</td><td>|·|·|</td></tr>
</table>

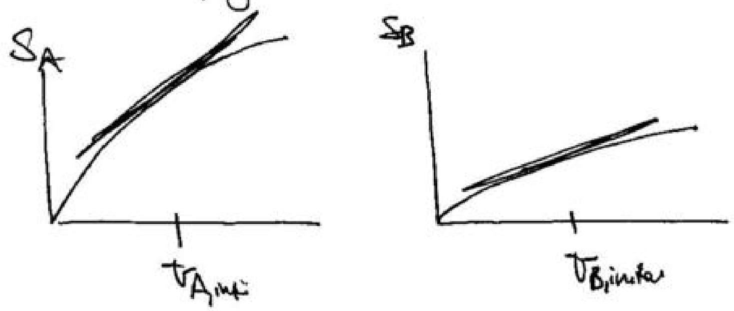
$q_{total} = \cancel{10} \; 6 = q_A + q_B$

$\Rightarrow \exists$ how many macro states? in $q_A$ we can have $0, 1, \cdots, 6 \Rightarrow 6+1$

macrostates. w/ ~~micro~~ macrostates being able to be distin.

(Prob 3,3)

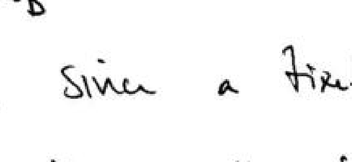
$$\frac{1}{T} = \frac{\partial S}{\partial U}$$




From the pictures

$\frac{\partial S_A}{\partial U_A} > \frac{\partial S_B}{\partial U_B}$ thus heat energy will flow from system B to system A. Since a fixed amount of heat flowing into system A will raise the total entropy of the entire system since the same amount flowing out of B will not be enough to decrease the total energy creation.

(Prob 3,4)




$\frac{\partial S}{\partial U}$ decreasing · $\frac{\partial S}{\partial U}$ increasing

If I were to add energy to this system. the amount of entropy increase in the system would increase, to the point that any amount of ~~heat~~ ~~thermal~~ thermal energy flowing into this solid would create much more entropy then that lost by the other body. Thus the entropy of the universe would increase.

I would think that you could put a mixing system up on that looked like S


... Not sure

Prob 2.8

$N_A = 10 \quad N_B = 10 \quad q = q_A + q_B = 20.$

(a) $q_A$ can be taking from 0 to 20 → 21 different macrostates

(b) # microstates $\Omega(N,q) = \binom{20+20-1}{20} = \binom{39}{20} = 6.89 \cdot 10^{10}$

(c) ~~To find~~ all the energy in system A would mean

$q_A = 20 \qquad q_B = 0.$

$\Omega_A = \binom{N_A+q_A-1}{q_A} \quad \Omega_B = 1$

$= \binom{29}{20} = 1.00 \cdot 10^7.$

∴ Prob All energy is in system A $= \dfrac{\Omega_A \cdot \Omega_B \text{(This instance)}}{\sum\limits_{\text{All instances}} \Omega_A \Omega_B}$

$= \dfrac{1.0 \cdot 10^7}{6.89 \cdot 10^{10}} = 1.45 \cdot 10^{-4}.$

(d) The prob of finding ½ energy in sol A ⇒ $q_A = 10$ → $q_B = 10$

∴ Prob $\dfrac{\Omega_A \Omega_B \text{(This instance)}}{\sum\limits_{\text{All inst}} \Omega_A \Omega_B} = \dfrac{\binom{N_A+q_A-1}{q_A}\binom{N_B+q_B-1}{q_B}}{\sum \cdot}$

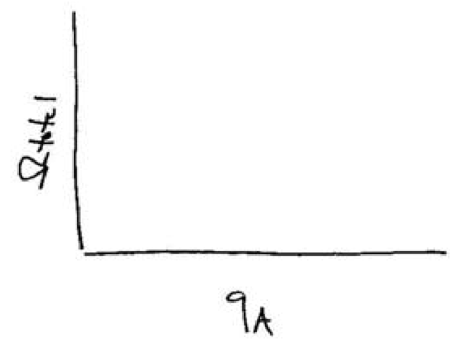
$= \dfrac{\binom{19}{10}^2}{6.89 \cdot 10^{10}} = .12$

(e) If the energy started entirely in one or the other solid one would have ~~to~~ ~~...~~ ~~...~~ ~~...~~ a $1/10^4$

$\frac{1}{10000}$ chance of finding All the energy in one or the other solid.

Prob 2.9 $q = q_A + q_B = 6 \qquad N_A = N_B = 3.$

| $q_A$ | $\Omega_A = \binom{q_A+N_A-1}{q_A}$ | $\Omega_B = \binom{q_B+N_B-1}{q_B}$ | $\Omega_{total} \equiv \Omega_A \Omega_B$ |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |
| ⋮ | | | |
| 6 | | | |




B Pseudo code

See Matlab code ...

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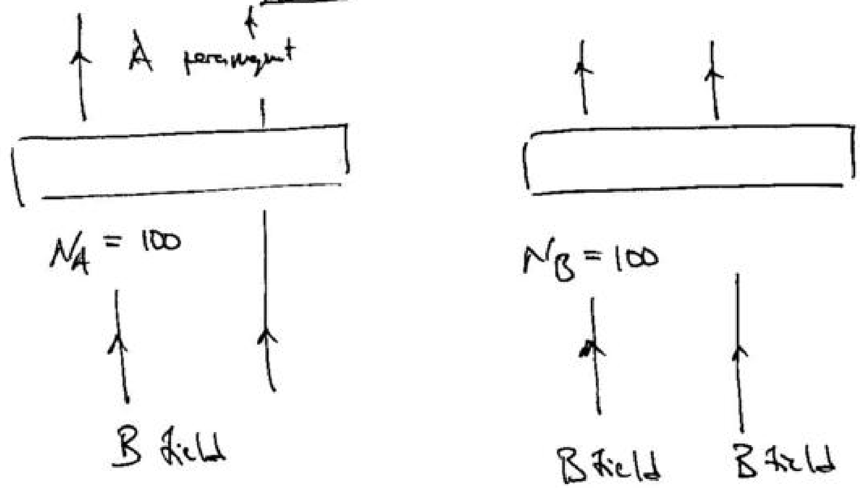
Prob 2.10




$q_{total} = 80.$

See matlab ...

(Prob 2.15) $N! \doteq N^N e^{-N}\sqrt{2\pi N}$

$N = 50$

$50! = \cdots$

$(50)^{50} e^{-50}\sqrt{2\pi(50)} = \cdots$

$$\left.\frac{N^N e^{-N}\sqrt{2\pi N}}{N!}\right|_{N=50} = .99$$

$\ln N! \cong N\ln N - N$

$\ln(N!) = \cdots$

$N\ln N - N = \cdots$

$$\frac{N\ln N - N}{\ln(N!)} \cong .98 .$$

(Prob 2.16) 1000 coins.

$$Pr = \frac{\binom{1000}{800}}{2^{1000}}$$

$\binom{1000}{800}$ ← choose 800 ~~the~~ coins from the 1000 to be heads.

$2^{1000}$ ← total # of outcomes for 1 flip

this is not! it is ~~log~~ $\ln(N!)$ !!

$$Pr = \frac{\frac{1000!}{800!\,500!}}{2^{1000}} \cong \frac{(1000\ln(1000) - 1000)}{\frac{(800\ln(500) - 800)(500(\ln 500) - 500)}{2^{1000}}}$$

$$\cancel{P_r} \doteq \cancel{\ln P_r = \ln(1000!)}$$

$$P_r = \frac{1000!}{2^{1000}\, 500!\, 500!} = \frac{1000!}{2^{1000}\,(500!)^2}$$

$$\ln P_r = \ln(1000!) - 2000 \ln 2 - 2\ln(500!)$$

$$\cong \cancel{1000 \ln(1000) - 1000 - 2000 \ln 2}$$

$$\cancel{- 500 \ln(500) + 500 - 500 \ln 500}$$

$$\cong 1000 \ln(1000) - 1000 - 2000 \ln 2$$

$$- 2(500 \ln(500) - 500)$$

$$= 1000 \ln(1000) - \cancel{1000} - 2000 \ln 2$$

$$- 1000 \ln(500) + \cancel{1000}$$

$$= 1000(\ln(1000) - \ln(500)) - 2000 \ln 2$$

$$= -693.1$$

$P_r = 9.33 \cdot 10^{-302}$ !! Can this be correct??

(b) $$P_r = \frac{\binom{1000}{400}}{2^{1000}} = \frac{\frac{1000!}{600!\,400!}}{2^{1000}}$$

$$\ln P_r = \ln(1000!) - \ln(600!) - \ln(400!) - 1000 \ln 2$$

$$\ln Pr \cong 1000 \ln(1000) - 1000$$
$$-600 \ln 600 + 600$$
$$-400 \ln(400) + 400$$
$$-1000 \ln 2$$
$$= -20.1$$

$$Pr = 1.7 \cdot 10^{-9}.$$

Prob 2.17

In the low temperature limit. $q \ll N$.

$$\Omega(N,q) = \binom{N+q-1}{q} = \frac{(N+q-1)!}{q!\,(N-1)!} = \frac{(N+q)!\,(N)}{(N+q)\,q!\,N!}$$

$$= \left(\frac{N}{N+q}\right)\cdot\frac{(N+q)!}{q!\,N!} \cong \frac{(N+q)!}{q!\,N!}$$

Now

$$\ln\Omega = \ln(N+q)! - \ln q! - \ln N!$$

$$\cong (N+q)\ln(N+q) - \cancel{(N+q)} - q\ln q + \cancel{q} - N\ln N + \cancel{N}$$

$$= (N+q)\ln(N+q) - q\ln q - N\ln N.$$

$$= (N+q)\ln\left[N\left(1+\frac{q}{N}\right)\right] - q\ln q - N\ln N$$

$$= (N+q)\left[\ln N + \ln\left(1+\frac{q}{N}\right)\right] - q\ln q - N\ln N$$

$$\cong (N+q)\left[\ln N + \frac{q}{N}\right] - q\ln q - N\ln N$$

$$= \cancel{N\ln N} + q\ln N + q + \frac{q^2}{N} - q\ln q - \cancel{N\ln N}$$

$$= q + \frac{q^2}{N} + q\ln\left(\frac{N}{q}\right) \cong q + q\ln\left(\frac{N}{q}\right) = q - q\ln\left(\frac{q}{N}\right)$$

for $q \ll N$ (low temperature limit)

$$\Omega(N,q) = e^{q} e^{-q\ln(\frac{q}{N})} = \left(\frac{N}{q}\right)^{q} e^{q} = \left(\frac{Ne}{q}\right)^{q}$$

Prob 2.18

$$\Omega(N,q) = \binom{N+q-1}{q} = \frac{(N+q-1)!}{(N-1)!\,q!} \qquad \left\{ N! \cong N^{N} e^{-N} \sqrt{2\pi N} \right\}$$

$$= \frac{(N+q)!\,N}{N!\,(N+q)\,q!} = \left(\frac{N}{N+q}\right) \frac{(N+q)!}{N!\,q!}$$

$$\underset{\text{large } q}{\overset{\text{large } N}{\cong}} = \left(\frac{N}{N+q}\right) \frac{(N+q)^{N+q} \cancel{e^{-(N+q)}} \sqrt{2\pi(N+q)}}{N^{N} \cancel{e^{-N}} \sqrt{2\pi N}\; q^{q} \cancel{e^{-q}} \sqrt{2\pi q}}$$

$$= \left(\frac{N}{N+q}\right) \frac{(N+q)^{N+q} \sqrt{N+q}}{N^{N} q^{q} \sqrt{2\pi} \sqrt{Nq}}$$

$$= \frac{\sqrt{N}}{\sqrt{N+q}} \; \frac{(N+q)^{N}}{N^{N}} \; \frac{(N+q)^{q}}{q^{q}} \; \frac{1}{\sqrt{2\pi}\sqrt{q}}$$

$$= \frac{\left(1+\frac{q}{N}\right)^{N} \left(1+\frac{N}{q}\right)^{q}}{\sqrt{2\pi q \left(\frac{N+q}{N}\right)}}$$

(Prob 2.19)

$$\Omega(N_\uparrow) = \frac{N!}{N_\uparrow!\,N_\downarrow!} \quad \text{v.s.} \quad \Omega(N,q) = \binom{q+N-1}{q}$$

$N = N_\uparrow + N_\downarrow$

$$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \binom{N_\uparrow + N_\downarrow}{N_\uparrow}$$

This is identical to $\Omega(N,q)$ where $N_\downarrow \equiv N-1$.

Thus in the limit where $N_\downarrow \ll N \iff N \ll q$ ?.

But basically the expressions are equivalent in some limit &

obtaining $\Omega \approx \left(\frac{Ne}{N_\downarrow}\right)^{N_\downarrow}$ should not be surprising.

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Prob 2.22

(a) $N$ oscil $N$ osc. total energy = $2N$.

the energy in the 1st solid can be $0, 1, 2, \cdots, 2N$.

or $2N+1$ possible macrostates

(b) Prob 2.18 gives $\Omega(N,q) \cong \dfrac{\left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N}{\sqrt{2\pi q(q+N)/N}}$ for each solid + for the combined system:

For the combined system one gets
~~$\Omega(4N, \; \Omega(2N, 2N)$~~

$$\Omega(2N, 2N) = \cancel{\frac{\left(\frac{3N}{2N}\right)^{2N}\left(\frac{3N}{2N}\right)^{2N}}{\sqrt{2\pi(2N)(3N/2N)}}} \quad \frac{\left(\frac{4N}{2N}\right)^{2N}\left(\frac{4N}{2N}\right)^{2N}}{\sqrt{2\pi(2N)\left(\frac{4N}{2N}\right)}}$$

$$= \frac{2^{2N}\cdot 2^{2N}}{\sqrt{2\pi\cdot 4N}} = \frac{2^{4N}}{\sqrt{8\pi N}} \checkmark$$

(c) From Prob 2.18 w/ $\Omega_1(N,N)\cdot\Omega_2(N,N) = \Omega(N,N)$

$$= \left(\frac{\left(\frac{2N}{N}\right)^N\left(\frac{2N}{N}\right)^N}{\sqrt{2\pi N\left(\frac{2N}{N}\right)}}\right)^2 = \left(\frac{2^{2N}}{\sqrt{2\pi\cdot 2\cdot N}}\right)^2$$

$$= \frac{2^{4N}}{4\pi N} \checkmark$$

Prob 2.20 $x = \frac{q}{2\sqrt{N}}$

$\therefore \Delta x = \frac{q}{\sqrt{N}}$ so if $N \hat{=} 10^{20}$.

$\Delta x \cong \frac{q}{10^{10}}$. What about the q?

If $10^{20}$ were to fit into 10 cm (width of a page)

our peak would have to be $\propto \frac{1}{10^{10}}$ of that # thus.

peak width would be $\frac{10\,cm}{10^{10}}$.

Prob 2.21 For

$$\Omega = \left(\frac{eq_A}{N}\right)^N \left(\frac{eq_B}{N}\right)^N = \left(\frac{e}{N}\right)^{2N} (q_A q_B)^N$$

$$z = \frac{q_A}{q} \qquad 1 - z = \frac{q - q_A}{q} = \frac{q_B}{q}.$$

$$\therefore \quad \Omega = \left(\frac{e}{N}\right)^{2N} q^{2N} \left(z(1-z)\right)^N = \left(\frac{e}{N}\right)^{2N} \frac{q^{2N}}{4^N} \left(4z(1-z)\right)^N$$

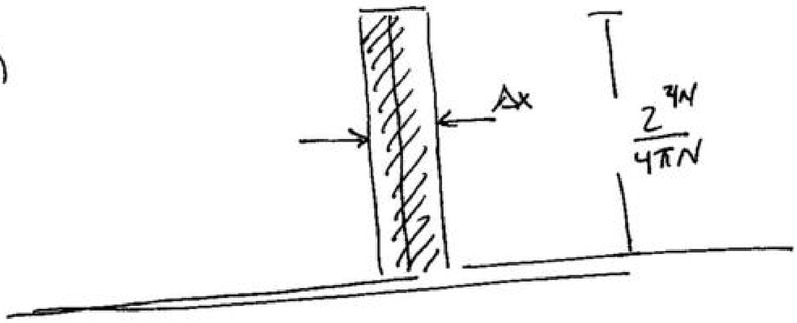
$$z \in (0,1) \qquad\qquad = C_{N,q} \left(4z(1-z)\right)^N$$

Pseudocode: z = linspace(0,1,1000); ns = [1,100, ...]

for N : 1 : length(ns)

$y_{\text{z}}^{\text{z}}$ = $(4z(1-z))^N$ figure

(d)




$$A_{total} = \frac{2^{4N}}{\sqrt{8\pi N}} = \Delta x \cdot \frac{2^{4N}}{4\pi N}$$

$$\Delta x = \frac{4\pi N}{\sqrt{8\pi N}} = \frac{\sqrt{4\pi N}}{\sqrt{2}}$$

For $N \cong 10^{20}$ $\quad \Delta x \propto 10^{10}$

$$\therefore \frac{\Delta x}{A_{total}} \propto \frac{\left(\frac{\sqrt{4\pi N}}{\sqrt{2}}\right)}{\left(\frac{2^{4N}}{\sqrt{8\pi N}}\right)} = \cancel{\frac{\sqrt{4\pi N}}{\sqrt{2}} \cdot \frac{\sqrt{8\pi N}}{2^{4N}}}$$

$$= \frac{\sqrt{4\pi N}}{\sqrt{2}} \cdot \frac{\sqrt{8\pi N}}{2^{4N}}$$

? Don't see how this is extremely small.

Prob 2.23

(a) $N = 10^{23}$

$$\Omega(N_\uparrow) = \binom{N}{N_\uparrow}$$

$$\Omega(N/2) = \binom{N}{N/2} = \frac{N!}{\left(\left(\frac{N}{2}\right)!\right)^2}$$

$$\ln \Omega = \ln N! - 2\ln\left(\tfrac{N}{2}\right)!$$

$$= N\ln N - N - 2\left(\tfrac{N}{2}\ln\left(\tfrac{N}{2}\right) - \tfrac{N}{2}\right)$$

$$= N\ln N - \cancel{N} - N\ln\left(\tfrac{N}{2}\right) + \cancel{N}$$

$$= N\left(\ln \frac{N}{N/2}\right) = N\ln 2$$

$$\therefore \quad \Omega \cong e^{(\ln 2)N}$$

So $\ln \Omega = \underline{\underline{N\ln 2}}$

(b) $1\,\text{yr} = 3.15 \cdot 10^7\,\text{s}$

$$\#_{\text{states}} = (10^9 \cdot 1/\text{s})(10 \cdot 10^9\,\text{yr})(3.15 \cdot 10^7\,\text{s}/\text{yr}) = 3.15 \cdot 10^{26}$$

log # states = 61.0.

(c) No. If the age of Universe is $10^{10}$ s one cannot expect to wait much longer than that.

Prob 2.24

(a) $\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!\,(N-N_\uparrow)!}$

$$\ln \Omega(N_\uparrow) \cong N\ln N - N - (N_\uparrow \ln N_\uparrow - N_\uparrow) - \left((N-N_\uparrow)\ln(N-N_\uparrow) - (N-N_\uparrow)\right)$$

$$= N\ln N - \cancel{N} - N_\uparrow \ln N_\uparrow + \cancel{N_\uparrow} - (N-N_\uparrow)\ln(N-N_\uparrow) + \cancel{(N-N_\uparrow)}$$

~~Thus when~~ $\cancel{N_\uparrow} \cong \cancel{\frac{N}{2}}$

~~[illegible]~~

$$\ln \Omega = N\ln N - N_\uparrow \ln N_\uparrow - (N-N_\uparrow)\ln(N-N_\uparrow)$$

when $N_\uparrow = \frac{N}{2}$

$$\ln \Omega = N\ln N - \frac{N}{2}\ln\left(\frac{N}{2}\right) - \left(\frac{N}{2}\right)\ln\left(\frac{N}{2}\right) = N\ln N - N\ln\left(\frac{N}{2}\right)$$

$$= N\left(\ln \frac{N}{N/2}\right) = N\ln 2$$

(b)

let $x \equiv N_\uparrow - \frac{N}{2}$, then $N_\uparrow = x + \frac{N}{2}$

$$\ln\Omega = N\ln N - \left(x+\frac{N}{2}\right)\ln\left(x+\frac{N}{2}\right) - \left(N-x-\frac{N}{2}\right)\ln\left(N-x-\frac{N}{2}\right)$$

$$= N\ln N - \left(x+\frac{N}{2}\right)\ln\left(x+\frac{N}{2}\right) - \left(\frac{N}{2}-x\right)\ln\left(\frac{N}{2}-x\right)$$

. . . .

$$\Omega = N^{N} N_{\uparrow}^{-N_{\uparrow}} N_{\downarrow}^{-N_{\downarrow}}$$

$$= \frac{N^N}{N_{\uparrow}^{N_{\uparrow}} N_{\downarrow}^{N_{\downarrow}}}$$

let $N_{\uparrow} = x + \frac{N}{2}$ then $N_{\downarrow} = N - N_{\uparrow} = N - x - \frac{N}{2} = \frac{N}{2} - x.$

$$\Omega = \frac{N^N}{\left(x+\frac{N}{2}\right)^{\left(x+\frac{N}{2}\right)}\left(\frac{N}{2}-x\right)^{\left(\frac{N}{2}-x\right)}}$$

$$= \frac{N^{x+\frac{N}{2}} \cdot N^{-x+\frac{N}{2}}}{\left(x+\frac{N}{2}\right)^{x+N/2}\left(\frac{N}{2}-x\right)^{\frac{N}{2}-x}}$$

$$= \left[\frac{1}{\frac{x}{N}+\frac{1}{2}}\right]^{x+\frac{N}{2}} \left[\frac{1}{\frac{1}{2}-\frac{x}{N}}\right]^{\frac{N}{2}-x}$$

$$= \left[\frac{1}{\frac{1}{2}+\frac{x}{N}}\right]^{x} \cdot \left[\frac{1}{\frac{1}{2}+\frac{x}{N}}\right]^{\frac{N}{2}} \cdot \left[\frac{1}{\frac{1}{2}-\frac{x}{N}}\right]^{\frac{N}{2}} \cdot \left[\frac{1}{\frac{1}{2}-\frac{x}{N}}\right]^{-x}$$

$$= \left(\frac{\frac{1}{2}-\frac{x}{N}}{\frac{1}{2}+\frac{x}{N}}\right)^{x} \cdot \left(\frac{1}{\frac{1}{4}-\frac{x^2}{N^2}}\right)^{N/2}$$

If $x=0$

$\Omega(0) = (4)^{N/2} = 2^N$. ? Not the same.

(c) $$P_1 = \frac{\binom{10^6}{510{,}000}}{2^{10^6}}$$

$$P_2 = \frac{\binom{10^6}{510{,}001}}{2^{10^6}}$$

Prob 225

(a) By flipping coins let every head denote a step forward & a tail denote a step backwards.

Then a string of T's & H's denotes a path in the space.

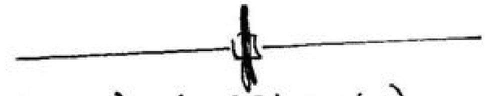
. . .

(Prob 2.25)

(a) Consider a string of L's + R's w/ L's representing steps to the left + R's representing steps to the right

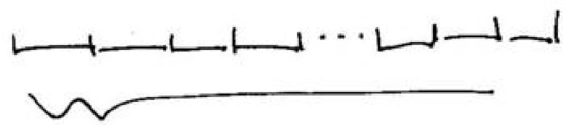
~~Most~~ Most likely location corresponds to the point w/ Maximal probability.

$P_n$ = prob n units to the right =



⇒ must have n more R's in my string of N (R's + L's) than L's.

$$= \frac{X}{2^N}$$



N - spots

N - string is ÷ up into 2

~~strings~~ (1) of length n. +

(2) of length $\underset{\uparrow}{N-n}$

has to be evenly ÷ into L's + R's

$\binom{N}{k}$

↑ have k Rights + N-k lefts

⇒ total distance $+k-(N-k)$

$2k-N$.

$$P_n = \frac{\binom{N-n}{\frac{N-n}{2}}}{2^N} = \frac{(N-n)!}{\left(\left(\frac{N-n}{2}\right)!\right)^2 2^N}$$

ignoring issues of ~~[illegible]~~ even/odd

N, n + if $\frac{N-n}{2}$ is fractional

or not.

$$\ln P_n = \ln (N-n)! - 2\ln\left(\left(\frac{N-n}{2}\right)!\right) - N\ln 2$$

$$\underset{\uparrow}{\cong} (N-n)\ln(N-n) - \cancel{(N-n)} - 2\left(\frac{N-n}{2}\right)\ln\left(\frac{N-n}{2}\right) + \cancel{2\left(\frac{N-n}{2}\right)} - N\ln 2$$

Stirling's approximation

$$= (N-n)\left\{\ln(N-n) - \ln\left(\frac{N-n}{2}\right)\right\} - N\ln 2$$

$$= (N-n)\left\{\ln\left(\frac{N-n}{\frac{N-n}{2}}\right)\right\} - N\ln 2$$

$$= (\cancel{N}-n)\ln 2 - \cancel{N\ln 2} = -n\ln 2$$

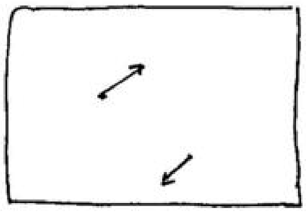
$P_n \cong e^{-n\ln 2}$ Max of this fn is at $n \equiv 0$

$\Rightarrow$ your expected location is $\underline{0}$.

(b) would it not be 0?

(c) ?

Prob 2.26



$\Omega_1 \propto A \cdot A_p$

$U = \frac{1}{2} m (v_x^2 + v_y^2) \Rightarrow p_x^2 + p_y^2 = 2mU.$ $\Delta x \cdot \Delta p_x \cong h$

$\Omega_1 = \frac{A A_p}{h^2}$. for 1 gas molecule in flat land!

w/ 2 molecules

$\Omega_2 = \frac{1}{2} \frac{A^2}{h^4} \times$ (Area of 4 dimensional hyper sphere of radius $\sqrt{2mU}$)

$\vdots$

$$\Omega_N = \frac{1}{N!} \frac{A^N}{h^{2N}} * \underbrace{(\text{Area of } 2N \text{ hyper sphere of radius } \sqrt{2mU})}_{\left.\frac{2\pi^N r^{2N-1}}{(N-1)!}\right|_{r=\sqrt{2mU}}}$$

$$\Omega_N = \frac{1}{N!}\frac{A^N}{h^{2N}} * \frac{2\pi^N (2m\tau)^{N-1/2}}{(N-1)!}$$

$$\Omega_N \cong \frac{1}{N!}\frac{A^N}{h^{2N}}\frac{2\pi^N}{N!}(2m\tau)^N \cdot \cong \cancel{\frac{A^N \pi^N}{(N!)^2 h^{2N}}}$$

$$\cong \frac{1}{(N!)^2}\frac{A^N}{h^{2N}}\pi^N (2$$

$$= \frac{(\pi A\, 2m\tau)^N}{(N!)^2\, h^{2N}} = \frac{(2\pi m A \tau)^N}{(N!)^2 h^{2N}}$$

Prob 2.27

$$\Omega_{total} = f(N)^2 (V_A V_B)^N (U_A U_B)^{3N/2}$$

$$\Omega(U, V, N) = f(N) V^N U^{3N/2}$$

$$\Omega(U, .99V, N) = (.99)^N \Omega(U, V, N)$$

For $N = 100, 10^4, 10^{23}$

$$(.99)^N = .366 \;,\; 2.2 \cdot 10^{-44} \;,\; 0.$$

i.e. w/ 100 molecule 36 xs out of 100 one finds the state searched for
$10^4$ " $\frac{2.2}{10^{44}}$ 2 out of $10^{44}$ times one finds the

Prob 2.28 $\Omega = 52!$

\# of arrangements $= 52!$

$$S = k \ln \Omega = k \ln 52! = k(156.36)$$

156.36 w/ $k = 1.381 \cdot 10^{-23}$ J/K $S = 2159 \cdot 10^{-21}$ J/K.

How determine it significant ?

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$$\Omega_N \cong \frac{1}{N!}\frac{V^N}{h^{3N}}\frac{\pi^{3N/2}}{(3N/2)!}\left(\sqrt{2mU}\right)^{3N}$$

$$S = k\ln\Omega = k\Big[-\ln N! + N\ln V - 3N\ln h + \frac{3N}{2}\ln\pi - \ln\left(\frac{3N}{2}!\right) + \frac{3N}{2}\ln(2mU)\Big]$$

$$\cong k\Big[-N\ln N + N + N\ln V - 3N\ln h + \frac{3}{2}N\ln\pi - \left(\frac{3N}{2}\right)\ln\left(\frac{3N}{2}\right) + \frac{3N}{2} + \frac{3N}{2}\ln(2mU)\Big]$$

Stirling's relation

$\{\ln N! = N\ln N - N$

$$= Nk\Big[-\ln N + 1 + \ln V - 3\ln h + \frac{3}{2}\ln\pi - \frac{3}{2}\ln\left(\frac{3N}{2}\right) + \frac{3}{2} + \frac{3}{2}\ln(2mU)\Big]$$

$$= Nk\Big[\ln\left(\frac{V}{N}\frac{\pi^{3/2}}{h^3}\frac{1}{\left(\frac{3N}{2}\right)^{3/2}}\cdot(2mU)^{3/2}\right) + \frac{5}{2}\Big]$$

$$= Nk\Big[\ln\left(\frac{V}{N}\left(\frac{\pi}{h^2}\frac{2}{3N}\cdot 2mU\right)^{3/2}\right) + \frac{5}{2}\Big]$$

$$= Nk\Big[\ln\left(\frac{V}{N}\left(\frac{4\pi}{3N}\frac{mU}{h^2}\right)^{3/2}\right) + \frac{5}{2}\Big]$$ eq 2.49 ✓

Prob 2,29 $S = k \ln \Omega$

$N_A = 300 \quad N_B = 200 \quad q_{total} = 100$ ← this is the same system as on pg 59.

$$\Omega(N, q) = \binom{q+N-1}{q}$$

$$\therefore \quad \Omega_{min} = 2.8 \cdot 10^{81} \qquad \Omega_{max} = 6.9 \cdot 10^{114}$$

$$S_{least} = 187.5 \qquad \Omega_{most} = 264.42$$

For entropy over long time scales $\Omega = 9.3 \cdot 10^{115} \rightarrow S = 267.02$

Prob 2,30

(a) $\Omega_N = \dfrac{2^{4N}}{\sqrt{8\pi N}} \qquad S = k \ln S = k(4N)\ln 2 - \frac{k}{2}\ln(8\pi N)$

If $N = 10^{23}$
$$S = (1.381 \cdot 10^{-23}\, J/K)(4)(10^{23})\ln 2 - \frac{(1.381 \cdot 10^{-23}\, J/K)}{2}\ln(8\pi \cdot 10^{23})$$
$$= 3.8 \ J/K$$

(b) $\underset{\text{most likely}}{\Omega_N} = \dfrac{2^{4N}}{4\pi N}$

$S = k(4N)\ln 2 - k\ln(4\pi N) = 3.8\ J/K.$

(c) One seems to get the same answer ~~as~~ ~~as~~ in ~~~~ ~~~~ either case

(d) The ~~usual~~ usual argument is that we have created entropy by inserting the partition. This creation of entropy would more than compensate for the difference observed.

Prob 2.31 See notes for pg 77

Prob 2.32 From Prob 2.26 $\Omega_N = \dfrac{(2\pi m A U)^N}{(N!)^2 h^{2N}}$

$$S = k\ln\Omega = \left(N\ln(2\pi m A U) - 2\ln(N!) - 2N\ln(h)\right)k$$

$$\cong \left(N\ln(2\pi m A U) - 2N\ln N + 2N - 2N\ln(h)\right)k$$

$$= Nk\left\{\ln(2\pi m A U) - 2\ln N + 2 - 2\ln(h)\right]$$

$$= Nk\left\{\ln\left(\frac{2\pi m A U}{N^2 h^2}\right) + 2\right]$$

$$= Nk\left\{\ln\left(\frac{A}{N}\left(\frac{2\pi m U}{N h^2}\right)\right) + 2\right].$$

$$U = \frac{3}{2} nRT$$

$$U = \frac{3}{2}(1)(8.314\ J/k\cdot mol)(300\ k) = 3741.0\ J.$$

$$\left\{ m = \frac{4}{6.022\cdot 10^{23}} = 6.64\cdot 10^{-27}\ kg \right.$$

$$V = \frac{nRT}{P} = \frac{(8.314\ J/k)(300\ k)}{10^5\ Pa} = 2.49\cdot 10^{-2}\ m^3$$

Then in Sackur-Tetrode eq:

$$\cancel{S = Nk\left[\ln\right.}$$

$$\frac{2.49\cdot 10^{-2}}{6.022\cdot 10^{23}}\left(\frac{4\pi\cdot 6.64\cdot 10^{-27}\cdot(3741.0)}{3\cdot 6.022\cdot 10^{23}\cdot(6.626\cdot 10^{-34})^2}\right)^{3/2}$$

$$= 3.24\cdot 10^5 \xrightarrow[\ln]{} 12.68$$

$$\therefore S = kN\Big[\underbrace{\ln(\quad) + \frac{5}{2}}_{12.68 + 2.5 = 15.18}\Big]$$

$$S = k(9.14\cdot 10^{24}) = 126.3\ J/k$$

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Prob 2,33 Argon is a noble gas $f=3$

$$U = N \cdot f\left(\tfrac{1}{2}kT\right) \qquad N = 6.022\cdot 10^{23}$$

$$PV = kNT \qquad m = \frac{39.948 \text{ g/mole}}{6.022\cdot 10^{23}\text{ atoms/mole}} = 6.63\cdot 10^{-23}\text{ g/atom.}$$

$$V = \frac{kNT}{P} \qquad \equiv \frac{M}{N_A}$$ what is $M$ called?

$$S(P,T,N) = Nk\left[\ln\left(\frac{kNT}{PN}\left(\frac{4\pi M}{3N N_A h^2}\,\frac{N\cdot f}{2}kT\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(\frac{kT}{P}\,\frac{2\pi M}{3N_A h^2}\, f\, kT\right)^{3/2} + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(\frac{2f\pi M}{3N_A h^2}\,\frac{(kT)^2}{P}\right)^{3/2} + \frac{5}{2}\right]$$

$$= 6.022\cdot 10^{23}\cdot(1.381\cdot 10^{-23}\text{ J/K})\left[\underbrace{\ln\left(\frac{2\cdot 3\cdot \pi(39.948\cdot 10^{-3})}{3(6.022\cdot 10^{23})(6.626\cdot 10^{-34})^2}\,\frac{(1.381\cdot 10^{-23}\cdot 300)^2}{10^5}\right)^{3/2}}_{-1.3} + \frac{5}{2}\right]$$

$S = 9.97$ J/K ? Should be larger .. Did you take the 3/2 power?

My guess would be that the molecular mass of Argon is about 10×s that of Helium.

Prob 2.34

quasi-static $\;\;\;$ isothermal expansion

$\Delta W = p\Delta V$ $\;\;\;$ T does not change

$$\Delta U = \Delta Q + \Delta W$$

$$\Rightarrow \Delta Q = \Delta U - \Delta W.$$
$$= \Delta U + p\Delta V.$$

$$\Delta S = \frac{\partial S}{\partial V}\Delta V + \frac{\partial S}{\partial U}\Delta U$$

$$= Nk\,\frac{1\left(\frac{1}{N}(\cdots)^{3/2}\right)}{\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right)}\Delta V + Nk\,\frac{\cancel{\frac{V}{N}}(\;\;)^{1/2}\cdot\frac{3}{2}\left(\frac{4\pi m}{3Nh^2}\right)}{\left(\cancel{\frac{V}{N}}(\;\;)^{3/2}\right)}\Delta U$$

$$= \frac{Nk}{V}\Delta V + \frac{Nk\,\frac{3}{2}\left(\frac{4\pi m}{3Nh^2}\right)}{\left(\frac{4\pi m U}{3Nh^2}\right)}\Delta U$$

$$= \frac{P}{T}\Delta V + \frac{3}{2}Nk\,\frac{1}{U}\Delta U. \qquad U = f\cdot N\cdot\frac{1}{2}kT$$

$$= \frac{P}{T}\Delta V + \frac{3}{2}\,\frac{\cancel{N}k}{\frac{f}{2}\cancel{N}kT}\Delta U \qquad f = 3 \text{ for a monatomic gas}$$

$$= \frac{P}{T}\Delta V + \frac{\Delta U}{T}$$

$$= \frac{P\Delta V + \Delta U}{T} = \frac{\Delta Q}{T} \checkmark.$$

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(Prob 2.35)

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$ Sackur-Tetrode eq

$T = 300K$ $\Rightarrow N = \frac{PV}{kT}$

$P = 10^5$ Pa

holding density fixed I will take as ~~mass~~ density, actually in Stat. Physics ~~mass~~ density usually means molar/atom density, i.e. $\left(\frac{V}{N}\right)$.

$U = \cancel{N} \cdot f \cdot \frac{1}{2}kT$

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m}{3\cancel{N}h^2}\frac{\cancel{N}f}{2}kT\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi f m}{3\cdot 2\cdot h^2}P\frac{V}{N}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$PV = NkT$

$P\left(\frac{V}{N}\right) = kT$

$P =$

$$S = N\cdot k\left[\ln\left(\frac{V}{N}\left(\frac{2\pi m\cdot f k}{3h^2}\cdot T\right)^{3/2}\right) + \frac{5}{2}\right]$$

$S$ is negative for a fixed $\frac{V}{N}$ when

$$\ln\left(\frac{V}{N}\left(\frac{2\pi}{3h^2}m\cdot f k\cdot T\right)^{3/2}\right) < -\frac{5}{2}$$

$$\Rightarrow T = \frac{\left(\dfrac{e^{-2.5}}{\frac{V}{N}}\right)^{2/3}}{\left(\frac{2\pi}{3h^2}m\cdot f\cdot k\right)}$$

$$\frac{V}{N} = \frac{kT}{P} = \frac{(1.381\cdot10^{-23}\,J/K)(300\,K)}{10^5\,Pa} = 4.14\cdot10^{-26}\,m^3$$

$$\left\{ \frac{J}{Pa} = \frac{kg\,m^2/s^2}{\frac{kg\,m/s^2}{m^2}} = m^3 \right\}$$

$$m_{He} = \frac{4\,g/mole}{6.022\cdot10^{23}\,atoms/mol} = 6.64\cdot10^{-24}\,g/atom.$$

$$\cancel{\frac{V}{N}\cdot\left(\frac{2\pi(6.64\cdot10^{-24}\cdot10^{-3}\,kg/atom)}{3(6.626\cdot10^{-34}\,J\cdot s)^2}\right)}$$

$$\frac{V}{N}\cdot\cancel{\frac{2\pi}{3h^2}m\cdot T\cdot k = (4.14\cdot10^{-26}\,m^3)}\left(\frac{\cancel{2\pi(6.64\cdot10^{-24}\cdot10^{-3}\,kg/atom)}\cdot\cancel{3}(1.381\cdot10^{-23}\,J/K)}{\cancel{3}(6.626\cdot10^{-34}\,J\cdot s)^2}\right)$$

$$\cancel{= 5.4\cdot10^{-8}}$$

$T$ ~~$= 1.8\cdot10^6$. What am I doing wrong~~?

$$\frac{2\pi}{3h^2}m\cdot T\cdot k = 1.31\cdot10^{18}$$

Then $T = 1.20\cdot10^{-2}$ K.

Prob 2.36

$$S_{\text{monatomic gas}} = Nk\left[\ln(\quad) + \tfrac{5}{2}\right]$$

$$S_{\text{einst solid}} = Nk\left[\ln(q/N) + 1\right]$$

$S_{Book}$

$\cancel{\text{ }}$: $1kg = 1000g$

$$N_{Book} = \frac{1000\,g}{12\,g/mol}\left(6.022 \cdot 10^{23}\ atoms/mol\right)$$

$$N_{moose} = \frac{400 \cdot 10^{3}\,g}{18\,g/mol} \cdot 6.022 \cdot 10^{23}\ atoms/mol.$$

$$N_{sun} = \frac{2 \cdot 10^{30} \cdot 10^{3}\,g}{2\,g/mol} \cdot 6.022 \cdot 10^{23}\ at/m$$

$$N_A \cdot k = 8.316\ J/K$$

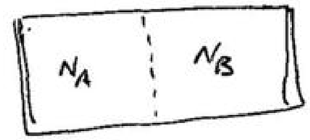
$$\therefore\ S_{Book} = 693.0\ J/K$$

$$S_{moose} = 1.84 \cdot 10^{5}\ J/K$$

$$S_{sun} = \cancel{10^{33}}\ 8.31 \cdot 10^{33}\ J/K$$

Prob 2.37

let: $x = \frac{N_A}{N}$, $1-x = \frac{N_B}{N}$




$\ni N_A + N_B = N$.

$$\Delta S_A = N_A k \ln\left(\frac{V_f}{V_i}\right) = N_A k \ln 2.$$

$$\Delta S_B = N_B k \ln 2$$

$$\Delta S = N_A k \ln 2 + N_B k \ln 2$$

$$= kN \left[ x \ln 2 + (1-x) \ln 2 \right]$$

... What you have calculated is for ... this is somehow for ? Don't see why this is wrong.

$$S = Nk \left[ \ln\left( \frac{V}{N} \left( \frac{4\pi m U}{3Nh^2} \right)^{3/2} \right) + \frac{5}{2} \right]$$

$$S_{total} = S_A + S_B$$

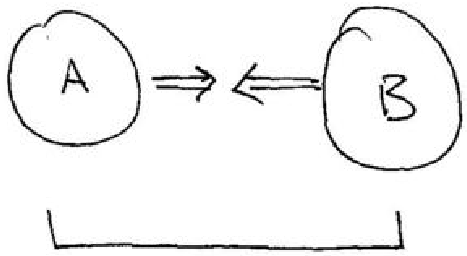
After adding $N_B$ molecules of molecule B.

$$= N_A k \left[ \ln\left( \frac{V}{N_A} \left( \frac{4\pi m U}{3N_A h^2} \right)^{3/2} \right) + \frac{5}{2} \right] + N_B k \left[ \ln\left( \frac{V}{N_B} \left( \frac{4\pi m U}{3N_B h^2} \right)^{3/2} \right) + \frac{5}{2} \right]$$

$$\Rightarrow \Delta S_{total} = \ \ldots$$

How do this calculation?

Prob 2,37




N. total # of particles of type A + B

w/ ~~#~~ $N_A + N_B = N - N_A$. $\quad x \equiv \frac{N_B}{N}$ ; $\frac{N_A}{N} = 1 - x$.

For an ideal gas the Sackur-Tetrode eq holds

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

~~$S_B = Nk$~~

~~$\Delta S_A = N_A k\left[\ln 2\right.$~~

Assuming that the Volume doubles $V_i$ w/o change in $U$.

So $$S_A^i = N_A k\left[\ln\left(\frac{V_i}{N_A}\left(\frac{4\pi m_A U}{3N_A h^2}\right)^{3/2} + \frac{5}{2}\right]$$

$$S_B^i = N_B k\left[\ln\left(\frac{V_i}{N_B}\left(\frac{4\pi m_B U}{3N_B h^2}\right)^{3/2} + \frac{5}{2}\right]$$

For simplicity lets assume $m_A = m_B \equiv m$

Then $$S_A^f = N_A k\left[\ln\left(\frac{2V_i}{N_A}\left(\frac{4\pi m U}{3N_A h^2}\right)\right) + \frac{5}{2}\right]$$

$$S_B^f = N_B k \left[ \ln\left( \frac{2V_i}{N_B} \left( \frac{4\pi m U}{3N_B h^2} \right) \right) + \frac{5}{2} \right]$$

So $\Delta S = S_A^f + S_B^f - S_A^i - S_B^i$

$= \Delta S_A + \Delta S_B$

For $\Delta S_A = N_A k \ln 2$

---

Think volumes as $\propto$ to $\qquad PV = kNT \qquad V \propto N.$

$\frac{V}{N} =$

By Sackur - Tetrode eq

$S = Nk \Big[ \ln\Big( \quad \ldots \quad ?$

(Prob 2.38) The # of states available for a mixed solid is

$\binom{N}{N_A}$ ?

$$\ln\binom{N}{N_A} = \ln\left(\frac{N!}{(N-N_A)!\,N_A!}\right) = \ln N! - \ln((N-N_A)!) - \ln N_A!$$

$$\cong N\ln N - \cancel{N} - (N-N_A)\ln(N-N_A) + (\cancel{N}-\cancel{N_A}) - N_A\ln N_A + \cancel{N_A}$$

$$= N\ln N - (N-N_A)\ln(N-N_A) - N_A\ln N_A$$

$N-N_A = N_B.$

$$= N\ln N - N_B\ln N_B - N_A\ln N_A$$

$$=$$

$$= N\left[\ln N - \frac{N_B}{N}\ln N_B - \frac{N_A}{N}\ln N_A\right]$$

$$= N\left[\ln N - (1-x)\ln\left(\frac{N_B}{N}\cdot N\right) - x\ln\left(\frac{N_A}{N}\cdot N\right)\right]$$

$$= N\left[\cancel{\ln N} - (1-x)\ln(1-x) - (1-x)\ln N - x\ln x - x\ln N\right]$$

$$= -N\left[x\ln x + (1-x)\ln(1-x)\right]$$

$$\therefore \Delta S = k\ln\Omega = -Nk\left[x\ln x + (1-x)\ln(1-x)\right]$$

$S_A = k \ln \Omega(N_A)$

$S_B = k \ln \Omega(N_B)$

Assuming that all particles are of the same type the mixture would have a total of $N$

~~$\Omega(N_A)$~~ = I would argue that when combined v.s. separated the combined system would have more degrees of freedom & that these additional degrees of freedom could be categorized as

... ?

$$\Delta S_{mixing} = k \ln \binom{N}{N_A} = k \ln\left(\frac{N!}{N_A!\,N_B!}\right)$$

$$\cong k\left[N \ln N - \cancel{N} - N_A \ln N_A + \cancel{N_A} - N_B \ln N_B + \cancel{N_B}\right]$$

$$= Nk\left[\ln N - \frac{N_A}{N}\ln N_A - \frac{N_B}{N}\ln N_B\right]$$

$$= Nk\left[\left(\frac{N_A}{N} + \frac{N_B}{N}\right)\ln N - \frac{N_A}{N}\ln N_A - \frac{N_B}{N}\ln N_B\right]$$

$$= Nk\left[\frac{N_A}{N}\ln\frac{N}{N_A} + \frac{N_B}{N}\ln\left(\frac{N}{N_B}\right)\right]$$

$$= Nk\left[x\ln x + (1-x)\ln(1-x)\right]$$

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Prob 2.39 $PV = nRT \qquad PV = NkT.$

$$U = N \cdot 7 \cdot \tfrac{1}{2} kT \qquad (T, P)$$

$$= \tfrac{7}{2} Nk \cdot T = \tfrac{7}{2} nRT = 3700\,J \text{ (same as before)}$$

$$+ \; V = .025\ m^3.$$

From pg 78 $S_{Helium} = 126\ J/k$ $\qquad m = \frac{4}{6.022 \cdot 10^{23}} = 6.64 \cdot 10^{-27}\ kg$

Assuming distinguishable molecules we get:

$$S = Nk\left[\ln\left(V\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{3}{2}\right]$$

$$= 1(8.314\ J/k)\left[\ln\left(.025\,m\left(\frac{4\pi \cdot 6.64 \cdot 10^{-27} \cdot 3700}{3 \cdot 6.02 \cdot 10^{23}(6.626 \cdot 10^{-34}\,J \cdot s)^2}\right)^{3/2}\right) + \frac{3}{2}\right]$$

$$= 8.314\ J/k \cdot 68.92$$

$= \cancel{687.2}\ J/k$ $\quad$ this is larger? what did I do wrong? ...

573.06 J/k $\quad$ maybe that is correct since distinguishable molecules would have more states available to them. + thus

I would think more micro states.

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Problem 2.39

indist molecules:

(1) $S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$

$U = N f \cdot \frac{1}{2}kT = \frac{3}{2}NkT$ $\quad PV = kNT$ $\quad \frac{V}{N} = \frac{kT}{P}$

$N = 6.022 \cdot 10^{23}$

$T = 300^\circ K$

$P = 10^5\ Pa$

distinguishable molecules

(2)

$S = Nk\left[\ln\left(V\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{3}{2}\right]$

w/ $U = \frac{7}{2}NkT$

$S_{ind} = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m \frac{7}{2}NkT}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$

$S_{dist} = Nk\left[\ln\left(V\left(4\pi m\right.\right.\right.$

$S_{ind} = Nk\left[\ln\left(V\left(\quad\right)^{3/2}\right) - \ln N + \frac{3}{2} + 1\right]$

$$S_{ind} = Nk\left[\ln\left(V\left(\quad\right)^{3/2}\right) + \frac{3}{2}\right]$$

$$\cancel{\quad} \quad Nk\left[1 - \ln N\right]$$

$$\Rightarrow S_{ind} = S_{dist} + Nk\left[1 - \ln N\right]$$

Now $S_{dist} = Nk\left[\ln\left(V\left(\frac{4\pi m \cdot \cancel{N}\frac{3}{2}kT}{3\cancel{N}h^2}\right)^{3/2}\right) + \frac{3}{2}\right]$

$$\Rightarrow S_{dist} = Nk\left[\ln\left(V\left(\frac{4\pi m \, \cancel{3} \cdot kT}{6h^2}\right)^{3/2}\right) + \frac{3}{2}\right]$$

Now for He

$$m = \frac{4.002 \, g/mol \, (1 \, mol)}{6.022 \cdot 10^{23} \text{ atoms}} =$$

Prob 2,40

(a) The once orderly salt crystals are now a solute in the soup.

(b)

.

:

In all cases one can view the state before as being more orderly, in that the atoms were arranged to have a distinctive purpose before the process & after words did not.

Except for cutting down a tree which might be more orderly than before, ...

Prob 2,41 With any building man creates or organization that he does the break down of molecular energy in the ~~food~~ food he eats more than compensates for the organization that comes about from the building that he does.

Prob 2.40

In all the microscopic everyday processes the amount of disorder measured by the # of available states has increased.

Prob 2.41

The same sort of things that come into place in 2.40 show here

Prob 2.42

(a) Assuming the black hole has mass M

constants involved M, c, Gravitational const

$$[F] = N = \frac{kg \cdot m}{s^2} \qquad F = -\frac{G M_1 M_2}{r^2}$$

Unit wise

$$\frac{kg \cdot m}{s^2} = \frac{(kg)^2 [G]}{m^2} \rightarrow [G] = \frac{m^3}{(kg) \cdot s^2}$$

Now given M, c, & G form a unit of length

$$\cancel{M^{p_1} c^{p_2} G^{p_3} = m} \quad [M]^{p_1} [c]^{p_2} [G]^{p_3} = m$$

$$(kg)^{p_1}\left(\frac{m^{p_2}}{s^{p_2}}\right)\frac{m^{3p_3}}{(kg)^{p_3}s^{2p_3}} = m \quad ✓$$

$$\Rightarrow (kg)^{p_1-p_3} \cdot \frac{1}{s^{(p_2+2p_3)}} \cdot m^{3p_3+p_2} = m$$

$$p_1 - p_3 = 0, \quad p_2 + 2p_3 = 0, \quad 3p_3 + p_2 = 1$$

$$p_1 = p_3 \qquad p_2 + 2p_1 = 0, \quad 3p_1 + p_2 = 1$$

$$3p_1 + p_2 = 1$$
$$2p_1 + p_2 = 0.$$

$$p_1 = \frac{\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix}} \qquad p_2 = \frac{\begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$p_1 = \frac{1}{(3-2)} \qquad p_2 = \frac{-2}{1} = -2$$
$$= +1$$

9-02-02 ∫

$$\therefore \quad \frac{MG}{c^2} \propto r$$

$$\frac{kg \cdot m}{s^2} \frac{m^2}{kg^2} = \frac{m^3}{s^2 kg}$$

$$r = \frac{(2 \cdot 10^{30}\,kg)(6.673 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2})}{(3 \cdot 10^8\,m/s)^2}$$

$$= 1.4 \cdot 10^3\,m$$

(b) ~~if~~

if $\ln N$ is simply a large # while $N$ is a very large # as formula the calculation of $S = N \ln N$.

(c) To maximize $S$ one would need to increase $N$ to as large an extent as possible, this would involve using (to keep the mass constant) as small ~~#~~ ~~par~~ w.r.t. mass particles as possible using near massless photons

$$Mc^2 = \text{total energy} = N \cdot e_{photon} \qquad e_{photon} = h\nu = \frac{hc}{\lambda}$$

$$= N \cdot \frac{hc}{\lambda} \cong N \cdot \frac{hc}{r_{BH.}}$$

$$\Rightarrow \quad N = \frac{Mc\,r_{B.H.}}{h}$$

Then $S_{BH} \cong K \cdot N_{BH.} = K \dfrac{Mc}{h} \dfrac{MG}{c^2}$

$$S_{BH} = \frac{kGM^2}{\hbar c}$$

d) $$\frac{(1{,}38\cdot 10^{-23}\ \mathrm{J/K})(2\cdot 10^{30}\ \mathrm{kg})^2(6{,}673\cdot 10^{-11}\ \mathrm{N\cdot m^2/kg^2})}{(6{,}626\cdot 10^{-34}\ \mathrm{J\cdot s})(3\cdot 10^{8}\ \mathrm{m/s})}$$

$$= 1{,}85\cdot 10^{52}\ \mathrm{J/K}$$

Prob 3.1 $T = \left.\frac{\partial U}{\partial S}\right|_{N,V} \cong \frac{U_{i+1} - U_{i-1}}{S_{i+1} - S_{i-1}}$

when $q_A = 1$

$$T_A = \frac{2\epsilon - 0\cdot\epsilon}{10.7\cdot k - 0\cdot k} = \frac{2}{10.7}\frac{\epsilon}{k} = 1.8\cdot10^{-1}\ \epsilon/k$$

$$T_B = \frac{98\epsilon - 100\epsilon}{185.3k - 187.5k} = \frac{-2\epsilon}{-2.2\,k} = \frac{2}{2.2}\ .909\ \epsilon/k$$

when $q_A = 60$

$$T_A = \cancel{\frac{7.7\cdot10^{69}\epsilon - 2.2\cdot10^{68}\epsilon}{160.9k - 157.4k}} = \cancel{\frac{7.4\cdot10^{69}\epsilon}{3.5k}} = 2.1\cdot 10^{69}\frac{\epsilon}{k}$$ ?

$$= \frac{59\epsilon - 61\epsilon}{157.4k - 160.9k} = \frac{-2\epsilon}{-3.5\,k} = \frac{2}{3.5}\frac{\epsilon}{k} = .5714\ \epsilon/k$$

$$T_B = \frac{39\epsilon - 40\epsilon}{103.5k - 107k} = \frac{-\epsilon}{-3.5k} = .285\ \epsilon/k$$

Assuming $\epsilon = .1\,eV$

$k = 8.617\cdot10^{-5}\ eV/k$

so $\frac{\epsilon}{k} = \frac{.1}{8.617\cdot10^{-5}}\frac{eV}{eV/k} = 1.16\cdot10^{3}\ k$

Prob 3,2

$$\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{N,V}$$

$A \cong B \qquad B \cong C \Rightarrow A \cong C.$

$$\frac{1}{T_A} = \frac{1}{T_B} \qquad \frac{\partial S_B}{\partial U_B} = \frac{\partial S_C}{\partial U_C}$$

$$\frac{\partial S_A}{\partial U_A} = \frac{\partial S_B}{\partial U_B}$$

Prob 3.5  Prob 2.17 gives

$$\Omega(N,q) = \left(\frac{Ne}{q}\right)^q \qquad \text{w/ } U = \epsilon q$$

$$\rightarrow q = \frac{U}{\epsilon}$$

$$\therefore \Omega(N,U) = \left(\frac{Ne}{U/\epsilon}\right)^{U/\epsilon}$$

$$= \left(\frac{Ne\epsilon}{U}\right)^{U/\epsilon}$$

$$S = S(N,U) = k\ln\Omega(N,U) = \frac{kU}{\epsilon}\ln\left(\frac{Ne\epsilon}{U}\right)$$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{V,N} = \frac{k}{\epsilon}\ln\left(\frac{Ne\epsilon}{U}\right) + \frac{kU}{\epsilon}\cdot\frac{1}{\left(\frac{Ne\epsilon}{U}\right)}\cdot\frac{Ne\epsilon}{U^2}(-1)$$

$$= \frac{k}{\epsilon}\ln\left(\frac{Ne\epsilon}{U}\right) + -\frac{k}{\epsilon}$$

$$\left(\frac{1}{T} + \frac{k}{\epsilon}\right)\frac{\epsilon}{k} = \ln\left(\frac{Ne\epsilon}{U}\right)$$

---

$$\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{V,N} = \left.\frac{\partial S}{\partial q}\right|_{V,N}\cdot\frac{dq}{dU}$$

$$= \frac{1}{\epsilon}\left.\frac{\partial S}{\partial q}\right|_{V,N.}$$

$$\therefore \frac{\epsilon}{T} = \left.\frac{\partial S}{\partial q}\right|_{V,N}$$

$U = \epsilon q$

$\frac{dq}{dU} = ?$

$1 = \epsilon\cdot\frac{dq}{dU}$

$\frac{dq}{dU} = \frac{1}{\epsilon}$

$$S(N,q) = k \ln \left(\frac{Ne}{q}\right)^q$$

$$= qk \ln\left(\frac{Ne}{q}\right)$$

$$= qk\left[\ln(Ne) - \ln q\right]$$

$$\frac{\partial S}{\partial q} = k\left[\ln(Ne) - \ln q\right] + qk\left(\frac{-1}{q}\right)$$

$$= k\left[\ln(Ne) - \ln q\right] - k$$

$$\frac{\epsilon}{kT} = \ln(Ne) - \ln q - 1$$

$$\frac{\epsilon}{kT} + 1 - \ln(Ne) = -\ln q$$

$$q = \exp\left\{\ln(Ne) - \frac{\epsilon}{kT} - 1\right\}$$

$$= Ne \cdot e^{-1} e^{-\epsilon/kT} = Ne^{-\epsilon/kT}$$

$$\Rightarrow U = N\epsilon e^{-\epsilon/kT}$$

Prob 3.6

Assuming $\Omega = U^{Nf/2}$

Then $S = k \ln \Omega = k \frac{Nf}{2} \ln U$.

$$\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{V,N} = \frac{kNf}{2U}$$

$$\rightarrow \quad U = kT \cdot \frac{Nf}{2}$$

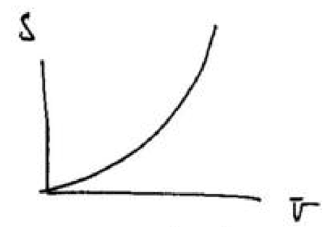
$$U = f \cdot N \cdot \frac{kT}{2}$$

if $U \ll 1 \implies T \ll J$ ... ?

Prob 3.7 $\quad S_{BH} = \frac{8\pi^2 G M^2 k}{hc} \quad$ if $Mc^2 = U$.

$$M^2 = \frac{U^2}{c^4}$$

$$\therefore \quad S_{BH} = \frac{8\pi^2 G U^2 k}{hc^5}$$



a miserly system

$$\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{V,N} = \frac{8\pi^2 G \cdot 2U \cdot k}{hc^5} = \frac{16\pi^2 G U \cdot k}{hc^5}$$

$$\frac{1}{T} = \frac{16\pi^2 G (Mc^2) k}{hc^5} = \frac{16\pi^2 G M k}{hc^3}$$

$$= \frac{16\pi^2 (6.673\cdot 10^{-11}\ N\cdot m^2/kg^2)(2\cdot 10^{30}\ kg)(1.381\cdot 10^{-23}\ J/K)}{(6.626\cdot 10^{-34}\ J\cdot s)(3\cdot 10^{8}\ m/s)^3} = 1.6\cdot 10^{7}\ K^{-1}$$

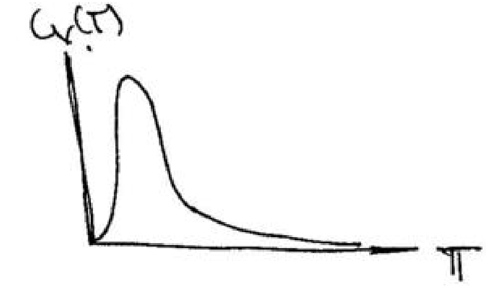
$T = 6.14\cdot 10^{-8}\ K$. ? seems low.

**Prob 3.8**

$$U = N\epsilon e^{-\epsilon/kT}$$

$$C_V \equiv \left.\frac{\partial U}{\partial T}\right|_{V,N} = N\epsilon e^{-\epsilon/kT}\left(\frac{-\epsilon}{kT^2}(-1)\right)$$

$$= \frac{N\epsilon^2}{kT^2} e^{-\epsilon/kT}$$




**Prob 3.9** each atom can then have 2 states CO + OC

thus we have a 2 state paramagnetic

so $\Omega(N) = \binom{N}{2} = \frac{N(N-1)}{2}$

so $S = k\ln\Omega = k\ln\left(\frac{N(N-1)}{2}\right)$ w/ $N \approx 10^{23}$

Prob 3.10

$T_c = 0°C$ $\quad T_f = 25°C$ $\quad T = 0 + 273 = 273k$

(a) $$\Delta S = +\frac{Q}{T} = \frac{l_{ice \to H_2O} \cdot m}{T}$$

$L = 333\ J/g$ melting ice.

$$\Delta S = \frac{(333\,J)(30\,g)}{273k} = 36.59\ J/k.$$

(b) Heat capacity of $H_2O$ is $\quad dS = \frac{C_V dT}{T}$

$1\ cal/k$ so for $30g$

$1\,cal = 4.186\,J$

$$dS = \frac{(30\ cal/k)\,dT}{T} =$$

$$dS = \frac{30(4.186)\ J/k\ dT}{T}$$

So $$\Delta S = \int_{T_i}^{T_f} C_V \frac{dT}{T} = 30(4.186)\ln\left(\frac{T_f}{T_i}\right)$$
$$= 30(4.186)\ln\left(\frac{273+25}{273+0}\right)$$
$$= 11.0\ J/k$$

(c) $\Delta S = - \frac{L \cdot m}{T_{kitchen}}$

$$= - \frac{(333\ J/g)(30g)}{(273 + 25K)} = -33.52\ J/K$$

For the 2nd step of the heating process the $H_2O$ warming up.
I would assume that the kitchen can be taken as a heat bath

So

$$\Delta S_{kitchen} = - \int_{T_i}^{T_f} \frac{C_v dT}{T} \cong \frac{-1}{T_{kit}} \int_{0°C}^{25°C} 30(4.186)\ J/K \cdot dT$$

$$= - \frac{30 \cdot 4.186 \cdot 25}{273+25} = -10.5\ J/K.$$

So in units of J/K

$$\Delta S_{total} = +36.59 + 11 - 33.52 - 10.5 = 3.53\ J/K \text{ increased}$$

Which is what I would expect

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(Prob 3.11)

$\boxed{A} \Leftrightarrow \boxed{B}$

50 liters, 55°C    25 liters, 10°C

For this problem we will Ignore the entropy of mixing + only consider the entropy of heat flow

The $H_2O$ at 55°C will give up heat to the 10°C $H_2O$.

The equilibrium temperature will be where w/ $C = 1 \, cal/gK$

1 liter = 1000 cc = 1000 g $H_2O$.

$$C \cdot m_1 \Delta T_1 + C m_2 \Delta T_2 = 0$$

$$\Rightarrow \quad (50 \cdot 1000 \cdot 10^{-3}\,kg)(T_f - 55°C) + (25 \cdot 1000 \cdot 10^{-3}\,kg)(T_f - 10°C) = 0$$

$$\Leftrightarrow \quad 50(T_f - 55) + 25(T_f - 10) = 0$$

$$2T_f - 2 \cdot 55 + T_f - 10 = 0$$

$$3T_f - 100 = 0$$

$$T_f = 33.3\,°C.$$

Then the amount of ~~heat~~ entropy that flows from Hot to cold would be

$$\Delta S_A = + \int_{55°C}^{33.3°C} \frac{C_v \cdot dT}{T} = + \int_{55°C}^{33.3°C} \frac{m C_v \, dT}{T} = m C_v \ln T_F / T_I$$

$$= 50 \cdot (1000\,g)(1 \, cal/g \cdot k)\left(\frac{4.186\,J}{1\,cal}\right) \ln\left(\frac{273 + 33.3}{273 + 55}\right) = -1.432 \cdot 10^4 \, J/k$$

$$\Delta S_B = 25(1000g)(1\ ^{cal}/_{g\cdot K})\left(\frac{4{,}186\ J}{1\ cal}\right)\ln\left(\frac{273+33{,}3}{273+10}\right)$$

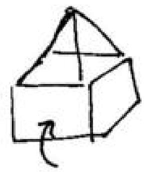
$$= 8{,}27\cdot 10^3\ ^{J}/_{K}$$

$$\Delta S_{total} = \Delta S_A + \Delta S_B = -6.04\cdot 10^3\ ^{J}/_{K}$$

How can this be negative? If I am to calculate the entropy of mixing How am I to do it?

Prob 3.12

$T_{outside} = 0°C = 273\,K$



$T_{inside} = 20°C = 293\,K$

$$\Delta S_{inside} = -\frac{Q}{T_{inside}} =$$

w/ $\Delta S$ + $Q$ in units of per unit time.

$$\Delta S_{outside} = +\frac{Q}{T_{outside}}$$

? What is Q? Heating cost in winter = \$300/month.

at rate of 50 cents per therm = $10^5$ Btu = $1054 \cdot 10^5$ J.

So at 50 cents per therm I am using

$$\frac{300}{.5} = 600 \text{ therms} = 6.32 \cdot 10^{10}\,J.$$

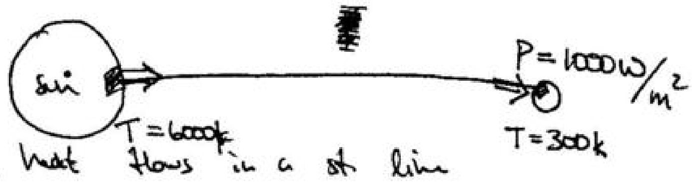
Then $$\Delta S_{inside} = -\frac{6.32 \cdot 10^{10}\,J/\text{month}}{293\,k} = -2.15 \cdot 10^{8}\,J/k\,\text{month}$$

$$\Delta S_{outside} = +\frac{6.32 \cdot 10^{10}\,J}{273\,k} = +2.316 \cdot 10^{8}\,J/k\,\text{month}$$

$\therefore$

$$\Delta S_{total} = +1.66 \cdot 10^{7}\,J/K\,\text{month}$$

Prob 3,13




(a) Assuming heat flows in a st line

The amount of heat leaving at temp $T_s$ + arriving at temp $T_e$ per unit time per unit area shall be

$$P = 1000\ W/m^2$$

$$= + \quad \Delta S_{created} = \frac{P(1\,year)\left(\frac{365\,days}{1\,year}\right)\left(\frac{24\,hr}{1\,day}\right)\left(\frac{3600\,s}{1\,hr}\right)}{300\,k}$$

$$= 1.05 \cdot 10^8\ J/k\,m^2 .$$

(b) So $(1.05 \cdot 10^8\ J/k/m^2)(1\,m^2)$

$\Rightarrow \quad 1.05 \cdot 10^8\ J/k$ of entropy is created in 1 year.

For a significant loss of entropy to occur due to plant growth one must have a plant w/ $S \cong kN$ N molecules per year

$$\Rightarrow \quad N = \frac{1.05 \cdot 10^8\ J/k}{1.381 \cdot 10^{-23}\ J/k} = 7.6 \cdot 10^{30} \quad \text{atoms or mole}$$

$\cong (1.26 \cdot 10^7)$ moles of "plant" growth

per year. What a hell of a plant!

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Prob 3.14

$$C_V = aT + bT^3$$

$a = \cdots$ , $b = \cdots$

$$dS = \frac{dQ}{T} = \frac{C_V\,dT}{T} = \frac{(aT + bT^3)}{T}\,dT = (a + bT^2)\,dT$$

$$S(T) = S_0 + aT + \frac{bT^3}{3}$$

By the 3rd law of thermodynamics $S(0) = 0 \Rightarrow S_0 = 0$.

$$\Rightarrow S(T) = aT + \frac{bT^3}{3}$$

Prob 3.15 Prob 1.55 . . .

Prob 3.16

(a) To remove a gigabyte of memory & have no record of what was there would result in a loss of $2^{30}$

1 kilobyte $= 2^{10}$
1 Mbyte $= (2^{10})^2 = 2^{20}$
1 Gbyte $= 2^{30}$

Guess $S = k \ln \Omega(N)$ w/ $N = 2^{30}$.

& $\Omega(N) = 2^{30}$.

$\therefore \Delta S = k(30) \ln 2$

$\Delta Q = T \Delta S = (300\,K)(1.381 \cdot 10^{-23}\, J/K)(30) \ln 2$ Not significant.

Prob 3.~~18~~

$\frac{29 \; \cancel{99} \; \cancel{100}}{103}$ Schroeder



$N_\uparrow$

100

99

98

$\vdots$

$- \; U = \mu B(N - 2N_\uparrow)$

$M = -\frac{U}{B}$

$\left\{ \frac{1}{T} = \frac{\partial S}{\partial U} \right\}$

$C_V = \frac{\partial U}{\partial T}\Big|_V$

$\Omega = \frac{N!}{N_\uparrow!\,(N - N_\uparrow)!}$

$- \; S/k = \ln \Omega(N_\uparrow)$

$N_\uparrow = 98 \quad N_\downarrow = 100 - 98 = 2 \quad U/\mu B = 100 - 2(98) = N_\downarrow - N_\uparrow = 2 - 98$

$= -96$

$\frac{M}{\mu N} = \frac{N_\uparrow - N_\downarrow}{N} = \frac{98 - 2}{100} = .96$

$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!\,N_\downarrow!} = \frac{100!}{98!\,2!} = \frac{100 \cdot 99}{2} = 50 \cdot 99 = 4950$

$$S/k = \ln \Omega(N_+) = \ln(4910) = \cdots$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = k \frac{\partial}{\partial U}(S/k) = \left(\frac{k}{\mu B}\right)\frac{\partial(S/k)}{\partial(U/\mu B)}$$

$$\Rightarrow T = \frac{\mu B}{k}\frac{\partial(U/\mu B)}{\partial(S/k)} \Rightarrow \frac{kT}{\mu B} = \frac{\partial(U/\mu B)}{\partial(S/k)} \cong \frac{\left(\frac{U}{\mu B}\right)_{i+1} - \left(\frac{U}{\mu B}\right)_{i-1}}{\left(\frac{S}{k}\right)_{i+1} - \left(\frac{S}{k}\right)_{i-1}}$$

$$\frac{-94-(-98)}{11.99-4.61} \cong \frac{4}{7.39} = 5.4 \cdot 10^{-1}$$

**e** $$C_V = \left.\frac{\partial U}{\partial T}\right|_V = k\frac{\partial(U/\mu B)}{\partial(kT/\mu B)} \cong k \cdots$$

$$\Rightarrow \frac{C_V}{k} \cong \frac{\left(\frac{U}{\mu B}\right)_{i+1} - \left(\frac{U}{\mu B}\right)_{i-1}}{\left(\frac{kT}{\mu B}\right)_{i+1} - \left(\frac{kT}{\mu B}\right)_{i-1}} = \frac{-94-(-98)}{.6-.47} = \frac{4}{.13}$$

$$\frac{C_V}{k} = 30.7$$

$$\frac{C_V}{Nk} = .307$$

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Prob 3,19

$$\frac{1}{T} = \frac{\partial S}{\partial U}\bigg|_{N,B} = \frac{\partial S}{\partial N_\uparrow}\frac{\partial N_\uparrow}{\partial U}$$

$$\frac{S}{k} = N\ln N - N_\uparrow \ln N_\uparrow - (N-N_\uparrow)\ln(N-N_\uparrow)$$

$$U = \mu B(N - 2N_\uparrow) \Rightarrow \quad U/\mu B = N - 2N_\uparrow$$

$$\frac{\partial U}{\partial N_\uparrow} = -2\mu B \qquad \hookrightarrow N_\uparrow = \frac{N - U/\mu B}{2}$$

$$\frac{\partial S}{\partial N_\uparrow} = -\ln N_\uparrow - \frac{N_\uparrow}{N_\uparrow} + \ln(N-N_\uparrow) - \frac{(N-N_\uparrow)}{N-N_\uparrow}(-1)$$

$$= -\ln N_\uparrow - \cancel{1} + \ln(N-N_\uparrow) + \cancel{1}$$

$$= -\ln N_\uparrow + \ln(N-N_\uparrow)$$

The

$$\frac{1}{T} = \left(\frac{-k}{2\mu B}\right)\left[\ln\left(\frac{N-N_\uparrow}{N_\uparrow}\right)\right]$$

$$\frac{1}{T} = \frac{-k}{2\mu B}\ln\left(\frac{N - \frac{1}{2}(N - U/\mu B)}{\frac{N-U/\mu B}{2}}\right) = \frac{-k}{2\mu B}\ln\left(\frac{2N - N + U/\mu B}{N - U/\mu B}\right)$$

$$\frac{1}{T} = \frac{-k}{2\mu B}\ln\left(\frac{N + U/\mu B}{N - U/\mu B}\right) = \frac{k}{2\mu B}\ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right) \quad \text{eq } 3.30$$

$$\frac{2\mu B}{Tk} = \ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right)$$

$$(N + U/\mu B)\, e^{\frac{2\mu B}{kT}} = N - U/\mu B$$

$$N\left(e^{\frac{2\mu B}{kT}} - 1\right) = -\frac{U}{\mu B} e^{\frac{2\mu B}{kT}} - \frac{U}{\mu B}$$

$$N = \frac{-\frac{U}{\mu B}\left(e^{\frac{2\mu B}{kT}} + 1\right)}{e^{\frac{2\mu B}{kT}} - 1} \Rightarrow N = -\frac{U}{\mu B}\left(\frac{e^{\frac{\mu B}{kT}} + e^{-\frac{\mu B}{kT}}}{e^{\frac{\mu B}{kT}} - e^{-\frac{\mu B}{kT}}}\right)$$

$$\Rightarrow N = -\frac{U}{\mu B}\,\frac{\cosh\left(\frac{\mu B}{kT}\right)}{\sinh\left(\frac{\mu B}{kT}\right)} \Rightarrow U = -\mu B N \tanh\left(\frac{\mu B}{kT}\right) \quad \text{eq } 3.31$$

$$M = -\frac{U}{B} = \mu N \tanh\left(\frac{\mu B}{kT}\right) \quad \text{eq } 3.32$$

$$C_B = \left.\frac{\partial U}{\partial T}\right|_{N,B} = \frac{-\mu B N}{\cosh^2\left(\frac{\mu B}{kT}\right)}\left(-\frac{\mu B}{kT^2}\right) = \frac{Nk\left(\frac{\mu B}{kT}\right)^2}{\cosh^2\left(\frac{\mu B}{kT}\right)} \quad \text{eq } 3.33$$

$$\left\{ \frac{d}{dx}\tanh(x) = \frac{d}{dx}\frac{\sinh(x)}{\cosh(x)} = \frac{\cosh(x)}{\cosh(x)} - \frac{\sinh(x)\sinh(x)}{\cosh^2(x)} \right.$$

$$= 1 - \left(\frac{\sinh x}{\cosh x}\right)^2 \qquad \cosh^2 - \sinh^2 = 1$$

$$= \frac{\cosh^2 - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

Prob 3,20 $B = 2{,}06\,T$ $T = 2{,}2\,k.$

$\mu = \mu_B = 5.788\cdot 10^{-5}\ eV/T$ $k = 8{,}617\cdot 10^{-5}\ eV/k$

$$U = -N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

$$\frac{U}{(N\mu B)} = -\tanh\left(\frac{\mu B}{kT}\right) = -\tanh\left(\frac{5{,}788\cdot 10^{-5}\cdot 2{,}06}{8{,}617\cdot 10^{-5}\cdot 2{,}2}\right)$$

$$\frac{M}{N\mu} = \tanh\left(\frac{\mu B}{kT}\right) = \quad \cdots$$

$$\frac{S}{k} = ? \qquad \frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{N,B}$$

$$\|$$

$$\frac{k}{2\mu B}\ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right) = \left.\frac{\partial S}{\partial U}\right|_{N,B}$$

$\rightarrow$ ~~S~~ $S = S(U)$ $\cdots$

$$\rightarrow \quad \frac{N_\uparrow}{N} = .99 \quad \Rightarrow \quad \frac{S}{k} = N\ln N - N_\uparrow \ln N_\uparrow - (N - N_\uparrow)\ln(N - N_\uparrow)$$

$\ast$ $\star$

$$\frac{S}{k} = N\ln N - N\frac{N_\uparrow}{N}\ln\left(N\frac{N_\uparrow}{N}\right)$$

$$- N\left(1 - \frac{N_\uparrow}{N}\right)\ln\left(N\left(1 - \frac{N_\uparrow}{N}\right)\right)$$

$$\frac{S}{k} = N\ln N - N\left(\frac{N_\uparrow}{N}\right)\left[\ln N + \ln\frac{N_\uparrow}{N}\right]$$

$$- N\left(1-\frac{N_\uparrow}{N}\right)\left[\ln N + \ln\left(1-\frac{N_\uparrow}{N}\right)\right]$$

... To obtain 99% magnetization

$$\frac{M}{N\mu} = .99 = \tanh\left(\frac{\mu B}{kT}\right)$$

$\Rightarrow$ make argument of $\tanh(x)$ closer to $+\infty$

$\Rightarrow$ increase B or lower the temp. or both

Prob 3,21

$$\frac{M}{N} = \mu \tanh\left(\frac{\mu B}{kT}\right)$$

$$= \left(5 \cdot 10^{-8} \frac{eV}{T}\right) \tanh\left(\frac{\left(5 \cdot 10^{-8} \frac{eV}{T}\right)(.63T)}{\left(8.617 \cdot 10^{-5} \frac{eV}{K}\right)(300K)}\right)$$

$$= 6.09 \cdot 10^{-14} \ \frac{eV}{T}$$

$$h f = E_{photon} = \mu B =$$

$$\frac{hc}{\lambda} = \mu B$$

$$\lambda = \frac{hc}{\mu B} = \frac{(4.136 \cdot 10^{-15} \ eV \cdot s)(2.998 \cdot 10^{8} \ m/s)}{\left(5 \cdot 10^{-8} \frac{eV}{T}\right)(.63T)}$$

$$= 39.3 \ m.$$

Prob 3,22 combine prob 3,23

Prob 3,23

$$\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{N,B}$$ See next pg.

$$\cancel{\frac{k}{2\mu B} \ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right)} = \left.\partial S/\partial U\right|_{N,B}$$

$$\frac{\partial (S/k)}{\partial (U/2\mu B)} = \ln(N - U/\mu B) - \ln(N + U/\mu B)$$

Prob 3,23

Show

$$S = Nk\left[\ln(2\cosh x) - x\tanh x\right] \qquad x = \frac{\mu B}{kT}$$

From $\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{N,B} = \frac{\partial S}{\partial T}\Big|_{N,B} \cdot \frac{dT}{dU}$.

$$U = -N\mu B\tanh\left(\frac{\mu B}{kT}\right)$$

$$\frac{dU}{dT} = \frac{Nk\,(\mu B/kT)^2}{\cosh^2(\mu B/kT)}$$

$$\frac{\partial S}{\partial T}\Big|_{N,B} = \frac{1}{T}\frac{dU}{dT} = \frac{Nk}{T}\,\frac{(\mu B/kT)^2}{\cosh^2(\mu B/kT)}$$

define $x = \frac{\mu B}{kT}$

$$dx = -\frac{\mu B}{kT^2}\,dT$$

$$dx = -\frac{\mu B}{kT}\,\frac{dT}{T}$$

$$dx = -x\,\frac{dT}{T}$$

$$\frac{1}{T} = -\frac{1}{x}\frac{dx}{dT} \checkmark$$

$$\Rightarrow \frac{\partial S}{\partial T}\Big|_{N,B} = Nk\,\frac{x^2}{\cosh^2(x)}\left(\frac{-1}{x}\right)\frac{dx}{dT}$$

$$= Nk\,\frac{x}{\cosh^2(x)}\,\frac{dx}{dT}$$

$$S(T) = Nk\left[-\ln(\cosh(x)) + x\tanh(x)\right] + S_0.$$

$$S(0) = \cancel{Nk}\ \underline{\underline{\ }}\ ?$$

$$\lim_{x\to+\infty}\left(x\tanh(x) - \ln(\cosh(x))\right) = \text{“}\infty - \infty\text{”}$$

$$= \ln\left[\lim_{x\to+\infty} e^{x\tanh(x) - \ln(\cosh(x))}\right]$$

$$= \ln\left[\lim_{x\to+\infty} \frac{e^{x\tanh(x)}}{\cosh(x)}\right] \qquad \frac{+\infty}{+\infty}$$

$$= \ln\left[\lim_{x\to+\infty} \frac{e^{x}}{\cosh(x)}\right] \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$= \ln\left[\lim_{x\to+\infty} \frac{e^{x}}{\frac{1}{2}(e^x + e^{-x})}\right] = \ln(2)$$

$\therefore\ S(0) = Nk\ln 2 + S_0 \equiv 0$ by 3rd law thermo $\Rightarrow S_0 = -Nk\ln 2$

$$\therefore\ S(T) = Nk\left[-\ln(\cosh(x)) + x\tanh(x) - \ln 2\right]$$

$$= -Nk\left[\ln(2\cosh(x)) - x\tanh(x)\right] \quad \text{w/ } x =$$

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Prob 3.24 For an einstein solid $q \gg N$ $\qquad U = q\epsilon.$

$$\frac{\partial U}{\partial q} = \epsilon$$

$$\Omega(N,q) = \left(\frac{eq}{N}\right)^N \qquad \frac{S}{k} = \ln \Omega(N,q)$$

$$\frac{1}{T} = T^{-1} \equiv \frac{\partial S}{\partial U} = \frac{\partial q}{\partial U}\frac{\partial S}{\partial q} = \frac{1}{\epsilon}\frac{\partial S}{\partial q}$$

$$\Rightarrow \frac{\epsilon}{T} = \frac{\partial S}{\partial q}$$

$$\Rightarrow \frac{T}{\epsilon} = \frac{\partial q}{\partial S} = \frac{\partial q}{\partial(S/k)\cdot k} \Rightarrow \frac{kT}{\epsilon} = \frac{\partial q}{\partial(S/k)} \cong \frac{q^{n+1} - q^{n-1}}{(\frac{S}{k})^{n+1} - (\frac{S}{k})^{n-1}}$$

$$C_B = \frac{\partial U}{\partial T} = \frac{\partial(q\epsilon)}{\partial(kT/\epsilon)\cdot(\frac{\epsilon}{k})} = \frac{\partial q}{\partial(kT/\epsilon)}k$$

$$\Rightarrow \frac{1}{N}\frac{C_B}{k} = \frac{1}{N}\frac{\partial q}{\partial(kT/\epsilon)}$$

entropy v.s. energy

+ C v.s. T

Prob 3.25

$$\Omega(N,q) \cong \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N$$

(a) $S = k \ln \Omega(N,q) = k\left[q \ln(q+N) - q \ln q + N \ln(q+N) - N \ln N\right]$

$\cancel{k[q \ln(q+N)}$

$$U = \epsilon q \qquad \frac{dU}{dq} = \epsilon$$

(b) $\dfrac{1}{T} = \dfrac{\partial S}{\partial U} =$

$$= \frac{\partial S}{\partial q} \cdot \frac{dq}{dU} = \frac{1}{\epsilon}\frac{\partial S}{\partial q} = \frac{k}{\epsilon}\left[\ln(q+N) + \frac{q}{q+N} - \ln q - 1 + \frac{N}{q+N}\right]$$

$$\Rightarrow \frac{1}{T} = \frac{k}{\epsilon}\left[\ln\left(\frac{q+N}{q}\right) \cancel{-1} + \cancel{1}\right]$$

$$\frac{1}{T} = \frac{k}{\epsilon}\ln\left(1+\frac{N}{q}\right) \Rightarrow T = \frac{\epsilon}{k}\frac{1}{\ln(1+\frac{N}{q})}$$

(c) $\dfrac{kT}{\epsilon} = \dfrac{1}{\ln(1+\frac{N}{q})} \Rightarrow \ln(1+\frac{N}{q}) = \dfrac{\epsilon}{kT}$

$$1 + \frac{N}{q} = e^{\epsilon/kT}$$

$$1 - e^{\epsilon/kT} = +\frac{N}{q}$$

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$$q = \frac{N}{1 - e^{\epsilon/kT}}$$

$$U = \frac{N\epsilon}{1 - e^{\epsilon/kT}}$$

$$C = \frac{\partial U}{\partial T} = \frac{N\epsilon}{(1-e^{\epsilon/kT})^2}(-e^{\epsilon/kT})(-\epsilon/kT^2)$$

$$= \frac{Nk\left(\frac{\epsilon^2}{k^2T^2}\right)e^{\epsilon/kT}}{\left(1-e^{\frac{\epsilon}{kT}}\right)^2}$$

(d) $\lim T \to +\infty$ $\quad \frac{0}{0} \quad e^x \cong 1 + x + \frac{x^2}{2}$

$Nk$

$$\lim_{T\to+\infty} C = \lim_{T\to+\infty} \frac{Nk\left(\frac{\epsilon}{kT}\right)^2}{\left(1-1-\left(\frac{\epsilon}{kT}\right)\right)^2} = Nk$$

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Prob 3,26

~~[illegible]~~ From prob 3,25

$$C = Nk\left(1 - \frac{1}{12}\left(\frac{\epsilon}{kT}\right)^2\right)$$

$$= nR\left(1 - \frac{1}{12}\left(\frac{\epsilon}{kT}\right)^2\right)$$

$Nk = nR$

At $T = 1000\,k$

$$C = \frac{5}{2}R$$

$$\frac{5}{2}R = 1 \cdot R\left(1 - \frac{1}{12}\left(\frac{\epsilon}{8.617 \cdot 10^{-5}\,\mathrm{eV/k})(1000)}\right)\right) \Rightarrow \epsilon = \cdots$$

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Prob 3,27

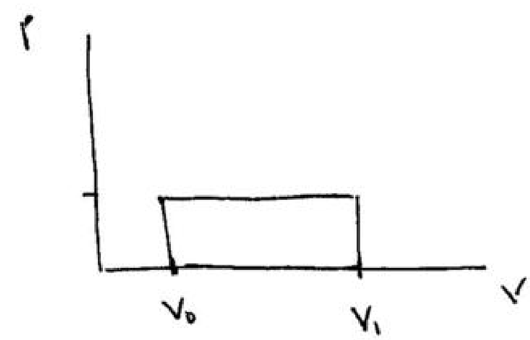
$dU = Q + W$

$dU = TdS - pdV$ at const entropy

$-p = \left.\frac{\partial U}{\partial V}\right|_S$

$p = -\left.\frac{\partial U}{\partial V}\right|_S$ did I already know this? I don't see how...

Prob 3,28




$W = P(V_1 - V_0)$  $\Delta U = Q + W$

For constant pressure processes like we have here

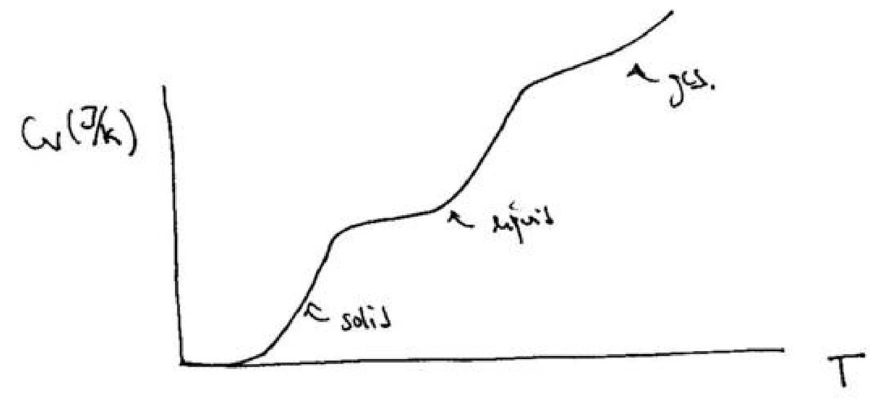
$(\Delta S)_p = \int_{T_i}^{T_f} \frac{C_p\,dT}{T} \cong C_p \ln\left(\frac{T_f}{T_i}\right)$

Assuming $PV = nRT$

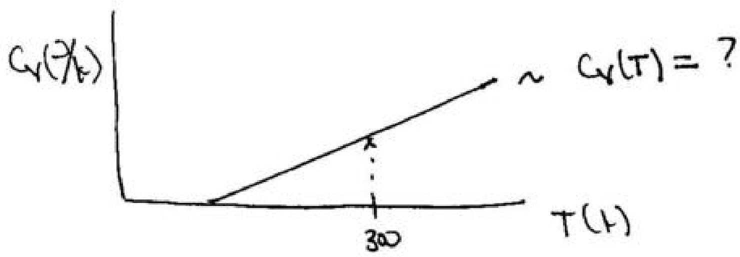
$\frac{T_f}{T_i} = \frac{\cancel{P_f} V_f}{\cancel{P_i} V_i} = 2$  $(\Delta S)_p \cong C_p \cdot \ln 2$.

What is $C_p$ = ?

Prob 3,29 I would assume it has a shape like that seen on pg 30




Prob 3,30




$$\left.\begin{array}{l} C_V(300k) = 7.5 \text{ J/k} \\ C_V(400k) = 10 \text{ J/k}. \end{array}\right\} \rightarrow C_V(T) = 7.5 + \frac{2.5}{(100)}(T-300)$$

$$\Delta S = \int_{298}^{500} \frac{C_V dT}{T} = \int_{298}^{500} \frac{(7.5 + 2.5 \cdot 10^{-2}(T-300))}{T} dT = 7.5 \ln\left(\frac{500}{298}\right)$$

$$+ 2.5 \cdot 10^{-2}\left[(500-298) - 300 \ln\left(\frac{500}{298}\right)\right]$$

$$= 3.88 + 5.05 - 3.88 = 5.05 \text{ J/k}$$

$$S(500k) = \Delta S_{298 \rightarrow 500} + \underset{2.38 \text{ J/k}}{S}$$

Prob 3.31 $\quad C_p = a + bT - \dfrac{c}{T^2} \qquad a = \cdots,\ b = \cdots,\ c = \cdots$

$$\Delta S_{298 \to 800} = \int_{298}^{800} \frac{C_p\,dT}{T} = \cancel{a(800-298) + \frac{b}{2}(800^2 - 298^2) + \frac{c}{T}}\Bigg|_{298}^{800}$$

$$= \cancel{3.4 \cdot 10^{3}\ \mathrm{J/k}} + \cancel{3.84 \cdot 10^{2}\ \mathrm{J/k}}$$

$$\approx \int_{T_i}^{T_f} \left( \frac{a}{T} + b - \frac{c}{T^3} \right) dT = a \ln T + b(T) + \frac{c}{2T^2} \Bigg|_{T_i}^{T_f}$$

$$= a \ln\left(\frac{T_f}{T_i}\right) + b(T_f - T_i) + \frac{c}{2}\left(\frac{1}{T_f^2} - \frac{1}{T_i^2}\right)$$

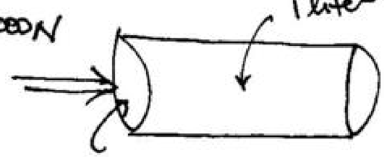
$$= 8.72 + 9.6 \cdot 10^{-1} - 3.1 \cdot 10^{0} = 6.57\ \mathrm{J/k}$$

$$\Delta S_{0 \to 298} = 8.53\ \mathrm{J/k}$$

$S(800\,k) \cong$ 15 J/k.

Prob 3.32

$F = 2000\,N$



1 liter $= 1000\ cm^3 = V$

$T = 300\,k$

$P = 10^5\ N/m^2 = 1\,atm.$

$A = .01\,m^2$

$\Delta x = 10^{-3}\,m$

Work applied by

(a) $W \cong \cancel{-}\, \Delta P \cdot A \cdot \Delta x$ done on gas

$= -(PA - F)\Delta x$

$= -(10^5 \cdot (.01) - 2000)(10^{-3})$

$= +1\,J.$

(b) Since the process occurred very quickly ... I am going to assume that it is ~~quasistatic~~ adiabatic + $\therefore$ no heat was added to the gas.

(c) $\Delta U = W + \cancel{Q}^{\,0}$

$\Delta U = 1\,J.$

(d) $dU = TdS - PdV \Rightarrow dS = \frac{1}{T}dU + \frac{P}{T}dV.$

Assuming again the adiabatic assumption ... + this not affecting ?? temperature ...

How calculate the change in entropy?

(Prob 3.33) Pr:

$$C_V = T \frac{\partial S}{\partial T}\bigg|_V$$

The thermodynamic identity is:

$$dU = T\,dS - p\,dV + \cancel{\mu\,dN}^{\,0}$$ Assuming no loss of particles

B. $C_V \equiv \frac{\partial U}{\partial T}\bigg|_V$

$$\therefore \quad dU\big|_V = T\,dS\big|_V$$

$$\Rightarrow \quad \frac{dU}{dT}\bigg|_V = T\frac{dS}{dT}\bigg|_V = C_V.$$

$$H = U + pV.$$

$$dH = dU + p\,dV + V\,dp \qquad C_p \equiv \frac{\partial U}{\partial T}\bigg|_p$$

$$dH\big|_p = dU\big|_p + p\,dV\big|_p.$$

$$\frac{\partial H}{\partial T}\bigg|_p = \frac{\partial U}{\partial T}\bigg|_p + p\frac{\partial V}{\partial T}\bigg|_p \Rightarrow \quad C_p = \frac{\partial H}{\partial T}\bigg|_p - p\frac{\partial V}{\partial T}\bigg|_p$$ Is this not correct?

$$dH = T\,dS - \cancel{p\,dV} + \cancel{p\,dV} + V\,dp$$

$dH = TdS + Vdp \quad \div dT$ at constant V

~~#####~~

$$\frac{\partial H}{\partial T}\Big|_V = T\frac{\partial S}{\partial T}\Big|_V + V\frac{\partial p}{\partial T}\Big|_V$$

$$\| \qquad \|$$

$$C_p = C_V + V\frac{\partial p}{\partial T}\Big|_V$$

from chase ... ?

Prob 3.34

(a) links

$\boxed{1}$ $\boxed{2}$ . . . $\boxed{k}$ $\boxed{k+1}$ . . . $\boxed{N}$

each link can point to the left or to the right, thus if I specify the # pointing left ~~there is still on~~ (and by doing so define a macrostate of the system) the # pointing right is determined but not its distribution of the . $N_+$ = # pointing right

$$\Omega(N_+) = \binom{N}{N_+}$$

$$S = k\ln\Omega(N_+) = k\ln\left(\frac{N!}{(N-N_+)!\,N_+!}\right) = k\Big[N\ln N - \cancel{N} - (N-N_+)\ln(N-N_+)$$

$$+ (N-N_+)$$

$$- N_+\ln N_+ + \cancel{N_+}\Big]$$

$$S = k\left[ N \ln N - (N - N_+) \ln (N - N_+) - N_+ \ln N_+ \right]$$

(b) $L = +\ell N_R - \ell N_L = \ell(N_R - N_L) = \ell(N_R - N + N_R)$

$= \ell(2N_R - N)$

(c)


w/ a tension force of F one would have a spring constant k?

$F = -kx \qquad U = \frac{1}{2}kx^2$

$x = -\frac{F}{k} \qquad U = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}\frac{1}{k}F^2$

$dU = \frac{F}{k}dF$

... I don't see how to derive this from 1st principles but only by analogy.

$dU = TdS - pdV$

$\cong \quad dU = TdS - F\,dL$

(d) $F = T \left.\frac{\partial S}{\partial L}\right|_U = T \frac{\partial S}{\partial N_R} \cdot \frac{dN_R}{dL}$

$\frac{dL}{dN_R} = 2\ell \qquad \frac{\partial S}{\partial N_R} = k\left[ \ln(N - N_R) + 1 - 1 - \ln N_R \right]$

$= k\left[ \ln(N - N_R) - \ln N_R \right]$

$$\therefore \quad F = \frac{Tk}{2l}\left[\ln(N-N_R) - \ln N_R\right]$$

By $\quad \frac{L}{l} = 2N_R - N \quad \Rightarrow \quad \frac{1}{2}\left(\frac{L}{l}+N\right) = N_R$ ✓

$$2\left(\frac{N_R}{N}\right) - 1 = \frac{L}{lN}$$ ✓

$$\therefore \quad F = \frac{kT}{2l}\left[\ln\left(\frac{N}{N_R} - 1\right)\right]$$

$$\frac{N_R}{N} = \frac{1}{2}\left(\frac{L}{lN}+1\right)$$ ✓

$$\frac{N}{N_R} = \frac{2}{\left(1+\frac{L}{lN}\right)}$$

$\therefore$

$$F = \frac{kT}{2l}\ln\left(\frac{2}{1+\frac{L}{lN}}\right) \neq \quad \text{if} \quad \frac{L}{lN} \ll 1$$

(e)

$$\frac{2}{1+\frac{L}{lN}} \cong 2\left(1-\frac{L}{lN}\right)$$

$$\cong \frac{kT}{2l}\ln\left(2\left(1-\frac{L}{lN}\right)\right)$$

$x \ll 1$

$\ln(1+x) \cong x$

$$= \frac{kT}{2l}\left(\ln\left(1-\frac{L}{lN}\right) + \ln 2\right)$$

$$\cong \frac{kT}{2l}\cdot\left(-\frac{L}{lN}\right) + \frac{kT}{2l}\ln 2$$

$$\therefore \quad F = -\frac{kT}{2l^2N}\cdot L + F_0$$

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(f) By $T \uparrow$ ~~then~~ the ~~bad~~ force in the ~~a~~ band increases

$\therefore$ it contracts,

This makes sense since the entropy $\left\{ \frac{1}{T} \equiv \frac{\partial S}{\partial U}\bigg|_{N,V} \right\}$

has to get ....

(g) By stretching I make the tension in the band increase.

Based on $F = -\frac{kT}{2l^2N} \cdot L + F_0$ I would expect that the

temperature would increase. ...

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~~9-100~~

9-10-02 /

$$\Omega(N,q) = \frac{(q+N-1)!}{q!(N-1)!}$$

$\{ N = \#$ of particles
$q =$ total energy shared among all particles.

If $N=3$ $q=3$

$$\Omega(3,3) = \frac{5!}{3!\,2!} = \frac{5\cdot 4}{2} = 10$$

$$\Omega(4,q) = \frac{(q+3)!}{q!\,3!} = \frac{(q+3)(q+2)(q+1)}{3\cdot 2} = 10$$

$$(q+3)(q+2)(q+1) = 60$$

If $q=2$

$5\cdot 4\cdot 3 = 60$ ✓.

$$\cancel{\Omega(N+1,q)} = \frac{(q+N)!}{q!\,N!} = \Omega(N,q) \cancel{=} \frac{(\cancel{q+N-1})!}{q!\,(N-1)!}$$

$\Omega(N,q) = \ldots$

$$\Omega(N+1,q) = \frac{(q+N+1-1)!}{q!\,N!} = \frac{(q+1+N-1)!}{q!\,(N-1+1)!}$$

$$= \frac{(q+1+N-1)}{N}\frac{(q+N-1)!}{q!\,(N-1)!} = \frac{q+N}{N}\,\Omega(N,q)$$

$$\Omega(N+1,q') = \Omega(N,q)$$

$$\parallel$$

$$\frac{(q'+N)!}{q'!\,N!} = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S = Nk\left[\ln\left(V\left(\frac{4\pi m U}{3h^2}\right)\frac{1}{N\cdot N^{3/2}}\right) + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(V\left(\frac{4\pi m U}{3h^2}\right)\right) - \frac{5}{2}\ln N + \frac{5}{2}\right]$$

$$\frac{\partial S}{\partial N} = k\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right] + Nk\left[-\frac{5}{2}\frac{1}{N}\right]$$

$$= k\left[\ln\left(\qquad\right) + \frac{\cancel{5}}{2}\right] - \frac{5}{\cancel{2}}k$$

$$\mu = -T\left.\frac{\partial S}{\partial N}\right|_{U,V} = -Tk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right)\right] \quad \text{eq } 3.63$$

Prob 3.35

3 ossilators & 4 units of energy

$$\mu \cong \frac{\Delta U}{\Delta N}\Big|_S = \frac{\quad}{1}$$

$$\Omega(N,q) = \frac{(q+N-1)!}{q!\,(N-1)!}$$

From this expression Adding 1 to N would require decreasing q. I would think it would require decreasing q by more then 1 since w/ q=3 decreasing q by 1 was all that was required.

P. 3.36 $N \gg 1 \quad q \gg 1$

(a) $$\mu \equiv -T \frac{\partial S}{\partial N}\Big|_{U,V}.$$

From problem 2.18 $\Omega(N,q) \cong \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N$

$$S = kq \ln\left(\frac{q+N}{q}\right) + kN \ln\left(\frac{q+N}{N}\right)$$

$$\frac{\partial S}{\partial N} = kq \frac{1}{\frac{q+N}{q}}\left(\frac{1}{q}\right) + k\ln\left(\frac{q+N}{N}\right) + kN \frac{1}{\frac{q+N}{N}} \cdot \left(\frac{-q}{N^2}\right)$$

$$\frac{\partial S}{\partial N} = k\frac{\cancel{q}}{\cancel{q}+N} + k\ln\left(\frac{q+N}{N}\right) + \frac{k(\cancel{-q})}{q+N}$$

$$= k\ln\left(\frac{q+N}{N}\right)$$

$$\mu = -Tk\ln\left(\frac{q+N}{N}\right) = -kT\ln\left(1+\frac{q}{N}\right)$$

$$= \cancel{kT\ln\left(\frac{N}{N+q}\right)} = kT\ln\left(\frac{N}{N+q}\right)$$

(b) $N \gg q$ $\qquad$ $N \ll q$

$$= kT\ln\left(\frac{N/q}{1+N/q}\right)$$

$$\cancel{\mu \cong -kT\ell}$$

$$\cong kT\ln\left(\frac{N}{q}\left(1-\frac{N}{q}\right)\right)$$

$$\mu \cong -kT\frac{q}{N} \qquad\qquad \mu \cong -kT\ln\left(\frac{q}{N}\right)$$

$$= kT\ln\left(\frac{N}{q}\right)$$

Prob 3.37

$$\mu = -T \frac{\partial S}{\partial N}\bigg|_{U,V} \qquad \mu = \frac{\partial U}{\partial N}\bigg|_{S,V}$$

If $U = N \cdot f \cdot \frac{1}{2}kT + N \cdot mg \cdot z$

$= N\left(\frac{f}{2}kT + mgz\right)$ For a monatomic gas $f=3$

$$U = N\left(\frac{3}{2}kT + mgz\right)$$

Deriving from $\mu = -T \frac{\partial S}{\partial N}\bigg|_{U,V}$

expect that Sackur-Tetrode eq does not change:

$S = \ldots$ eq 3.62

so

$$\mu = -kT \ln\left[\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right]$$ eq 3.63 still holds

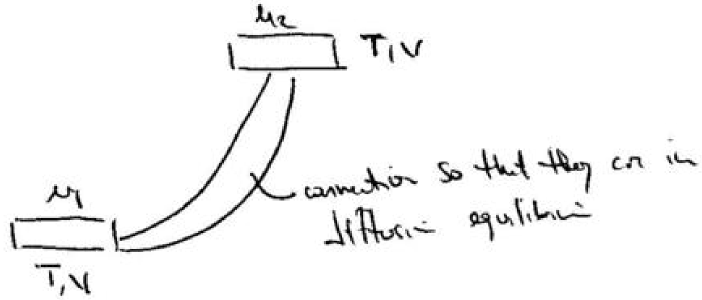
$\Rightarrow$

$$\mu = -kT \ln\left[\frac{V}{N}\left(\frac{4\pi m}{3h^2}\cdot\left(\frac{3}{2}kT + mgz\right)\right)^{3/2}\right]$$

$$= -kT \ln\left[\frac{V}{N}\left(\frac{2\pi m kT}{h^2} + \frac{4\pi m}{3h^2}\cdot mgz\right)^{3/2}\right]$$

(a) ... Don't see how to get.

(b)




$\Rightarrow \quad \mu_1 = \mu_2$

$$-kT \ln\left[\frac{V}{N_0}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right] = -kT \ln\left[\frac{V}{N(z)}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right] + mgz$$

$$\frac{\cancel{V}}{N(0)}\cancel{\left(\quad\right)^{3/2}} = \frac{\cancel{V}}{N(z)}\cancel{\left(\quad\right)^{3/2}} \cdot \exp\left\{-\frac{mgz}{kT}\right\}$$

$$\Rightarrow \quad N(z) = N_0 \exp\left\{-\frac{mgz}{kT}\right\}$$

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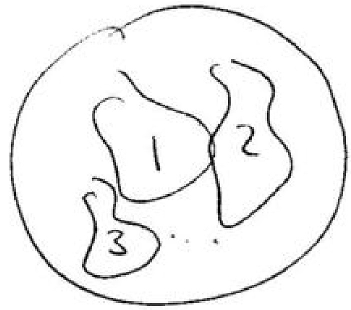
3.38




$$\mu_1 = -T\frac{\partial S}{\partial N_1}\Big|_{U,V,N_2,\dots} \qquad \mu_2 = -T\frac{\partial S}{\partial N_2}\Big|_{U,V,}$$

By an ideal gas we assume no ~~no~~ interactions among the particles. $\Rightarrow$ $\Omega(N) = \Omega(N_1)\,\Omega(N_2)\cdots\Omega(N_n)$ but they are distinguishable & thus for ~~no~~ no division is needed.

$$\Rightarrow \quad S = \sum_{k=1}^{n} S_k .$$

w/ each gas obeying the Sack. - Tetrod eq.

$$\Rightarrow \quad \mu = \sum_k \mu_k \qquad \text{w/} \quad \mu_k = -kT \ln\left[\frac{V}{N_k}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right]$$

$$P_k V = k N_k T$$

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3.39 Prob 2.32 I got.

$$S = Nk \left\{ \ln\left(\frac{A}{N}\left(\frac{2\pi m U}{Nh^2}\right)\right) + 2 \right\}$$

$$dU = TdS - pdV + \mu dN \quad ; \quad dU = TdS - \tau dA + \mu dN$$

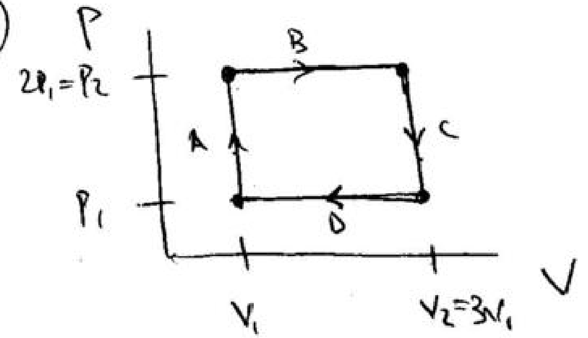
$$T = \left.\frac{\partial U}{\partial S}\right|_{V,N} \rightarrow \frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{V,N}$$

$$p = -\left.\frac{\partial U}{\partial V}\right|_{S,N}$$

$$dS = \frac{dU}{T} + \frac{\tau}{T} dA - \frac{\mu}{T} dN$$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{A,N} \quad ; \quad \frac{\tau}{T} = \left.\frac{\partial S}{\partial A}\right|_{U,N} \quad ; \quad -\frac{\mu}{T} = \left.\frac{\partial S}{\partial N}\right|_{T,A}$$

4.1




$$dU = TdS - pdV + \mu dN$$

$$\therefore dU = -pdV$$

$$W_A = 0 = W_C$$

$\{ W > 0$ if gas does it.

$\Delta V > 0 \; ; \; p > 0.$

$\therefore \Delta W = p\Delta V$

$$W_B = + \int_{V_i}^{V_f} p\,dV$$

$$= p_B \Delta V = 2p_1 2V_1 = 4p_1V_1$$

$PV = nRT$

$$W_D = p_D \Delta V = p_1(-2V_1) = -2p_1V_1$$

$$W_T = \sum W_i \quad \Leftarrow \text{simply mechanical processes} \ldots$$

For an ideal Gas $U = U(T) \Rightarrow \frac{\partial U}{\partial V} = 0$ i.e. does not depend at all on mechanical processes...

So $\Delta U_A = \frac{3}{2} kN \cdot \Delta T_A$

$$U = N \cdot \frac{3}{2} kT \quad + \quad PV = NkT$$

$$U(P,V) = N \cdot \frac{3}{2} k \left( \frac{PV}{Nk} \right) = \frac{3}{2} PV$$

$$\Delta U = \frac{3}{2} \Delta (PV)$$

$\Delta U = Q + W_{us}$ $\qquad \Delta U = Q - W_{gas} \Rightarrow Q = \Delta U + W_{gas}$

$\Delta U_A = \frac{3}{2} V_1 \Delta P = \frac{3}{2} V_1 P_1$ ✓ $\qquad \Delta U_C = \frac{3}{2}(3V_1)(-P_1)$ ✓

$\Delta U_B = \frac{3}{2}(2P_1)(2V_1)$ ✓ $\qquad \Delta U_D = \frac{3}{2}P_1(-2V_1)$ ✓

$Q >$ when absorbed into gas. $\qquad Q = \underline{\Delta U} + W_{gas}$

$Q_A = \frac{3}{2} V_1 P_1 + 0$
$= \frac{3}{2} V_1 P_1$

$Q_C = -\frac{9}{2} V_1 P_1 + 0 = -\frac{9}{2} V_1 P_1$

$Q_B = 6P_1V_1 + 4P_1V_1$
$= 10 P_1 V_1$

$Q_D = -3P_1V_1 - 2P_1V_1 = -5P_1V_1$

~~Results~~

total heat absorbed / given off

$= \left(\frac{3}{2} + \frac{20}{2} - \frac{9}{2} - \frac{10}{2}\right) V_1 P_1 = \frac{2}{2} P_1 V_1 > 0$

heat is absorbed.

total work done $= 2P_1V_1 > 0$ work is done by the gas

$\therefore$ efficiency of cycle $= \frac{W}{Q} = \frac{2P_1V_1}{P_1V_1} = 2$ !!??

(b) Assume that the correct eel would
have given $\frac{1}{4}$.
cal. max/min temp when lapsing and cycle

then compt $1 - \frac{T_{min}}{T_{max}}$

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Prob 4.2

$T_h = 500°C$
$T_c = 20°C$

(a) $e_{max} = 1 - \frac{20}{500} = 1 - \frac{1}{25} = 1 - .04 = .96$

(b) $e'_{max} = 1 - \frac{20}{600} = 1 - \frac{1}{30} \approx 1 - .03 = .97$

$\frac{1}{30} > .03$

$\frac{1}{3} = .33\ldots \quad -\frac{1}{30} < -.03$

Change in efficiency of 1%

∴ Produce 1% more energy

Assuming the plant produces 1GW (kw/hrs of power) orginally

it now ~~...~~ produces an addition $10^9(.01) = 10^7$ Watts

~~(.01)~~ =

~~...~~ . . . of power (prob. per day)

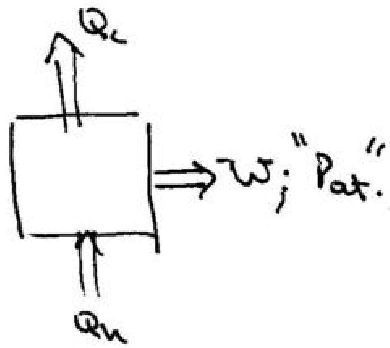
$= 4.16 \cdot 10^5$ w/hr additional + assuming there is a demand

for this addition power you make

$(4.16 \cdot 10^2 \text{ kw/hr})(.05 \$/(\text{kw/hr})) = \$20.8$

Prob 4.3 $P_{at} = 10^9$ W.

$e = .4$




$$e \equiv \frac{W}{Q_h}$$

$$Q_h = Q_c + W.$$

(a) What is $Q_c$?

$$e = \frac{W}{Q_c + W} = \frac{\dot{W}}{\dot{Q}_c + \dot{W}} = \frac{1}{\frac{\dot{Q}_c}{\dot{W}} + 1}$$

$$\Rightarrow \quad \frac{\dot{Q}_c}{\dot{W}} + 1 = \frac{1}{e}$$

$$\dot{Q}_c = \dot{W}\left(\frac{1}{e} - 1\right) = (1.5)\cdot 10^9 \text{ W}.$$

(b) Plant dumps $1.5 \cdot 10^9$ W of power

$= 1.5 \cdot 10^9$ J of heat per second.

w/ a flow rate (vol/time) of $100 \text{ m}^3/\text{s}$

~~we have 1 m$^3$ of $H_2O$ absorb~~ each second we have

100 $m^3$ of $H_2O$ absorbing $1.5 \cdot 10^9$ J of heat

$$C_p(H_2O) = 75.3\ \text{J}/\text{K}\cdot\text{mol}$$

$$V = 18.068\ \text{cm}^3/\text{mol}$$

$$C_p(H_2O) = 75.3\ \text{J}/\text{K}\cdot\text{mol} \cdot \frac{1\ \text{mole}}{18.068\ \text{cm}^3} \cdot \left(\frac{1\,\text{cm}}{10^{-2}\,\text{m}}\right)^3$$

$$= 4.16 \cdot 10^6\ \frac{\text{J}}{\text{K}\cdot\text{m}^3}$$

Thus in 1 second

$$1.5 \cdot 10^9\ \text{J} = \left(4.16 \cdot 10^6\ \frac{\text{J}}{\text{K}\cdot\text{m}^3}\right)(100\ \text{m}^3)(T_f - 300\,\text{K})$$

$$T_f = 303.6$$

By ≈ 3.6 degr

(c) Have to dispose of $1.5 \cdot 10^9$ J of heat per second.

For evaporation cham amount of heat liberated would be

$$\ell = 2260\ \text{J}/\text{g} = 40.6\ \text{kJ}/\text{mol}$$

Require to know $\text{m}^3/\text{s}$ or vol flow rate of fluid $\equiv q$

to evaporate $(1.5 \cdot 10^9\ \text{J}/\text{s})\ell = q$ ——

$1\,mol = 18\,g.$

$1\,g = 1\,cm^3$

$1\,mol = 18\,cm^3 = 18\,(10^{-2})^3\,m^3$

$$\ell = 40.6\,\frac{kJ}{mol} \cdot \frac{1\,mol}{18 \cdot (10^{-2})^3\,m^3} = \frac{40.6 \cdot 10^3}{18} \cdot (10^6)\ J/m^3$$

$$\therefore \quad (1.5 \cdot 10^9\ J/s) = q\left(\frac{40.6 \cdot 10^3 \cdot 10^6}{18}\ J/m^3\right)$$

$\Rightarrow \quad q = .665\ m^3/s$ required $H_2O$ evaporation rate

The fraction this represents of the river is

$= 6.65 \cdot 10^{-3} = .6\ \%.$

9
12
21

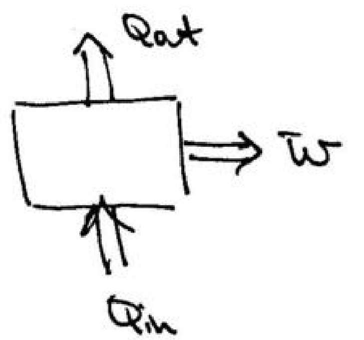
Prob 4.4

22°C

4°C.

(a) $e_{max} = 1 - \frac{T_c}{T_h} = \cancel{1 - \frac{4}{22}} = 1 - \frac{277}{295} = .061$

(b) $\dot{W} = 10^9$ Watts




$10^9 J = V c_p (T_h - T_c)$

$c_p = 4.16 \cdot 10^6 \frac{J}{K \cdot m^3}$

$10^9 J = (4.16 \cdot 10^6 \frac{J}{K \cdot m^3})\, V\, (18)$

$V = 13.3\ m^3$ each second ... doesn't seem unreasonable ...

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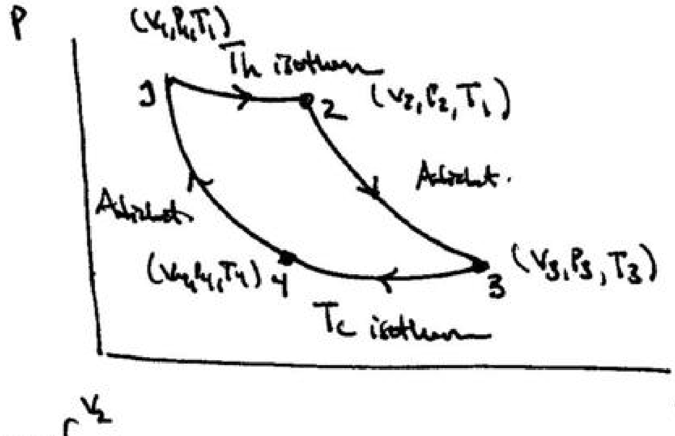
Prob 4.5




$PV = NkT$

$U = N \cdot f \cdot \frac{1}{2}kT$

$= N\frac{f}{2}kT$

1→2:

$$W_{12} = \int_{V_1}^{V_2} P\,dV = NkT\int_{V_1}^{V_2}\frac{dV}{V} = NkT\ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta U_{12} = N\frac{f}{2}k\,\Delta T_{12} = 0 \quad \text{since } T_1 = T_2$$

$$Q_h = \Delta U + W = W = NkT\ln\left(\frac{V_2}{V_1}\right) > 0 \Rightarrow \text{heat absorbed}$$

2→3:

For an Adiabatic process $dQ = 0$.

$$dU = \cancel{dQ}^{\,0} - dW = -p\,dV$$

$\parallel$

$$\cancel{N}\frac{f}{2}\cancel{k}\,dT = -\frac{\cancel{Nk}T}{V}dV$$

$$\frac{f}{2}\frac{dT}{T} = -\frac{dV}{V} \Rightarrow V^{\gamma}P = \text{const.} \Rightarrow P = cV^{-\gamma}$$

$$\gamma = \left(\frac{f+2}{f}\right) \qquad = \frac{P_2V_2^{\gamma}}{V^{\gamma}}$$

$$W_{23} = \int_2^3 P\,dV = \int_2^3 cV^{-\gamma}dV$$

$$W_{23} = C_1 \frac{V^{-r+1}}{-r+1}\Bigg|_2^3$$

$$= \cancel{\frac{C_1}{1-r}\left(\frac{1}{V^{1-r}}\right)}\Bigg|_2^3 = \frac{C_1}{1-r}$$

$$= \frac{C_1}{1-r} V^{1-r}\Bigg|_2^3 = \frac{C_1}{1-r}\left(V_3^{1-r} - V_2^{1-r}\right)$$

$$= \frac{P_2 V_2^{r}}{1-r}\left(V_3^{1-r} - V_2^{1-r}\right)$$

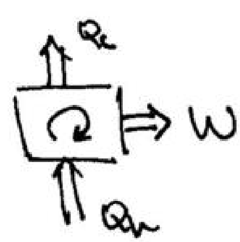
$$= \frac{1}{1-r}\left(P_3 V_3^{r} \cdot V_3^{1-r} - P_2 V_2\right)$$

$$= \frac{1}{1-r}\left(P_3 V_3 - P_2 V_2\right) \qquad PV = NkT$$

$$= \frac{1}{1-r}\left(NkT_3 - NkT_2\right) = \frac{Nk}{1-r}(T_3 - T_2) = \frac{Nk}{1-r}(T_3 - T_1)$$

- - - -

$$e = \frac{W}{Q_h} \equiv 1 - \frac{Q_c}{Q_h}$$




$$W = Q_h - Q_c$$

Since the $Q_h$ comes from the $T_h$ isotherm

& the $Q_c$ comes from the $T_c$ isotherm

$$Q_h = NkT_h \ln\left(\frac{V_2}{V_1}\right)$$

Along path 3→4 the same arguments hold &

$$W_{34} = NkT_c \ln\left(\frac{V_4}{V_3}\right) = Q_c \quad \to > 0 \Rightarrow \text{heat absorbed...}$$

want it to be positive to represent heat expelled.

~~[illegible] [illegible]~~ ∴ $Q_c = NkT_c \ln\left(\frac{V_3}{V_4}\right)$

$$\therefore \; e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}\frac{\ln\left(\frac{V_4}{V_3}\right)}{\ln\left(\frac{V_2}{V_1}\right)}$$

express $\frac{V_4}{V_3}$ in terms of $V_2$ + $V_1$'s. + $T_h$ + $T_c$

along an adiabat. (i) $PV^{\gamma} = \cancel{\phantom{xx}}\, P_0V_0^{\gamma}$ (ii) $VT^{f/2} = V_0T_0^{f/2}$

for an ideal gas $\left\{\gamma = \frac{f+2}{f}\right\}$ ∴ $\frac{V}{V_0} = \left(\frac{T_0}{T}\right)^{f/2}$.

∴ $\cancel{\frac{V_4}{V_1} = \left(\frac{T_h}{T_c}\right)^{f/2}}$ & $\cancel{\frac{V_3}{V_2} = }$

$$\frac{V_4}{V_1} = \left(\frac{T_h}{T_c}\right)^{f/2} \qquad \frac{V_3}{V_2} = \left(\frac{T_h}{T_c}\right)^{f/2}$$

Thus: $V_4 = V_1 \left( \frac{T_h}{T_c} \right)^{f/2}$

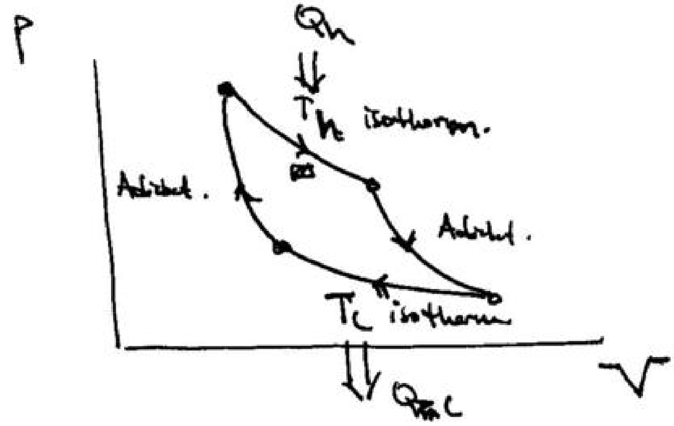
$V_3 = V_2 \left( \frac{T_h}{T_c} \right)^{f/2}$

$$\therefore \frac{\ln\left(\frac{V_4}{V_3}\right)}{\left[-\ln\left(\frac{V_2}{V_1}\right)\right]} = \frac{\ln\left(\frac{V_1 \left(\frac{T_h}{T_c}\right)^{f/2}}{V_2 \left(T_h/T_c\right)^{f/2}}\right)}{\left[-\ln\left(\frac{V_2}{V_1}\right)\right]} = +1$$

$$\therefore e = 1 - \frac{T_c}{T_h}.$$

Prob 4.6

(a)




$$Q_h = NkT \ln\left(\frac{V_f}{V_i}\right)$$

Assume an amount of heat $Q_h$ flows into the cycle.

$$\Delta S_{\text{isotherm 1}} = -\frac{Q_h}{T_h} + \frac{Q_h}{T_{hw}}$$

$$\Delta S_{\text{isotherm 2}} = -\frac{Q_c}{T_{cw}} + \frac{Q_c}{T_c}$$

$$\Delta S_{\text{total}} \geq 0$$

$$\Rightarrow \quad -\frac{Q_h}{T_h} + \frac{Q_h}{T_{hw}} - \frac{Q_c}{T_{cw}} + \frac{Q_c}{T_c} \geq 0$$

$$Q_h\left(\frac{1}{T_{hw}} - \frac{1}{T_h}\right) + Q_c\left(\frac{1}{T_c} - \frac{1}{T_{cw}}\right) \geq 0$$

Since the process is cyclical $\Delta S_{\text{total}} \equiv 0$ $\therefore$ equality in the above holds.

(b) $W = Q_h - Q_c$

$$\dot{W} = \dot{Q}_h - \dot{Q}_c$$

$$\Rightarrow \quad P = k(T_h - T_{hw}) - k(T_{cw} - T_c)$$

Eliminating $T_{cw}$ from pt (a) gives,

$$\frac{Q_h}{Q_c}\left(\frac{1}{T_{hw}} - \frac{1}{T_h}\right) + \frac{1}{T_c} = \frac{1}{T_{cw}}$$

$$\Rightarrow \quad T_{cw} = \frac{1}{\frac{Q_h}{Q_c}\left(\frac{1}{T_{hw}} - \frac{1}{T_h}\right) + \frac{1}{T_c}}$$

$$\therefore \cancel{P = k(T_h - T_{hw}) - \frac{k}{\quad}}$$

$$\Rightarrow \quad P(T_{hw}) = k(T_h - T_{hw}) - k\left[\frac{1}{\frac{Q_h}{Q_c}\left(\frac{1}{T_{hw}} - \frac{1}{T_h}\right) + \frac{1}{T_c}} - T_c\right]$$

(c) $$P'(T_{hw}) = -k - k\left[\frac{1}{\left(\frac{Q_h}{Q_c}\left(\frac{1}{T_{hw}} - \frac{1}{T_h}\right) + \frac{1}{T_c}\right)^2} \cdot \left(\frac{Q_h}{Q_c}\right)\left(\frac{-1}{T_{hw}^2}\right)\right] \overset{\text{set}}{=} 0$$

$$+1 = + \frac{\left(\frac{Q_h}{Q_c}\right)\left(\frac{1}{T_{hw}}\right)^2}{(\qquad\qquad)^2}$$

$$\left(\frac{Q_h}{Q_c}\right)^2\left(\frac{1}{T_{hw}}-\frac{1}{T_h}\right)^2+\frac{2}{T_c}\left(\frac{Q_h}{Q_c}\right)\left(\frac{1}{T_{hw}}-\frac{1}{T_h}\right)+\frac{1}{T_c^2}=\left(\frac{Q_h}{Q_c}\right)\left(\frac{1}{T_{hw}}\right)^2$$

$$\Rightarrow\left(\frac{Q_h}{Q_c}\right)^2\left[\frac{1}{T_{hw}^2}-\frac{2}{T_{hw}T_h}+\frac{1}{T_h^2}\right]$$

(d) $e = 1 - \sqrt{\frac{T_c}{T_h}}$

$$e = 1 - \sqrt{\frac{273+25}{273+600}} = .415$$

$$e_{const} = 1 - \frac{273+25}{273+600} = .65$$

(Prob 4.7) Because you need a place to dump the $Q_h$ heat. Dumping it in the middle of the room would be counter productive

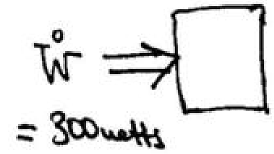
(Prob 4.8) No. Most internal refrigerators take heat from the internal compartment + put it ~~to~~ ~~then~~ into the room. & with the door open the heat ~~would~~ removed would /could find its way back into the fridge creating an ?? cycle

(Prob 4.9) $COP \leq \frac{T_c}{T_h - T_c} = \frac{1}{\frac{T_h}{T_c} - 1} = \frac{1}{\frac{298}{293} - 1} = 58.9$

$T_c \cong 273 + 20 = 293$

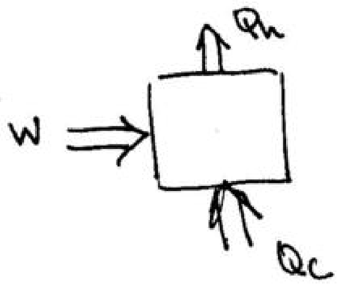
$T_h \cong 273 + 25 = 298$

(Prob 4.10)




Then it must expell ~~heat~~ 300 watts of heat at $T_c$ to $T_h$

Energy-flow diagram



Thus the heat flowing in should be

$\dot{Q}_c$ so an energy eqn would be

$\dot{Q}_h = \dot{Q}_c + \dot{W}$

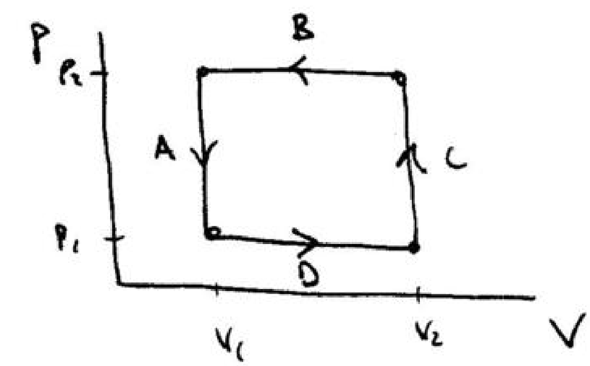
Not sure how to solve . . .

Prob 4.11

$$COP \leq \frac{T_c}{T_h - T_c} = \frac{1}{\frac{T_h}{T_c} - 1}$$

$$COP \leq \frac{1}{\frac{1}{.01} - 1} = \frac{1}{99} = 1.01 \cdot 10^{-2}$$

Prob 4.12




$\Delta S = 0.$

$\Delta W =$ Area under cyclic graph

$= (V_2 - V_1)(P_2 - P_1)$

~~path~~: ~~4~~: $U = N \cdot \frac{f}{2} kT$ ; $PV = NkT$

path A: $\Delta U = Q - W$.

$$W_A = \int p\,dV = 0$$

$$\Delta U_A = N \cdot \frac{f}{2} k \Delta T = N\frac{f}{2} k \left(\frac{P_1 V_1}{Nk} - \frac{P_2 V_2}{Nk}\right)$$

$$= \frac{f}{2}(P_1 V_1 - P_2 V_2)$$

$$Q_A = \Delta U + W = \frac{f}{2}(P_1 V_1 - P_2 V_2) = \frac{f V_1}{2}(P_1 - P_2) < 0$$

~~path B~~: path D:

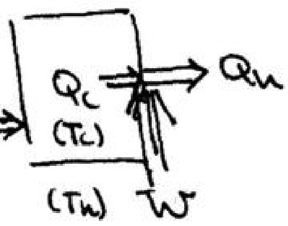
⋮

Since this is a cyclic path $\Delta U_{ADCB} = 0$

$$\therefore \quad W = Q$$

I don't see why this won't work as an engine except one pumps as much heat as ~~??~~ work that one puts in...

(Prob 4.13)




$= \cancel{k}T_c$

$k(T_h - T_c) = \dot{Q}_{in}$

to obtain a steady state situation the energy balance would be

$\dot{Q}_{in} = \dot{Q}_c$

$Q_h = W + Q_c$

$\dot{Q}_h = \dot{W} + \dot{Q}_c$

$= \dot{W} + k(T_h - T_c)$

$\dot{W} = \dot{Q}_h - k(T_h - T_c)$

... How do?

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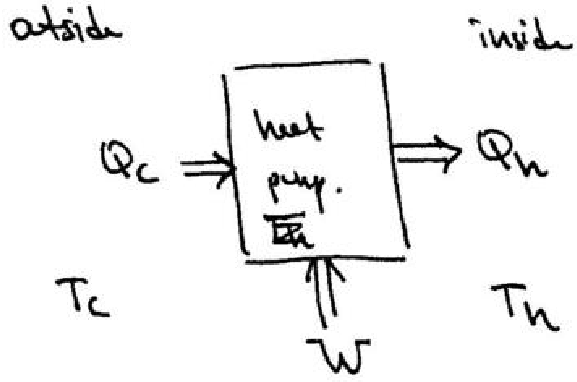
Prob 4.14




(a) $\text{COP} = \frac{Q_h}{W}$ because this ratio represents the benefit amount of heat pumped into the room ÷ amount of work required to obtain this

(b) energy cons requires

$$W + Q_c = Q_h$$

$$1 + \frac{Q_c}{W} = \frac{Q_h}{W} = \text{COP} \qquad \text{COP} \geq 1 \qquad \text{yes.}$$

(c) By usage ~~the entropy~~ ~~increase input at~~ $T_c$ ~~is~~

$$\frac{Q_c}{T_c} \cancel{+ \text{ entropy}} \text{ decrease } \cancel{\text{is}} \; \frac{\cancel{Q_h}}{T_h}$$

$\Delta S \geq 0$ ~~region~~ $+ \frac{Q_c}{T_c} =$ entropy **lost** by environment.

$\frac{Q_h}{T_h}$ = entropy gained by room

$$\Delta S = -\frac{Q_c}{T_c} + \frac{Q_h}{T_h} \geq 0$$

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c} \quad \div \quad \boxed{\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}}$$

$$\therefore \quad \frac{W}{Q_c} + 1 = \frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}$$

$\|$

?

since

$$1 + \frac{Q_c}{W} = \frac{Q_h}{W}$$

$$\cancel{\frac{Q_c}{W} = \frac{Q_h}{W} - 1}$$

$$\cancel{\frac{W}{Q_c} = \frac{1}{\frac{Q_h}{W} - 1}}$$

$$\therefore \quad \frac{1}{\frac{Q_h}{W} - 1} + \frac{\frac{Q_h}{W} - 1}{\frac{Q_h}{W} - 1} \geq \frac{T_h}{T_c}$$

$\frac{Q_h}{W}$ . . .

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$$\cancel{\frac{\frac{Q_h}{W}}{\frac{Q_h}{W}-1}} \geq \frac{T_h}{T_c}$$

Since

$$COP = \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c} = \frac{\frac{Q_h}{Q_c}}{\frac{Q_h}{Q_c}-1}$$

$$= 1 + \frac{1}{\frac{Q_h}{Q_c}-1} \leq 1 + \frac{1}{\frac{T_h}{T_c}-1}$$

$$\therefore \quad COP \leq \frac{\frac{T_h}{T_c}-1+1}{\frac{T_h}{T_c}-1} = \frac{\frac{T_h}{T_c}}{\frac{T_h}{T_c}-1} \cdot \frac{T_c}{T_c}$$

$$COP \leq \frac{T_h}{T_h - T_c}$$

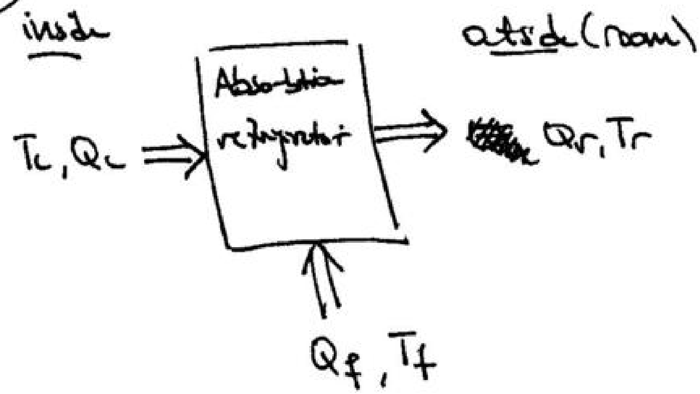
(d) $T_h = 20°C = 293K$

$T_c = \cancel{10°C}\ 0°C = 273K \qquad COP \leq 14.65.$

$$COP = \frac{Q_h}{W}.$$

converting electrical work into (directly) into heat results in a $COP = 1$ where one can see from above values greater than 1 are easily obtained.

Prob 4.15




(a) $cop \equiv \dfrac{Q_c}{Q_f}$ this is the classic benefit/cost ratio.

(b) $Q_r = Q_c + Q_f$

$$\frac{Q_r}{Q_f} = \frac{Q_c}{Q_f} + 1 \Rightarrow \frac{Q_c}{Q_f} = \frac{Q_r}{Q_f} - 1 \overset{?}{\geq} 1$$

$Q_r \geq 2Q_f$ ... don't know

how to tell if this is possible ...

(c) ~~By~~ entropy ~~thesis~~

$$\Delta S = -\frac{Q_c}{T_c} - \frac{Q_f}{T_f} + \frac{Q_r}{T_r} \geq 0.$$

$$-\frac{1}{T_c}\frac{Q_c}{Q_f} - \frac{1}{T_f} + \frac{Q_r}{Q_f}\cdot\frac{1}{T_r} \geq 0$$

$$-\frac{1}{T_c}\frac{Q_c}{Q_f} - \frac{1}{T_f} + \frac{1}{T_r}\left(\frac{Q_c}{Q_f}+1\right) \geq 0$$

$$\left(-\frac{1}{T_c}+\frac{1}{T_r}\right)\frac{Q_c}{Q_f} - \frac{1}{T_f} + \frac{1}{T_r} \geq 0$$

$$\left(\frac{-T_r+T_c}{T_rT_c}\right)\frac{Q_c}{Q_f} \geq \frac{1}{T_f}-\frac{1}{T_r}$$

$$\cancel{\left(\frac{T_c - T_r}{T_rT_c}\right)}$$

$$-\frac{(T_r-T_c)}{T_rT_c}\frac{Q_c}{Q_f} \geq \frac{T_r-T_f}{T_f\cdot T_r} = -\frac{(T_f-T_r)}{T_f\cdot T_r}$$

$$\frac{Q_c}{Q_f} \cancel{\geq} \leq \frac{\cancel{T_r}T_c}{(T_r-T_c)}\frac{(T_f-T_r)}{T_f\cdot\cancel{T_r}} = T_c\frac{(T_f-T_r)}{(T_r-T_c)}$$

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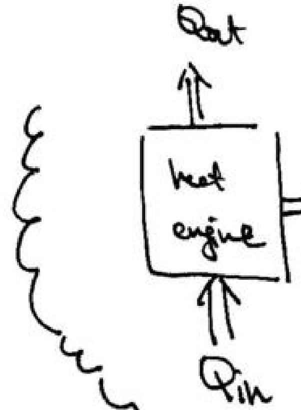
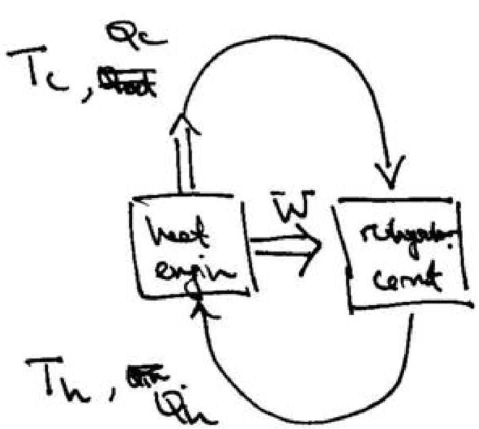
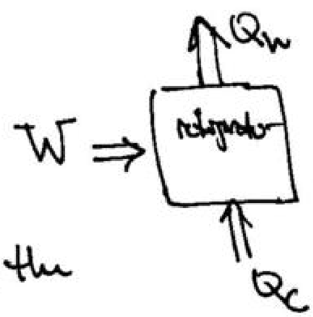
Prob 4.16

$$e_{carnot} = 1 - \frac{T_c}{T_h}$$




Assume $e_{wax} \geq 1 - \frac{T_c}{T_h}$




Thus we use the heat engine to drive the refrigerator.

Lets check that the energy's add up.

heat engine : $Q_h = Q_c + W$

refrig : $Q_c + W = Q_h$ ✓.

$e_{carnot} = 1 - \frac{T_c}{T_h}$

Entropys add up :

$\frac{Q_c}{T_c}$ $\frac{Q_h}{T_h}$

looks like $\Delta S = 0$.

Check that no work is req

$$\frac{W}{Q_h} \equiv e_{heating} \gtrsim 1 - \frac{T_c}{T_h}$$

$$e_{refrig} \equiv \frac{Q_c}{W}$$

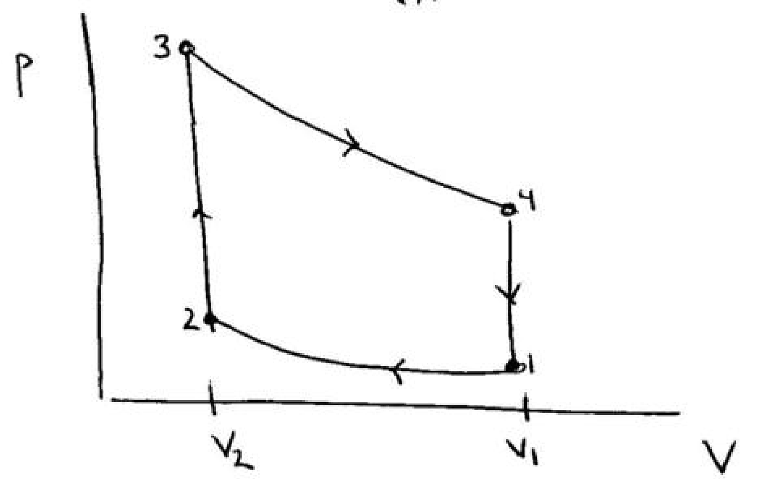
~~Thus~~ $W > e_{heat} \cdot Q_h$

$$e_{refrig} > \frac{Q_c}{e_{heat} Q_h} \quad \ldots$$

Prob 4.17 Hook the systems as shown in problem 4.16.
But I can not see how to show the analytic statement ...

Prob 4.18

Pv: $e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ for the otto cycle




$PV^\gamma = \text{const}$

$e = \frac{W}{Q_h}$

Work done by the gas on the expansion portion of the curve $3 \to 4$.

$$W = \int_3^4 P\,dV = C\int_3^4 V^{-\gamma}\,dV = C\left.\frac{V^{-\gamma+1}}{1-\gamma}\right|_3^4 = \frac{C}{1-\gamma}\left(V_4^{1-\gamma} - V_3^{1-\gamma}\right)$$

$\gamma > 1$ $W_{34} > 0$ ✓

$$W_{34} = \frac{1}{1-\gamma}\left(P_4V_4^{\gamma}\cdot V_4^{1-\gamma} - P_3V_3^{\gamma}V_3^{1-\gamma}\right) = \frac{1}{1-\gamma}\left(P_4V_4 - P_3V_3\right)$$

Work done on the compression part of the cycle $1 \to 2$

$$W = \int_1^2 P\,dV = \frac{C}{\cancel{\gamma}-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$$

$V_2 < V_1$

$$= \frac{-C}{\gamma-1}\left(\frac{1}{V_2^{1-\gamma}} - \frac{1}{V_1^{1-\gamma}}\right) < 0. ✓$$

$V_2^{1-\gamma} < V_1^{1-\gamma}$

$$= \cancel{\frac{1}{\gamma-1}}\left(\cancel{P_2V_2} - \right.$$

$\frac{1}{V_1^{1-\gamma}} < \frac{1}{V_2^{1-\gamma}}$

$$W_{12} = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

$$\therefore W_{total} = \frac{1}{1-\gamma}\left[P_4V_4 - P_3V_3 + P_2V_2 - P_1V_1\right]$$

$Q_h$ = heat brought in

$$\& \quad \Delta U = Q_h + -\cancel{W}^{0}$$

$$\|$$

$$\frac{f}{2}Nk\Delta T = Q_h$$

$$N\frac{f}{2}k\left[T_3 - T_2\right] = \frac{N \cdot f \cdot k}{2}\left[\frac{P_3V_3}{Nk} - \frac{P_2V_2}{Nk}\right]$$

$$= \frac{f}{2}(P_3V_3 - P_2V_2) = Q_h$$

Now $\gamma = \dfrac{f+2}{f}$

$\Rightarrow \gamma f - f = 2$

$(\gamma-1)f = 2$

$f = \dfrac{2}{\gamma-1} \qquad \therefore Q_h = \dfrac{1}{\gamma-1}(P_3V_3 - P_2V_2)$

Now

$$e = \frac{W}{Q_h} = \frac{\frac{1}{1-\gamma}(P_4V_4 - P_3V_3 + P_2V_2 - P_1V_1)}{\frac{1}{\gamma-1}(P_3V_3 - P_2V_2)}$$

$$e = \frac{-(P_4V_4 - P_3V_3 + P_2V_2 - P_1V_1)}{P_3V_3 - P_2V_2}$$

$$= 1 - \frac{(P_4V_4 - P_1V_1)}{P_3V_3 - P_2V_2}$$

Now $P_4V_4^r = P_3V_3^r$

$$\left.\begin{matrix} 1 \leftrightarrow 2 \\ 3 \leftrightarrow 4 \end{matrix}\right\} \Rightarrow \begin{matrix} 1 \leftrightarrow 4 \\ 2 \leftrightarrow 3 \end{matrix} \; ?$$

$$P_4V_4 = \frac{P_3V_3^r}{V_4^{r-1}}$$

$$= \frac{P_3V_2^r}{V_1^{r-1}}$$

$$e = 1 - \frac{P_4V_1 - P_1V_1}{P_3V_2 - P_2V_2} = 1 - \left(\frac{V_1}{V_2}\right)\left(\frac{P_4 - P_1}{P_3 - P_2}\right)$$

$$P_4 = \left(\frac{P_3V_3^r}{V_4^r}\right) = P_3\left(\frac{V_2}{V_1}\right)^r$$

$$e = 1 - \left(\frac{V_1}{V_2}\right)\left[\frac{P_3\left(\frac{V_2}{V_1}\right)^r - P_1}{P_3 - P_2}\right] = 1 - \left(\frac{V_1}{V_2}\right)\left[\frac{P_3\left(\frac{V_2}{V_1}\right)^r - P_1}{P_3 - P_1\left(\frac{V_1}{V_2}\right)^r}\right]$$

$$P_2 = \frac{P_1V_1^r}{V_2^r}$$

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$$e = 1 - \left(\frac{V_1}{V_2}\right)\left[ \cancelto{1}{\frac{P_3\left(\frac{V_2}{V_1}\right)^r - P_1}{P_3\left(\frac{V_2}{V_1}\right)^r - P_1}} \right] \frac{\left(\frac{V_2}{V_1}\right)^r}{1}$$

$$e = 1 - \frac{V_1}{V_2} \cdot \left(\frac{V_2}{V_1}\right)^r$$

$$= 1 - \left(\frac{V_2}{V_1}\right)^{r-1} \quad \text{eq 4.10.}$$

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Prob 4.19

$$e = \frac{W}{Q_h}$$

For very large compression ratios the amount of time the piston spends sliding increases, Thus the influence of friction is more.

I would expect the engine to be most efficient when opperating at low power by analogy w/ the carnot engine which opperates very slowly.

I don't understand why/how one can burn more fuel & increase the power but the efficiency does not change.

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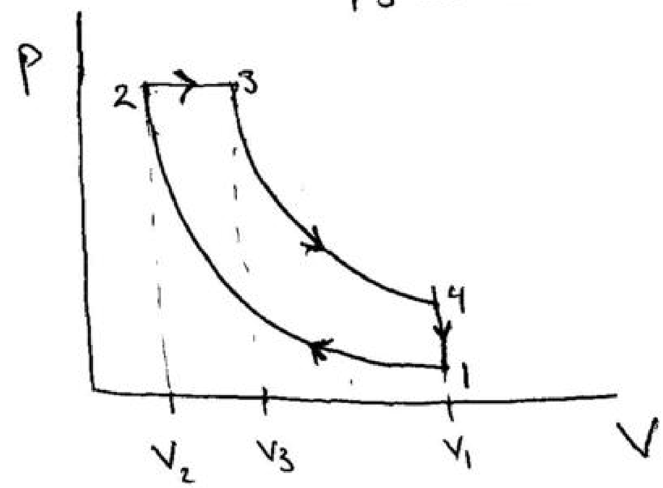
Prob 4.20




$$e = ? = e\left(\frac{V_1}{V_2}, \frac{V_3}{V_2}\right)$$

For the diesel engine the heat flows into the engine during $2 \to 3$

The work done during the entire cycle = area

$$W_{23} = \int_2^3 p\,dV = p(V_3 - V_2)$$

$$W_{34} = \int_3^4 p\,dV = \frac{c}{1-\gamma}\left( V^{-\gamma+1} \right|_3^4 = \frac{1}{1-\gamma}\left(P_4V_4 - P_3V_3\right)$$

$$W_{41} = 0 \quad ; \quad W_{12} = \int_1^2 p\,dV = \frac{1}{1-\gamma}\left(P_2V_2 - P_1V_1\right)$$

$$\therefore W_{total} = P_3V_3 - P_2V_2$$

$$+ \frac{1}{1-\gamma}\left(P_4V_4 - P_3V_3\right)$$

$$+ \frac{1}{1-\gamma}\left(P_2V_2 - P_1V_1\right)$$

$$= \frac{-1}{1-\gamma} P_1V_1 + \left[-1 + \frac{1}{1-\gamma}\right] P_2V_2$$

$$\left[\frac{-1}{1-\gamma} + 1\right] P_3V_3$$

$$+ \frac{1}{1-\gamma} P_4V_4$$

$$\Rightarrow W_{total} = -\frac{P_1V_1}{1-\gamma} + \frac{-1+\gamma+1}{1-\gamma} P_2V_2 + \frac{-1+1-\gamma}{1-\gamma} P_3V_3$$

$$+ \frac{1}{1-\gamma} P_4V_4$$

$$= -\frac{1}{1-\gamma} P_1V_1 + \frac{\gamma}{1-\gamma} P_2V_2 - \frac{\gamma}{1-\gamma} P_3V_3 + \frac{1}{1-\gamma} P_4V_4$$

The heat that enters enters during the $2 \rightarrow 3$ portion:

For this process $\Delta U = Q - W$. $\Rightarrow Q = \Delta U + W$.

Now: $U = N \cdot f \cdot \frac{1}{2} kT$

$$\Delta U = N \cdot f \cdot \frac{1}{2} k \Delta T = \cancel{N} \cdot f \cdot \frac{1}{2} \cancel{k} \left( \frac{P_f V_f}{\cancel{N} k} - \frac{P_i V_i}{\cancel{N} k} \right)$$

$$= \frac{f}{2}(P_f V_f - P_i V_i)$$

Now $\gamma = \frac{f+2}{f}$

$$f \gamma = f + 2$$

$$(\gamma - 1) f = 2$$

$$f = \frac{2}{\gamma - 1}$$

$$= \frac{1}{\gamma-1}(P_f V_f - P_i V_i)$$

Now $Q_{23} = \Delta U_{23} + W_{23}$

$$= \frac{1}{\gamma-1}(P_3 V_3 - P_2 V_2) + P_3 V_3 - P_2 V_2$$

$$= \left(\frac{1}{\gamma-1} + 1\right) P_3 V_3 + \left(\frac{-1}{\gamma-1} - 1\right) P_2 V_2$$

$$= \frac{1+\gamma-1}{\gamma-1} P_3 V_3 - \left(\frac{1}{\gamma-1} + 1\right) P_2 V_2$$

$$= \frac{\gamma}{\gamma-1} P_3 V_3 - \frac{\gamma}{\gamma-1} P_2 V_2$$

$\therefore$

$$e = \frac{W}{Q} = \frac{\frac{-1}{1-\gamma} P_1 V_1 + \frac{\gamma}{1-\gamma} P_2 V_2 - \frac{\gamma}{1-\gamma} P_3 V_3 + \frac{1}{1-\gamma} P_4 V_4}{-\frac{\gamma}{\gamma-1} P_2 V_2 + \frac{\gamma}{\gamma-1} P_3 V_3}$$

$$= \frac{\frac{1}{\gamma-1} P_1 V_1 - \frac{\gamma}{\gamma-1} P_2 V_2 + \frac{\gamma}{\gamma-1} P_3 V_3 - \frac{1}{\gamma-1} P_4 V_4}{-\frac{\gamma}{\gamma-1} P_2 V_2 + \frac{\gamma}{\gamma-1} P_3 V_3}$$

$$e = 1 + \frac{\frac{1}{r-1} P_1V_1 - \frac{1}{r-1} P_4V_4}{-\frac{r}{r-1} P_2V_2 + \frac{r}{r-1} P_3V_3}$$

$$= 1 + \frac{P_1V_1 - P_4V_4}{-r P_2V_2 + r P_3V_3}$$

$$= 1 - \frac{1}{r}\left[\frac{P_1V_1 - P_4V_4}{P_2V_2 - P_3V_3}\right]$$

$V_1/V_2$ ; $V_3/V_2$.

$$= 1 - \frac{1}{r}\left[\frac{P_1V_1 - \frac{P_3V_3^r}{V_1^r}\cdot V_1}{\frac{P_1V_1^r}{V_2^r}\cdot V_2 - P_3V_3}\right]$$

$$P_4 = \frac{P_3V_3^r}{V_4^r} = P_3\frac{V_3^r}{V_1^r}$$

$$P_1V_1^r = P_2V_2^r = P_3V_2^r$$

$$= 1 - \frac{1}{r}\frac{V_1}{V_2}\left[\frac{P_1 - P_3\left(\frac{V_3}{V_1}\right)^r}{P_1\left(\frac{V_1}{V_2}\right)^r - P_3\left(\frac{V_3}{V_2}\right)}\right]$$

$$\Rightarrow P_3 = P_1\left(\frac{V_1}{V_2}\right)^r$$

$$P_2 = P_1\frac{V_1^r}{V_2^r}$$

$$= 1 - \frac{1}{r}\frac{V_1}{V_2}\cdot\frac{1}{\left(\frac{V_1}{V_2}\right)^r}\left[\frac{P_1 - P_3\left(\frac{V_3}{V_2}\cdot\frac{V_2}{V_1}\right)^r}{P_1 - P_3\,\frac{V_3}{V_2}\cdot\left(\frac{V_2}{V_1}\right)^r}\right]$$

$$= 1 - \frac{1}{r}\left(\frac{V_1}{V_2}\right)\left(\frac{V_2}{V_1}\right)^r\left[\frac{P_1/P_3 - \left(\frac{V_3}{V_2}\right)^r\left(\frac{V_2}{V_1}\right)^r}{P_1/P_3 - \left(\frac{V_3}{V_2}\right)\left(\frac{V_2}{V_1}\right)^r}\right]$$

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Can I express $\frac{P_1}{P_3} = F\left(\frac{V_1}{V_2}, \frac{V_3}{V_2}\right)$.

$$P_1 = P_2\left(\frac{V_2}{V_1}\right)^r = P_3\left(\frac{V_2}{V_1}\right)^r$$

$\therefore$

$$e = 1 - \frac{1}{r}\left(\frac{V_2}{V_1}\right)^{r-1} \cdot \left[ \frac{\left(\frac{V_2}{V_1}\right)^r - \left(\frac{V_3}{V_2}\right)^r\left(\frac{V_2}{V_1}\right)^r}{\left(\frac{V_2}{V_1}\right)^r - \left(\frac{V_3}{V_2}\right)\left(\frac{V_2}{V_1}\right)^r} \right]$$

$$e = 1 - \frac{1}{r}\left(\frac{V_2}{V_1}\right)^{r-1} \cdot \cancel{\left(\frac{V_2}{V_1}\right)^r} \left[ \frac{1 - \left(\frac{V_3}{V_2}\right)^r}{1 - \left(\frac{V_3}{V_2}\right)} \right]$$

$$e = 1 - \frac{1}{r}\left(\frac{V_2}{V_1}\right)^{r-1} \left[ \frac{1 - \left(\frac{V_3}{V_2}\right)^r}{1 - \left(\frac{V_3}{V_2}\right)} \right].$$

V.S. $\quad \frac{V_3}{V_2} > 1 \quad \left(\frac{V_3}{V_2}\right)^r > \left(\frac{V_3}{V_2}\right) \quad \frac{1 - \left(\frac{V_3}{V_2}\right)^r}{1 - \left(\frac{V_3}{V_2}\right)} < 1$

$$\therefore \frac{\left(\frac{1-(\ )^r}{1-(\ )}\right)}{r} < 1 \qquad \therefore e_{diesel} > e_{otto}\ ?$$

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take $\frac{V_2}{V_1} = 18^{-1}$ $e_{otto} = .719$

w/ air as working substance.

$e_{diesel} = .631.$

$\frac{V_3}{V_2} = 2$ $\frac{V_1}{V_2} = 18$

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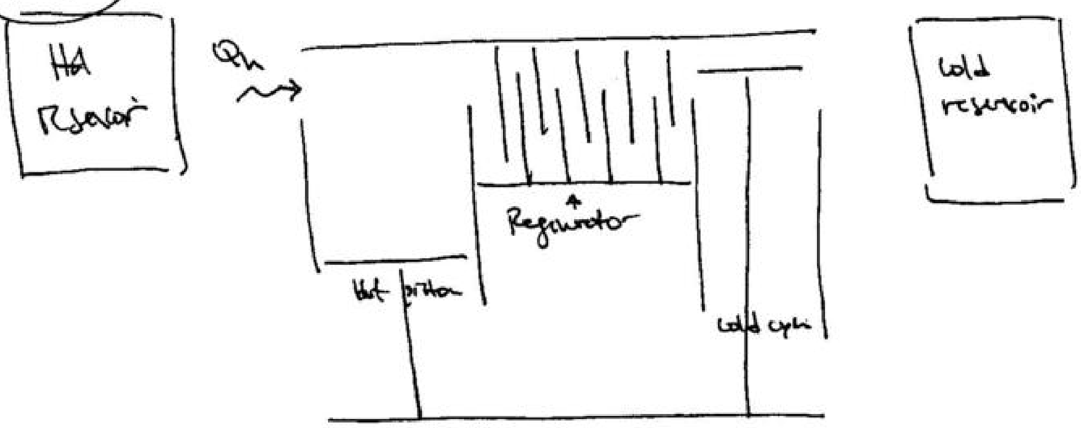
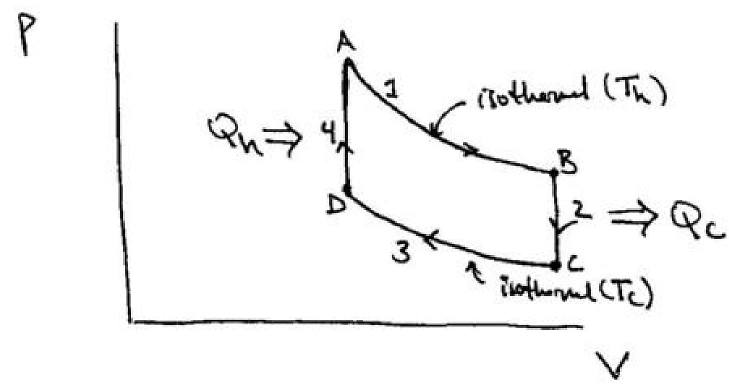
Prob 4.21




"power stroke"

(a)




$PV = \text{const}$

$P = \frac{C}{V}.$

(b) $e = \frac{W}{Q_h}$

$Q_h = \Delta U - \cancel{W}^{\,0}.$

$= \frac{f}{2} N \cdot k \Delta T \cdot$

$W_{AB} = \int_{AB} p\,dV = NkT_h \int_{AB} \frac{dV}{V} = NkT_{hot} \ln\left(\frac{V_B}{V_A}\right)$

$W_{CD} = NkT_{cold} \ln\left(\frac{V_D}{V_C}\right)$

$W_{tota} = \sum W_{\square}$

$$e = \frac{\cancel{Nk}\left[-T_h \ln\left(\frac{V_L}{V_R}\right) + T_c \ln\left(\frac{V_L}{V_R}\right)\right]}{\left(\frac{f}{2}\right)\cancel{Nk}(T_h - T_c)}$$

$$e = \left(\frac{2}{f}\right)\frac{\left[T_h \ln\left(\frac{V_R}{V_L}\right) - T_c \ln\left(\frac{V_R}{V_L}\right)\right]}{(T_h - T_c)}$$

$$\gamma = \frac{f+2}{f}$$

$$\gamma f = f + 2$$

$$(\gamma - 1)f = 2$$

$$f = \frac{2}{\gamma - 1}$$

$$\frac{f}{2} = \frac{1}{\gamma - 1}$$

$$e = (\gamma - 1)\left[\quad \cdots \quad\right]$$

$$e = (\gamma-1)\left[\frac{T_h}{T_h - T_c}\ln\left(\frac{V_R}{V_L}\right) - \frac{T_c}{T_h - T_c}\ln\left(\frac{V_R}{V_L}\right)\right]$$

$\Rightarrow e = (\gamma - 1)\ln\left(\frac{V_R}{V_L}\right)$ This doesn't seem correct ...

(c) What is the regenerator's purpose? To simply hold heat?

(d) ?

$$\Delta U = Q - P dV.$$

$$dU = dQ - P dV$$

$$dU + P dV = dQ$$

$$\rightarrow d(U + PV)\Big|_P = dQ.$$

$$dH\Big|_P = dQ. \quad \Rightarrow \quad \Delta H\Big|_P = dQ$$

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} \qquad Q_h = W + Q_c$$

$$= 1 - \frac{Q_c}{Q_h}$$

$$= \cancel{1 - \frac{H_4 - H_1}{H_3 - H_2}} = \cancel{1 - \frac{H_4 - H_1}{H_2 - H_1}}$$

$$= \cancel{H_2}$$

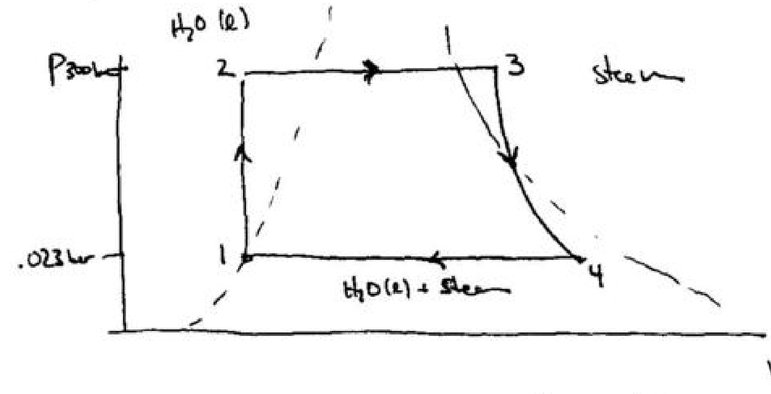
Now $Q_h = \Delta H_{23} = H_3 - H_2$

$Q_c = \Delta H_{41} = H_4 - H_1$

$$\therefore \quad e = 1 - \frac{H_4 - H_1}{H_3 - H_2} \approx 1 - \frac{H_4 - H_1}{H_3 - H_1}$$

$H_4 = ?$

look up entropy at pt ~~B~~ 3 + then using that entropy, look up H from table 4.1 (sat. $H_2O$ steam)




$H_1 = 84$ kJ $\qquad H_3 = 3444$ kJ

entropy at 3 = $6.233$ kJ/K → pt 4 in $H_2O$ (l) + steam at

$$\overset{H_2O}{x(.297)} + \overset{\text{steam}}{(1-x)(8.667)} = 6.233$$

$$(.297 - 8.667)x = 6.233 - 8.667 =$$

$$x = .2908 \text{ of } H_2O$$

$$+ \ 1-x = .71 \text{ of steam}$$

This mix has

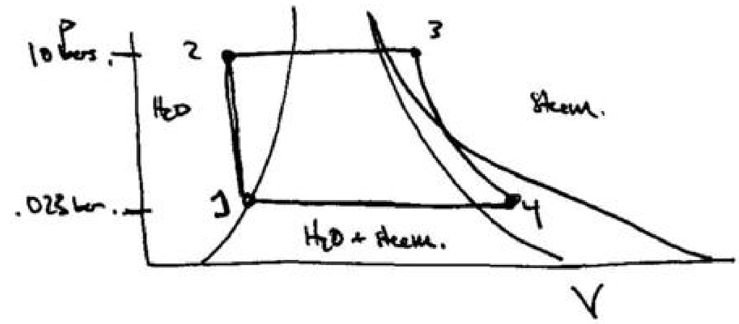
$$H_4(\text{kJ}) = .29(84) + (.71)(2538) = 1826.3$$

$$e = 1 - \frac{(1826.3 - 84)}{(3444 - 84)} = .481 \ldots$$

$$e_{carnot} = \cancel{1 - \frac{T_h}{T_c}} \ 1 - \frac{T_c}{T_h} = 1 - \frac{20+273}{600+273} = .66$$

Prob 4.22

$$e \cong 1 - \frac{H_4 - H_1}{H_3 - H_1}$$




$H_1 = 84$ kJ. $H_3 = 3051$ kJ

$H_4 = ?$

$S_3 = S_4 = 7.123 = .297\, x_{H_2O} + 8.667(1 - x_{H_2O})$

$\rightarrow$

$x_{H_2O} = \cdots$

$\cancel{8.667} +$

$-8.37\, x_{H_2O} = -1.544$

$x_{H_2O} = .184 \leftarrow$

$1 - x_{H_2O} = .81 \leftarrow$

Then $H_4 = (.184)84 + (.81)(2538) = \cdots$

Prob 4.23 $H \equiv U + PV$. $\Delta H = \Delta U + P\Delta V$

$H_2 = H_1 + \Delta H$

?

$dH = dQ$

$\frac{dH}{T} = \frac{dQ}{T} = dS$

Prob 4,24

(a) $e \cong 1 - \dfrac{H_4 - H_1}{H_3 - H_1}$ $P_{max} = 300$ bars

$T_{max} = 800$ then $H_2 + H_3$ change

$H_3 = 3081$ kJ

$e \cong 1 - \dfrac{1824 - 84}{3081 - 84} = .419$

(b) $P_{max} = 100$ bars. then $H_2 + H_3$ change

$H_3 = 3625$

$e \cong 1 - \dfrac{1824 - 84}{3625 - 84} = .508$

(c) $T_{min} = 10^\circ C \Rightarrow H_1 + H_4$ change.
($H_2O$) $p_{min} = ~~.025~~$ .01 bar

$H_1 = 42$ kJ.

w/ $S_3 = 6.233$ kJ/k
$S_4$ still

$x_{H_2O}(.151) + (1 - x_{H_2O})(8.901) = 6.233$
$\Rightarrow x_{H_2O} = .304$ $1 - x_{H_2O} \cong .7$

$H_4 = .3(42) + .7(2520)$

$= 1776 \text{ kJ}.$

$$e \cong 1 - \frac{1776 - 84}{3444 - 84} = .496$$

P 4.25 in going from pt 3 full steam to $H_2O$ + steam an ~~the~~ increase in entropy is to be expected. Since steam has so much more entropy than $H_2O$, this increasing entropy prob. shows itself as an increase in the amount of steam present. Thus $x_{H_2O}$ (mass fraction of liquid $H_2O$) is less than expected

$\Rightarrow$ more steam $\Rightarrow$ causes the ~~entropy~~ enthalpy of $H_4$ to increase lowering the ~~heat~~ efficiency of the steam engine.

P 4.26 $\dot{W} = 10^9 \text{ W}.$

$$\left\{ B = -V \frac{\partial P}{\partial V} \right\}$$

$$.4 \cong e = \frac{\dot{W}}{\dot{Q}_h}$$

$$\dot{Q}_h = H_3 \times H_2 \approx H_3 \times H_1$$
$$= (l_{kg}) \dot{M}$$

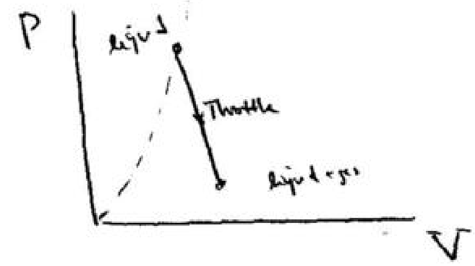
$$l = 2260 \text{ J/g}$$
$$= 2260 \cdot 10^3 \frac{\text{J}}{\text{kg}}$$

$$\underline{\dot{m}} = \frac{(\dot{W}/e)}{\ell}$$

P4.27 ? T + P don't increase at the same rate. P is increasing faster than T + ∴ applying more pressure "squeezes" the entropy.

P.4.28 ?

Prob 4.29 When throttled $H_1 = H_2$




$H_{liquid}(12 \text{ bars}) = 116 \text{ kJ}$

$= H(1 \text{ bar}) = x H_{liquid} + (1-x) H_{gas}$

$= x(16) + (1-x)(231)$

$\Rightarrow x = .53$ fraction of liquid $\quad 1-x = .4651$ fraction vaporizes

$T_{final} = -26.4\,°C$

Prob 4.30

(a) ~~S(10 bars)~~

Assuming all initially is gas $S(1 \text{ bar}) = .94 \text{ kJ/K}$

then compressing adiabatically to 10 bars requires the final temperature of the superheated (gaseous) reactant

$S = S(T)$

$$S = .907 + \frac{(.943 - .907)}{(50-40)}(T-40) \Rightarrow S = .94 \text{ gives } T = 322.16 \text{ K} = 49.16\,°C$$

(b) At pt 2 $H_2 = $ ~~[illegible]~~

$$274 + \frac{(284-274)}{(50-40)}(49.16-40)$$

$$= 283.16 \text{ kJ} \qquad T = 49.16°C$$

At pt 1 $H_1 = 231$ kJ $\qquad T = -26.4°C$

Assuming we have
all vapor

At pt 3 $H_3 = 105$ kJ $\qquad T = 39.4$

Assuming we have
all liquid

At pt 4: Throttling requires $\underline{H}_3 = \underline{H}_4$ $\qquad T = -26.4$

$H_3 = 105 = x(16) + (1-x)\cdot 231$

$\Rightarrow x = .586$ (liquid) $\qquad 1-x = .4139$ (vapor)

$$\therefore \text{COP} = \frac{H_1 - H_3}{H_2 - H_1} = \frac{231 - 105}{283.16 - 231} = 2.415 > 1 \;?$$

$$e_{carnot} = 1 - \frac{T_c}{T_h} = 1 - \left(\frac{273 - 26.4}{273 + 49.16}\right) =$$

. . .
Both temperatures seem a bit to large/small

(c) .4139

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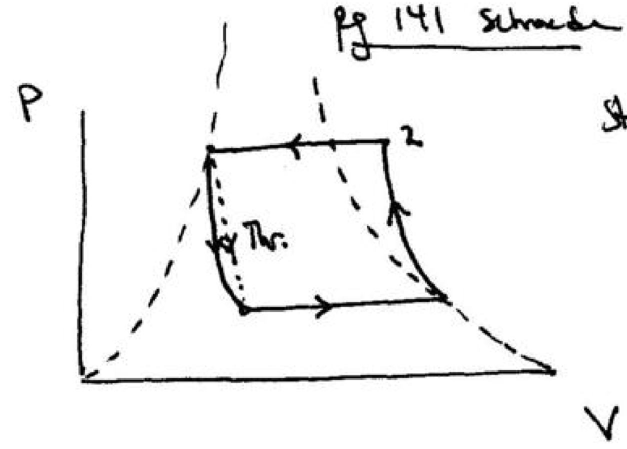
P. 4.31




Standard refrigeration cycle ...

?

P. 4.32 I would prob. have it operate between 1bar - 12 bars pressure difference ... As the other problems did.

Prob 4.33

(a) $T_i = 300k$

$P_i = 100 bars$

$H_i = ~~\text{}~~\ 8174\ J$

$$= \frac{(1800-8717)}{(200-300)}(T-300) + 8717 = 8174$$

$\Rightarrow T = 281.3\ k.$

(b) $H_i = 4442\ J.$

$$\cancel{= \frac{(4442+1946)}{(200-100)}(T-100)+1946}$$

$$= \frac{(1800-2856)}{(200-100)}(T-100) + 2856$$

$\rightarrow 4442 = 29.44(T-100) + 2856$

$T = 153.87\ k$

(c) $H_i = -1946$ $T_{final} = 77\ k$

$= x(-3407) + (1-x)(2161)$

$\Rightarrow -1946 = -5.56 \cdot 10^3 x + 2161 \Rightarrow x = .737$ liquid

(d) ~~For perfect heater~~

$$H_{f} = (.00\cdots01)(-3407) + (.99\cdots9)(2161)$$
limit

$$\rightarrow 2161\ J$$

$$\Rightarrow \quad 2161\,J = \frac{4442+1946}{(200-100)}(T-100) + -1946$$

$$= 63.8(T-100) - 1946$$

$$T = \underline{164}.37\ k$$

(e) The temperature after throttling would increase. because we have to get an increased entropy

Prob 4.34

(a) Constant enthalpy process happen whenever ~~one~~ ~~gas~~ transformations take place at constant pressure ... ~~I believe this process~~ + no heat flows into or out of the system.

$$H = U + pV$$
$$dH = dU + p\,dV$$

Since the heat flow by the heat exchanger is self contained. I claim that this is a constant enthalpy process

$$H_{in} = x H_{liquid} + (1-x) H_{gas}$$
$$= x(H_{liquid} - H_{gas}) + H_{gas}$$

$$X = \frac{H_{in} - H_{gas}}{H_{liquid} - H_{gas}}$$ ~~so.~~ ~~How get~~

$$= \frac{H_{in} - H_{out}}{H_{liq} - H_{out}} = \frac{H_{out} - H_{in}}{H_{out} - H_{liq}}$$ since $H_{out} = H_{gas}$

(c) 1 bar — 100 bars

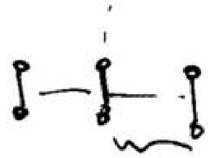
$T_{in} = 300\,K$ $T_{out} = 300\,K$

$$x = \frac{8717 - 8174}{8717 - (-3407)} = .0447 \sim 4\%$$

if $T = 200\,k$

$$x = \frac{8800 - 4442}{8800 - (-3407)} = .147 \sim 15\%$$

P. 4.35



$\left(\frac{\mu_0}{4\pi}\right)\frac{\mu}{r^3}$ $\mu_0 = 4\pi\cdot10^{-7}$

$\mu = 1.9\cdot10^{-24}\,J/T$

1 nm

$x=0$

(a) $2\sum_{P \neq r_i}\left(\frac{\mu_0}{4\pi}\right)\frac{\mu}{r_i^3} = 2\frac{\mu_0}{4\pi}\mu\sum_{i=1}^{\infty}\frac{1}{(10^{-9})^3 i^3}$ $i^{-3} \to \frac{i^{-2}}{-2}$

$\cong 2\frac{\mu_0\mu}{4\pi}(10^9)^3\left(\frac{-1}{2i^2}\right|_1^\infty$

$\cong \cancel{2}\frac{\mu_0\mu}{4\pi}\frac{(10^9)^3}{\cancel{2}}$

$\overset{?}{=} 10^{-7}\cdot(10^{-24}\cdot9)(10^{27}) = 9\cdot10^{-4}\,J/T$ units?

(b) $B = 1T$

From argument given on pg 145

$\frac{M}{\mu N} = .57$ w/ M fixed & lowering B to that point

due to molecules themselves or about 10,000

so from $T_i = 300$ to $T_f = \frac{3}{100} = .03$ K

(c) ?

(d) Again this argument about difficulty of getting very low temp & very stable systems?

Prob 4.36

$$p = \frac{h}{\lambda} \qquad U = N \cdot f \left(\tfrac{1}{2} k_B T\right)$$

recoil energy is loss of momentum derived from laser

$$KE = \frac{1}{2}\frac{p^2}{m}$$

$$\frac{f}{2} k_B T = \frac{1}{2}\frac{1}{m}\left(\frac{h^2}{\lambda^2}\right) \qquad \text{for solids } f = 6.$$

$$T = \frac{1}{f}\frac{1}{m k_B}\left(\frac{h^2}{\lambda^2}\right)$$

$$= \frac{1}{6} \frac{1}{(1.419 \cdot 10^{-22} \cdot 10^{-3}\ \mathrm{kg/atom})}$$

$$\frac{85.46\ \mathrm{g/mol}}{6.022 \cdot 10^{23}\ \mathrm{atoms/mol}} = 1.419 \cdot 10^{-22}\ \mathrm{g/atom}$$

$$T = \frac{1}{6(1.419 \cdot 10^{-22} \cdot 10^{-3}\ \mathrm{kg/atom})(1.381 \cdot 10^{-23}\ \mathrm{J/K})} \frac{(6.626 \cdot 10^{-34}\ \mathrm{J \cdot s})^2}{(780 \cdot 10^{-9})^2}$$

$$= 6.136 \cdot 10^{-11}\ \mathrm{K}.$$

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Prob 4.37 One certainly cannot go as low as $T=0$.

Don't fully understand question asked, ...

$$V_{(1)} = \frac{(1)(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}{10^5\ \text{Pa}}$$

$$\cong \frac{2400\ \text{J}}{10^5\ \text{Pa}}$$

$$\Delta V = \frac{2400}{10^5}\left(\frac{\text{J}}{\text{Pa}}\right)$$

So $P\Delta V \cong 2.4\ \text{kJ}$ How get 4 kJ?

Prob 5.1

$P = 1\text{atm} = 10^5\ \text{Pa}$

$T = 300\text{K}$

$V = \frac{(1)RT}{P}$ ✓

$U_{\text{thermal}} = 7 \cdot N \cdot \frac{1}{2} kT = $ ✓.

$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$

$H = U + PV$

$F = U - TS$

$G = U + PV - TS$ .

Prob 5.2

$N_2 + 3H_2 \rightarrow 2NH_3$

$\Delta H = ?$

$\cancel{\Delta H(N_2 \rightarrow 2N) = 0}$

$\cancel{\Delta H(H_2 \rightarrow 2H) = 0}$

$\Delta H(NH_3 \rightarrow N + 3H) = \Delta H(NH_3 \rightarrow \cancel{\tfrac{1}{2}N_2} + \tfrac{3}{2}H_2)$

$=$

$\Delta H(NH_3) = -46.11$ kJ

$S(NH_3) = 192.45$ J/K

$=$ $\Delta G$(Formation of $NH_3$ from $N_2 + H_2$) $= 2(-46.11$ kJ$)$ $+ T\Delta S$

$= -92.22$ kJ $+ 298\,(192.45$ J$)\cdot 2$

$=$ $2248$ kJ. $\neq -16.$ kJ.

What's wrong?

P 5.3 $Pb + PbO_2 + 4H^+ + 2SO_4^{2-} \to 2PbSO_4 + 2H_2O$

$\Delta H = -316$ kJ/mol

$\Delta H =$ ~~$\Delta H$~~ $2(-920) + 2(-285.83)$

$-0-(-277.4)-0-2(-909.3)$

$= -315.6$ kJ/mol ✓.

$\Delta G = 2(-813) + 2(-237.13) - 0 - (-217.33) - 4(0) - 2(-744.53)$

$= -393.8$ kJ.

P. 5.4 Electrolosis of $H_2O$ requires

$\Delta G = 237$ kJ of electrical work input by U.S.

(Additional "work" can come from heat flow inward)

$$\frac{237 \text{ kJ/mol}}{2 \cdot 6.02 \cdot 10^{23} \text{/mol}} = 1.96 \cdot 10^{-19} \text{ J}$$

$\{1 \text{J} = 6.2 \cdot 10^{18} \text{ eV}\}$ $= 1.22$ eV.

$\Rightarrow$ ~~Batt.~~ 1.22 Volts. would think that this would be the minimum

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Prob 5.5

(a) $CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$

$$\Delta H = -393.51 + 2(\underbrace{-241.82}) - (-74.81) - 2(0) \qquad (kJ)$$

the assumption that this is a gas maybe incorrect if the reaction takes place at standard temp + pressure. because in that range the $H_2O$ would be a liquid...

$$= \cdots$$

$$\Delta G = (-394.36) + 2(-228.57) - (-50.72) - 2(0) \qquad (kJ)$$

$$= \cdots$$

So

$$\Delta H = -802.34 \text{ kJ}$$

$$+ \ \Delta G = -800.78 \text{ kJ}$$

(b) From eq 5.8 $\Delta G \leq W_{other}$

$\therefore$ electrical energy available $\overset{?}{\leq}$ $-800.78$ kJ

(c) $\Delta G = \Delta H - T\Delta S$ w/ $T\Delta S$ entering as heat

$$\therefore T\Delta S = \Delta H - \Delta G = -802.34 - (-800.78) = 1.56 \text{ kJ}.$$

(d) Thus 8 electrons go around $\forall$ ~~...~~ reaction step

Thus obtaining $\dfrac{800.78 \text{ kJ}}{8 \cdot 6.02 \cdot 10^{23}} = 1.66 \cdot 10^{-19} \text{ J} = 1.037 \text{ eV}$

$\Rightarrow$ 1 volt cell

$1 \text{ J} = 6.2421 \cdot 10^{18} \text{ eV}$

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Prob 5.6

(a) $C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$

$$\Delta H = 6(-393.51) + 6(-285.83) - (-1273) - 6\cdot 0$$
$$= -2803 \text{ kJ}$$

$$\Delta G = 6(-394.36) + 6(-237.13) - (-910) - 6\cdot 0$$
$$= -2878.94 \text{ kJ}.$$

(b) Max work a muscle can perform is 2878.94 kJ work (electrical)

(c) $\Delta G = \cancel{\phantom{xx}}\ \cancel{\phantom{xx}}\ \Delta H - T\Delta S$

$\Rightarrow T\Delta S = \Delta H - \Delta G = -2803 + 2878.94 = 75.94 \text{ kJ}.$

comes from heat absorbed into the cell.

(d) ?

(e) The work the muscle could do would be less than 2878.94 kJ

& the heat absorbed ... less also?

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Prob 5.7

$$\langle F \rangle = 4 \cdot 10^{-12}\ N.$$

$$d \cong 11 \cdot 10^{-9}\ m$$

$$W = (4 \cdot 10^{-12} \cdot 11 \cdot 10^{-9})\ J = (4.4 \cdot 10^{-20})\ J$$

$$e = \frac{W}{W_{max}} = \frac{(4.4 \cdot 10^{-20})\ J}{\frac{(2878.94 \cdot 10^{3}\ J)}{6.02 \cdot 10^{23}}} = 9.2 \cdot 10^{-3}$$

(Prob 5.8) $G = U + PV - TS$

$$dG = dU + dP \cdot V + P\,dV - dT \cdot S - T\,dS$$

$$= \cancel{T\,dS} - \cancel{P\,dV} + \mu\,dN + V \cdot dP + \cancel{P\,dV} - S\,dT - \cancel{T\,dS}$$

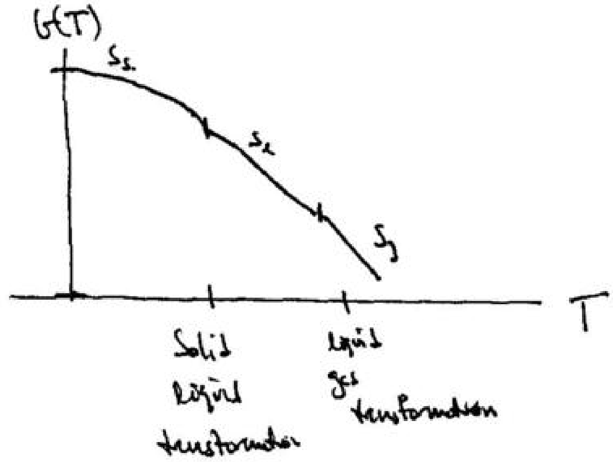
$$= V\,dP - S\,dT + \mu\,dN$$

$$= -S\,dT + V\,dP + \mu\,dN \qquad \text{eq } 5.23$$

$$\left.\frac{\partial G}{\partial T}\right|_{P,N} = -S \;;\; \left.\frac{\partial G}{\partial P}\right|_{T,N} = V \;;\; \left.\frac{\partial G}{\partial N}\right|_{T,P} = \mu$$

$$S = -\left.\frac{\partial G}{\partial T}\right|_{P,N} \;;\; V = \left.\frac{\partial G}{\partial P}\right|_{T,N} \;,\; \mu = \left.\frac{\partial G}{\partial N}\right|_{T,P}$$

(Prob 5.9) $dG = -S\,dT + V\,dP + \mu\,dN$ $\qquad S \geq 0$




$P = \text{const}$ + $N = \text{const}$

$\Rightarrow \quad dG = -S\,dT$

$S_{solid} \leq S_{liquid} \leq S_{gas}$

Prob 5.10 $\Delta T = 5°C = 5K$

$$\Delta G \cong -S\Delta T + V\cancel{\Delta P}^{0} + \mu\cancel{\Delta N}^{0}$$

$$\underset{H_2O\ S.T.P.}{S} = 69.91\ J/K$$

$\therefore\ \Delta G \cong (-69.91)(5) = -349.55\ J$

$$\Delta G = 0 = -349.55J + V\Delta P$$

$$V\Delta P = 349.55\ J$$

$$\|$$

$(18.068\ cm^3)(\Delta P) =$ $1cm = 10^{-2}m$

$$(18.068(10^{-2})^3 m^3)\,\Delta P = 349.55\ J$$

$\Delta P = 1.934 \cdot 10^7\ J/m^3$ $J = N \cdot m$

$= 1.934 \cdot 10^7\ Pa.$ $1atm = 1.013 \cdot 10^5\ Pa$

$= 190.9\ atm.$ $\Rightarrow 1Pa = 9.87 \cdot 10^{-6}\ atm$

Prob 5.11 $H_2O \rightarrow H_2 + \frac{1}{2}O_2$ $\Delta H = 286$ kJ

$\Delta G = 237$ kJ

$dG = -S\,dT + V\,dP + \mu\,dN$

$S_{H_2O} = 70\ J/K$ $S_{H_2} = 131\ J/K$ $S_{O_2} = 205\ J/K$ } at P = 1 atm, T = 298

$G_{H_2O} = 237$ kJ $G_{H_2} = 0$ $G_{O_2} = 0$

(a) $\Delta G_{H_2} \cong -S\Delta T = (-131)(50) = -6.55$ kJ

$\Delta G_{H_2O} \cong -S\Delta T = -(70)(50) = -3.5$ kJ

$\Delta G_{O_2} \cong -(205)(50) = -10.25$ kJ

$\therefore$ $G_{H_2} = -6.55$ kJ $G_{O_2} = -10.25$ kJ $G_{H_2O} = 237 - 3.5 = 233.5$ kJ

(b) Then $\Delta G_{cycle} = G_{H_2} + \frac{1}{2}G_{O_2} - G_{H_2O}$

$= -6.55 + \frac{1}{2}(-10.25) - (233.5$

.... is there a sign error?

Prob 5.12

$$dU = T\,dS - p\,dV + \mu\,dN$$

$$\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial S}\right) = \frac{\partial}{\partial S}\left(\frac{\partial U}{\partial V}\right)$$

$$\left.\frac{\partial T}{\partial V}\right|_S = -\left.\frac{\partial p}{\partial S}\right|_V$$

$$dH = T\,dS + V\,dp + \mu\,dN$$

Note: $\frac{\partial}{\partial p}\left(\frac{\partial H}{\partial S}\right) = \frac{\partial}{\partial S}\left(\frac{\partial H}{\partial p}\right)$

$$\Rightarrow \left.\frac{\partial T}{\partial p}\right|_S = \left.\frac{\partial V}{\partial S}\right|_p$$

$$dF = -S\,dT - P\,dV + \mu\,dN$$

$$\frac{\partial}{\partial V}\left(\frac{\partial F}{\partial T}\right) = \frac{\partial}{\partial T}\left(\frac{\partial F}{\partial V}\right)$$

$$\left. -\frac{\partial S}{\partial V}\right|_T = -\left.\frac{\partial p}{\partial T}\right|_V \Rightarrow \left.\frac{\partial S}{\partial V}\right|_T = \left.\frac{\partial p}{\partial T}\right|_V$$

$$dG = -S\,dT + V\,dp + \mu\,dN$$

$$\frac{\partial}{\partial p}\left(\frac{\partial G}{\partial T}\right) = \frac{\partial}{\partial T}\left(\frac{\partial G}{\partial p}\right)$$

$$\frac{\partial}{\partial p}(-S)\Big|_T = \frac{\partial}{\partial T}(V)\Big|_p$$

$$\Rightarrow -\frac{\partial S}{\partial p}\Big|_T = \frac{\partial V}{\partial T}\Big|_p$$

Prob 8.13 $\beta \equiv \frac{1}{V}\frac{\partial V}{\partial T}\Big|_{p?}$

relate $V \sim S$ Then by the 4th identity

$\beta = \frac{1}{V}\left(-\frac{\partial S}{\partial p}\right)\Big|_T$ How do I know $\frac{\partial S}{\partial p} = 0$ when $T=0$?

Prob 8.14 Prob 3.33 $\Rightarrow$ $C_V = T\frac{\partial S}{\partial T}\Big|_V$

$$dS = \frac{\partial S}{\partial T}\Big|_V dT + \frac{\partial S}{\partial V}\Big|_T dV = \frac{C_V}{T}dT + \frac{\partial S}{\partial V}\Big|_T dV$$

$$dV = \frac{\partial V}{\partial T}\Big|_p dT + \frac{\partial V}{\partial p}\Big|_T dp$$

$$\text{so } dS = \cancel{\frac{C_V}{T}dT}\ \frac{C_V}{T}dT + \frac{\partial S}{\partial V}\Big|_T\left(\frac{\partial V}{\partial T}\Big|_p dT + \frac{\partial V}{\partial p}\Big|_T dp\right)$$

$$dS = \left( \frac{C_V}{T} + \frac{\partial S}{\partial V}\Big|_T \frac{\partial V}{\partial T}\Big|_P \right) dT + \left( \frac{\partial S}{\partial V}\Big|_T \frac{\partial V}{\partial P}\Big|_T \right) dP$$

if $dP = 0$

$$\frac{\partial S}{\partial T}\Big|_P = \frac{C_V}{T} + \frac{\partial S}{\partial V}\Big|_T \frac{\partial V}{\partial T}\Big|_P \qquad \frac{\partial S}{\partial T}\Big|_P =$$

$$Q_P = \cancel{\frac{\Delta S}{T}}\Big|_P \qquad \Delta S|_P = \frac{Q|_P}{T} \Rightarrow Q_P = T\Delta S$$

$$C_P = \frac{Q_P}{\Delta T} = \frac{1}{T}\frac{\partial S}{\partial T}\Big|_P \qquad C_P = \frac{Q_P}{\Delta T} = T\frac{\partial S}{\partial T}\Big|_P$$

$$\Rightarrow \frac{\partial S}{\partial T}\Big|_P = \frac{1}{T} C_P.$$

$\therefore$

$$\frac{C_P}{T} = \frac{C_V}{T} + \frac{\partial S}{\partial V}\Big|_T \frac{\partial V}{\partial T}\Big|_P$$

$$C_P - C_V = T \frac{\partial S}{\partial V}\Big|_T \frac{\partial V}{\partial T}\Big|_P$$

$$\frac{\partial}{\partial V}\left(\frac{\partial F}{\partial T}\right) = \frac{\partial}{\partial T}\left(\frac{\partial F}{\partial V}\right)$$

$$-\frac{\partial S}{\partial V}\Big|_T = -\frac{\partial P}{\partial T}\Big|_V$$

(c) Now $\beta \equiv \frac{1}{V}\frac{\partial V}{\partial T}\Big|_P$ $\Rightarrow \frac{\partial S}{\partial V}\Big|_T = \frac{\partial P}{\partial T}\Big|_V$

thermal expansion coefficient

$\therefore$

$$C_P - C_V = T V \beta \frac{\partial S}{\partial V}\Big|_T$$

$$\cancel{\beta_T} \approx \kappa_T \equiv -\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T$$

$$C_p - C_v = TV\beta \left.\frac{\partial P}{\partial T}\right|_V$$

considering P, V, + T as variables ...

$$\left.\frac{\partial P}{\partial V}\right|_T \cdot \left.\frac{\partial V}{\partial T}\right|_P \cdot \left.\frac{\partial T}{\partial P}\right|_V = -1 \Rightarrow \left.\frac{\partial P}{\partial T}\right|_V =$$

$$= - \left.\frac{\partial P}{\partial V}\right|_T \left.\frac{\partial V}{\partial T}\right|_P$$

$\therefore$ ~~$C_p - C_v = TV\beta\left(-\frac{\partial P}{\partial}\right)$~~

$$= -V\left.\frac{\partial P}{\partial V}\right|_T \cdot \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P$$

$$= \frac{\beta}{\kappa_T}$$

$$\therefore C_p - C_v = \frac{TV\beta^2}{\kappa_T}$$

(d) For an ideal gas $PV = NkT \Rightarrow V = \frac{NkT}{P}$

$$\beta \equiv \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P = \frac{1}{V}\left(\frac{Nk}{P}\right) = \frac{1}{T}$$

$$\kappa_T = -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_T = -\frac{1}{V}\left(\frac{NkT}{P^2}\right)(-1) = \frac{NkT}{VP \cdot P} = \frac{1}{P}$$

$$C_p = \left.\frac{\partial H}{\partial T}\right|_V \qquad H = \left(\frac{f+2}{2}\right)NkT \qquad \left.\frac{\partial H}{\partial T}\right|_V = \left(\frac{f+2}{2}\right)Nk$$

$$C_V = \left.\frac{\partial U}{\partial T}\right|_V \qquad U = f \cdot N \cdot \frac{1}{2}kT$$

$$\frac{\partial U}{\partial T} = fN\frac{1}{2}k$$

$$C_p - C_v = \left(\frac{f+2}{2}\right)Nk - \frac{f}{2}Nk = Nk$$

$$\stackrel{?}{=} \frac{T\cdot V\cdot(1/T)^2}{(1/P)} = \frac{PV}{T} = Nk \checkmark$$

(e) $C_p - C_v \geq 0$

$C_p \geq C_v$.

(f) $\underline{H_2O\ (25°C)}$

$\beta = 2.57\cdot 10^{-4}\ K^{-1}$

$\kappa_T = 4.52\cdot 10^{-10}\ Pa^{-1}$

$V = 18.068\cdot 10^{-6}\ m^3$

$\underline{Hg\ (25°C)}$

$\beta = 1.81\cdot 10^{-4}\ K^{-1}$

$\kappa_T = 4.04\cdot 10^{-11}\ Pa$

$V = 14.81\cdot 10^{-6}\ m^3$

So

$$\frac{TV\beta^2}{\kappa_T} = .744\left(\frac{K\cdot m^3\cdot K^{-2}}{Pa^{-1}}\right) \qquad \frac{TV\beta^2}{\kappa_T} = 3.578\ (J/K)$$

$$K^{-1} m^3 \left(\frac{N}{m^2}\right)$$

$$K^{-1} m N$$

$$\frac{J}{K}$$

By $\approx$ 300% / 380.9%

(9) $C_p - C_V = \frac{TV\beta^2}{k_T}$

$\beta = \beta(T)$

when T is small the difference btween $C_p$ + $C_V$ is small.

but for larger T's the difference is larger

Prob 5,15 $C_p = \left.\frac{\partial H}{\partial T}\right|_p$ $C_V = \left.\frac{\partial U}{\partial T}\right|_V$

$$C_p - C_V = \left.\frac{\partial H}{\partial T}\right|_p - \left.\frac{\partial U}{\partial T}\right|_V \qquad H = U + PV.$$

$$= \left.\frac{\partial}{\partial T}(U + PV)\right|_p - \left.\frac{\partial U}{\partial T}\right|_V$$

$$= \left.\frac{\partial U}{\partial T}\right|_p + \left.\frac{\partial (PV)}{\partial T}\right|_p - \left.\frac{\partial U}{\partial T}\right|_V$$

$$= \left.\frac{\partial U}{\partial T}\right|_p + \left.\frac{P\partial V}{\partial T}\right|_p - \left.\frac{\partial U}{\partial T}\right|_V$$

Fr $dU = S\,dT - p\,dV + \mu\,dN$

Don't see?

Prob 5.16

$$\kappa_T = \kappa_S + \frac{TV\beta^2}{C_P} \qquad \kappa_S = -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_S$$

$$\kappa_T = -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_T$$

~~Consider~~ $V = V(\cancel{T}, S)$

$$\cancel{dV = \left.\frac{\partial V}{\partial T}\right|_S dT + \frac{\partial V}{}}$$

~~Consider~~ $\cancel{V = V(P,T)}$

$$\cancel{dV = \left.\frac{\partial V}{\partial P}\right|_T dP + \left.\frac{\partial V}{\partial T}\right|_P dT}$$

$$C_V = T\left.\frac{\partial S}{\partial T}\right|_V \qquad C_P = T\left.\frac{\partial S}{\partial T}\right|_P$$

consider $V = V(P,S)$

$$dV = \left.\frac{\partial V}{\partial P}\right|_S dP + \left.\frac{\partial V}{\partial S}\right|_P dS$$

Now consider $S$ to be a function of $P$ + $T$

$$dS = \left.\frac{\partial S}{\partial P}\right|_T dP + \left.\frac{\partial S}{\partial T}\right|_P dT$$

$$dV = \left.\frac{\partial V}{\partial P}\right|_S dP + \left.\frac{\partial V}{\partial S}\right|_P \left.\frac{\partial S}{\partial P}\right|_T dP + \cancel{\left.\frac{\partial V}{\partial S}\right|_P \left.\frac{\partial S}{\partial T}\right|_P dT}^{\,0 \text{ T const}}$$

$$\left.\frac{\partial V}{\partial P}\right|_T = \left.\frac{\partial V}{\partial P}\right|_S + \left.\frac{\partial V}{\partial S}\right|_P \left.\frac{\partial S}{\partial P}\right|_T$$

$$-V\kappa_T = -V\kappa_S + \left.\frac{\partial V}{\partial S}\right|_p \cdot \left.\frac{\partial S}{\partial P}\right|_T$$

Now $\left.\frac{\partial S}{\partial P}\right|_T \cdot \left.\frac{\partial T}{\partial S}\right|_p \cdot \left.\frac{\partial P}{\partial T}\right|_S = -1$

$$\Rightarrow \quad \left.\frac{\partial S}{\partial P}\right|_T = \frac{-1}{\left.\frac{\partial T}{\partial S}\right|_p \cdot \left.\frac{\partial P}{\partial T}\right|_S}$$

$$-V\kappa_T = -V\kappa_S - \frac{\partial V/\partial S|_p}{\left.\frac{\partial T}{\partial S}\right|_p \left.\frac{\partial P}{\partial T}\right|_S} \qquad \beta \equiv \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_p$$

$$\left.\frac{\partial V}{\partial S}\right|_p = \left.\frac{\partial V}{\partial T}\right|_p \cdot \left.\frac{\partial T}{\partial S}\right|_p = V\beta \left.\frac{\partial T}{\partial S}\right|_p$$

$$-V\kappa_T = -V\kappa_S - \frac{V\beta}{\left.\frac{\partial P}{\partial T}\right|_S}$$

$$\kappa_T = \kappa_S + \frac{\beta}{\left.\frac{\partial P}{\partial T}\right|_S}$$

$$C_p = T\left.\frac{\partial S}{\partial T}\right|_p \qquad \left.\frac{\partial P}{\partial T}\right|_S \left.\frac{\partial T}{\partial S}\right|_p \cdot \left.\frac{\partial S}{\partial P}\right|_T = -1$$

$$\kappa_T = \kappa_S + \frac{-\beta}{\left(\frac{\partial T}{\partial S}\Big|_p \cdot \frac{\partial S}{\partial p}\Big|_T\right)^{-1}}$$

$$\kappa_T = \kappa_S - \beta \left(\frac{\partial T}{\partial S}\right)_p \left(\frac{\partial S}{\partial p}\Big|_T\right) \qquad \frac{C_p}{T} = \frac{\partial S}{\partial T}\Big|_p$$

$$= \kappa_S - \frac{\beta T}{C_p} \cdot \frac{\partial S}{\partial p}\Big|_T$$

Show $\frac{\partial S}{\partial p}\Big|_T = -\beta V$ $\qquad \beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_p$

$$= -\frac{\partial V}{\partial T}\Big|_p$$

Is this a maxwell relation?

$$dU = T\,dS - p\,dV + \mu\,dN$$

$$\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial S}\right) = \frac{\partial}{\partial S}\left(\frac{\partial U}{\partial V}\right)$$

$$\|$$

$$\frac{\partial}{\partial V}(T)\Big|_S = \frac{\partial}{\partial S}(-p)\Big|_V \quad \Rightarrow \quad \frac{\partial T}{\partial V}\Big|_S = -\frac{\partial p}{\partial S}\Big|_V$$

$$\frac{\partial V}{\partial T}\Big|_S = -\frac{\partial S}{\partial p}\Big|_V$$

everything is correct except the constants ...

$$dF = \cancel{dU} - S\,dT - P\,dV + \mu\,dN$$

$$\frac{\partial S}{\partial V} \cancel{=} \frac{\partial P}{\partial T}$$

$$dG = -S\,dT + V\,dP + \mu\,dN$$

$$\frac{\partial S}{\partial P} = \frac{\partial V}{\partial T}$$

$$\frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\right) = \frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\right)$$

$$\Rightarrow \frac{\partial}{\partial P}(-S)\Big|_T = \frac{\partial}{\partial T}(V)\Big|_P \Rightarrow \frac{\partial S}{\partial P}\Big|_T = -\frac{\partial V}{\partial T}\Big|_P \quad \text{yes!!}$$

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(Prob 5.17) $\cancel{\#}$ $H = \frac{NI}{L}$ $H = \frac{B}{\mu_0} - \frac{M}{V}$

(a) $dH = \frac{N}{L} dI$

$dH = \frac{dB}{\mu_0} - \frac{dM}{V}$

?

$\nabla \circ D = \rho$ ·

$\nabla \circ H = 0$ ·

$\nabla \times D = \frac{\partial H}{\partial t}$ ·

$\nabla \times H = J + \frac{\partial D}{dt}$

(b) $W_{total} = VH\,dB$

$= VH$

$$\left\{ dH = \frac{dB}{\mu_0} - \frac{dM}{V} \Rightarrow dB = \mu_0\left(dH + \frac{dM}{V}\right) \right.$$

$W_{total} = VH\mu_0\left(dH + \frac{dM}{V}\right)$

$= VH\mu_0 dH + H\mu_0 dM$

$= V\mu_0 d\left(\frac{H^2}{2}\right) + H\mu_0 dM$

$W_{total} - V\mu_0 d(H^2/2) = H\mu_0 dM$

(c) ?

(d) By subtracting of the self inductance ... ?

(8.18) I would imagine that a ~~big~~ large portion of the bricks potential → kinetic energy would be transformed into the thermal motion of the atoms that make up the brick + the earth. The energy has be distributed from many molecules moving in uniform ~~mot~~ motion to many molecules moving in random arrangements.

(8.19) $F = U - TS$ ?

(8.20) ~~$\Delta F =$~~ $S = k \ln 4 = (1.38 \cdot 10^{-23}\ J/K) \ln 4$

$= 1.913 \cdot 10^{-23}\ J/K$

$F = 10.2 \cdot (1.602 \cdot 10^{-19}\ J)$

$- T(1.913 \cdot 10^{-23}\ J/K) > 0$

$10.2(1.602 \cdot 10^{-19}) > (1.913 \cdot 10^{-23})T$ its positive

$T < 8.54 \cdot 10^{4}\ K$

5.21

$C$ is extensive

$c = \frac{C}{m}$ is intensive

5.22 eq 5.40 $\mu(T,P) = \mu^0(T) + kT \ln(P/P^0)$

The formulas in section 3.5

$$\mu = -T \left.\frac{\partial S}{\partial N}\right|_{U,V} \quad \text{+} \quad \mu = \left.\frac{\partial U}{\partial N}\right|_{S,V}$$

For an ideal gas $S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$

$$\frac{\partial S}{\partial N} = \frac{S}{N} + Nk\left[\frac{1}{\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right)}\left[\left(\frac{-V}{N^2}\right)\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right.\right.$$

$$\left.\left. + \frac{V}{N}\cdot\frac{3}{2}\left(\frac{4\pi m U}{3Nh^2}\right)^{1/2}\cdot\left(\frac{4\pi m U}{3N^2h^2}\right)(-1)\right]\right]$$

$$= \frac{S}{N} + Nk\left[(-1/N) + \frac{3/2(-1/N)}{(1)}\right]$$

$$= \frac{S}{N} + Nk\left[\frac{5}{2}\frac{1}{N}\right] = \frac{S}{N} + \frac{5}{2}k \quad \ldots$$

In section 3.5 we derived

$$\mu = -kT \ln\left[\frac{V}{N}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right]$$ for a monatomic ideal gas

$PV = kNT$

$kT = \frac{PV}{N}$

$$\mu = -kT \ln\left[\frac{V}{N}\left(\frac{2\pi m}{h^2}\frac{PV}{N}\right)^{3/2}\right]$$

? This does not look like what we are looking for.

(5.23) $\Phi \equiv U - TS - \mu N$

$H = U + PV$

$F = U - TS$

$G = U + PV - TS$

(a) $d\Phi = dU - dT\cdot S - T\cancel{dS} - d\mu\cdot N - \mu \cancel{dN}$

$\quad\| $

$T\cancel{dS} - p\,dV + \mu \cancel{dN}$

$= -p\,dV - S\,dT - N\,d\mu$ ~~manN~~

$$\Rightarrow \quad p = -\left.\frac{\partial \Phi}{\partial V}\right|_{T,\mu} \qquad S = -\left.\frac{\partial \Phi}{\partial T}\right|_{V,\mu} \qquad N = -\left.\frac{\partial \Phi}{\partial \mu}\right|_{V,T}$$

(b) $\cancel{dS}_{reservoir} = \cancel{+\frac{dU_R}{T_R}}$

$dU = TdS - PdV + \mu dN$

$dS = \frac{1}{T}dU + \frac{P}{T}dV - \frac{\mu}{T}dN$

For reservoir exchanging particles & energy but not volume $dS = \frac{dU_r}{T} - \frac{\mu}{T}dN_r$

$= -\frac{dU}{T} + \frac{\mu}{T}dN$

Now change in reservoirs' entropy is expressed in terms of system variables

$dS_{total} = dS_{system} + dS_{reservoir}$

$= dS + -\frac{dU}{T} + \frac{\mu}{T}dN$

$= -\frac{1}{T}(-TdS + dU - \mu dN)$

$= -\frac{1}{T}(dU - TdS - \mu dN) > 0$

$=$ If $d\Phi = dU - TdS - \mu dN$ then $\Phi$ would have to decrease.
?

(c) ?

$\mu = ?$

$\Phi_1 = -13.6\text{ eV} - (8800\text{K})\cdot 0 - \cancel{\alpha}$

$\Phi_2 = 0 - (\ \ )\cdot 0$

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(5.24) C(graphite) $\Delta_f G = 0$ kJ / mole

C(diamond) $\Delta_f G = 2.9$ kJ

$\therefore$ graphite is stable at s.t.p

$$\left(\frac{\partial G}{\partial P}\right)_{T,N} = V \qquad dG = -S\,dT + V\,dP + \sum_i \mu_i\, dN_i$$

C(graphite) $V = 5.30\ \text{cm}^3 > V_{diam}$

C(diamond) $V = 3.42\ \text{cm}^3$

Assuming V constant $1\,\text{cm}^3 = 10^{-6}\,\text{m}^3$

$$G = V \cdot P + G_0$$

$$\cancel{G_{graphite}(P) = (5.3)(10^{-6}\,\text{m}^3)P + 0}$$

$$\cancel{G_{diamond}(P)} =$$

$$G_{graphite}(P) = (5.3(10^{-6})\,\text{m}^3)P + 0$$

$$G_{diamond}(P) = (3.42(10^{-6})\,\text{m}^3)P + 2900\ \text{J}$$

$$(5.3 - 3.42)10^{-6}\,P = 2900\ \text{J/m}^3$$

$$\rightarrow P = 1.5 \cdot 10^9\ \text{Pa} \qquad 1\ \text{bar} = 10^5\ \text{Pa}$$

$$= 15\ \text{kilobars} \qquad 1\ \text{kilobar} = 10^8\ \text{Pa}$$

(5.25) $\frac{kJ}{kbar} = \frac{10^3\ J}{10^3\ bar} = \frac{1\ N\cdot m}{10^5\ N/m^2} = 10^{-5}\ m^3$

(5.26) C (graphite) $S = 5.74\ J/K$

C (diamond) $S = 2.38\ J/K$

this entropy is at 1 atm, $T = 273\,K$ & ~~...~~ implys that graphite would be the more stable form under these conditions.

would the entropy change w/ pressure such that at a higher pressure the entropies switch ~~v~~ values (which one is greater?)

?

(5.27)(a) $\left\{ \kappa = -\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T \right\} \Rightarrow \frac{\partial V}{\partial P}\Big|_T = -V\kappa$

$$\frac{\partial G}{\partial P}\Big|_{T,N} = V \cong V_0 + \frac{\partial V}{\partial P}\Big|_T (V - V_0)$$

$$= V_0 - V_0\kappa_0(V - V_0) \pm \cancel{\ldots}$$

$$= V_0 + V_0\kappa_0(V_0 - V) > V_0$$

$\Rightarrow$ Thus the slope of Graphite is <u>steeper</u> than originally predicted

$\therefore$ transformation pressure should happen at a <u>lower</u> pressure.

(b) $\kappa_{graphite} = 3 \cdot 10^{-6}\ bar^{-1}$ ⇒ ∀ atmosphere of pressure increase graphite shrinks by 3 one millionth.

$$|||$$

$$-\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T = \kappa \text{ const.} \Rightarrow \frac{\partial V}{\partial P}\Big|_T = -V\kappa$$

$$\frac{\partial G}{\partial P}\Big|_{T,N} = V \qquad = V(P,T_0) = -V\kappa P$$

$$= -V\kappa P$$

$$\Rightarrow G \cong \int^P -V_0\kappa_0 P'\,dP' = -V_0\kappa_0\frac{P^2}{2} + G_0 \qquad \text{equate + solve...}$$

(5.28)

| | $\Delta H_f$ | $\Delta_f G$ (kJ) | S | $C_p$ | V ($cm^3$) |
|---|---|---|---|---|---|
| $CaCO_3$ (calcite) | −1206.9 | −1128.8 | 92.9 | 81.88 | 36.93 |
| $CaCO_3$ (aragonite) | −1207.1 | −1127.8 | 88.7 | 81.25 | 34.15 |

(a) Arguing that the form w/ the smallest Gibbs free energy is the stablest form gives calcite as the stable form.

(b) ~~Again~~ as $\frac{\partial G}{\partial P}\Big|_{T,N} = V$ Assuming the volume does not change w/ P. i.e. the compressibility is negligible over the range of pressures considered ...

$$G = VP + G_0.$$

$$G_{calcite} = (36.93 \cdot 10^{-6}\ m^3)P + -1128.8\ kJ$$

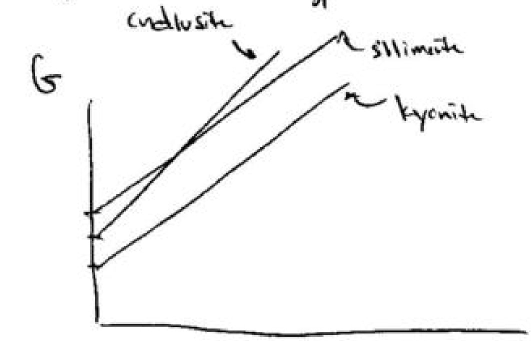
$$G_{aragonite} = (34.15 \cdot 10^{-6}\ m^3)P + \ldots$$

(5.29)

| | $\Delta H_f$ | $\Delta_f G$ (1.73) | S (7.0) | $C_p$ | V |
|---|---|---|---|---|---|
| $Al_2SiO_5$ | | | | | |
| kyanite | −2594.29 | −2443.88 | 83.81 | 121.71 | 44.09 |
| andalusite | −2590.27 | −2442.66 | 93.22 | 122.72 | 51.53 |
| sillimanite | −2587.76 | −2440.99 | 96.11 | 124.52 | 49.90 |

(a) The phase w/ the lowest gibbs free energy is the stablest form

⇒ kyanite




$$\left.\frac{\partial G}{\partial P}\right|_{V,T} = V$$

then kyanite starts as the smallest & stays that way for all pressures.

(b) $dG = -SdT + VdP + \sum_i \mu_i dN$

$$\left.\frac{\partial G}{\partial T}\right|_{P,N} = -S$$

$$G(T_2) - G(T_1) = -\int_{T_1}^{T_2} S(T)\, dT$$

(c) $\Delta G(T_2) = \Delta G(T_1) - \Delta S(T_1)(T_2 - T_1)$

$$\Delta G_{kyanite}(T) = -2443.88 - 83.81(T-273)$$

$$\Delta G_{andalusite}(T) = -2442.66 - 93.22(T-273)$$

$$\Delta G_{\text{Sill.manide}} = -2440. - 96.11(T-273)$$

stablest form is the one w/ $\Delta G$ smallest.

(d) $$\left.\frac{\partial S}{\partial T}\right|_V = \frac{C_V}{T}$$

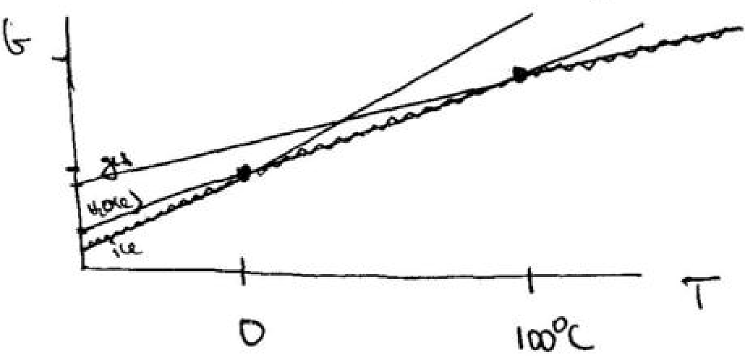
$$\left.\frac{\partial S}{\partial T}\right|_P = \frac{C_P}{T}$$

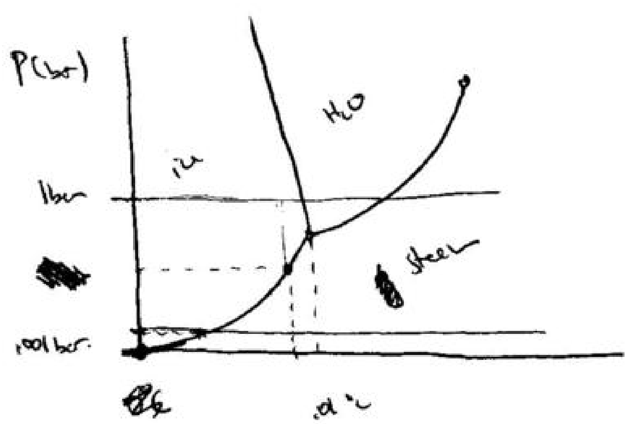
$$\left.\frac{\partial S}{\partial T}\right|_P \cong \frac{120\ \text{kJ}}{T}$$

don't know how to comment on the validity of constant enthalpy approximation.

(5.30)

ice ⟶ $H_2O(l)$ ⟶ gas




At .001 bar the temperatures where there would be no liquid phase

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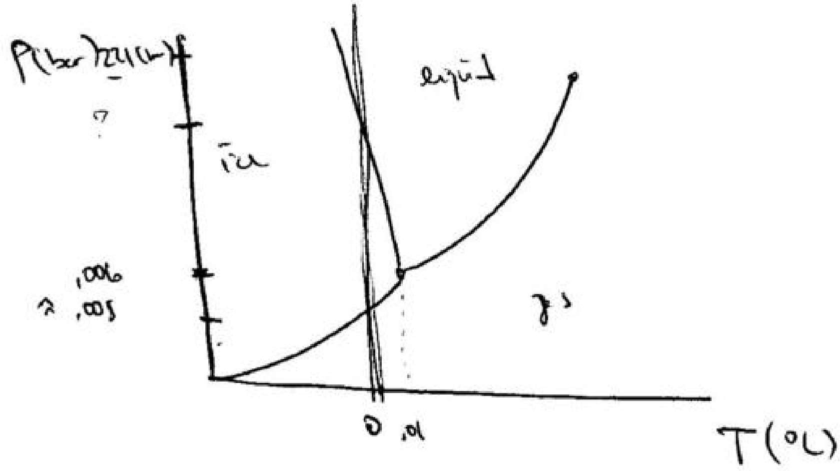
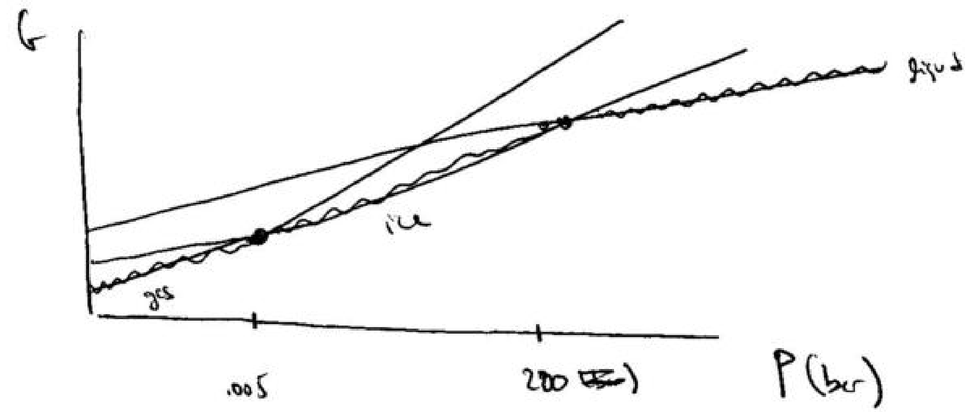
(5.31)




At high temp. there might be no ice phase



5.32

$$\frac{dP}{dT} = \frac{L}{T\Delta V} \quad \text{per mole}$$

$\rho_{ice} = 917\ \mathrm{kg/m^3}$

$\rho_{H_2O} = 1\ \mathrm{g/cm^3}$

$= 10^{-3}\ \mathrm{kg} / 10^{-6}\ \mathrm{m^3} = 1000\ \mathrm{kg/m^3}$

Change /mole into /kg

$L_{ice \to H_2O} = 333\ \mathrm{J/g}$

$1\ \text{mole} = M\ \text{kg}$ $\quad M$ = molecular weight in kg's

$$\frac{x}{\text{mole}} = \frac{x}{1\ \text{kg}} \cdot \frac{M\ \text{kg}}{1\ \text{mole}}$$

$$\Delta S = \frac{L}{T}$$

$$\frac{dP}{dT} = \frac{S_s - S_e}{V_s - V_e}$$

$$= \frac{L}{T(V_s - V_e)}$$

$$\Leftrightarrow \quad \frac{dP}{dT} = \frac{L}{T\Delta(\rho^{-1})} \quad \text{per kg}$$

$$= \frac{L}{T\left(\frac{1}{\rho_s} - \frac{1}{\rho_e}\right)}$$

$$= \frac{L}{T}\frac{1}{\left(\frac{1}{917} - \frac{1}{1000}\right)} = \frac{L}{T}(1.1 \cdot 10^4)\ \mathrm{kg/m^3}$$

... I know $S_s < S_e \Rightarrow S_s - S_e = -\frac{L}{T}$ w/ minus sign

$$\therefore \quad \frac{dP}{dT} = -\frac{L}{T}(1.1 \cdot 10^4\ \mathrm{kg/m^3})$$

(b) $\frac{dP}{dT} \cong -\frac{333\ \text{J/kg} \cdot 10^3}{(273\ \text{K})}\left(1.1 \cdot 10^4\ \text{kg/m}^3\right)$

$= \cancel{1.34 \cdot 10^4\ \text{Pa/K}}$ $-1.341 \cdot 10^7\ \text{Pa/K} = -132.4\ \text{atm/K}$

At $P = 1$ atm. $T = 0°C$.

$$P(-1°C) \cong P(0°C) + \left.\frac{dP}{dT}\right|_{0°C} \cdot (-1-0) + O(\Delta T^2)$$

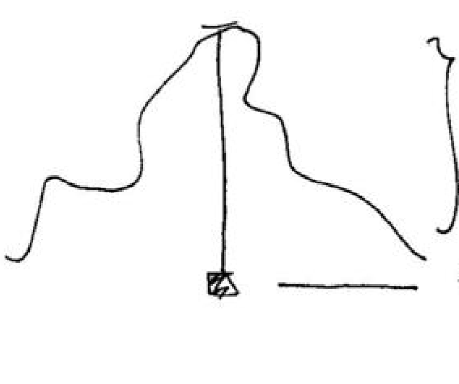
$= 1\ \text{atm} + (132.4\ \text{atm})(+1)$

$= \cancel{131.4\ \text{atm}}\ 133.4$ atm.

$f(x+h) = f(x) + f'(x)h + O(h^2)$

$f(x) = f(x_0) + f'(x_0)(x-x_0) + O((x-x_0)^2)$

(c) ~~Assume that the density~~



$\rho_{liq} h g = P = 133.4$ atm

$h = 1.5$ km

(d) ~~xxxx~~ 150 lbs = 68.1 kg = Mass of skater

Force = Mg

Area = (12 inch)(1 cm) = $30.48 \cdot 10^{-4}\,m^2$

$$P = \frac{(68.1)(9.8)}{30.48 \cdot 10^{-4}} = 2.19 \cdot 10^5\ Pa = 2.16\ atm.$$

$$\Delta P = 2.16\ atm - 1\ atm \cong \left.\frac{dP}{dT}\right|_{1atm} (\Delta T)$$

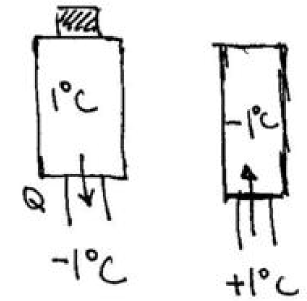
would give the change in melting temperature due to the additional weight

$$\left.\frac{dP}{dT}\right|_{1atm} = -132.4\ atm/K$$

$\Delta T = -8.76 \cdot 10^{-3}$ K a negible change.

~~Everts~~ ~~exp.~~ ~~that~~ How do we know that the changes are not very, very slight?

(5.33)





$$e \equiv \frac{W}{Q_{in}} = \frac{\text{Work}}{Q_{removed}}$$

Clausius-Clapeyron relation:

$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$

The device has to absorb a finite amount of heat between upon


freezing, $L_{ice \to liqui}$

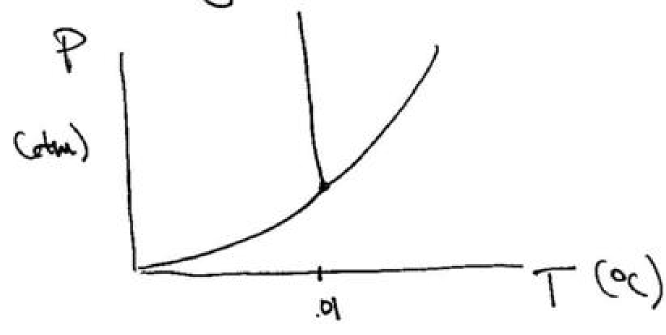
For $H_2O$

$L_{solid \to liquid}$

$L_{sl} = +333\ J/g$ $\qquad$ $L_{lg} = +2260\ J/g$

The machine has to remove 333 J/g of heat

~~while on melting~~



(1) w/ weight <u>on</u> the amount of heat that must be ~~added~~ removed is .

i) enough to bring the system from 1°C to the freezing point.

ii) To freeze the liquid $H_2O$.

iii) To bring the liquid from the freezing point to −1°C

(2) The amount of work <u>done</u> by the system under these conditions is (ignoring the expansion of the liquid & solid when not changing state

$$\Leftrightarrow \beta \equiv \frac{1}{V}\frac{\partial V}{\partial T}\Big|_p \ll 1 \quad \text{i.e. the thermal}$$ )

$$W = p \cdot \Delta V_{ls} = \frac{Mg}{A_{piston}} \cdot \Delta V_{ls},$$

(3) The ...

Assume the weight has mass $m_w$, + the mass of the working substance $m_{H_2O}$.

$$-Q_{removed} = c_p m_{H_2O}(T_f - 274) + c_p m_{H_2O}$$

$$= c_p m_{H_2O}(T^*_f - T_{0_i}) + m_{H_2O}L + c_p m_{H_2O}(T_f - T^*)$$

$$W = m_w g$$

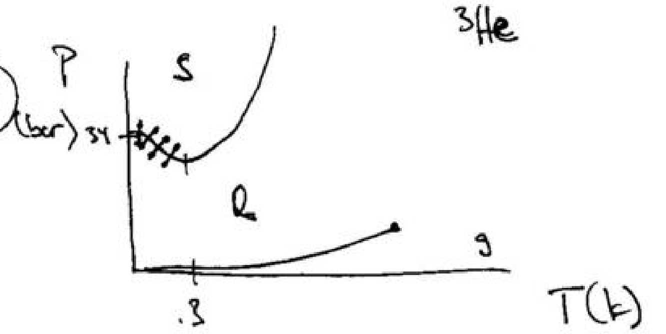
? Don't -

$\cancel{e} =$

$$e = \frac{P_0 \Delta V}{m_{H_2O} \cdot L} = \frac{P_0}{m_{H_2O}}\left(\frac{\Delta V}{L}\right) = \frac{P_0}{m_{H_2O}} \cdot \frac{dT}{T\,dP}$$

Don't see how to do this problem?

S,34




$\frac{dP}{dT} < 0$

(a) $\frac{dP}{dT} = \frac{L}{T\Delta V} < 0 \quad \rightarrow \quad \Delta V < 0$

$\Delta V \equiv V_l - V_s < 0$ $\qquad \frac{L}{T} = S_l - S_s$

$V_l < V_s \quad \therefore$ liquid is more dense

I as assuming that the liquid phase has more entropy.

(b) $\frac{dP}{dT} = \frac{S_s - S_l}{V_s - V_l}$ $\qquad$ As $T \to 0$.

$$\left.\begin{aligned} S_s &= S_s(T) \to 0 \\ S_l &= S_l(T) \to 0 \end{aligned}\right\} \text{By 3rd law of thermodynamics}$$

$\therefore \quad S_s - S_l \to 0$

$+ \quad \frac{dP}{dT} \to 0.$

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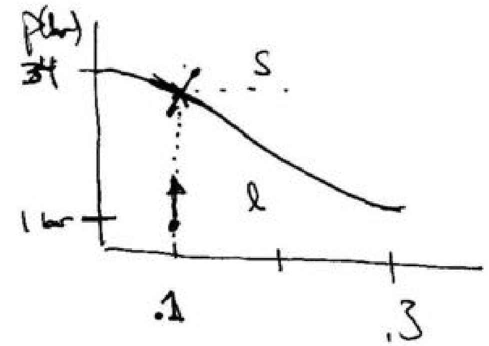
(c)




$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$

Adiabatic compression $\Rightarrow$ no heat change & $\Leftrightarrow$ $dS = 0$.

$\therefore \quad \frac{dP}{dT} \cong \frac{\Delta S}{\Delta V} =$ $\qquad dS = \frac{dQ}{T}$

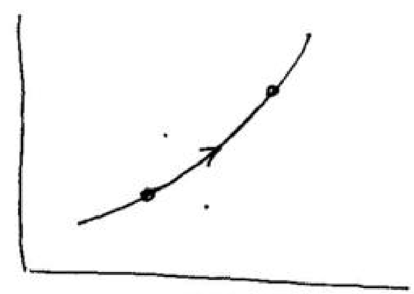
$$\frac{dP}{dT} = \frac{S_s - S_0}{\Delta V} < 0$$

w/ $S_0$ = entropy at room temp & pressure.

Since we require the phase change to also take place adiabatically it will have to not involve changing the entropy from $S_0$. $\Rightarrow$ $\frac{dP}{dT} \cong 0$. Since the only place on the phase diagram where this happens is at $T = 0$ the system should move towards that state i.e. The temperature will <u>decrease</u>.

5.35

$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$




$$= \frac{L(T,P)}{T\Delta V(T,P)}$$

$$\cong \frac{L}{T\,\cancel{\Delta V}\,V_{gas}}$$ L constant

$\Delta V = V_f - V_i$
$= V_{gas}$

$$V = \frac{NkT}{P} = \frac{nRT}{P}$$

$$\Rightarrow \frac{dP}{dT} = \frac{L}{T\left(\frac{NkT}{P}\right)} = \frac{L}{T\left(\frac{nRT}{P}\right)} = \frac{PL}{nRT^2}$$

$$\Rightarrow \frac{dP}{P} = \frac{dT}{T^2}\left(\frac{L}{nR}\right)$$

$$\ln P = -\frac{1}{T}\left(\frac{L}{nR}\right) + C \rightarrow P = C_0 \exp\left\{-\frac{L}{nRT}\right\}$$

5.36

$T = 50°C \Rightarrow T = 50 + 273 = 323\,K$

$T = 100°C \Rightarrow T = 373\,K$

From Fig 5.11. $L\,(kJ/mol) \cong \frac{1}{2}(42.92 + 40.66) = 41.790\ kJ/mol$

$=$

$R = 8.314\ J/mol \cdot K$

So $P = C_0 \exp\left\{ - \frac{41.79 \cdot 10^3\ J/mol}{(8.314\ J/mol \cdot K)\,T} \right\}$

$\Rightarrow P = C_0 \exp \{$

Fitting equation at the left end point:

$0.1234\ \text{bar} = C_0 \exp \left\{ \frac{-41.79 \cdot 10^3\ J/mol}{(8.314\ J/mol \cdot K)(323\,K)} \right\}$

$\Rightarrow C_0 = \ldots$

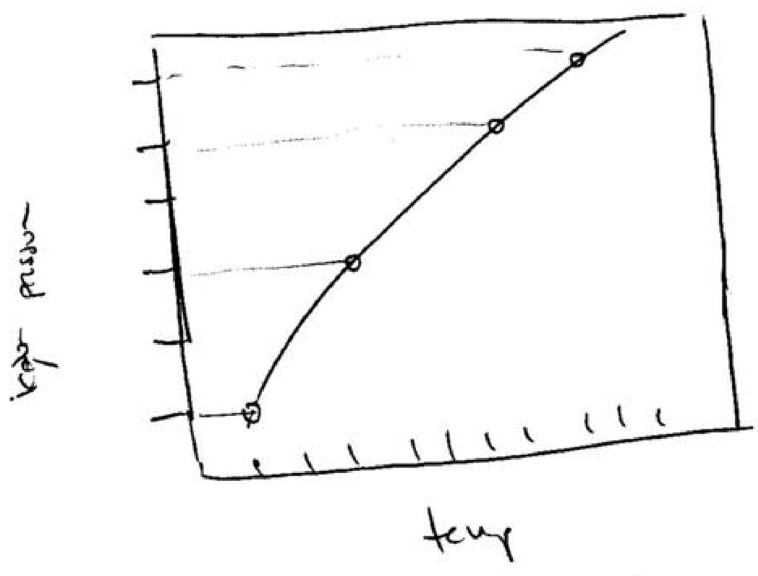
Then plot $P(T)$ for $T \in [323\,K, 373\,K]$

Prob 1.16 asks for pressures at given heights

& derives $P(z) = P_0 \exp\left\{ \frac{-mg}{RT} z \right\}$

$\Rightarrow P(z) = (1\,atm)\exp\left\{ -z \cdot 9.822 \cdot 10^{-5} \right\}$

& at heights = 4700 ft, 1430 m
19,150 ft, 3090 m
14,500 ft, 4420 m
8850 m
$\Rightarrow P(\text{height}) =$




As the height ↗ the atmospheric pressure ↘ & the temperature at which $H_2O$ boils decreases

I would think this would mean that it gets easier to cook in the mountains.

(5,37) $\frac{dP}{dT} = \frac{L}{T\Delta V}$ By Clausius Clapeyron

$V_{calcite} = 36.93 \text{ cm}^3$ $\Rightarrow$ $\Delta V = 2.78 \text{ cm}^3$

$V_{aragonite} = 34.15 \text{ cm}^3$

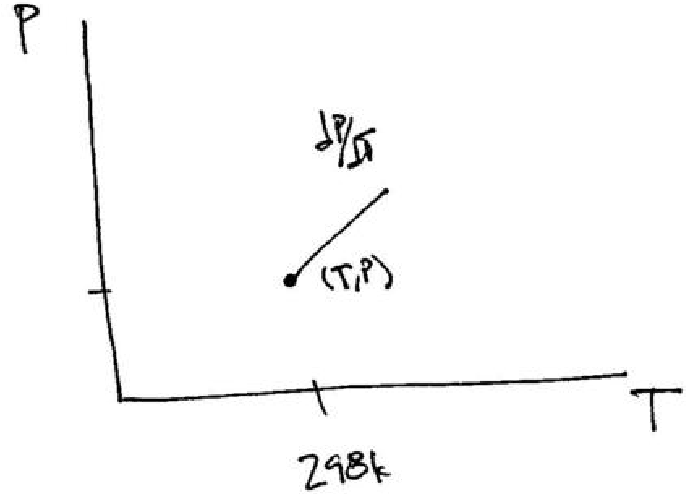
$T = 298\text{k}$

$L = \cancel{\frac{\Delta S}{T}}$ $\Delta S = \frac{L}{T} \Rightarrow L = T\Delta S$

$\therefore$ $S_{cal} = \cdots$ $\Delta S = \cdots$

$S_{ara} = \cdots$

$\frac{dP}{dT} = \frac{T\Delta S}{T\Delta V} = \frac{\Delta S}{\Delta V}$



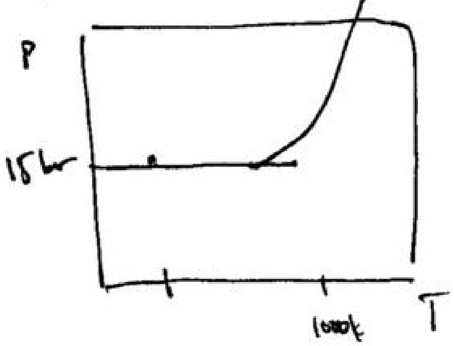

(5,38) $\Delta S = \cdots$

$L = T\Delta S$

$\frac{dP}{dT} = \frac{L}{T\Delta V} = \frac{T\Delta S}{T\Delta V} = \frac{\Delta S}{\Delta V} = \ldots 20$

predicted by graph.

From 5,17



For low temp: $\Delta S \approx 0$ between the graphite phase + the diamond phase

+ At high temp $\Delta S \propto \Delta V$

Prob 5.39

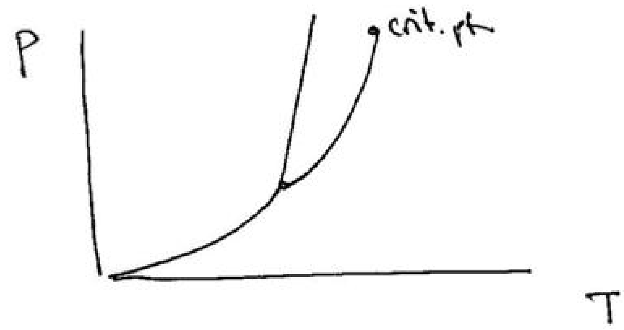
$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$

By Clausius Clapeyron Eq

w/ $L = T\Delta S$

$$\therefore \frac{dP}{dT} = \frac{\Delta S}{\Delta V}$$

"typical" phase diagrams look like



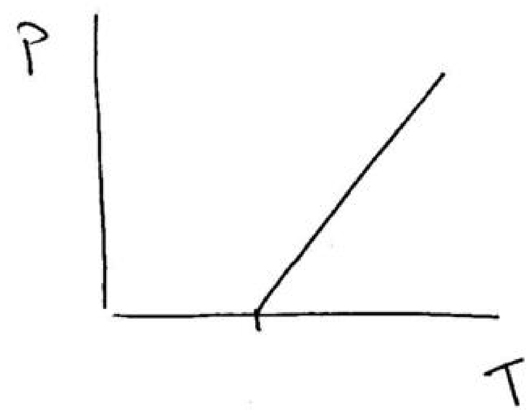

5.40

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V}$$ Assumed independent of T & P. at S.T.P.

$$= \frac{((133.5 + 41.84) - 207.4)\ J/k}{((60.4 + 22.69) - 100.07)\ cm^3} = 1.88\ J/cm^3 \cdot k$$

$$\Rightarrow \quad P - P_0 = \left(\frac{\Delta S}{\Delta V}\right)_0 (T - T_0)$$

$\Rightarrow$ P



(5.41)

initially → pump gas into top.

diffusion of particles occurs across this surface


$\therefore$ Chemical potentials must change to match each other

$$d\mu_l = d\mu_g$$

eq 5.40 $\quad \mu(T,P) = \mu^\circ(T) + kT\ln(P/P^\circ)$ chem potential of ideal gas.

$$d\mu_g = d\left(\mu^\circ(T) + kT\ln\left(P/P^\circ\right)\right)$$

$$? = \frac{kT}{P_v}$$

$P_v$ ← vapor pressure

Assuming $\quad P_v(P) = P_v(P_v)\,e^{(P-P_v)V/NkT}$

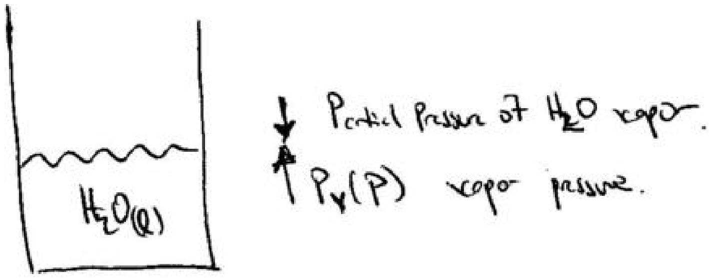
$25^\circ C \approx \quad P_v(P_v) = 1\,atm.$

$PV = NkT$

$$?\ \left(\frac{V}{NkT}\right) \approx \frac{\cancel{V}}{10^{23}(1.38\cdot10^{-23})}$$

$$\frac{\cancel{V}\ (1\text{ liter})}{(6.022\cdot10^{23})(1.38\cdot10^{-23}\,J/K)(300\,K)}$$

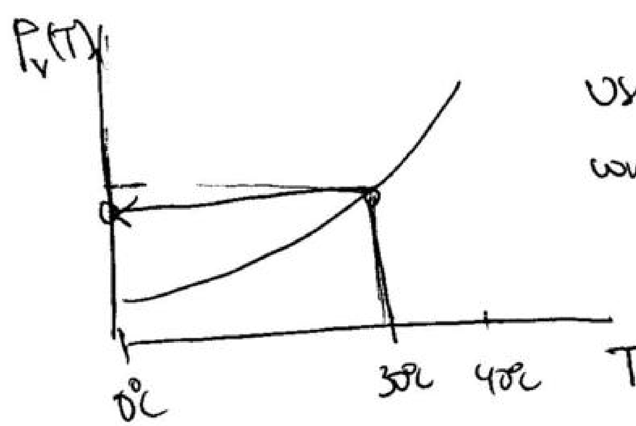
5.42




$$\text{rel Humidity} = \frac{PP_{H_2O}}{P_{vl}}$$

$$P_v = P_0 e^{-L/RT}$$ (vapor pressure eq.)

Fig 5.11 gives $\{L_i\}, \{T_i\}, \{P_{v_i}\}$




Use to get $P_v(30°C)$ then compute $PP_{H_2O}$ knowing rel hum = 90%
= 40%.

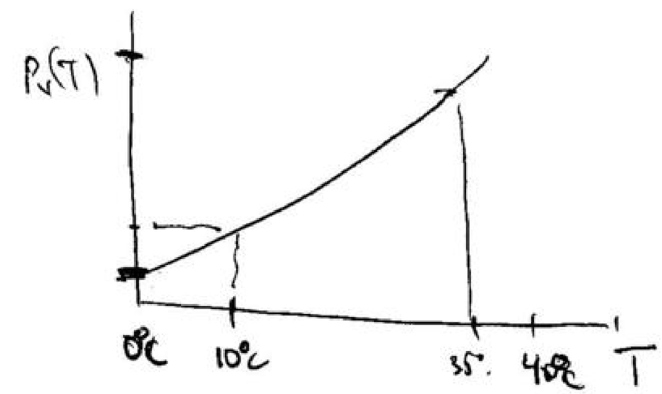
5.43

rel Hum = .9

$35°C \Longrightarrow$ | $10°C$

rel Hum = .9 | rel Hum = ?

Since we can see our breath ⇒




$$\text{rel Hum} \equiv \frac{PP_{H_2O}}{P_v(T)}$$

$$.9 = \frac{PP_{H_2O}}{P_v(35°C)}$$

↖ read from graph in prob. 5.42

$\Rightarrow PP_{H_2O} = \ldots$

Then relative Humidity $\equiv \dfrac{PP_{H_2O}}{P_v(10°C)} = \ldots$

$$\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P}$$

$$\frac{dT}{dz}\cdot\frac{dz}{dP} = \frac{2}{f+2}\cdot\frac{T}{P}$$

$$\frac{dT}{dz} = \left(\frac{dP}{dz}\right)\cdot\frac{2}{f+2}\frac{T}{P}$$

5.44

$z = ?$

$T(z) = 25°C + -10°C\,(z)$

$$\frac{dT}{dz} = -10°C/km$$

dry adiabatic lapse rate.

$z = 0$

$T = 25°C$

rel Hum = .5

$$\equiv \frac{PP_{H_2O}}{P_v(25°C)} = .5$$

Assuming I have a graph of $P_v(T)$.

Then I compute $PP_{H_2O} = \ldots$

Require

$$\frac{PP_{H_2O}}{P_v(25 - 10z)} \stackrel{?}{=} 1$$

solve for z.

8.45

$$\left(\frac{f}{f+2}\right)\frac{T}{P}\,dP$$

(a) From $\frac{dP}{dT} = \frac{L}{T\Delta V}$

$$\cong \frac{L}{TV_g}$$

$PV = kNT$

Adiabatic expansion w/ phase change...

?

$$dT = \frac{2}{7}\frac{T}{P}\,dP - \frac{2}{7}\frac{L}{nR}\,dn_w$$

(b) $\frac{n_w}{n} = F(T, P)$

$$\frac{dn_w}{n} = \left.\frac{\partial F}{\partial T}\right|_P dT + \left.\frac{\partial F}{\partial P}\right|_T dP$$

$$\cancel{dT = \left(\frac{2}{7}\frac{T}{P} + \frac{\partial F}{\partial P}\Big|_T\right) dP - \frac{2}{7}\frac{L}{R}}$$

$$dT = \frac{2}{7}\frac{T}{P}\,dP - \frac{2}{7}\frac{L}{R}\frac{\partial F}{\partial T}\Big|_P dT - \frac{2}{7}\frac{L}{R}\frac{\partial F}{\partial P}\Big|_T dP$$

$\Rightarrow$

$$\left(1 + \frac{2}{7}\frac{L}{R}\frac{\partial F}{\partial T}\Big|_P\right) dT = \left(\frac{2}{7}\frac{T}{P} - \frac{2}{7}\frac{L}{R}\frac{\partial F}{\partial P}\Big|_T\right) dP$$

?

5.46 $\sigma = \dfrac{G_{\text{boundary}}}{A}$ $\qquad \sigma = 0.073\ \mathrm{J/m^3}$

(a)

$$G = N_\ell \mu_\ell + (N - N_\ell)\mu_v$$

At eq $\mu_\ell = \mu_v \equiv \mu$

$\therefore G = N\mu$

$$N_\ell = \frac{\left(\frac{4}{3}\pi r^3\right)}{v_\ell}$$

$$\therefore G = \frac{\left(\frac{4}{3}\pi r^3\right)}{v_\ell}\mu_\ell + \left(N - \frac{\frac{4}{3}\pi r^3}{v_\ell}\right)\mu_v$$

$$G = \left(\frac{\frac{4}{3}\pi r^3}{v_e}\right)\mu_e + \left(N - \frac{\frac{4}{3}\pi r^3}{v_e}\right)\mu_v$$

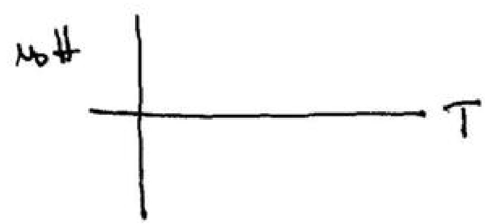
$$+ (4\pi r^2)\sigma$$

(c) ? Thought $\mu_e = \mu_v$ ?

(d) ?

(5.47) $dG_M = -S\,dT - \mu_0 M\,dH$

$= -S\,dT - M(\mu_0\,dH)$




At a phase boundary $\cancel{dG_M = 0}$ $\Delta G_{M,\text{total}} = 0$

$\Rightarrow \quad dG_1 = dG_2$

$\Rightarrow -S_1\,dT - M_1(\mu_0\,dH) = -S_2\,dT - M_2(\mu_0\,dH)$

$\Rightarrow \cancel{\frac{dT}{\mu_0\,dH} = \frac{-(S_1 - S_2)}{M_1 - M_2}} = \frac{-(L)}{T\Delta M}$

$dT(-S_1 + S_2) = (M_1 - M_2)(\mu_0\,dH)$

$\frac{dT}{(\mu_0\,dH)} = \cancel{\frac{-S_1 + S_2}{}} \quad \cancel{M_1 - M_2} \quad \left(\frac{M_1 - M_2}{-S_1 + S_2}\right)$

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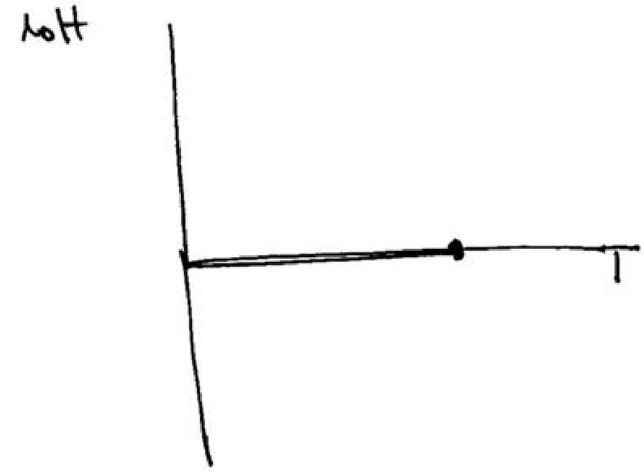
$$\therefore \quad \frac{dT}{\mu_0 dH} = -\frac{(M_1 - M_2)}{(S_1 - S_2)}$$

$$? \quad \frac{\mu_0 dH}{dT} = -\frac{L}{T\Delta M}$$




(b) ?

(c) ?

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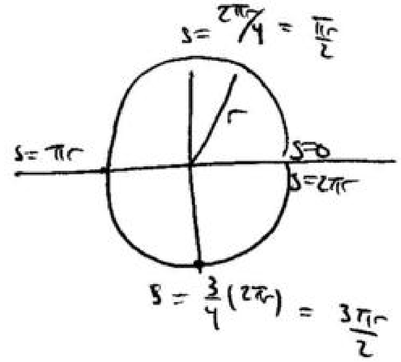
Prob A.20




$\psi = \psi(s)$

p well defined.

sin

?

Prob A.21

$|\vec{L}|^2 = l(l+1)\hbar^2 \qquad l = 1, 2, 3, \ldots$

$E = \frac{-2\pi^2 m_e e^4 k^2}{h^2} \frac{1}{n^2} \qquad n = 1, 2, 3,$

$L_t = m\hbar \qquad -l \le m \le l$

$n = 1$

$l = 1, \ldots \qquad l = 2$

$m = -1, 0, +1 \qquad m = -2, -1, 0, +1, +2$

?

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5.48

$$P(V) = \frac{NkT}{V-Nb} - \frac{aN^2}{V^2}$$

(1) $$\frac{dP}{dV} = \frac{(-1)NkT}{(V-Nb)^2} + \frac{2aN^2}{V^3} = 0$$ see.

(2) $$\frac{d^2P}{dV^2} = \frac{2NkT}{(V-Nb)^3} - \frac{6aN^2}{V^4} = 0$$ don't see

From eq (1) $$NkT = \frac{2aN^2}{V^3}(V-Nb)^2$$

So from eq (2)

$$\frac{2}{(V-Nb)^3}\cdot\frac{2a\cancel{N^2}}{\cancel{V^3}}(V-Nb)^2 - \frac{6a\cancel{N^2}}{V^4} = 0$$

$$\frac{4}{(V-Nb)} - \frac{6}{V} = 0$$

$$2V - 3(V-Nb) = 0$$

$$2V - 3V + 3Nb = 0$$

$$-V + 3Nb = 0 \qquad \underline{\underline{V_c = 3Nb}}$$

Then from (1) $$(-1)\frac{\cancel{N}kT}{(3Nb-Nb)^2} + \frac{2aN^{\cancel{2}}}{(3Nb)^3} = 0$$

$$\Rightarrow \quad \frac{-kT}{4N^{\cancel{2}}b^2} + \frac{2a\cancel{N}}{27N^3b^3} = 0$$

$$\Rightarrow T = \frac{8(1)a}{27bk} \checkmark$$

Then
$$P_c = \frac{N(\frac{8a}{27b})}{(3Nb - Nb)} - \frac{aN^2}{(9N^2b^2)}$$

$$= \frac{8aN}{27b}\frac{1}{(2Nb)} - \frac{a}{9b^2}$$

$$= \frac{4a}{27b^2} - \frac{a}{9b^2} = \frac{4a-3a}{27b^2} = \frac{a}{27b^2} \checkmark$$

5.49

| | $N_2$: $a = 4\cdot10^{-49}\,J\cdot m^3$, $b = 6\cdot10^{-29}\,m^3$ | $H_2O$: $a = 16\cdot10^{-49}\,J\cdot m^3$, $b = 6\cdot10^{-29}\,m^3$ |
|---|---|---|
| $V_c/N$ | | |
| $P_c$ | | |
| $kT_c$ | | |

| | He: $a = \frac{1}{40}\cdot4\cdot10^{-49}\,J\cdot m^3$, $b = ?$ |
|---|---|
| $V_c/N$ | |
| $P_c$ | |
| $kT_c$ | |

8.80 $\frac{PV}{NkT} = \frac{V}{V-Nb} - \frac{aN^2}{V(NkT)}$ For van der Waals gas.

(*) $\frac{PV}{NkT} = \frac{V}{V-Nb} - \frac{aN}{VkT}$

$= \frac{1}{1-\frac{Nb}{V}} - \frac{aN}{VkT}$ $\quad \left\{ \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \right\}$

$= 1 + \frac{Nb}{V} + \frac{N^2b^2}{V^2} + \frac{N^3b^3}{V^3} + \cdots$

$-\frac{aN}{VkT}$

$= 1 + \frac{N}{V}\left(b - \frac{a}{kT}\right) + \frac{N^2b^2}{V^2} + \frac{N^3b^3}{V^3} + \cdots$

- - -

From eq (*) At critical point

$\frac{P_cV_c}{NkT_c} = \frac{3Nb}{2Nb} - \frac{aN}{(3Nb)(\frac{8}{27}\frac{a}{b})}$

$= \frac{3}{2} - \frac{9}{8} = \frac{3}{12} = \frac{1}{4}$ independent of a or b.

5.57 $V_c = 3Nb$ ; $P_c = \frac{1}{27}\frac{a}{b^2}$ ; $kT_c = \frac{8}{27}\frac{a}{b}$

$$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = NkT$$

let $t = \frac{T}{T_0}$ ; $v = \frac{V}{V_0}$ ; $p = \frac{P}{P_0}$

$$\Rightarrow \left(pP_0 + \frac{aN^2}{V_0^2 v^2}\right)(V_0 v - Nb) = NkT_0 t$$

$$\Rightarrow t = \frac{P_0}{NkT_0}\left(p + \frac{aN^2}{V_0^2 P_0 v^2}\right)V_0\left(v - \frac{Nb}{V_0}\right)$$

$$\Rightarrow t = \frac{P_0 V_0}{NkT_0}\left(p + \frac{aN^2}{V_0^2 P_0 v^2}\right)\left(v - \frac{Nb}{V_0}\right)$$

take $\underline{V_0 = Nb}$ + $\frac{aN^2}{V_0^2 P_0} = \frac{aN^2}{N^2 b^2 P_0} \overset{set}{=} 1 \Rightarrow \underline{P_0 = \frac{a}{b^2}}$

+ then $\frac{P_0 V_0}{NkT_0} \overset{set}{=} 1$

$$\frac{(a/b^2)(Nb)}{NkT_0} = 1 \Rightarrow \underline{kT_0 = \frac{a}{b}}$$ + dimensionless eq becomes:

$$t = \left(p + \frac{1}{v^2}\right)(v - 1)$$

with regard to scaling let $V_0 = 3Nb$, $P_0 = \frac{1}{27}\frac{a}{b^2}$, $kT_0 = \frac{8}{27}\frac{a}{b}$

Then

$$t = \left(\frac{P_0 V_0}{NkT_0}\right)\left(P + \frac{aN^2}{V_0^2 P_0 V^2}\right)\left(V - \frac{Nb}{V_0}\right)$$

$$\Rightarrow t = \frac{\left(\frac{1}{27}\right)\left(\frac{a}{b^2}\right)(3Nb)}{\cancel{Nk}\,N\left(\frac{8}{27}\frac{a}{b}\right)}\left(P + \frac{aN^2}{9N^2b^2\left(\frac{1}{27}\frac{a}{b^2}\right)V^2}\right)\left(V - \frac{Nb}{3Nb}\right)$$

$$= \frac{1}{9}\cdot\frac{27}{8}\left(P + \frac{3}{V^2}\right)\left(V - \frac{1}{3}\right)$$

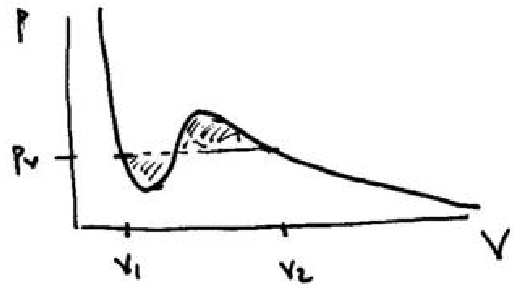
$$= \frac{3}{8}\left(P + \frac{3}{V^2}\right)\left(V - \frac{1}{3}\right) \quad \text{independent of } a \,\&\, b\,!!$$

(5.52) if $t = .95 = \frac{3}{8}\left(P + \frac{3}{V^2}\right)\left(V - \frac{1}{3}\right)$

$$\Rightarrow \quad P = \frac{t}{\left(V - \frac{1}{3}\right)}\left(\frac{8}{3}\right) - \frac{3}{V^2}$$

$$P = \frac{8t}{(3V-1)} - \frac{3}{V^2}$$

plotting... to be done ... assuming




$$0 = \int_{V_1}^{V_2} \left(P_v - p(V)\right) dV$$

$$= P_v(V_2 - V_1) - \int_{V_1}^{V_2} p(V)\,dV$$

$$\Rightarrow P_v = \frac{1}{V_2 - V_1} \int_{V_1}^{V_2} p(v)\, dv \quad \ldots$$

$$= \frac{1}{V_2 - V_1} \int_{V_1}^{V_2} \left( \frac{8t}{(3v-1)} - \frac{3}{v^2} \right) dv$$

$$= \frac{1}{V_2 - V_1} \left[ \frac{8t}{(3v-1)^2} \frac{1}{3} - \frac{-3}{(-1)v} \right|_{V_1}^{V_2}$$

From 5.87 w/ $p = \frac{8t}{(3v-1)} - \frac{3}{v^2}$

$$0 = \int_{loop} V\, dP \qquad V = V(P)$$

$$\frac{dP}{dV} = \frac{8t\,(3)}{(3v-1)^2} - \frac{6}{V^3}$$

$$= \int_{loop} V \frac{dP}{dV} \cdot dV$$

$$0 = \int_{loop.} \left( \frac{24t\, V}{(3v-1)^2} - \frac{6}{v^2} \right) dv$$

How do I compute numerically the maxwell construction?

From 5.86:

$$G = -NkT \ln(V - Nb) + \frac{(NkT)(Nb)}{V - Nb} - \frac{2aN^2}{V} + C(T)$$

$$Nb = \frac{V_c}{3} \qquad \frac{G}{NkT} = -\ln\left(V - \frac{V_c}{3}\right) + \frac{V_c/3}{(V - V_c/3)} - \frac{2aN^2}{(NkT)V} + \frac{C(T)}{NkT}$$

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$$\frac{G}{NkT} = -\ln\left(v - \frac{v_c}{3}\right) + \frac{v_c/3}{(v - v_c/3)} - \underbrace{\frac{2aN}{kTv}}_{\text{How handle this term?}} + C'(T)$$

take $v_c = 1$

Don't understand?

5.54

$$F = U - TS$$

$$dF = dU - TdS - SdT$$

$$\left\{ \begin{array}{l} \cancel{dF = pdV} \\ dU = TdS - pdV + \mu dN \end{array} \right.$$

$$= T\cancel{dS} - pdV - T\cancel{dS} - SdT$$

$$= -pdV - SdT$$

$$dG = -SdT + VdP + \mu dN$$

At a fixed temperature

$$F = -\int^{V} p\,dV + C(T)$$

$$= -\int \left( \frac{8t}{(3v-1)} - \frac{3}{v^2} \right) dv + C(T)$$

$$= -\frac{8t}{3} \ln(3v-1) + 3\frac{v^{-1}}{-1} + C(T)$$

$$= -\frac{8t}{3} \ln(3v-1) - \frac{3}{v} + C(T)$$

The free energy of a system minimizes, maximizes during transformation?

(5.55) Van der Waals in reduced form is:

$$T = \frac{3}{8}\left(p + \frac{3}{v^2}\right)\left(v - \tfrac{1}{3}\right)$$

$$\Rightarrow \quad p(v) = \frac{\frac{8}{3}T}{v - \frac{1}{3}} - \frac{3}{v^2}$$

Then the critical values are defined when $p'(v) = p''(v) = 0$

thus $$p'(v) = -\frac{\frac{8}{3}T}{(v-\frac{1}{3})^2} + \frac{3\cdot 2}{v^3} = 0 \qquad (1)$$

$$p''(v) = \frac{\frac{16}{3}T}{(v-\frac{1}{3})^3} - \frac{9\cdot 2}{v^4} = 0 \qquad (2)$$

By eq (1) $\dfrac{\frac{8}{3}T}{(v-\frac{1}{3})^2} = \dfrac{6}{v^3}$ + putting this into eq (2) gives

$$\frac{\cancel{2}}{(v-\frac{1}{3})}\,\frac{\overset{2}{\cancel{6}}}{\cancel{v^3}} - \frac{\overset{3}{\cancel{9}}\cdot\cancel{2}}{\cancel{v^4}} = 0 \Rightarrow \frac{2}{(v-\frac{1}{3})} - \frac{3}{v} = 0$$

$$\Rightarrow \quad 2v - 3\left(v - \tfrac{1}{3}\right) = 0$$

$$\Rightarrow \quad 2v - 3v + 1 = 0 \Rightarrow v_{crit} = 1$$

From eq (1) $$\frac{-\frac{\overset{2}{\cancel{8}}}{\cancel{3}}T}{\frac{4}{3}\cdot\frac{\cancel{4}}{\cancel{9}}} + \frac{6}{1} = 0$$

$$\Rightarrow \quad -\frac{2T}{\frac{1}{3}} + 6 = 0 \Rightarrow T_{cit} = 1$$

Then $P_{crit} = \dfrac{\frac{8}{3}}{\frac{2}{3}} - 3 = 1$

By taylor thm:

$$P(v) \cong P(1) + P'(1)(v-1) + \frac{P''(1)}{2!}(v-1)^2 + \frac{P'''(1)}{3!}(v-1)^3 + O((v-1)^4)$$

$$P(1) = \frac{\frac{8}{3}T}{\frac{2}{3}} - 3 = 4T-3$$

$$P'(v) = -\frac{\frac{8}{3}T}{(v-1/3)^2} + \frac{6}{v^3} \; ; \; P'(1) = -\frac{\frac{8}{3}T}{\frac{4}{9}} + 6 = -\frac{2T}{\frac{1}{3}} + 6$$

$$= -6T + 6$$

$$= -6(T-1)$$

$$P''(v) = +\frac{\frac{16}{3}T}{(v-1/3)^3} - \frac{18}{v^4} \; ; \; P''(1) = \frac{\frac{16}{3}T}{(\frac{8}{27})} - 18 = \frac{2T}{\frac{1}{9}} - 18$$

$$= 18(T-1)$$

$$P'''(v) = -\frac{\frac{48}{3}T}{(v-1/3)^4} + \frac{72}{v^5} \; ; \; P'''(1) = \frac{-\frac{48}{3}T}{\frac{16}{81}} + 72$$

$$= \frac{-\frac{12}{3}T}{\frac{4}{9\cdot 9}} + 4\cdot 9\cdot 2$$

$$= \frac{-T}{\frac{1}{99}} + 4\cdot 9\cdot 2 = -9\cdot 9\cdot T + 4\cdot 9\cdot 2 = -9(9T-8)$$

Thus

$$P(v) \cong (4T-3) - 6(T-1)(v-1) + 9(T-1)(v-1)^2 - \frac{3}{2}(9T-8)(v-1)^3 + O((v-1)^4)$$

Assuming uniformity in all variables towards their critical values then:

$$T-1 = O(\varepsilon)$$

$$v-1 = O(\varepsilon)$$

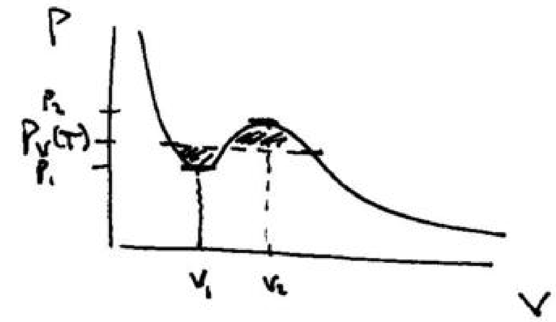
each term in the taylor expansion is of order

$$O(1) + O(\varepsilon^2) + O(\varepsilon^3) + O(\varepsilon^{3}) + O(\varepsilon^4)$$

Thus the $O(\varepsilon^3)$ term is the smallest. + we will drop it.

(b) Thus

$$P(v) = (4T-3) - 6(T-1)(v-1) - \frac{3}{2}(9T-8)(v-1)^3 + O((v-1)^4)$$




What are the $v_1$ + $v_2$ ?

$$P'(v) = -6(T-1) - \frac{9}{2}(9T-8)(v-1)^2 + \ldots = 0.$$

$$\Rightarrow -12(T-1) - 9(9T-8)(v-1)^2 = 0$$

$$-4(T-1)-3(9T-8)(V-1)^2=0$$

$$(V-1)^2 = -\frac{4}{3}\frac{(T-1)}{(9T-8)}$$

for

$\frac{8}{9} < T < 1$

$$-\frac{(T-1)}{(9T-8)} > 0$$

$$\Rightarrow V = 1 \pm \sqrt{\frac{4}{3}\frac{(1-T)}{(9T-8)}}$$

Then $P_v(T)$ is given by: ? How do I do these calculations?

given $P_v(T)$

$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$ By Clausius clap. How would I get

$$(V_g - V_e) \propto \cancel{\epsilon_g} - (T_c - T)^\beta \ ?$$

$$\Rightarrow \Delta V = \frac{L}{T(\frac{dP_v}{dT})}$$

$$L = T\Delta V \frac{dP(T)}{dT}$$

⋮

(f) $\kappa = \cancel{\quad} -\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T$

$$\frac{\partial P}{\partial V}\Big|_T = -6(T-1) - \frac{9}{2}(9T-8)(V-1)^2$$

$\cancel{\text{so}} =$ $T \to 1^+$ $\quad \kappa \to -6(T-1)^{1}$

$T \to 1^-$ $\quad \kappa \to -6(T-1)^{1}$,

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5.56 $\Delta S_{mixing}(x) = -R\left[x\ln x + (1-x)\ln(1-x)\right]$

$$\frac{d\Delta S_{mixing}}{dx} = -R\left[\ln x + \frac{x}{x} + (-1)\ln(1-x) + \frac{(1-x)(-1)}{1-x}\right]$$

$$= -R\left[\ln x - \ln(1-x)\right]$$

$$= -R\ln\left(\frac{x}{1-x}\right) = R\ln\left(\frac{x-1}{x}\right) = R\ln\left(1-\frac{1}{x}\right)$$

$$\left.\begin{array}{l}\dfrac{d\Delta S_{mixing}(0)}{dx} = R\ln(-\infty) = \text{complex } \infty \\ \dfrac{d\Delta S_{mixing}(1)}{dx} = R\ln(0) = -\infty\end{array}\right\} \text{Both } \infty$$

5.57 $x = \dfrac{N_A}{N_A+N_B}$ $\quad 1-x = \dfrac{N_B}{N_A+N_B}$ $\quad N_A+N_B = 100$

$$\Delta S_{mixing}(x) = -R\left[x\ln x + (1-x)\ln(1-x)\right]$$

$x = \dfrac{N_A}{100}$ $\quad N_A = 0$ $\quad \Delta S_{mixing}(0) = 0$

$$\Delta S_{mixing}(1) = -R\left[\tfrac{1}{100}\ln\left(\tfrac{1}{100}\right) + \left(\tfrac{99}{100}\right)\ln\left(\tfrac{99}{100}\right)\right]$$

$nR = Nk$

$\Delta S_{increase} = \Delta S_{mixing}(1) - \Delta S_{mix}(0) =$

$R = \left(\dfrac{N}{n}\right)k$

$\Delta S_{mixing}(2) = -R\left[\ \cdots\ \right]$

5.88

(a) $U_0 \cdot n \cdot \frac{N}{2}$

(b) let $x = \frac{N_B}{N}$. When mixed a given atom will have $x \cdot n$ of its neighboring molecules of type B + $(1-x)n$ of its neighbors of type A.

(on average)

Then a molecule of type A will have potential energy of

$$\frac{1}{2}\Big[\underbrace{(x \cdot n U_0 + (1-x)\, n U_{AB})}_{\text{potential energy of each B centered bond}}\, xN + \underbrace{(x n U_{AB} + (1-x) n U_0)}_{\text{potential energy of each A centered bond}}(1-x)N\Big] = U_{total}$$

$\frac{1}{2}$: Avoid double counting.

$xN$: # of B centered Bonds

$(1-x)N$: # of each A centered bonds

$$\Rightarrow U_{total} = \frac{1}{2}N\left[x^2 n U_0 + x(1-x)\, n U_{AB} + x(1-x)\, n U_{AB} + (1-x)^2 n U_0\right]$$

$$= \frac{1}{2}Nn\left[(x^2 + (1-x)^2)U_0 + 2x(1-x)U_{AB}\right]$$

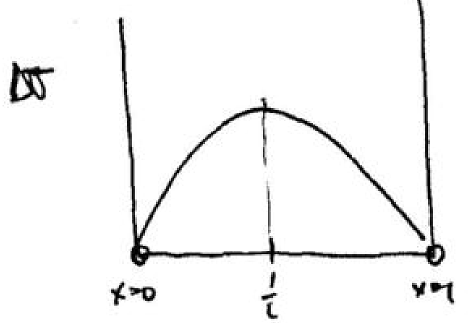
$$= \frac{1}{2}Nn\left[(2x^2 - 2x + 1)U_0 + 2x(1-x)U_{AB}\right]$$

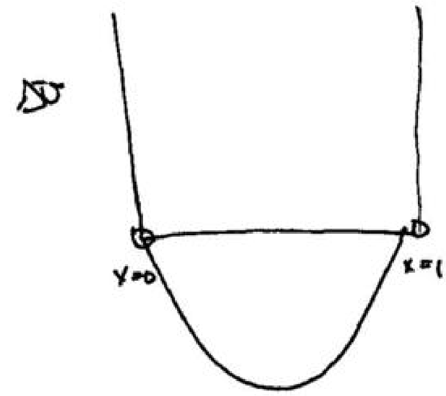
(c) $\Delta U_{potential} = U_{mixed} - U_{unmixed}$

$$= \frac{1}{2} N n \left[ 2x(x-1) U_0 + 2x(1-x) U_{AB} \right] \qquad 0 \le x \le 1$$

$$= \frac{1}{2} N n \; x(1-x) \left[ U_{AB} - U_0 \right] \qquad 1 - 2x = 0.$$

Assume $U_{AB} - U_0 > 0$ $\qquad$ $U_{AB} - U_0 < 0$




(d) $\frac{d\Delta U_{pot}}{dx} = G(1-x) - Gx = G(1-2x) \neq \infty.$

$nR = Nk$

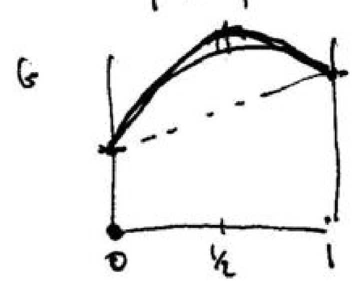
$N = \frac{nR}{k}$

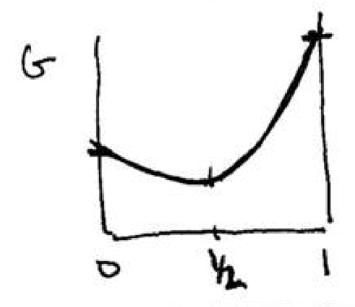
(e) $G = (1-x) G_A^o + x G_B^o + RT \left[ x \ln x + (1-x) \ln(1-x) \right]$

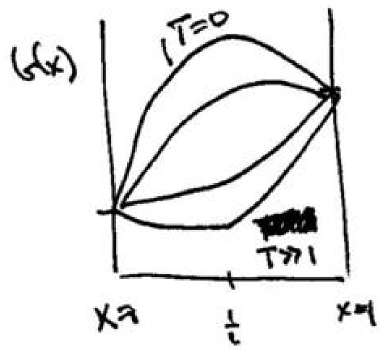
$$+ \frac{1}{2} N n x(1-x)(U_{AB} - U_0)$$

$$\Rightarrow G = (1-x) G_A^o + x G_B^o + \frac{1}{2} N n x(1-x)(U_{AB} - U_0) + RT \left[ x \ln x + (1-x) \ln(1-x) \right]$$

$T \ll 1$ $\qquad$ $T \gg 1$



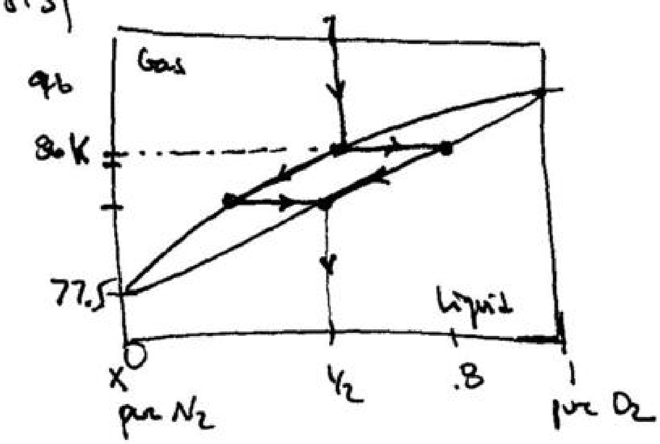



(f) How get Solubility gap?

(g) ?

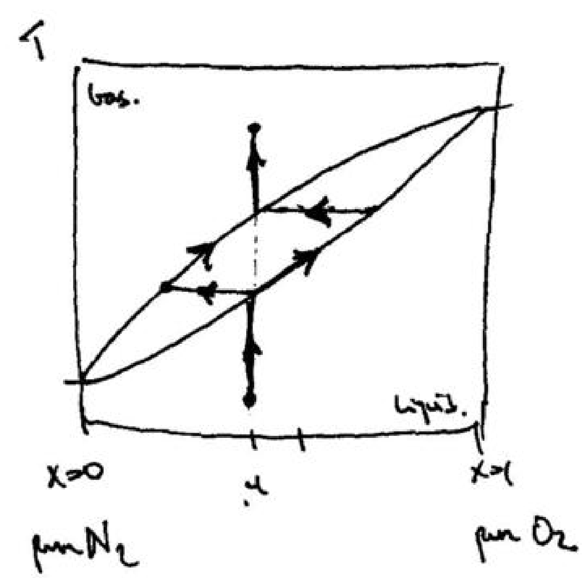
(h) ?

**8.59** From Fig 8.31




From the diagram, the gas cools as a mixture until about 86 K. A horizontal line intersects the liquid line at about x=.8, so the liquid that forms is 80% $O_2$. As the temperature is cooled further, the fractional amount of $O_2$ in the gas + the liquid decrease. ~~[illegible]~~ until about 82 K, at which point the gibbs free energy of the entire liquid phase is lower than the gas phase + one has entire liquid.

**8.60**



As the liquid heats up, ~~[illegible]~~ at around 80.5K a liquid + gas phase forms, initially the ~~[illegible]~~ gas is of concentration $x \cong .2$ or 20% $O_2$ + 80% $N_2$. The concentration of $O_2$ increases in both the gas + the liquid until about $T = 84K$ when the mixture becomes entirely gas.

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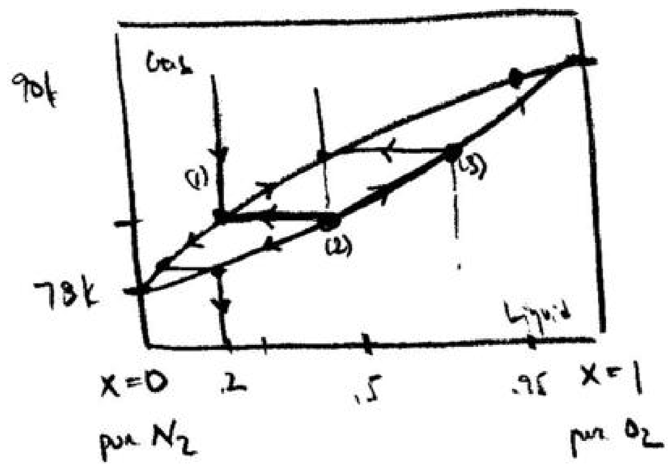
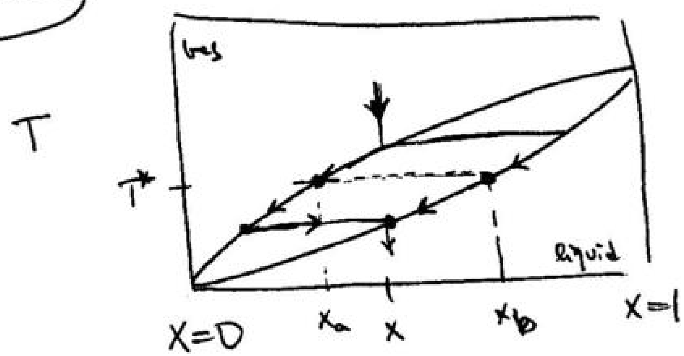
(5.61) $O_2$ 95% pure

Assume air is 79% $N_2$ + 21% $O_2$




Given initial gas (air) cool it until liquid 1st begins to form. This liquid is $\approx$ .5 % $O_2$ ! Then holding the temperature constant, pump off the gas above the liquid. Then let the temperature increase again, the liquid will then increase in % of $O_2$ w/ each cycle in this ~~manner~~ manner.

Prob 5.62




It

$$\frac{\text{Amt liquid}}{\text{Amt gas}} = \frac{x - x_a}{x_b - x}$$

$$\Rightarrow (\text{Amt of liquid})(x_b - x) = (\text{Amt of gas})(x - x_a)$$

$$\Rightarrow A_L x_b - A_L x = A_G x - A_G x_a$$

$$(A_G + A_L)\,x = A_L x_b + A_G x_a$$

$$\Rightarrow x = \frac{A_L}{A_G + A_L} x_b + \frac{A_G}{A_G + A_L} x_a$$

$$\Rightarrow x = \frac{A_G}{A_G + A_L} x_a + \frac{A_L}{A_L + A_G} x_b \quad \ldots \quad ?$$

5.63

$$G = U + PV - TS$$

$$dG = dU + V\,dP - S\,dT \Rightarrow \frac{\partial G}{\partial P} = V > 0$$

∴ As $P \uparrow$ Both phases Gibbs free energy would increase.

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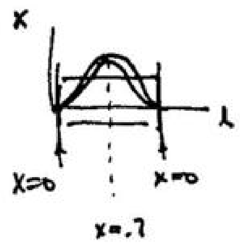
Since the volume phase has $V_{gas} > V_{liquid}$

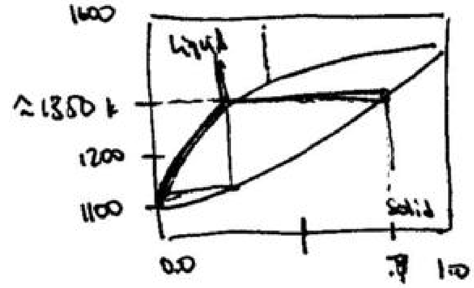
$\frac{\partial G_{gas}}{\partial P} > \frac{\partial G_{liquid}}{\partial P}$ + the gass' gibbs free energy

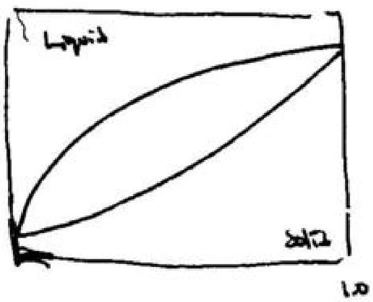
I would expect the change in pressure to make the sliver in the T-x diagram smaller.

(5.64)




(a) ?

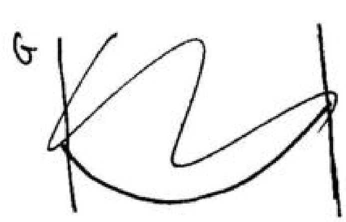


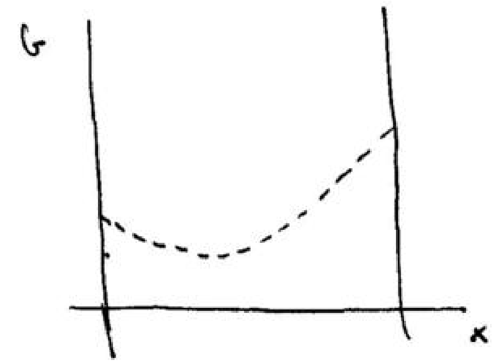



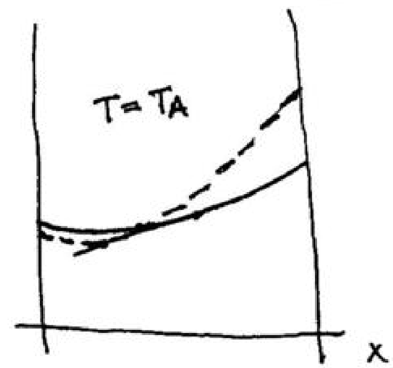

(b) ?

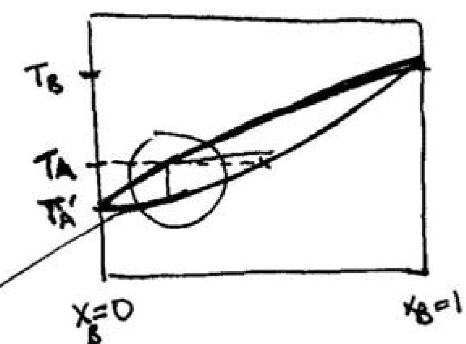
(5.65) Free energy of liquid (substantial positive energy) + graph is much flatter

similar structure as in Fig 5.30 but w/ actual temp corresponding different than shown





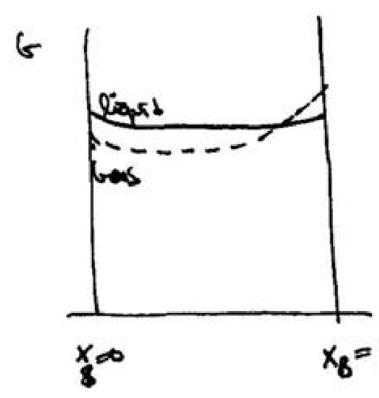



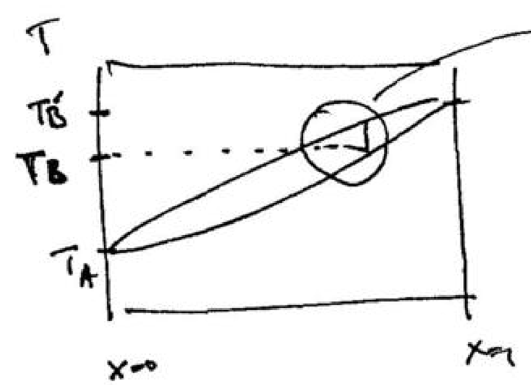




Not sure what happens?

something happens here since below $T_A$ the pure substance A

5.66




Not sure how the cut off goes...

(8.67) $\mu \equiv \frac{\partial G}{\partial N}$

Since $G = (1-x)G_A^\circ + xG_B^\circ + RT\left[x\ln x + (1-x)\ln(1-x)\right]$

(a)

$\frac{\partial}{\partial N_A} =$

$= \frac{\partial x}{\partial N_A}\frac{\partial}{\partial x}$.

$x = \frac{N_B}{N_A+N_B}$

$\frac{\partial x}{\partial N_A} = \frac{-N_B}{(N_A+N_B)^2} = \frac{-x}{N}$.

$\frac{\partial x}{\partial N_B} = \frac{1}{(N_A+N_B)} - \frac{N_B}{(N_A+N_B)^2}$

$= \frac{N_A+N_B-N_B}{(N_A+N_B)^2} = \frac{N_A}{(N_A+N_B)^2}$

$= \frac{(1-x)}{N}$

$\therefore$

$\mu_A = \frac{\partial G}{\partial N_A} = -\frac{x}{N}\frac{\partial G}{\partial x}$

$= \frac{-x}{N}\Big[-G_A^\circ + G_B^\circ$

$+ RT\left[\ln x + \cancel{1} - \ln(1-x) - \cancel{1}\right]$

$= \frac{-x}{N}\left[-G_A^\circ + G_B^\circ + RT\left(\ln x - \ln(1-x)\right)\right]$ ?

$nR = Nk$

$\frac{R}{N} = \frac{k}{n}$

Assume $\mu_A = \mu_A^\circ + kT\ln(1-x)$

& $\mu_B = \mu_B^\circ + kT\ln x$

(b) $\mu_A(T) = \mu_B(T)$

$\Rightarrow \mu_A^\circ + kT\ln(1-x) = \mu_B^\circ + kT\ln x$

$\frac{\mu_A^\circ - \mu_B^\circ}{kT} = \ln x - \ln(1-x) = \ln\left(\frac{x}{1-x}\right)$

Assume $\mu_B = \mu_B^0 + kT \ln(1-x)$

Then equality of chemical potential $\Rightarrow$

$$\mu_A = \mu_B$$

$$\mu_\ell^0 + kT \ln(1-x_\ell) = \mu_g^0 + kT \ln(1-x_g)$$

$\{nR = kN\}$

$$\underbrace{\frac{\mu_\ell^0 - \mu_g^0}{kT}}_{} = \ln\left(\frac{1-x_\ell}{1-x_g}\right)$$

~~Assum~~ Assuming $\mu_\ell^0 \equiv \frac{G_\ell^0}{N_A}$

$\mu_g^0 = \frac{G_g^0}{N_A}$ we get

$$\overset{?}{\exp}\left\{\frac{\Delta G_{\ell g}}{RT}\right\} = \frac{1-x_\ell}{1-x_g}$$

$$(1-x_g)\, e^{\frac{\Delta G}{RT}} = (1-x_\ell)$$

$$\& \quad e^{\frac{\Delta G}{RT}} - 1 =$$

(c) Assuming: $\Delta G^0_{\{A,B\}} = \Delta H^0 - T\Delta S^0$ gives:

$$\frac{1-x_\ell}{1-x_g} = \exp\left\{\frac{\Delta H_A^0 - T_A \Delta S_A}{RT}\right\}$$

$$\frac{x_\ell}{x_g} = \exp\left\{\frac{\Delta H_B^0 - T\Delta S_B}{RT}\right\}$$

(5.68) $w_e = .67$ plumbers ~~solder~~ solder.
$w_{tin} = .33$

As it cools 1st $\beta$ solidify's then tin, the solid Pb might ~~sink~~ to
$\beta$ = Pb
the bottom allowing the tin to solidify on top of it, ~~protecting~~ preventing its contact w/ $H_2O$.

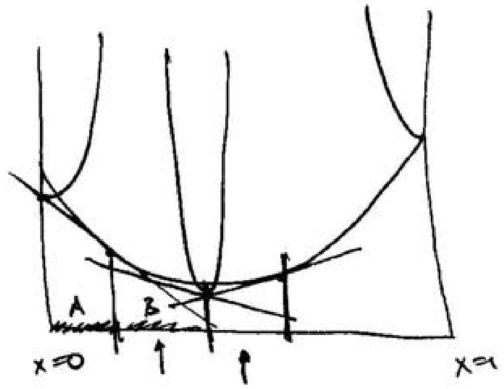
(5.69) Some salt gets into the ~~snow~~ ice lowering its freezing point, so that for the given composition the $H_2O$ salt mixture would stay a liquid rather than a solid.

If the temp is super low, then one ~~would~~ ~~still~~ would/could end up w/ a mixture that is required to be a Solid still.

(5.70) When you add salt to the ice bath you lower the ~~temperature~~ freezing temperature of the ~~ice~~ ice – $H_2O$ mixture. ~~This~~ ~~allows~~ ~~allows~~ ~~the ice salt mixture to absorb colder more~~

Some how by lowering the freezing temperature we are able to extract heat from the ~~ice~~ ~~cream~~, ~~causing~~ ~~it to~~ ~~the~~ milk sugar mixture of ice cream, causing it to freeze.

5.71




$$\frac{\partial G}{\partial T} = -S$$

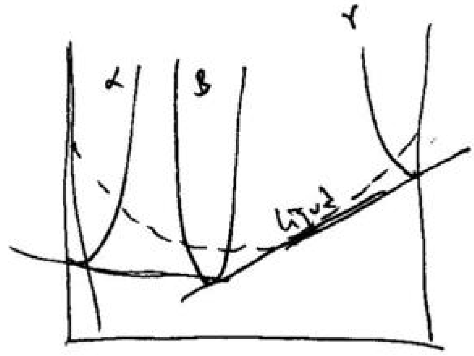
$$S_{liquid} > S_{solid}$$

$$\frac{\partial G_{liquid}}{\partial T} < \frac{\partial G_{solid}}{\partial T}$$

$$\left\{ -S_{liq} < -S_{solid} \right\}$$

$$S_{liq} > S_{solid} \quad ✓$$

5.72



$\Omega \propto N_A$

$S \propto k \ln \Omega$

$dS = k \ln N_A$

$\cancel{dG} = f(T,P)$

$dG = \underbrace{dU + PdV}_{} - Tk \ln N_A - \underbrace{T\,dS}_{\text{ind of } N_A}$

$\quad = f(T,P) - Tk \ln N_A$

÷ by # duplicated states = 2

$dG = 2f(T,P) - 2kT \ln N_A + kT \ln 2$

$dG = N_B f(T,P) - N_B kT \ln N_A + kT \ln N_B!$

$\quad = N_B f(T,P) - N_B kT \ln N_A$ $\qquad N_B \ln N_B - N_B$

$G = N_A \mu_0(T,P) + N_B f(T,P) - N_B kT \ln N_A + N_B kT \ln N_B - N_B kT$ eq 5.68

$\mu_A = \left.\frac{\partial G}{\partial N_A}\right|_{T,P,N_B} = \mu_0(T,P) - \frac{N_B kT}{N_A}$

$\mu_B = \left.\frac{\partial G}{\partial N_B}\right|_{\dots} = f(T,P) - kT \ln N_A + kT \{ \ln N_B \}$

$\quad = f(T,P) + kT \ln\left(\frac{N_B}{N_A}\right)$

molality = $\dfrac{\text{moles solute}}{\text{kilograms of solvent}}$ "$\dfrac{mol}{kg}$"

$$N_B = N_{Avg} \cdot n_B$$

$$N_A = N_{Avg} \cdot n_A$$

$$\text{kilogram solvent} = M \cdot n_A = M \cdot \frac{N_A}{N_{Avg}}$$

$$m = \frac{n_B}{M \cdot n_A} = \frac{N_B / \cancel{N_{Avg}}}{M\, N_A / \cancel{N_{Avg}}} = \left(\frac{1}{M}\right) \frac{N_B}{N_A}$$

$\therefore$

$$\mu_B = f(T,P) + kT \ln (M \cdot m)$$

$$= \mu^0(T,P) + kT \ln(m_B)$$

(5.73) let $N_A = rN_A$

$N_B = rN_B$

$$G' = rN_A\mu_0(T,P) + rN_B f(T,P) - rN_B kT\ln(rN_A)$$

$$+ rN_B kT\ln(rN_B) - rN_B kT$$

$$= rN_A\mu_0(T,P) + rN_B f(T,P) - rN_B kT\ln(N_A) - rN_B \cancel{kT}\ln(r)$$

$$+ rN_B kT\ln(N_B) + rN_B \cancel{kT}\ln(r) - rN_B kT$$

$$= r\Big\{ \underbrace{N_A\mu_0(T,P) + N_B f(T,P) - N_B kT\ln(N_A) + N_B kT\ln(N_B)}_{G} - N_B kT \Big]$$

$= rG.$

w/o $N_B kT\ln(N_B) - N_B kT$ or $kT\ln(N_B!)$ term we wouldn't have the cancellation shown.

(5.74) 5.69 5.70 $G = N_A\mu_A + N_B\mu_B$

Now $N_A\mu_A + N_B\mu_B = N_A\mu_0(T,P) - N_B kT + N_B f(T,P) + kTN_B\ln(N_B/N_A)$

$$= N_A\mu_0(T,P) - N_B kT + N_B f(T,P) + kTN_B\ln(N_B) - kTN_B\ln(N_A)$$

Yes

5.75 5.68 dilute solution

$$G = N_A \mu_0(T,P) + N_B f(T,P) - N_B kT \ln(N_A) + N_B kT \ln N_B - N_B kT$$

5.61 ideal mixture

$$G = (1-x) G_A^\circ + x G_B^\circ + RT\left[x \ln x + (1-x)\ln(1-x)\right]$$

$$G_A^\circ = N_A \mu_A^\circ(T,P) \qquad x = \frac{N_B}{N_A+N_B}\ ; 1-x = \frac{N_A}{N_A+N_B}$$

$$G_B^\circ = N_B \mu_B^\circ(T,P) \qquad \text{if } N_B \ll N_A$$

$$x \simeq \frac{N_B}{N_A} \ \ \& \ \ 1-x \cong 1$$

Then $G = N_A \mu_A^\circ(T,P) + \frac{N_B}{N_A} \cdot N_B \mu_B^\circ(T,P)$

$$+ RT\left[\frac{N_B}{N_A} \ln\left(\frac{N_B}{N_A}\right) + ( \quad \cdots \quad \right]$$

$\overset{?}{n} RT = NkT$

$$\mu_A = \mu_0(T,P) - \frac{N_B kT}{N_A} \qquad 5.69$$

A ~~solute~~ solvent, B ~~solvent~~ solute

$$\mu_B = f(T,P) + kT\ln\left(\frac{N_B}{N_A}\right) \qquad 5.70$$

$$\mu_0(T,P_2) = \mu_0(T,P_1) - \frac{N_B kT}{N_A} \qquad P_2 > P_1$$

$$\mu_0(T,P_2) \approx \mu_0(T,P_1) + (P_2-P_1)\frac{\partial \mu_0}{\partial P}$$

$$(P_2-P_1)\frac{\partial \mu_0}{\partial P} = -\frac{N_B}{N_A}kT$$

$$\frac{\partial \mu_0}{\partial P} = \frac{\partial(G/N)}{\partial P} = \frac{1}{N}\frac{\partial G}{\partial P}$$

$$= \frac{V}{N}$$

$$(P_2-P_1)\frac{V}{N_A} = \frac{N_B kT}{N_A}$$

$$P_2 - P_1 = \frac{N_B kT}{V} = \frac{n_B RT}{V}$$

$$dG = dU + PdV - TdS$$

$$\cancel{dG = dU + PdV - dT}$$

But $dU = TdS - PdV + \mu dN$

$\therefore$

$\cancel{dG = TdS - PdV}$

$$G = U + PV - ST$$

$$H = U + PV$$

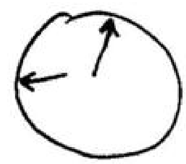
$$dG = dU + PdV + VdP - SdT - TdS$$

$$= TdS - PdV + \mu dN$$

$$= VdP - SdT + \mu dN$$

$$\left.\frac{\partial G}{\partial P}\right|_{T,N} = V$$

$$\frac{n_B}{V} = \left(\frac{1}{200}\right)\left(\frac{1\,\text{mole}}{18\,\text{cm}^3}\right)\left(\frac{100\,\text{cm}}{1\,\text{m}}\right)^3 = \ldots$$



$$P_2 - P_1 = \frac{n_B RT}{V}$$

$$= (278\ \text{mol/m}^3)(\cancel{8.31}\ 8.314\ \text{J/mol}\cdot\text{K})(300\text{K}) \cong 7 \cdot 10^5\ \text{N/m}^2$$

$10^5 \cong 1$ Atm

5.86 Veg sea $H_2O$

(a)


$$P_2 - P_1 = \frac{n_B RT}{V}$$

$$35\,g\ NaCl = \frac{35\,g}{(23+35)\,g/mol} = \left(\frac{35}{58}\right) mol$$

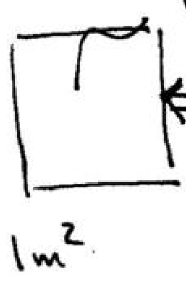
$$V = 1\,kg\ (H_2O) \qquad 1\,g = 1\,cm^3$$

$$= 1000\,g\ \frac{1\,cm^3}{1\,g} \cdot \left(\frac{1\,m}{100\,cm}\right)^3 = \frac{10^3}{10^6} = 10^{-3}\,m^3$$

So $\frac{n_B}{V} = \left(\frac{35/58}{10^{-3}}\right) \frac{mol}{m^3}$

(b)

1 liter sea $H_2O$. $= \square\,m^3$



$\Leftarrow P > P_2 = P_1 + \frac{n_B}{V} \cdot RT$

Assuming a side of $1\,m^2$.

$$W \geq [P(1\,m^2)] \cdot \Delta l$$

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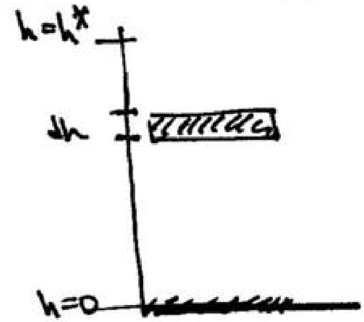
(5.77) $(\text{grams}/\text{liter}) \cdot M^{-1}$ moles/l

$C\,(\text{grams}/l)$ M = molecular weight

g/mol.

$P_2 - P_1 = \int_{h=0}^{h^*} \Delta h \cdot A \cdot \rho g h$



$= \rho g$ $F = m \cdot g$ ; $P = \dfrac{F}{A}$

$= V \cdot \rho \cdot g$

$F = \Delta h \cdot \rho \cdot A \cdot g$ $\therefore$ $P_2 - P_1 = \Delta h \cdot g \cdot \rho = \dfrac{n_B RT}{V}$

plotting [H] v.s. $\Delta h$ $\dfrac{\Delta h \cdot g \cdot \rho}{RT} = \dfrac{n_B}{V}$ = moles/liter

$\dfrac{[H]}{M}$ = moles/liter

$\therefore \dfrac{\Delta h \cdot g \cdot \rho}{RT} = \dfrac{[H]}{M}$

(5.78) ?

$$-(T-T_0)\left[\underbrace{\left(\frac{S}{N}\right)_{liq} - \left(\frac{S}{N}\right)_{gas}}\right] = \frac{N_B kT}{N_A}$$

let $N = N_A$ $\quad \frac{-L}{N_A T_0}$

$$\Rightarrow \quad T - T_0 = \frac{N_B kT T_0}{L} \cong \frac{N_B k T_0^2}{L} \qquad \text{eq } 5.90$$

$NaCl = 23 + 35 = 58$ g/mol

Avg Atomic mass $= \frac{1}{2}(23 + 35) = \frac{58}{2} \cong 25 + 1.5 \doteq 26.5$

$$T - T_0 = \frac{(1.2\text{ mol})(8.314\text{ J/mol·K})(373\text{ K})^2}{2260\text{ kJ}} \qquad N_B k = n_B R$$

$= .6$ K

$$P - P_0 = -\frac{N_B}{N_A} P_0 = -\frac{n_B}{n_A} P_0 = -\frac{1.2}{\left(\frac{1000}{18}\right)} P_0$$

B is solvent

A is solute.

(Prob 5.79)

1 teaspoon = 1 [?] amt of salt

$\cong$ 10 grams NaCl

$= \frac{40}{29+...} = \frac{10}{23+35}$ moles $= \frac{10}{} \equiv n_B$

$$T - T_0 = \frac{n_B (R)(373K)^2}{2260\ kJ} < .6\,k.$$ has negligible effect

(Prob 5.80) eq 5.90 $T - T_0 = \frac{n_B R T_0^2}{L}$

Clausius - Clapeyron: $\frac{dP}{dT} = \frac{L}{T\Delta V} = \frac{\Delta S}{\Delta V}$

w/ $\frac{P}{P_0} = 1 - \frac{n_B}{n_A}$ Raoult's law

$$\frac{dP}{dT} = \frac{L}{T\Delta V} \Rightarrow dT = \left(\frac{T\Delta V}{L}\right) dP$$

Assuming $\frac{T\Delta V}{L}$ is a constant as our transformation takes place from $T_0 \to T$, $P_0 \to P$

one gets $T - T_0 = \frac{T_0 \Delta V_0}{L}(P - P_0) = \frac{T_0 \Delta V_0}{L}\left(-\frac{n_B}{n_A} P_0\right)$ By Raoult's law

But $\{ PV = nRT \}$ + Assuming

$$T - T_0 = -\frac{T_0}{L\, n_A} (P\Delta V)\, n_B$$

A is solvent

$$= -\frac{T_0}{L n_A} n_B (n_A R T_0) =$$

$$= -n_B \frac{R T_0^2}{L}$$

off by a minus sign? why?

(Prob 5.81)


$\{ T - T_0 < 0 \}$

By eq $\mu_{A,solid}(P,T) = \mu_{A,liq}(P,T)$

$$\parallel$$

$$\mu_0(T,P) - \frac{N_B}{N_A} kT \qquad \text{(By eq 5.69)}$$

Holding the pressure fixed to investigate changes in temperature only we get

$$\mu_{A,solid}(P,T) = \mu_{A,solid}(P,T_0) + (T - T_0) \frac{\partial \mu_{A,solid}}{\partial T}(P,T_0)$$

$$\mu_0(P,T) \cong \mu_0(P,T_0) + (T - T_0) \frac{\partial \mu_0}{\partial T}(P,T_0)$$

chem potential of ~~solution~~ of solvent only in liquid phase

Then we get

$$\mu_{A,solid}(\cancel{P},T_0) + (T-T_0)\frac{\partial \mu_{A,solid}}{\partial T} = \mu_0(\cancel{P},T_0) + (T-T_0)\frac{\partial \mu_0}{\partial T}(P,T) - \frac{N_B}{N_A} kT$$

$$d\left\{\frac{\partial \mu}{\partial T} = \frac{\partial}{\partial T}\left(\frac{G}{N}\right) = \frac{1}{N}\frac{\partial G}{\partial T} = -\frac{S}{N}\right\}$$

$$G = U + PV - TS$$

$$dU = TdS - pdV$$

$$dG = dU + PdV + VdP - TdS - SdT$$

$$= T\cancel{dS} - p\cancel{dV} + p\cancel{dV} + VdP - T\cancel{dS} - SdT$$

$$= VdP - SdT$$

$$\therefore \quad \frac{(T-T_0)}{N_A}\Big[\underbrace{S_{A,solid} - S_{A,liquid}}_{(-L/T)}\Big] = -\frac{N_B}{N_A}kT$$

$$T - T_0 = \frac{N_B kT^2}{L} \quad \ldots \text{ what went wrong?}$$

(Prob 5.82) One would get the exact same result as for the boiling point elevation as in the text?

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5,83 Equilibrium conditions: equality of gibbs free energies

(a) $2\mu_H = \mu_{H_2}$

(b) $2\mu_{CO} + \mu_{O_2} = 2\mu_{CO_2}$

(c) $\mu_{CH_4} + 2\mu_{O_2} = 2\mu_{H_2O} + \mu_{CO_2}$

(d) $\mu_{H_2SO_4} = 2\mu_{H^+} + \mu_{SO_4}$

(e) $2\mu_p + 2\mu_n = \mu_{^4He}$

5,84 eq 5,108

$$\frac{P_{NH_3}^2 (P^0)^2}{P_{N_2} P_{H_2}^3} = K \equiv 6.9 \cdot 10^{-5}$$

$$\frac{\left(\frac{P_{NH_3}}{P^0}\right)^2}{\left(\frac{P_{N_2}}{P_0}\right)\left(\frac{P_{H_2}}{P_0}\right)^3} = K$$

?

5,85 $k = e^{-\Delta G/RT}$

$$\ln k = -\frac{\Delta G}{RT}$$

$$\frac{d}{dT}(\ln k) = +\frac{\Delta G}{RT^2} - \frac{1}{RT}\frac{d\Delta G}{dT}$$

$dU = -p\,dV + S\,dT$

$G = U + PV - ST$

$\Delta G = -p\cancel{dV} + S\cancel{dT} + p\cancel{dV} + V\,dP - S\cancel{dT} - T\,dS$

$= V\,dP - T\,dS$

$$\frac{d}{dT}(\ln k) = \frac{1}{RT^2}\left(\Delta G - T\frac{d\Delta G}{dT}\right)$$

$$=$$

Is $\frac{d\Delta G}{dT} = -S$ yes.

Then $\Delta G - T\frac{d\Delta G}{dT} = \Delta G + ST = \Delta H$

$$\therefore \quad \frac{d}{dT}(\ln k) = \frac{\Delta H}{RT^2}$$

5.86 $N_2 + 3H_2 \leftrightarrow 2NH_3$

$T_1 = 373\ k$

$T_2 = 273 + 500 = 773\ k$

$$\Delta H^\circ = 2\Delta_f H(NH_3) - 3\Delta_f \overset{0}{\cancel{H}}(H_2) - \Delta_f \overset{0}{\cancel{H}}(N_2)$$

$$= 2(-46.11) \approx -90\ kJ$$

eq 5.69 $H_2O$

$$\mu_A = \mu_0(T,P) - \frac{N_B}{N_A} kT$$

eq 5.72 for $H^+$ + $OH^-$

$$\mu_B = \mu^\circ(T,P) + kT \ln m_B \qquad m = \frac{\text{moles solute}}{\text{kilogram solvent}}$$

$$\mu^\circ_{H_2O}(T,P) = \mu^\circ_{H^+} + kT \ln m_{H^+} + \mu^\circ_{OH^-} + kT \ln m_{OH^-}$$

$$-N_A\left(\underbrace{\mu^\circ_{H^+} + \mu^\circ_{OH} + - \mu^\circ_{H_2O}}\right) = N_A kT \ln (m_{H^+} m_{OH^-})$$

$$-\Delta G^\circ = RT \ln (m_{H^+} m_{OH^-})$$

$$pH \equiv -\log_{10} m_{H^+}$$

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(8.87)

$$H_2SO_4 \rightarrow H^+ + HSO_4^- \qquad k = 10^2$$

$$HSO_4^- \longleftrightarrow H^+ + SO_4^{2-} \qquad k = 10^{-1.9}$$

(a) Based on eq 8.114

$$m_{H^+} m_{HSO_4^-} = 10^2 \Rightarrow \underset{\parallel}{m_{H^+}} = 10 = m_{HSO_4^-}$$

$\equiv \dfrac{\text{moles } \{H^+, HSO_4^-\}}{\text{kilograms solvent.}}$ quite a ~~few~~ few moles !!

I would assume at a pH equivalent to this

$$pH \cong -\log_{10} m_{H^+} \cong -\log_{10}(10) = -1.$$

(b) If $m_{HSO_4^-} = 5 \cdot 10^{-5}$ mol/kg

Then ?

(c) $k$ for that reaction is $10^{-14} \Rightarrow m_{H^+} = 10^{-7} \ll m_{H^+}$ due to sulfuric acid.

(d) ?

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5.88 $\frac{\partial(\Delta G^{\circ})}{\partial P}$

$$\left.\frac{\partial(\Delta G)}{\partial P}\right|_T = V$$

Show that for volumes of liquids in solution this # V is small.

$$G = U + PV - ST$$

$$\Delta G = \Delta U + \Delta P \cdot V + V\Delta P - \Delta S \cdot T - S\Delta T$$

$$= -P\cancel{\Delta V} + S\cancel{\Delta T} + \cancel{\Delta P V} + V\Delta P - \Delta S \cdot T - \cancel{S\Delta T}$$

$$= V\Delta P - \Delta S \cdot T$$

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$$\mu_{gas} = \mu_{solute}$$

$$\parallel$$

$$\mu^\circ_{gas}(T) + kT\ln(P/P^\circ) = \mu^\circ_{solute}(T,P) + kT\ln m_B$$

$$N_A(\mu^\circ_{solute} - \mu^\circ_{gas}) = N_A kT\left(\cancel{B}\ \ln(P/P^\circ) - \ln(m_B)\right)$$

$$\Delta G^\circ = RT\ln\left(\frac{P/P^\circ}{m}\right)$$

$$\frac{m}{P/P^\circ} = e^{-\Delta G^\circ/RT}$$

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$\Delta H^\circ = -11.7\,kJ$

Van't Hoff equation:

$$\frac{d(\ln K)}{dT} = \frac{\Delta H^\circ}{RT^2}$$

$\Rightarrow$ if $\Delta H^\circ$ is independent of temperature

~~Ideal~~ $\ln K(T_2) - \ln K(T_1) = \frac{\Delta H^\circ}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\Rightarrow \ln(K(372\,K)) - \ln(1/780) = \frac{(-11.7\cdot 10^3 J)}{(8.314\, J/K)}\left(\frac{1}{273} - \frac{1}{373}\right)$

$\Rightarrow K(372\,K) = \ldots$

How calculate $K(272\,K)$? Can I not just use 5.124?

5.90 $\Delta G^\circ = -1307.67 - (-856.64) - 2(-237.13)$ kJ

(a) $T = 273 + 25$ K

$\exp\left(\frac{-\Delta G^\circ}{RT}\right) = \cdots$

Then $\frac{m}{P/P^\circ} = \ldots$ what is the corresponding P expression?

think $P/P^\circ \cong 1$, but why?

(b) compute $K(372\,K)$ by ~~van't~~ van't Hoff eq.

$+ \Delta H^\circ$

(5.91) Using eq 5.123 calculate m, i.e.:

$$\frac{m}{(P/P^0)} = e^{-\Delta G^0/RT}$$

$P = 3.4\cdot 10^{-4}$ bar

m = ... Molality of Carbonic acid

Given that "neutral" $H_2O$ has this much.

$$\Delta G^0 = -623.08 - 1(-237.13) - 1(-394.36) \quad kJ$$

$$= 8.41 \ kJ$$

$$k = e^{-\Delta G^0/RT} = .03167$$

$$\Rightarrow m = 1.07\cdot 10^{-5} \ \frac{\text{moles}}{\text{kg solvent}}$$

Then the entirely aqueous transformation is governed by:

$$\cancel{m_{H^+}\cdot m_{HCO_3^-} = e^{-\Delta G^0/RT}}$$

w/ $\cancel{\Delta G}$ $\quad \mu_{H_2CO_3} = \mu_{H^+} + \mu_{HCO_3^-}$

$$\mu^0_{H_2CO_3} + kT\ln(m_{H_2CO_3}) = \mu^0_{H^+} + kT\ln(m_{H^+}) + \mu^0_{HCO_3^-} + kT\ln(m_{HCO_3^-})$$

$$N_A\left(\mu^{\circ}_{H_2CO_3}-\mu^{\circ}_{H^+}-\mu^{\circ}_{HCO_3^-}\right) = $$

$$= N_A kT \ln\left[\frac{m_{H^+} m_{HCO_3^-}}{m_{H_2CO_3}}\right]$$

$$\Rightarrow \exp\left(-\frac{\Delta G^{\circ}}{RT}\right) = \frac{m_{H_2CO_3}}{m_{H^+} m_{HCO_3^-}}$$

$$\|$$

Assuming that $m_{H^+} = m_{HCO_3^-}$ ∴

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$$\mu_H = \mu_p + \mu_e$$

$$\parallel$$

$$\mu_H^\circ + kT\ln\left(\frac{P_H}{P_0}\right) = \mu_p^\circ + kT\ln\left(\frac{P_p}{P_0}\right) + \mu_e^\circ + kT\ln\left(\frac{P_e}{P_0}\right)$$

$$\Rightarrow \mu_p^\circ + \mu_e^\circ - \mu_H^\circ = kT\ln\left[\frac{P_H/P_0}{\left(\frac{P_p}{P_0}\right)\left(\frac{P_e}{P_0}\right)}\right]$$

$$= kT\ln\left[\frac{P_0\,P_H}{P_p\,P_e}\right] \quad \text{eq } 5.126 \;\checkmark$$

$$\mu = -kT\ln\left[\frac{V}{N}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right]$$

$$PV = NkT$$

$$\frac{V}{N} = \frac{kT}{P}$$

5.92

(a) $\frac{N_p}{N_H^\circ}$

$$N_H^\circ = N_H(T) + N_p(T)$$

$PV = kNT$    $P = \frac{kNT}{V}$

$N = \frac{PV}{kT}$

$P_{H^\circ}$ ?

(b) ?

(c) ?

(d)

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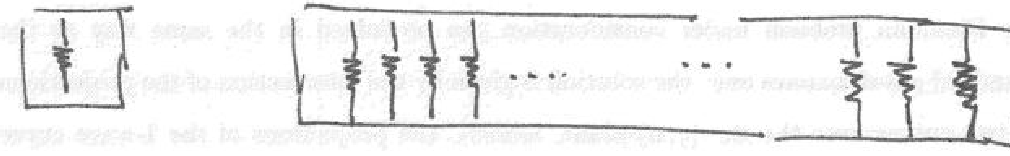
(6.1)



$$\Omega(N,q) = \binom{q+N-1}{q}$$ Multiplicity of an einstein solid w/ $N$ oscillators & $q$ units of energy

$$\Omega_{AB} = \Omega\binom{q+N_A+N_B-1}{q}$$

$N_A = 1$ ; $N_B = 100$ ; $q = 500$ ; ? I should be able to do this problem...

(6.2) $$P(S) = \frac{1}{Z} e^{-E(S)/kT}$$

$$P(E) = ?$$

since there are $\Omega(E)$ multiplicities of states w/ energy $E$ one gets

$$P(E) = \frac{\Omega(E)}{Z} e^{-E/kT}$$

~~$\Omega(E) = k \ln$~~

$$S = k \ln \Omega(E)$$

w/ $E$ = energy of system in state $S$.

$$\Omega = \exp\{S/k\}$$

so $$P(E) = \frac{1}{Z} e^{\frac{S}{k}} e^{-\frac{E}{kT}} = \frac{1}{Z}\exp\left\{-\frac{(E-TS)}{kT}\right\}$$

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$$P(E) = \frac{1}{Z} \exp\left\{ -\frac{F}{kT} \right\}$$

$$P(E) = \frac{1}{Z} e^{-F/kT}$$

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(6.3) 2 grand states

$E_0 = 0\ eV$

$E_1 = 2\ eV$

$$Z = \sum_s e^{-E(s)/kT}$$

$$= e^{-0} + e^{-2eV/kT} = 1 + e^{-\frac{2}{kT}}$$

$[kT] = eV$

(6.4)

$$Z = \sum e^{-E(s)/kT} \qquad E(s) =$$

$$= \oint \sum_s P(s) = \int_0^\infty P(s)\,ds = \int_0^\infty e^{-s/kT}\,ds = \cdots$$

(6.5)

(a)

$$Z = \sum_s e^{-\frac{E(s)}{kT}}$$

$$= e^{\frac{-.05}{kT}} + 1 + e^{\frac{.05}{kT}} = \cdots$$

$T = 300K$

(b)

$$P(s_1; E_1 = -.05\ eV) = \frac{e^{-.05/kT}}{Z} \qquad P(s_3; E_3 = +.05\ eV) = \frac{e^{\frac{+.05}{kT}}}{Z}$$

$$P(s_2; E_2 = 0) = \frac{1}{Z}$$

(c) Z will change since E has changed 01-05-03 2

The prob. will not change.

Prob 6.6

$$\frac{P(s_2)}{P(s_1)} = \frac{e^{-E(s_2)/kT}}{e^{-E(s_1)/kT}} = \exp\left\{-\frac{(E(s_2)-E(s_1))}{kT}\right\}$$

$$= \exp\left\{-\frac{(10.2\,eV)}{(8.62\cdot 10^{-5}\,eV/K)(293\,K)}\right\} = \ldots$$

Then since 1st excited state has 4 levels all w/ the same energy we get

4 x (this #)

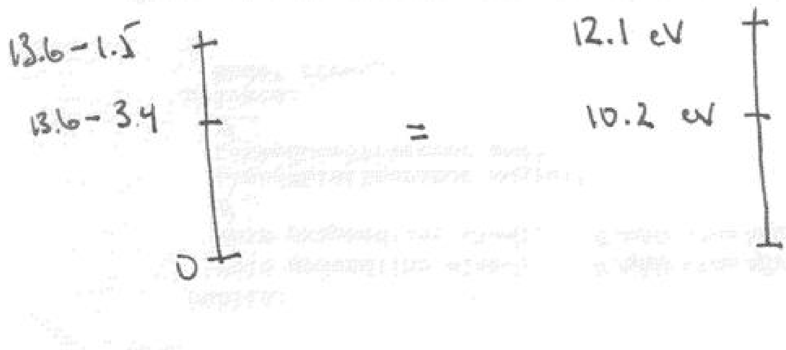
Same w/ $T = 9500\,K$.

(6.7) Think both $p(s_2)$ + $p(s_1)$ would get multiplied by a constant # 2

(6.8) The methods of this section apply to a given system emersed in a "universe" at another a temperature T.

Don't know?

(6.9) $Z = \sum_s e^{-E(s)/kT} =$



(a)

$$Z = 1 + e^{\frac{-10.2}{kT}} + e^{-\frac{12.1}{kT}}$$

$T = 5800\,k$

is there a multiplicity coefficient here?

From Appendix A: $E(n) = -\frac{13.6\text{ eV}}{n^2} + 13.6\text{ eV}$

b)

How do I know

$$= 13.6\,\text{eV}\left[1 - \frac{1}{n^2}\right]$$

$$\sum_n \exp\left\{\frac{13.6}{kT}\left(1 - \frac{1}{n^2}\right)\right\} \quad \text{diverges?}$$

$$= \exp\left\{\frac{13.6}{kT}\right\} \sum_n \exp\left\{-\frac{13.6}{kTn^2}\right\} \quad \lim_{n\to\infty} \exp\left\{-\frac{13.6}{kTn^2}\right\} \neq 0$$

$\therefore$ Series diverges

(c) $R_n = a_0 n^2$

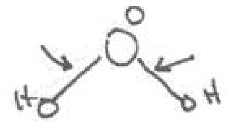
$dV$ for large $n$'s then becomes

$$dV = a_0\left((n+1)^2 - n^2\right) = a_0\left(\cancel{n^2} + 2n + 1 - \cancel{n^2}\right) = a_0(2n+1) = \Theta(n)$$

$\therefore P\,dV = \Theta(n)$ for large $n$

$\therefore$ P-V work ~~associated~~ ~~by~~ that the atmosphere does when an atom transfers levels is not negligible for high energy level transfers. Don't see ~~why~~ where this P-V work effects $E(s)$?

6.10



$f_0 = 4.8\cdot 10^{13}$ Hz $\qquad E(s) = \frac{(2s+1)hf}{2}$

(a) $$Z = \sum_s e^{-\frac{E(s)}{kT}} = \sum_{s=0}^{\infty} \exp\left\{ \frac{\left(\frac{2s+1}{2}\cdot hf_0\right)}{kT} \right\} = \cdots$$

$$P(s_1) = \frac{\exp\left\{\frac{hf_0}{kT}\right\}}{Z}$$

Think in the partition formulation if we had degeneracy at a given energy level then we must add in that # of $\exp\{-\frac{E(s)}{kT}\}$ factors.

(b) ...

6.11 $E = m\mu B \qquad \mu = 1.03\cdot 10^{-7}\,\frac{eV}{T} \qquad B = 0.63\,T$

$$Z = \sum_s e^{-E(s)/kT} = e^{+\frac{3}{2}\mu B/kT} + e^{\frac{1}{2}\mu B/kT} + e^{-\frac{1}{2}\mu B/kT} + e^{-\frac{3}{2}\mu B/kT}$$

$$P\left(-\tfrac{3}{2}\right) = \frac{e^{+\frac{3}{2}\frac{\mu B}{kT}}}{Z}, \ldots$$

reversing $B \leftrightarrow -B \Leftrightarrow -T$

(6.12) $\frac{P(s^*)}{P(s_0)} = \frac{3}{10}$ By experimental observation.

$= \frac{3e^{-\frac{E(s^*)}{kT}}}{e^{-\frac{E(s_0)}{kT}}} = .3$ By Boltzmann statistics + knowledge of degeneracy of 1st excited state

$\rightarrow \exp\left\{-(E(s^*)-E(s_0))/kT\right\} = .1$

$\Rightarrow \exp\left\{\frac{-4.7\cdot 10^{-4}\,eV}{kT}\right\} = .1 \Rightarrow T = ..$

(6.13) $E_n - E_p = (2.3\cdot 10^{-30}\,kg)\,c^2$

$T = 10^{11}\,K.$

$\frac{P(n)}{P(p)} = \exp\left\{-\frac{(E_n-E_p)}{kT}\right\} = \dots =$ fraction of nucleons that were neutrons.

w/ no degeneracy.

(6.14)

$- - \circ - -$ molecule of air at z

$- - \circ - -$ molecule of air at $z=0$ (sea level)

$\frac{N(z)}{N(0)} = \frac{P(z)}{P(0)} = \exp\left\{-\frac{(E(z)-E(0))}{kT}\right\}$

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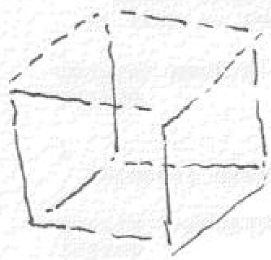
Then w/ $E(z) = mgz$ potential energy

$$N(z) = N_0 \exp\left\{-\frac{mgz}{kT}\right\}$$ to relate to pressure

$$PV = kNT$$

$$\Rightarrow N = \frac{PV}{kT}$$

What does V represent? A fixed volume that one moves up to height z



then
$$\frac{P(z)V}{kT} = \frac{P(0)V}{kT} \exp\left\{-\frac{mgz}{kT}\right\}$$

Same as eq 1.16 —

6.15

(a) $\bar{E} = \dfrac{4 \cdot 0eV + 3 \cdot 1eV + 2 \cdot 4eV + 1 \cdot 6eV}{10}$

(b) $P(0eV) = \frac{4}{10}$ $P(4eV) = \frac{2}{10}$

$P(1eV) = \frac{3}{10}$ $P(6eV) = \frac{6}{10}$

(c) $\bar{E} = (\frac{4}{10}) \cdot 0eV + (\frac{3}{10}) \cdot 1eV + (\frac{2}{10}) \cdot 4eV + (\frac{1}{10}) \cdot 6eV$

6.16 $Z = \sum_s e^{-\frac{E(s)}{kT}} = \sum_s e^{-E(s)\beta}$

$\dfrac{\partial Z}{\partial \beta} = \cancel{\sum_s -\beta} \sum_s -E(s) e^{-E(s)\beta}$

since $\bar{E} = \frac{1}{Z} \sum_s E(s) e^{-E(s)\beta}$

we see that $\cancel{\sum}$ $\bar{E} = \dfrac{-1}{Z}\dfrac{\partial Z}{\partial \beta} = -\dfrac{\partial \ln Z}{\partial \beta}$

6.17 (a) $E_i - \bar{E} = \left\{ \begin{array}{l} 7-3 = 4 \\ 4-3 = 1 \\ 4-3 = 1 \\ 0-3 = -3 \\ 0-3 = -3 \end{array} \right\} \Delta E_i \, ; \, \Delta E_i^2 = \left\{ \begin{array}{l} 16 \\ 1 \\ 1 \\ 9 \\ 9 \end{array} \right.$

$$\overline{(\Delta E_i)^2} = \frac{1}{5}(16+1+1+9+9) = \frac{36}{5} = 7.2 \equiv \sigma_E^2$$

$$\sigma_E = \sqrt{7.2}$$

$$\sigma^2 = \text{Avg}(E_i - \bar{E})^2$$

$$= \text{Avg}_i(E_i^2 - 2E_i\bar{E} + \bar{E}^2)$$

$$= \overline{E^2} - 2\bar{E}^2 + \bar{E}^2 = \overline{E^2} - \bar{E}^2$$

(d) $\sigma^2 = \ldots$

Prob 6.18 $\quad Z = \sum_s \exp(-E(s)\beta) \qquad \overline{E^2} = \sum_s E^2(s) \frac{e^{-E(s)\beta}}{Z}$

$$= \frac{1}{Z}\sum_s E(s)^2 e^{-E(s)\beta}$$

$$\frac{\partial Z}{\partial \beta} = \sum_s -E(s) e^{-E(s)\beta}$$

$$\frac{\partial^2 Z}{\partial \beta^2} = \sum_s E^2(s) e^{-E(s)\beta}$$

$$\therefore \quad \overline{E^2} = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$$

since $\sigma^2 = \overline{E^2} - \bar{E}^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2} - \left(-\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right)^2$

Since $\bar{E} = \frac{1}{Z}\frac{\partial Z}{\partial \beta}$

$$\sigma^2 = \frac{1}{Z}\frac{\partial}{\partial \beta}(\bar{E}) - \bar{E}^2$$

Something is fishy... try again

$$\sigma_E^2 = \overline{E^2} - (\bar{E})^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2} - \left(-\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right)^2$$

$$= \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2} - \frac{1}{Z^2}\left(\frac{\partial Z}{\partial \beta}\right)^2$$

$$= \frac{\partial}{\partial \beta}\left(\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right) = \frac{\partial}{\partial \beta}\left(-\bar{E}\right)$$

w/ $\beta = \frac{1}{kT}$

$$\frac{\partial}{\partial \beta} = \frac{\partial T}{\partial \beta}\cdot\frac{\partial}{\partial T} \qquad \frac{\partial \beta}{\partial T} = \frac{-1}{kT^2}$$

$$\therefore \sigma_E^2 = \left(\frac{1}{kT^2}\right)^{-1}\frac{\partial \bar{E}}{\partial T} = \left(\frac{1}{kT^2}\right)^{-1} C = kT^2 C$$

$$\sigma_E = T\sqrt{kC} = kT\sqrt{C/k}$$ ✓

(6.19) $\frac{\sigma_E}{\bar{E}}$ = fractional fluctuation in Energy

Prob 6.19 Einstein Solid high temp limit $\Omega(N,q) \cong \left(\frac{eq}{N}\right)^N$

$$S = Nk\left[\ln(q/N) + 1\right]$$

$$U = NkT$$

$$C_V = Nk$$

Based on prob 6.18

$$\sigma_E = kT\sqrt{\frac{C}{k}} = kT\sqrt{N} = \cancel{\text{standard deviation of}}$$

Standard deviation of the energy of system

Fractional fluctuation in energy $= \frac{\sigma_E}{U} = \frac{kT\sqrt{N}}{NkT} = \frac{1}{\sqrt{N}}$

$F(N=1) = 1$

$F(N=10^4) = 10^{-2}$

$F(N=10^{20}) = 10^{-10}$.

6.20

$$1 + x + x^2 + \cdots$$

(a) $|x| < 1$

$$1-x \;\overline{)\; 1 + 0x + 0x^2 + 0x^3 + 0x^4 + \cdots}$$

$$\underline{1 - x}$$

$$x + 0x^2 + 0x^3 + \cdots$$

$$\underline{x - x^2}$$

$$+x^2 + 0x^3 + 0x^4 + \cdots$$

$$\underline{+x^2 - x^3}$$

$$x^3$$

(b) $$Z = \sum_s e^{-E(s)\beta}$$

$$E(s) = shf \qquad s = 0, 1, 2, \ldots$$

$$= \sum_{s=0}^{\infty} e^{-shf\beta} = \sum_{s=0}^{\infty} \left(e^{-hf\beta}\right)^s = \frac{1}{1-e^{-hf\beta}} \cdot \frac{e^{+hf\beta/2}}{e^{+hf\beta/2}}$$

$$= \frac{e^{hf\beta/2}}{2\sinh(hf\beta)}$$

(c) $$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{Z}\frac{\partial}{\partial \beta}\left(\frac{1}{1-e^{-hf\beta}}\right)$$

$$= \frac{-1}{Z}\frac{1}{(1-e^{-hf\beta})^2}(+hf) = \frac{-1}{Z} Z^2 hf$$

$$= -Zhf = \frac{-hf}{1-e^{-hf\beta}}$$

(d) $U = N\overline{E} = \dfrac{-h f N}{1-e^{-hf\beta}}$

hd 3,25

(e) $C = \dfrac{\partial \overline{E}}{\partial T} = \dfrac{\partial}{\partial T}\left(\dfrac{-hfN}{1-e^{-hf\beta}}\right) = \ldots$

(6.21) $E_n \approx \epsilon(1.03n - 0.03n^2) \qquad n = 0,1,2\ldots$

$$Z = \sum_s e^{-E(s)/kT} = \sum_{n=0}^{\infty} e^{-\epsilon\frac{(1.03n-0.03n^2)}{kT}}$$ partition fn

$$\overline{E} = \frac{-1}{Z}\frac{\partial Z}{\partial \beta} \qquad + \qquad \frac{\partial Z}{\partial \beta} = \sum_{n=0}^{\infty} e^{-\epsilon(1.03n-0.03n^2)\beta}\left(-\epsilon(1.03n-0.03n^2)\right)$$

$$= -\epsilon\sum_{n=0}^{\infty}(1.03n-0.03n^2)\,e^{-\epsilon(1.03n-0.03n^2)\beta}$$

$$C = \frac{\partial \overline{E}}{\partial T} = \ldots$$

do summations only up to $n = 15$.

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(6.22) $\mu_z = -j\delta_\mu, (-j+1)\delta_\mu, (-j+2)\delta_\mu, \ldots, (j-1)\delta_\mu, j\delta_\mu$

$\delta_\mu$ constant $= \Delta\mu_z$

$\{-5, 4\}$
$\{-2, 1$
$\underbrace{-2, -1, 0, 1,}_{4}$

(a)
$$\begin{array}{r} 1 + x + x^2 + \ldots \quad x^{n-1} + x^n \\ 1-x \overline{\big) 1 + 0x + 0x^2 + 0x^3 + \cdots + 0x^{n-1} + 0x^n + 0x^{n+1}} \\ \underline{1 - x} \\ x \\ \underline{x - x^2} \\ x^2 + 0x^3 + \cdots + 0x^n + x^{n+1} \\ \vdots \quad x^3 + 0x^4 + \ldots \\ x^n + x^{n+1} \\ \underline{x^n - x^{n+1}} \\ 2x^{n+1} \quad ? \end{array}$$

5 do w/ $\infty$ series ...

(b) $$Z = \sum e^{-E(s)\beta} = \sum_{s=-j}^{j-1} e^{-s\delta_\mu\beta} = \sum_{s=0}^{2j-1} e^{-\delta_\mu\beta(s-j)}$$

$$= e^{+\delta_\mu\beta j} \sum_{s=0}^{2j-1} e^{-\delta_\mu\beta s} = e^{\delta_\mu\beta j} \frac{1 - e^{-\delta_\mu\beta(2j)}}{1 - e^{-\delta_\mu\beta}}$$

$$= \frac{e^{\delta\beta j} - e^{-\delta\beta j}}{1 - e^{-\delta\beta}} \cdot \frac{e^{\delta\beta/2}}{e^{\delta\beta/2}}$$

$$= e^{d\beta(j+1/2)} - e^{-d\beta(j\mp 1/2)}$$

(6.22)

$\mu_z = -j\delta_\mu, (-j+1)\delta_\mu, \ldots, (j-1)\delta_\mu, j\delta_\mu$

(a) ✓

(b) $Z = \sum_s e^{-E(s)\beta}$

$E(s) = -\mu_z(s)B = -s\delta_\mu B$

$\therefore \quad Z = \sum_{s=-j}^{+j} \exp\{-s\delta_\mu B\beta\}$ let $b = \beta\delta_\mu B$

$= \sum_{s=-j}^{+j} \exp\{-sb\} = \sum_{s=0}^{2j} \exp\{-(s-j)b\}$

$= \sum_{s=0}^{2j} e^{-sb} \cdot e^{jb} = e^{jb} \sum_{s=0}^{2j} e^{-sb}$

$= e^{jb}\left(\frac{1-e^{-b(2j+1)}}{1-e^{-b}}\right) = \frac{e^{jb} - e^{-b(j+1)}}{1-e^{-b}} \cdot \frac{e^{b/2}}{e^{b/2}}$

$= \frac{e^{b(j+1/2)} - e^{-b(j+1/2)}}{e^{b/2} - e^{-b/2}} = \frac{\sinh(b(j+1/2))}{\sinh(b/2)}$

(c)

$$\overline{\mu_z} = \sum_s \mu_z(s) P(s) = \frac{1}{Z}\sum_s \mu_z(s) e^{-E(s)\beta}$$

$$\mu_z(s) = s\delta_\mu \qquad s = -j, -j+1, -j+2, \ldots, j-1, j$$

$$E(s) = -\mu_z(s)B = -s\delta_\mu B$$

$$\overline{\mu_z} = \frac{1}{Z}\sum_{s=-j}^{+j} (s\delta_\mu) e^{+s\delta_\mu B\beta} = \frac{\delta_\mu}{Z}\sum_{s=-j}^{+j} s e^{sb} \qquad b = \delta_\mu B\beta$$

$$= \frac{\delta_\mu}{Z}\sum_{s=-j}^{+j}\frac{\partial}{\partial b} e^{sb} = \frac{\delta_\mu}{Z}\frac{\partial}{\partial b}\sum_{s=-j}^{+j} e^{sb}$$

$$= \frac{\delta_\mu}{Z}\frac{\partial}{\partial b}\left[\sum_{s=0}^{2j} e^{(s-j)b}\right] = \frac{\delta_\mu}{Z}\frac{\partial}{\partial b}\left[e^{-jb}\sum_{s=0}^{2j} e^{sb}\right]$$

$$= \frac{\delta_\mu}{Z}\frac{\partial}{\partial b}\left[e^{-jb}\left(\frac{1-e^{b(2j+1)}}{1-e^{b}}\right)\right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[ \frac{e^{-jb} - e^{b(j+1)}}{1-e^{b}} \right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\Big[$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[ \frac{e^{-b(j+1/2)} - e^{b(j+1/2)}}{e^{-b/2} - e^{b/2}} \right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[ \frac{\sinh(b(j+1/2))}{\sinh(b/2)} \right]$$

$$= \frac{\delta\mu}{Z}\cdot\left[ \frac{(j+1/2)\cosh(b(j+1/2))}{\sinh(b/2)} - \frac{\sinh(b(j+1/2))\cosh(b/2)\cdot 1/2}{(\sinh(b/2))^2} \right]$$

$$= \delta\mu\left[ (j+1/2)\frac{\cancel{\sinh(b/2)}}{\sinh(b(j+1/2))}\,\frac{\cosh(b(j+1/2))}{\cancel{\sinh(b/2)}} - \frac{1}{2}\frac{\cancel{\sinh(b/2)}}{\sinh(b(j+1/2))}\,\frac{\cancel{\sinh(b(j+1/2))}\cosh(b/2)}{(\sinh(b/2))^2} \right]$$

$$= \delta\mu\left[ (j+1/2)\coth(b(j+1/2)) - \frac{1}{2}\coth(b/2) \right]$$

Then $M = N\bar{\mu}_z$

$$= N\delta\mu\left[ (j+1/2)\coth(b(j+1/2)) \ldots \right]$$

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(d) $T \to 0$ $M \to 0$ why is this expected...?

(e) $M \sim \frac{1}{T}$ does this not contradict the plots obtained?

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} \cong$$

$$\cosh(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \cdots$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\sum_{k=0}^{\infty} \frac{x^k}{k!} - \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2}(1-(-1)^k)\frac{x^k}{k!}$$

$$= \sum_{k=1,3,5,\ldots}^{\infty} \frac{x^k}{k!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

Thus

$$\coth(x) \approx \frac{1 + x^2/2 + x^4/4!}{x + x^3/3! + \cdots} = \frac{1}{x + x^3/3!} + \frac{x^2/2}{x + x^3/3!} + \cdots$$

$$\cong \cancel{\frac{1}{x} + \frac{x}{2} + O(x^2)}$$

$$= \frac{1}{x(1+x^2/6)} + \frac{x}{2}\frac{1}{(1+x^2/6)} + O(x^3)$$

$$= \frac{1}{x}\left[\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{6^k}\right] + \frac{x}{2}\left[\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{6^k}\right] + O(x^3)$$

$$= \frac{1}{x}\left[1 - \frac{x^2}{6} + O(x^4)\right] + \frac{x}{2}\left[1 - O(x^2)\right]$$

$$= \frac{1}{x} - \frac{x}{6} + \frac{x}{2} + O(x^2)$$

$$= \frac{1}{x} + x\left[\frac{1}{2} - \frac{1}{6}\right] + O(x^2)$$

$$= \frac{1}{x} + x\left[\frac{2}{6}\right] + O(x^2)$$

$$= \frac{1}{x} + \frac{x}{3} + O(x^2)$$

Then $T \to \infty \Rightarrow \beta \to 0 \Rightarrow b \to 0$

$$M \sim N\delta_\mu\left[(j+1/2)\cdot\left[\frac{1}{b(j+1/2)} + \frac{b(j+1/2)}{3}\right] - \frac{1}{2}\left[\frac{1}{b/2} + \frac{b/2}{3}\right]\right]$$

$$M \sim N\delta_\mu \left[ \cancel{\tfrac{1}{6}} + \frac{b}{3}(j+\tfrac{1}{2})^2 - \cancel{\tfrac{1}{6}} - \frac{b}{12} \right]$$

$$= \frac{N\delta_\mu}{3}\left[ b(j+\tfrac{1}{2})^2 - \frac{b}{4} \right]$$

$$= \frac{N\delta_\mu b}{3}\left[ (j+\tfrac{1}{2})^2 - (\tfrac{1}{2})^2 \right] \sim \frac{1}{T} \quad \text{curie's law.}$$

(7) $M = N\delta_\mu \left[ \cosh(b) - \frac{1}{2}\cosh(\tfrac{b}{4}) \right]$ ..

(6.23) $\epsilon = .00024\ eV$

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$Z_{rot}(300k) = \sum_{j=0}^{\infty} (2j+1) e^{\frac{-j(j+1)(2.4\cdot 10^{-4})\,eV}{(8.617\cdot 10^{-5}\,eV/k)(300k)}}$$

$$=$$

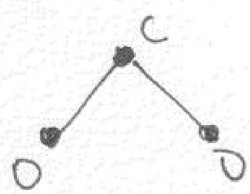
$$Z_{rot} = \frac{kT}{\epsilon} \quad \text{Approx} \ldots$$

(6.24)

$Z_{rot} \cancel{\approx} kT$ $\quad O_2$ ~~is~~ has 2 indistinguishable atoms

$$Z_{rot} \sim \frac{kT}{2\epsilon}$$

(6.25)




$Z_{rot} \approx \frac{kT}{\epsilon}$ if analysis holds ...

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6.26 $kT \ll \epsilon$

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$\cong 1 + 3e^{-2\epsilon/kT} = 1 + 3e^{-2\epsilon\beta}$$

$$\overline{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{Z}\frac{\partial}{\partial \beta}\left(1+3e^{-2\epsilon\beta}\right) = -\frac{1}{Z}\left(3(-2\epsilon)e^{-2\epsilon\beta}\right)$$

$$= -\frac{1}{Z}(-2\epsilon)\left(3e^{-2\epsilon\beta}\right)$$

$$= -\frac{1}{Z}(-2\epsilon)(Z-1) \quad \doteq \cdots + 2\epsilon\left(1 - \frac{1}{Z(\beta)}\right)$$

$$\overline{C}_V = \frac{\partial \overline{E}}{\partial T} = \frac{\partial \beta}{\partial T}\frac{\partial \overline{E}}{\partial \beta} \qquad \beta = \frac{1}{kT}$$

$$= -\frac{\beta}{T}\frac{\partial \overline{E}}{\partial \beta} \qquad \frac{\partial \beta}{\partial T} = \frac{-1}{kT^2} = -\frac{1}{T}\beta$$

$$\therefore \cancel{\overline{E}} \quad \overline{C}_V = -\frac{\beta}{T}\left(\frac{1}{Z^2}\frac{\partial Z}{\partial \beta}\right)$$

$$\overline{C}_V(T \to 0) \overset{?}{=} 0$$

$$\overline{C}_V =$$

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(6.26)

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

low temp limit $\frac{\epsilon}{kT} \gg 1$

$$\cong 1 + 3e^{-2\epsilon/kT} = 1 + 3e^{-2\epsilon\beta}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{Z}\frac{\partial}{\partial \beta}(1+3e^{-2\epsilon\beta}) = -\frac{1}{Z}(3(-2\epsilon)e^{-2\epsilon\beta})$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}}$$

$$\bar{C}_V = \frac{\partial \bar{E}}{\partial T} \; ; \quad \bar{C} = \frac{\partial \bar{E}}{\partial T} = \frac{\partial \beta}{\partial T}\frac{\partial \bar{E}}{\partial \beta}$$

$$\beta = \frac{1}{kT}$$

$$\frac{\partial \beta}{\partial T} = \frac{-1}{kT^2}$$

$$= -\frac{\beta}{T}\frac{\partial \bar{E}}{\partial \beta}$$

$$= -\frac{1}{T}\beta$$

$$k\left(\frac{\delta E}{kT}\right)^2 = C$$

so

$$\frac{\partial \bar{E}}{\partial \beta} = \frac{-12\epsilon^2 e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}} - \frac{6\epsilon e^{-2\epsilon\beta}\cdot(-6\epsilon e^{-2\epsilon\beta})}{(1+3e^{-2\epsilon\beta})^2}$$

$$= \frac{-12\epsilon e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}} + \frac{36\epsilon^2 e^{-4\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left[-2(1+3e^{-2\epsilon\beta}) + 6\epsilon e^{-2\epsilon\beta}\right]$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left(\cancel{-2-6+6\epsilon} -2 - 6e^{-2\epsilon\beta} + 6\epsilon e^{-2\epsilon\beta}\right)$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left(-2-6(1+\epsilon)e^{-2\epsilon\beta}\right)$$

$$= -\frac{12\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left(1+3(1+\epsilon)e^{-2\epsilon\beta}\right)$$

to simplify keeping only the largest T dependent term $\Rightarrow \beta \ll 1$

$$\Rightarrow \bar{E} = \frac{6\epsilon e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}} \simeq 6\epsilon \cancel{e^{-2\epsilon\beta}(1-3e^{-2\epsilon\beta})}$$

$$= 6\epsilon\cancel{(e^{-2\epsilon\beta} - 3e^{-4\epsilon\beta})}$$

$$C \equiv \frac{\partial \bar{E}}{\partial T} = -\frac{\beta}{T}\frac{\partial \bar{E}}{\partial \beta} = -\frac{\beta}{T}6\epsilon\left(-2\epsilon e^{-2\epsilon\beta} + 12\epsilon e^{-4\epsilon\beta}\right)$$

$$\cancel{\bar{E}} \simeq 6\epsilon \cancel{e}^{-2\epsilon\beta}$$

Using the full expression:

(6.27)

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT} \quad \text{v.s.}$$

$$Z_{rot} = \int_0^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT} \, dj = \frac{kT}{\epsilon}$$

...

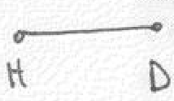
(6.28)

$$Z_{rot} \cong \sum_{j=0}^{6} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$\bar{E} = -\frac{1}{Z} \frac{\partial Z}{\partial \beta} \qquad 0 \le \frac{kT}{\epsilon} \le 3 \quad \Rightarrow \quad \frac{1}{3} \le \frac{\epsilon}{kT} \le \infty$$

...

(6.29) $\epsilon = 0.0057 \text{ eV}$




Assume $\frac{kT}{\epsilon} \ll 1$ $\qquad \frac{\epsilon}{kT} \gg 1$

$$\Rightarrow \quad T \ll \frac{\epsilon}{k} = \frac{0.0057 \text{ eV}}{(8.617 \cdot 10^{-5} \text{ eV/k})}$$

(6, 30) $\epsilon = .0076$ eV

singlet (1) triplet (3)

$\circ\!\!-\!\!\!-\!\!\!-\!\!\circ$ $\circ\!\!\longrightarrow$

parahydrogen orthohydrogen.

(a) j even

$$Z_{rot} = \sum_{j=0}^{\infty} (4j+1)\, e^{-2j(2j+1)\epsilon/kT}$$

$\epsilon$ known

$\frac{\epsilon/k}{T}$

. . .

$$(b) \quad Z_{rot} = \sum_{j=0}^{\infty} (2(2j+1)+1)\, e^{-(2j+1)((2j+1)+1)\epsilon/kT}$$

. . .

(c) $\frac{1}{4}$(para hy) + $\frac{3}{4}$(ortho hy)

$$Z_{rot\,mix} \overset{?}{=} \frac{1}{4} Z_{rot_{para}} + \frac{3}{4} Z_{rot_{ortho.}}$$

$$(d) \quad Z = \sum_{j=0}^{\infty} (4j+1)\, e^{-2j(2j+1)\epsilon/kT} + \sum_{j=0}^{\infty} 3\cdot(2(2j+1)+1)\, e^{-(2j+1)(2j+2)\epsilon/kT}$$

. . .

(e)

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det 9 tot

3 anti 6 symm

(6.3) $E = c|q|$

Steps in Equip thm

1) Assume energy levels are discrete $q_i$

2) Compute the partition function for a system w/ this energy

$$Z = \sum_q e^{-\beta E(q)} = \sum_q e^{-\beta c|q|} = \frac{1}{\Delta q}\sum_q e^{-\beta c|q|}\Delta q$$

$$\cong \frac{1}{\Delta q}\int_{-\infty}^{+\infty} e^{-\beta c|q|}\,dq = \frac{2}{\Delta q}\int_0^{\infty} e^{-\beta c q}\,dq = \frac{2}{\Delta q}\frac{e^{-\beta c q}}{(-\beta c)}\Bigg|_0^{\infty}$$

$$= \frac{2}{\Delta q(-\beta c)}(0-1) = \frac{2}{\Delta q \beta c}$$

Now:

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{\left(\frac{2}{\Delta q\beta c}\right)}\frac{\partial}{\partial\beta}\left(\frac{2}{\Delta q\beta c}\right)$$

$$= \frac{-\cancel{\Delta q}\beta \cancel{c}}{\cancel{2}}\left(\frac{\cancel{2}}{\cancel{\Delta q c}}\right)\left(-1\beta^{-2}\right) = \beta\cdot\beta^{-2} = \beta^{-1} = \underline{\underline{kT}}$$

6.32

(a) $Z = \sum_q e^{-\beta q} = \int_{-\infty}^{\infty} e^{-\beta q(x)} dx$

This is the definition of

$\bar{x} = \sum_i x_i P(x_i)$ or if the variable $x$ is given continuously

w/ $P(x_i) = \dfrac{e^{-\beta U(x_i)}}{Z}$

$\bar{x} = \int_{\mathbb{R}} x P(x)\, dx = \int x \dfrac{e^{-\beta U(x)}}{Z} dx = \ldots$

(b) For stability $F = -\frac{dU}{dx}$

B ~~must~~ be this If a linear term was not zero

$F = -\frac{1}{dx}\left[ (x-x_0)\frac{dU}{dx}\Big|_{x_0} + \frac{(x-x_0)^2}{2}\frac{d^2U}{dx^2}\Big|_{x_0} + \cdots \right]$

$\approx -\frac{dU}{dx}\Big|_{x_0} - (x-x_0)\frac{d^2U}{dx^2}\Big|_{x_0} + O((x-x_0)^2)$

gives constant force $\rightarrow\leftarrow$ to point $x_0$ being a stationary point.

If $\left.\frac{dU}{dx}\right|_{x_0} = 0$ ~~then~~ & $\left.\frac{d^2U}{dx^2}\right|_{x_0} > 0$ then

~~[illegible]~~.

w/ $F \approx -(x-x_0)\left.\frac{d^2U}{dx^2}\right|_{x_0}$

$F < 0 \quad x > x_0$
$F > 0 \quad x < x_0$
} point $x_0$ is a stable equilibrium point.

Assume $U(x) \approx U(x_0) + \frac{1}{2}(x-x_0)^2 \left.\frac{d^2U}{dx^2}\right|_{x_0}$

Then

$$\bar{x} = \frac{\int e^{-\beta\left(U(x_0) + \frac{1}{2}(x-x_0)^2 U''(x_0)\right)} \cdot x \, dx}{\int e^{-\beta\left(U(x_0) + \frac{1}{2}(x-x_0)^2 U''(x_0)\right)} dx}$$

$$= \frac{\int_{-\infty}^{\infty} x e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)} dx}{\int e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)} dx} = \frac{\int_{-\infty}^{\infty} (x-x_0) e^{-\beta\frac{1}{2}(x-x_0)^2 U''(x_0)} dx + x_0 \int_{-\infty}^{\infty} e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)} dx}{\int e^{-\beta}}$$

$$= \frac{\int_{-\infty}^{+\infty} (x-x_0)\, e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)}\, dx}{\int_{-\infty}^{\infty} e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)}\, dx} + x_0$$

$= x_0$ ✓

(c) Ass $u(x) \simeq u(x_0) + \frac{1}{2}(x-x_0)^2 u''(x_0) + \frac{1}{6}(x-x_0)^3 u'''(x_0)$

$$\bar{x} = \frac{\int x\, e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 u'''(x_0)}\, dx}{\int e^{-\beta (x-x_0)^2 u''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 u'''(x_0)}\, dx}$$

$$= x_0 + \frac{\int (x-x_0)\, e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 u'''(x_0)}\, dx}{\int e^{-\beta (x-x_0)^2 u''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 u'''(x_0)}\, dx}$$

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$$= x_0 + \int x e^{-\frac{\beta}{2}x^2 U''(x_0)} \cdot \left[1 - \frac{\beta}{6} x^3 U'''(x_0) + O(x^6)\right] dx$$

$$e^x = \left\{\sum \frac{x^k}{k!}\right\} \qquad \int e^{-\frac{\beta}{2}x^2 U''(x_0)} \left[1 - \frac{\beta}{6}x^3 U''(x_0) + \frac{\beta^2}{36}x^6 U'''(x_0)^2 + O(x^9)\right]$$

$$= x_0 + \cancel{\int x e^{-\left(\frac{\beta}{2}\right)x^2 U''(x_0)}}^{\,0} - \frac{\beta}{6}\int x^4 e^{-\frac{\beta}{2}x^2 U''(x_0)} dx$$

$$\int e^{-\frac{\beta}{2}x^2 U''(x_0)} dx$$

Now

$$\int e^{-\frac{\beta}{2}x^2 U''(x_0)} dx$$

let $v^2 = \frac{\beta}{2} U''(x_0) x^2 \qquad v = \sqrt{\frac{\beta U''(x_0)}{2}}\, x$

$$\int e^{-v^2} \frac{\sqrt{2}}{\sqrt{\beta U''(x_0)}} dv \qquad dv = \sqrt{\phantom{x}}\, dx$$

$$= \frac{\sqrt{2\pi}}{\sqrt{\beta U''(x_0)}}$$

Now

$$\int x^4 e^{-\frac{\beta}{2}U''(x_0)x^2} dx$$

let $v = \sqrt{\frac{\beta U''(x_0)}{2}}\, x \qquad x = \sqrt{\frac{2}{\beta U''(x_0)}}\, v$

$$= \int \left(\frac{2}{\beta U''(x_0)}\right)^2 v^4 e^{-v^2} \sqrt{\frac{2}{\beta U''(x_0)}}\, dv$$

$$= \left(\frac{2}{\beta U''(x_0)}\right)^{5/2} \underbrace{\int v^4 e^{-v^2} dv}_{\frac{3\sqrt{\pi}}{4}}$$

$$\therefore \bar{x} = x_0 + \frac{-\frac{\beta}{6}\left(\frac{3\sqrt{\pi}}{4}\right)\left(\frac{2}{\beta U''(x_0)}\right)^{5/2}}{\sqrt{\pi}\left(\frac{2}{\beta U''(x_0)}\right)^{1/2}}$$

$$= x_0 - \frac{\beta}{6}\frac{3}{4}\left(\frac{2}{\beta U''(x_0)}\right)^2 = x_0 - \frac{1}{8}\cdot\frac{4}{\beta\,(U''(x_0))^2}$$

$$= x_0 - C'\cdot kT$$

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B (d) compute $U''(x) = \cdots$

(6.33)

$V_{mp}$ = velocity at which $D(v)$ is largest.

$$= \sqrt{\frac{2kT}{m}}$$

$$m_{O_2} = \frac{2 \cdot 16\text{ g}}{6.022 \cdot 10^{23}} = \frac{2 \cdot 16 \cdot 10^{-3}\text{ kg}}{6.022 \cdot 10^{23}}$$

$$kT = (1.381 \cdot 10^{-23}\,\text{J/K})(300\,\text{K}) =$$

$$\bar{V} = \sqrt{\frac{8kT}{\pi m}} = \cdots$$

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

(6.34) $$D(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}}$$

$$m_{N_2} = \frac{2 \cdot 14 \cdot 10^{-3}\text{ kg}}{6.022 \cdot 10^{23}} \cong \cdots \frac{2 \cdot 14}{6} \cdot 10^{-26}\text{ kg}$$

$$kT = (1.381 \cdot 10^{-23}\,\text{J/K})\,T =$$

$\cancel{\frac{m}{kT}}$ = spaced in units of $\tilde{v} = \sqrt{\frac{kT}{m}}$ say

(6.35)

$$D(v)dv = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}} dv$$

$$\frac{dD}{dv} = \left(\frac{m}{2\pi kT}\right)^{3/2}\cdot 4\pi\left[2v e^{-\frac{mv^2}{2}\cdot\beta} + v^2\left(\cancel{\frac{mv^2}{2}} - \frac{2mv\beta}{2}\right)e^{-\frac{mv^2}{2}\beta}\right] \overset{\text{set}}{=} 0$$

$$= 2v \cancel{+} - v^3 m\beta = 0$$

$v=0$ or $2 = v^2 m\beta$

$$v = \sqrt{\frac{2}{m\beta}} = \sqrt{\frac{2kT}{m}} \checkmark$$

(6.36) eq 6.51

$$\bar{v} = \sum_{\text{all } v} v D(v) dv = \int_0^{\infty} v\, C\, 4\pi v^2 e^{-\frac{mv^2}{2}\cdot\beta} dv$$

$$= 4\pi C \int_0^{\infty} v^3 e^{-\frac{mv^2}{2}\beta} dv \qquad \text{let } u = \sqrt{\frac{m\beta}{2}}\, v$$

$$dv = \sqrt{\frac{2}{m\beta}}\, du$$

$$= 4\pi C \int_0^{\infty} \left(\frac{2}{m\beta}\right)^{3/2} e^{-u^2} u^3 \left(\frac{2}{m\beta}\right)^{1/2} du$$

$$= 4\pi C \left(\frac{2}{m\beta}\right)^2 \int_0^\infty v^3 e^{-v^2}\, dv$$

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(6.37)

$$\overline{v^2} = \int_0^\infty v^2 D(v)\,dv = \int_0^\infty v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2}\beta}\,dv$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^4 e^{-\frac{mv^2}{2}\beta}\,dv$$

let $u = \sqrt{\frac{m}{2}\beta}\,v \qquad v = \left(\frac{2}{m\beta}\right)^{1/2} u$

$$dv = \left(\frac{2}{m\beta}\right)^{1/2} du$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty \left(\frac{2}{m\beta}\right)^{4/2}\left(\frac{2}{m\beta}\right)^{1/2} u^4 e^{-u^2}\,du$$

$$= \cancel{4\pi\left(\frac{1}{2\pi}\right)^{3/2}\left(\frac{m}{\beta}\right)}$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2}{m\beta}\right)^{5/2}\int_0^\infty u^4 e^{-u^2}\,du$$

6.38

$$f = \int_0^{300\,m/s} D(v)\,dv = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{300\,m/s} v^2 e^{-\frac{mv^2}{2}\beta}\,dv$$

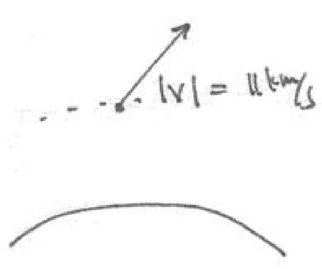
~~Set~~ $u = \left(\frac{m\beta}{2}\right)^{1/2} v \qquad v =$

$$f = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{\left(\frac{m\beta}{2}\right)^{1/2}\cdot 300} \left(\frac{2}{m\beta}\right)^{2/2} \cdot \left(\frac{2}{m\beta}\right)^{1/2} e^{-u^2} u^2\,du$$

$$= \cancel{\left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{\frac{300}{\left(\frac{2}{m\beta}\right)^{1/2}}}}$$

$$= \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \left(\frac{2}{m\beta}\right)^{3/2} \int_0^{\frac{300}{\left(\frac{2}{m\beta}\right)^{1/2}}} u^2 e^{-u^2}\,du$$

6.39




$N_2 =$

$$m_{N_2} = \frac{2.14\cdot 10^{-3}\,kg}{6.022\cdot 10^{23}}$$

$T = 1000\,K.$

$$P = \int_{11\cdot 10^3\,m/s}^{\infty} D(v)\,dv$$

But "several" of these particles will not be pointing in the outward direction

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$$P = \int_{11\cdot 10^3 \, m/s}^{\infty} \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}} \, dv = \ldots$$

(b) $m_{H_2} = \frac{2\cdot 10^{-3}}{6.022\cdots}$

$m_{He} = \underline{\qquad}$

Based on $P_{N_2}$, $P_{H_2}$; $P_{He}$ one should find proportionally the same # of molecules.

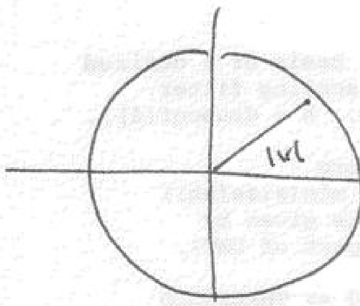
(c)

$$P = \int_{2.4\cdot 10^3 \, m/s}^{\infty} D(v)\, dv \approx 1.$$

it is very possible ~~that~~ that molecule will be able to escape

(6.40) Don't see how? Must be inelastic collisions

(6.41)



$$D(v) \propto \left(\begin{array}{c}\text{prob molec}\\ \text{have vector } \vec{v}\end{array}\right) \times \left(\begin{array}{c}\text{\# of vectors } \vec{v}\\ \text{w/ } |\vec{v}| = v\end{array}\right)$$

$$\propto e^{-\frac{mv^2}{2kT}} \cdot 2\pi v$$

$$D(v) = 2\pi C\, v e^{-\frac{mv^2}{2kT}}$$

$$1 = 2\pi C \int_0^\infty v e^{-\frac{mv^2}{2kT}}\, dv$$ solve for $C$

most likely vector ~~$\frac{dD}{dv}$~~ $= 0$ are w/ maximum $D(v)$

$\vec{v} = 0$

most likely speed ~~$D(v)$~~ $\frac{dD(v)}{dv} = 0 \Rightarrow v = \cdots$

6.42 $Z_{h.o.} = \frac{1}{1-e^{-\beta\epsilon}}$

(a) $F = -kT \ln Z$

$F = -kT \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right)$ is there a power of $N$ + a Nonrelativistic factor of $N!$?

Think yes

$Z_{total} = \frac{Z_{ho}^{N}}{N!}$

$F = -kTN \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right)$

(b) $S = -\frac{\partial F}{\partial T}\Big|_{V,N} = -kN \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right) - kTN\left(1-e^{-\beta\epsilon}\right) \cdot$

$\frac{\partial}{\partial T}\left(1-e^{-\beta\epsilon}\right)$

$= -kN \ln\left(\frac{1}{\quad}\right) - kTN\left(1-e^{-\beta\epsilon}\right) \cdot -\frac{1}{kT^2} \frac{\partial}{\partial \beta}\left(1-e^{-\beta\epsilon}\right)$

$\left\{ \frac{\partial \beta}{\partial T} = -\frac{1}{kT^2} \right\}$

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(6.43)

$$S = -k \sum_s P(s) \ln P(s)$$

(a) Isolated system $P(s) = \frac{1}{\Omega}$ ? Don't see

$$S = -k \sum_s \frac{1}{\Omega} \ln(1/\Omega)$$ know $\sum_s P(s) = 1$

$$= \cancel{k \sum_s \frac{\ln(\Omega)}{\Omega}}$$ Can I factor out $\ln(1/\Omega)$ to get

$$S = -k \ln(1/\Omega) \sum_s 1/\Omega = +k \ln \Omega \quad \checkmark.$$

(b) $P(s) = \frac{e^{-E(s)\beta}}{Z}$ $\qquad \sum_s P(s) = 1$

$$S = -k \sum \frac{e^{-E(s)\beta}}{Z} \ln\left(\frac{e^{-E(s)\beta}}{Z}\right)$$

$$= -\frac{k}{Z} \sum e^{-E(s)\beta} \ln\left(\frac{e^{-E(s)\beta}}{Z}\right)$$

How show?

(6.44)

$$Z_{total} = \frac{Z_1^N}{N!}$$

$$\{\ln N! \approx N\ln N - N\}$$

$$F = -kT\ln(Z_{total}) = \cancel{\ } -kTN\ln Z_1 + kT\ln N!$$

$$\approx -kTN\ln Z_1 + kTN\ln N - kTN$$

$$= kTN\left[-\ln Z_1 + \ln N - 1\right]$$

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{V,T} = kT\left[-\ln Z_1 + \ln N - 1\right] + kTN\left[\frac{1}{N}\right]$$

$$= kT + kT\left[-\ln Z_1 + \ln N - 1\right]$$

$$= kT\left[-\ln Z_1 + \ln N\right]$$

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(6.45)

$$S = -\frac{\partial F}{\partial T}\Big|_{V,N}$$

of $F = -NkT\left[\ln V - \ln N - \ln v_Q + 1\right] + F_{int}$

$$\frac{\partial F}{\partial T}\Big|_{V,N} = -Nk\left[\ln V - \ln N - \ln v_Q + 1\right] - NkT\left[ - \frac{1}{v_Q}\frac{\partial v_Q}{\partial T}\right] + \frac{\partial F_{int}}{\partial T}\Big|_{V,N}$$

$*$ $$\frac{1}{v_Q}\frac{\partial v_Q}{\partial T} = \left(-\frac{3}{2}\frac{v_Q}{T}\right)\frac{1}{v_Q} = -\frac{3}{2}\cdot\frac{1}{T}$$

$$v_Q = \left(\frac{h}{\sqrt{2\pi mkT}}\right)^3 = \left(\frac{h}{\sqrt{2\pi mk}}\right)^3 \cdot T^{-3/2}$$

$$f = x^p$$

$$\frac{\partial f}{\partial x} = p\frac{f}{x}$$

$\therefore$ $$S = -\frac{\partial F}{\partial T}\Big|_{V,N} = +Nk\left[\ln V - \ln N - \ln v_Q + 1\right]$$

$$+ NkT\left(\frac{3}{2}\cdot\frac{1}{T}\right) + -\frac{\partial F_{int}}{\partial T}\Big|_{V,N}.$$

$$\Rightarrow S = +Nk\left[\ln V - \ln N - \ln v_Q + 1 + \frac{3}{2}\right] - \frac{\partial F_{int}}{\partial T}\Big|_{V,N}$$

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$$S = +Nk\left[\cancel{\phantom{x}}\ \ln\left(\frac{V}{V_Q \cdot N}\right) + \frac{5}{2}\right] - \frac{\partial F_{int}}{\partial T}\bigg|_{V,N} \quad \text{eq } 6.92 \checkmark$$

$$= Nk\,\cancel{\left[\ln\right(}$$

$$\mu = \frac{\partial F}{\partial N}\bigg|_{T,V} = \frac{\partial}{\partial N}\bigg|_{T,V}\left[-NkT\left[\ln V - \ln N - \ln v_Q + 1\right] + F_{int}\right]$$

$$= -kT\left[\ln V - \ln N - \ln v_Q \cancel{+1}\right] - NkT\,\cancel{\left[-\frac{1}{N}\right]} + \frac{\partial F_{int}}{\partial N}\bigg|_{T,V}$$

$$= \cancel{kT}\ \ kT\left[\ln\left(\frac{N v_Q}{V}\right)\right] + \frac{\partial F_{int}}{\partial N}\bigg|_{T,V}$$

$$F_{int} = -kT \ln Z$$

$$\frac{\partial F_{int}}{\partial N} = -\frac{kT}{Z}\frac{\partial Z_{int}}{\partial N} \qquad \cancel{Z_{int}} = \frac{\cancel{Z_{int}}^N}{N!}$$

$$= -kT\left[\ln\left(\frac{V}{N v_Q}\right)\right] + \frac{\partial}{\partial N}(-kT \ln Z_{int})$$

$$= -kT\left[\ln\left(\frac{V}{N v_Q}\right) + \frac{\partial (N \ln Z_{int})}{\partial N}\right]$$

w/ $Z_{int}$ independent of $N$.

$$\Rightarrow \mu = -kT\left[\ln\left(\frac{V Z_{int}}{N V_Q}\right)\right] \qquad \text{eq } 6.93 \checkmark$$

(6.46) $\dfrac{V Z_{int}}{N V_Q}$

$V \sim 1 \text{ liter} = 10^{-3}\,m^3$

$N \sim 6.23 \cdot 10^{23}$

$V_Q \sim (2 \cdot 10^{-11})^3 \sim 8 \cdot 10^{-33}$
Nitrogen

$Z_{int} \sim$ Don't know. what values this has? $\gg 1$

$$\frac{(10^{-3})(?)}{(10^{23})(10^{-33})} \gg 1$$

(6.47) $Z_{1d} = \dfrac{L}{l_Q}$ $\qquad l_Q = \sqrt{\dfrac{2\pi m k T}{h^2}}$

Assume we have freeze out of translational degrees of freedom would be when

$Z_{1d} \sim 1$ or $Z_{1d} \ll 1$

$$\Rightarrow L \sim l_Q \Rightarrow \sqrt{\frac{2\pi m k T}{h^2}} = L$$

$$\frac{2\pi m k T}{h^2} = L^2$$

$$\Rightarrow T = \frac{h^2 L^2}{2\pi m k} \Rightarrow T = \frac{(6.626\cdot 10^{-34}\,J\cdot s)^2 (10^{-2}\,m)^2}{2\pi\left(\frac{14\cdot 10^{-3}\,kg}{6.022\cdot 10^{23}}\right)(1.381\cdot 10^{-23}\,J/k)}$$

$$\Rightarrow T = 21\cdot 10^{-23}\,k.$$

(6.48)  $Z_{int} = Z_e \cdot Z_{rot}$

$$F_{int} = -kTN \ln Z_{int} = -kTN \ln(Z_e Z_{rot})$$

From eq 6.92 we get

$$S = Nk\left[\ln\left(\frac{V}{N v_Q}\right) + \frac{5}{2}\right] - \frac{\partial}{\partial T}\left(-kTN\ln(Z_e Z_{rot})\right)$$

$$\cancel{kN\ln(Z_e Z_{rot})}$$

$$= \cancel{Nk\left[\ln\left(\frac{V Z_e Z_{rot}}{N v_Q}\right)\right.} \quad \text{w/ } Z_{rot} \cong \frac{kT}{\epsilon}$$

$$\Rightarrow S = Nk\left[\ln\left(\frac{V}{N v_Q}\right) + \frac{5}{2}\right] + kN\ln(Z_e Z_{rot})$$

$$+ \frac{kN\cancel{T}}{Z_e \cancel{Z_{rot}}} \cdot Z_e \cdot \frac{k}{\epsilon}$$

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$$\Rightarrow S = Nk\left[\ln\left(\frac{VZ_eZ_{rot}}{NV_Q}\right) + \frac{5}{2}\right] + kN$$

$$= Nk\left[\ln\left(\frac{VZ_eZ_{rot}}{NV_Q}\right) + \frac{7}{2}\right]$$

~~For~~ ~~$PV = NkT$~~ ~~$\frac{V}{N} = \frac{kT}{P}$~~ ~~$Nk = \frac{PV}{T}$~~

$\cancel{S=}$ 1 mole $\Rightarrow N = N_A = 6.022 \cdot 10^{23}$

Also $PV = NkT$ $\Rightarrow \frac{V}{N} = \frac{kT}{P}$

$Z_{rot} = \frac{kT}{\epsilon}$ $\epsilon_{O_2} \cong 0.00018$ eV (pg 236 Schroeder)

(b) $\mu = -kT\ln\left(\frac{VZ_eZ_{rot}}{NV_Q}\right)$ ...

(6.49) $U, H, F, G, S, \mu$ $\quad \epsilon_{N_2} = .00025\ eV$

$T = 273\,k$

$P = 1\,atm$

... Not sure

(6.50) $G = U - TS + PV \qquad dU = TdS - pdV$

so $G = U_{int} + \frac{3}{2}NkT - T\left(Nk\left[\ln\left(\frac{V}{NvQ}\right) + \frac{5}{2}\right] - \frac{\partial F_{int}}{\partial T}\right) + PV$

$\overset{?}{=} N\left(-kT \ln\left(\frac{V Z_{int}}{N\, vQ}\right)\right)$

what is $Z_{int}$?

?

(6.51) $Z_{tr} = \frac{1}{h^3}\int d^3r\, d^3p\, e^{-E_{tr}/kT} \qquad E_{tr} = +\left(\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}\right)$

$= \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$

$= \frac{1}{h^3}\underbrace{\int d^3r}_{V} \cdot \int d^3p\, e^{-E_{tr}/kT}$

$= \frac{V}{h^3}\int d^3p \exp\left\{\frac{-1}{2m(kT)}(p_x^2 + p_y^2 + p_z^2)\right\}$

$= \frac{V}{h^3}\prod_{i=1}^{3}\int dp\, e^{-\frac{p^2}{2mkT}}$

let $v^2 = \frac{p^2}{2mkT} \Rightarrow v = \frac{p}{\sqrt{2mkT}}$

$\Rightarrow p = \sqrt{2mkT}\, v.$

$dp = \sqrt{2mkT}\, dv$

So $$Z_{tr} = \frac{V}{h^3}\left(\sqrt{2mkT}\int_{-\infty}^{\infty} dv\, e^{-v^2}\right)^3$$

$$= \frac{V}{h^3}(2mkT)^{3/2}\left(\sqrt{\pi}\right)^3$$

$$= V \cdot \left(\frac{2\pi mkT}{h^2}\right)^{3/2} = \frac{V}{\left(\frac{h^2}{2\pi mkT}\right)^{3/2}}$$

define $v_Q = \left(\frac{h^2}{2\pi mkT}\right)^{3/2}$ ✓.

6.52 $E = pc$ vs. $E = \frac{p^2}{2m}$

$$Z_1 = \frac{1}{h}\int dx\,dp\, e^{-E/kT} = \frac{1}{h}\cdot L\int_{-\infty}^{+\infty} dp\, e^{-p/ckT}$$

$v = \frac{p}{ckT}$

$dp = ckT\,dv$

$$= \frac{L}{h}\cdot ckT\int_{-\infty}^{\infty} dv\, e^{-v}.$$

$$= \frac{L\cdot ckT}{h}\left(-e^{-v}\right|_{-\infty}^{+\infty} \quad \times$$

in $E = pc$ $p$ is $|p|$

$$\therefore Z_1 = \frac{L}{h}\int_0^{\infty} \quad \cdots \quad = \frac{L\cdot ckT}{h}\left(-e^{-v}\right|_0^{\infty}$$

$$= -\frac{LckT}{h}(0-1) = \frac{LckT}{h}$$

$$\Rightarrow \frac{L}{\left(\frac{h}{ckT}\right)} \;;\; l_Q = \frac{h}{ckT}$$

Check units $[l_Q] = \frac{J\cdot s}{\left(\frac{m}{s}\right)\left(\frac{J}{\cancel{K}}\right)(\cancel{K})} = \frac{s^2}{m}$ ? error somewhere

6.53 $H_2 \leftrightarrow 2H$

$$k = \frac{[H]^2}{[H_2]} \cong \frac{P_H^2}{P_{H_2} \cdot P^\circ}$$

P's = partial press

... Not sure.

(7.1) $Z = \quad \ldots ?$

$$Z = 1 + e^{-(\epsilon-\mu)/kT}$$

$$P = \frac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}} \qquad \text{w/ } \mu = \mu(P^{\circ}_{O_2}), \quad \epsilon = -.7\,eV.$$

$$PV = NkT \quad \text{so} \quad \mu(P^{\circ}) = -kT \ln\left(\frac{Z_{int}\, kT}{V_Q P^{\circ}}\right)$$

plot $P(P^{\circ}) = \cdots$

(7.2) $$Z = 1 + 2e^{-(-.55\,eV - \mu(P_{O_2}))/kT} + e^{-(-1.3\,eV - \mu(P_{O_2}))/kT}$$

Then

$$P_{\text{zero sites occupied}}(P_{O_2}) = \frac{1}{Z(P_{O_2})}$$

$$P_{\text{one site occupied}}(P_{O_2}) = \frac{2e^{-(-.55\,eV - \cdots)/kT}}{Z(P_{O_2})}$$

$$P_{\text{two sites occ}} = \frac{e^{-(1.3\,eV \cdots)/kT}}{Z(P_{O_2})}$$

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(7.3) $Z = 1 + e^{-(\epsilon - \mu)/kT}$

$\epsilon$ = ionization energy for H atom

$\mu$ = chemical potential of ideal ~~gas~~ gas

(7.4) $Z = 1 + e^{-(\epsilon' - \mu)/kT} + e^{-(\epsilon'' - \mu)/kT}$

How change chemical potential of gas?

(7.5)

(a) $P_{\text{ionized}} = 1 - P_{\text{original donor site occupied}} = 1 - \dfrac{2e^{-(I - \mu)/kT}}{1 + 2e^{-(I - \mu)/kT}}$

2 for 2 electron spin states

(doesn't change the ionization energy)

(b) $\mu = -kT \ln\left(\dfrac{Z_{int}}{V_Q} \cdot \dfrac{V}{N}\right)$

(c) ?

(d) P ..

(7.6) $$p(s) = \frac{e^{-(E(s)-\mu N(s))/kT}}{Z}$$

$$\frac{\partial p}{\partial N} = 0$$ solve for $N$

$$p(s)\cdot\frac{\mu}{kT} - \frac{p(s)}{Z}\frac{\partial Z}{\partial N} = 0$$

$$\Rightarrow \frac{\mu}{kT} - \frac{1}{Z}\frac{\partial Z}{\partial N} = 0$$

$$\sigma_N^2 = \langle N^2\rangle - \langle N\rangle^2 = \frac{(kT)^2}{Z}\frac{\partial^2 Z}{\partial\mu^2} - \frac{kT}{Z}\frac{\partial Z}{\partial\mu}$$

$$=$$

...

(7.7) Prob 5.23 is $\Phi = U - TS - \mu N$

$F = -kT\ln Z$ $\quad\quad$ $\Phi = -kT\ln \mathcal{Z}$

show $\Phi = U - TS - \mu N$ + $\Phi = -kT\ln\mathcal{Z}$ satisfy same

D.E. w/ initial conditions.

7.8

$N_s = 10.$

(a) $Z \equiv \sum_s e^{-\beta E(s)} = \sum_{s=1}^{10} e^{-\beta E_s} = 10$ since each term is 1

(b) $Z = 10^2$

(c) Bosons: can occupy the same state & identical

$\tilde{A}$ $\tilde{B}$
x x

$Z = \frac{(Z_1)(Z_1)}{2} = \frac{Z_1^2}{2}$

(d) Fermions: cannot occupy the same state.

$Z = \frac{Z_1(Z_1-1)}{2}$

(e) $Z = \frac{1}{2!} Z_1^2$

(f) $P_{\text{Fermions}} = 0$ can't happen

$P_{\text{Bosons}} = \frac{?}{\left(\frac{Z_1^2}{2}\right)}$

7.9 $N_2$

$$V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^{3}$$

$$m_{N_2} = \frac{2\ g/mole\ (14)}{6.022 \cdot 10^{23}\ molecules/mole}$$

$$PV = N \cdot kT$$

$$\frac{V}{N} = \frac{kT}{P}$$

$$\frac{V}{N} \gg V_Q$$ Boltzman statistics regime.

$$\frac{kT}{P} \gg \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$

$$(2\pi m k T)^{3/2} \cdot kT \gg h^3 P$$

Density constant $\Rightarrow \frac{1}{V} = \rho_N$

$$\Rightarrow \left(\frac{N}{V}\right) = \frac{P}{kT}$$

$$(2\pi m k T)^{3/2} \gg h^3 \left(\frac{P}{kT}\right)$$

↑

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7.10

(a)

| | Identical Fermions | Identical Bosons | Distinguishable Part |
|---|---|---|---|
| $E_{ground}$ | $\epsilon_1+\epsilon_2+\epsilon_3$ | $3\epsilon_1$ | $\epsilon_1+\epsilon_2+\epsilon_3$ v.s. $3\epsilon_1$ |

(b) $q$: ?

(c) ?

(d) ?

(7.11) $\bar{n}_{FD} = \dfrac{1}{e^{(\epsilon-\mu)/kT}+1}$

(a) $\epsilon - \mu = -.1\ eV$

$kT = (8.617 \cdot 10^{-5}\ eV/k)(300k)$

$\hat{=} 24 \cdot 10^{-3}\ eV$

$\hat{=} 2.4 \cdot 10^{-2}\ eV$

(b) $\epsilon - \mu = -.01\ eV$

(c) $\epsilon - \mu = 0$

$\bar{n}_{FD} = \dfrac{1}{2}$

(d) $\epsilon - \mu = .01\ eV$

(e) $\epsilon - \mu = 1\ eV$

(7.12)

$\mu + x$ ——•—— $= \epsilon_B$

$\mu$ ——

$\mu - x$ —— $= \epsilon_A$

$$P_{B\ \text{occupied}} = \frac{e^{-(\epsilon_B-\mu)/kT} + \cancel{e}}{e^{-(\epsilon_B-\mu)/kT} + 1 + e^{-(\epsilon_A-\mu)/kT}} = \frac{e^{-x/kT}}{1+e^{-x/kT}+e^{+x/kT}}$$

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$$P_{A \text{ unoccupied}} = \frac{e^{-(\epsilon_B-\mu)/kT} + 1}{e^{-x/kT} + e^{+x/kT} + 1} = \frac{e^{-x/kT} + 1}{e^{-x/kT} + e^{+x/kT} + 1} \quad ?$$

7.13 $$\bar{n}_{BE} = \frac{1}{e^{(\epsilon-\mu)/kT} - 1} \qquad Z = \frac{1}{1 - e^{-(\epsilon-\mu)/kT}} \quad \text{Bosons}$$

(a) $\epsilon - \mu = .001$ eV

(b) $\epsilon - \mu = 0.01$ eV

(c) $\epsilon - \mu = .1$ eV

(d) $\epsilon - \mu = 1$ eV

$$P = \frac{1}{Z} e^{-(n\epsilon - n\mu)/kT} = \frac{e^{-n(\epsilon-\mu)/kT}}{Z} \qquad n = 0, 1, 2, 3.$$

7.14 For large energy $\{\epsilon - \mu\}$ all statistics should obey <u>Boltzmann</u> statistics $\Rightarrow$ expect each to be the boltzmann distribution

$$\bar{n}_{\text{Boltzmann}} = e^{-(\epsilon-\mu)/kT}$$

$$\bar{n}_{BE} = \frac{e^{-(\epsilon-\mu)/kT}}{1 - e^{-(\epsilon-\mu)/kT}} = e^{-(\epsilon-\mu)/kT} \sum_{k \geq 0} e^{-((\epsilon-\mu)/kT)\cdot k}$$

$$\bar{n}_{BE} \cong e^{-(\epsilon-\mu)/kT}\left[1+e^{-(\epsilon-\mu)/kT}+\text{H.O.T.}\right]$$

$$\bar{n}_{FD} = \frac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}} = e^{-(\epsilon-\mu)/kT}\sum_{n\geq 0}(-1)^n e^{-(\epsilon-\mu)n/kT}$$

$$\cong e^{-(\epsilon-\mu)/kT}\left[1-e^{-(\epsilon-\mu)/kT}\right]$$

$\therefore$ ~~BH~~ Both statistics have relative error of

$$\left|e^{-(\epsilon-\mu)/kT}\right| \ll .01$$

$$\Rightarrow -\frac{(\epsilon-\mu)}{kT} \ll \ln(10^{-2})$$

$$(\epsilon-\mu) \gg \underline{2kT\ln(10)}$$

How tell if violated?

(7.15) 7.31 $\bar{n}_{Boltzmann} = e^{-(\epsilon-\mu)/kT}$

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$$\cancel{\sum \bar{n}_{Boltz}} \quad \sum_{n\geq 0} e^{-(\epsilon-\mu)n/kT} = N \quad \text{why? Don't see?}$$

$$= \underbrace{\sum_{n\geq 0} e^{-\frac{\epsilon n}{kT}}}_{Z_1} \cdot \sum_{n\geq 0} e^{+\mu n/kT} = N$$

$$\sum_{n\geq 0} e^{+\mu n/kT} = \frac{1}{1-e^{\mu/kT}}$$

$$\rightarrow \quad \frac{Z_1}{N} = 1 - e^{\mu/kT}$$

$$e^{\mu/kT} = 1 - \frac{Z_1}{N}$$

$$\mu = kT \ln\left(1 - \frac{Z_1}{N}\right) \quad ?$$

(7.16)

(a) ?

(b) ?

(c) ?

(d)

(7.17) ? $q=0$ $q=1$

Energy

All in zero state

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7.18

$$Z = e^{-0(\epsilon-\mu)/kT} + e^{-1(\epsilon-\mu)/kT} + e^{-2(\epsilon-\mu)/kT}$$

$$\bar{n} = \sum_n n P(n) = 0 \cdot P(0) + 1 \cdot P(1) + 2P(2)$$

$$= \frac{e^{-(\epsilon-\mu)/kT}}{Z} + \frac{e^{-2(\epsilon-\mu)/kT}}{Z}$$

plot picture like Fig 7.6

(7.19)

$\rho_{Cu} = ?$

$m_{Cu} = 63.546 \ g/mol \qquad V = 7.12 \ cm^3/mol$

$$\rho_{Cu} = \frac{63.546 \ g/mol}{7.12 \ cm^3/mol} \cong 9 \ g/cm^3$$

$$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3} = \cdots$$

$$m = \frac{63.546 \ g/mol}{6.022 \cdot 10^{23}} = \cdots$$

$$T_F = \frac{\varepsilon_F}{k} = \cdots$$

$$P = -\frac{\partial}{\partial V}\left[\frac{3}{5}N \cdot \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}\right] = \frac{2N\varepsilon_F}{5V} = \cdots$$ degeneracy pressure

$$B = -V\left.\frac{\partial P}{\partial V}\right|_T = \frac{10}{9}\frac{U}{V} =$$

$$= -V\frac{\partial}{\partial V}\left[\frac{3}{5}N\frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}\right] = \cdots$$

(7.20) ~~[illegible]~~ $\nu = 10^{32}\,{}^1/_{m^3}$

To determine the statistics to use in dealing w/ this system

consider $\frac{V}{N} = \nu^{-1} = 10^{-32}\ m$

v.s. $V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3 = \left(\frac{(6.626\cdot 10^{-34}\ J\cdot s)}{\sqrt{2\pi(9.109\cdot 10^{-31}\ kg)(1.381\cdot 10^{-23}\ J/K)(\cancel{[illegible]})10^7\ K}}\right)^3$

$V_Q = 1.304\cdot 10^{-32}$

since $V_Q \sim \frac{V}{N}$ I would argue that ⇒ use degenerate

Fermi gas.

(7.21) $\frac{N}{V} = .18\ (10^{-15}\ m)^{-3} = (.18)\ 10^{45}\ m^{-3}$

$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3}$

Since we can hold 4 nucleons

$N = \underline{\underline{4}} \times (\text{vol of } 1/8 \text{ sphere}) = 4\cdot\left(\frac{1}{8}\right)\frac{4}{3}\pi\, n_{max}^3 = \frac{2}{3}\pi\, n_{max}^3$

↓

Also $\varepsilon_F = \frac{h^2 n_{max}^2}{8mL^2}$

$n_{max} = \left(\frac{3}{2}\frac{N}{\pi}\right)^{1/3}$

So that ~~[illegible]~~

$$\varepsilon_F = \frac{h^2}{8mL^2}\left(\frac{3}{2}\frac{N}{\pi}\right)^{2/3}$$

$$= \frac{h^2}{8mV^{2/3}}\left(\frac{3}{2}\frac{N}{\pi}\right)^{2/3}$$

$$T_F = \frac{\varepsilon_F}{k}$$

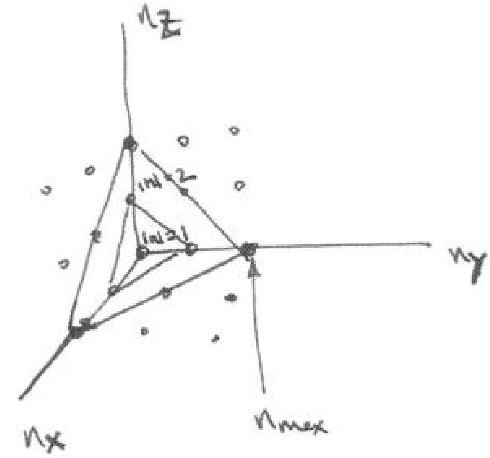
7.22 $\epsilon = pc = \frac{h}{\lambda}c$ $\qquad \lambda_n = \frac{2L}{n}$

$\therefore \epsilon_n = c\frac{h}{2L}n$ $\qquad p_n = \frac{h}{\lambda_n} = \frac{h\cdot n}{2L}$

w/ 3 degrees of mot freedom

$$\epsilon_{\vec{n}} = \frac{ch}{2L}(n_x + n_y + n_z)$$

$$\therefore \varepsilon_F = \frac{ch}{2L}(n_{max})$$




$N$ = Vol of pyramid w/ side of length $n_{max}$

= Vol of tetrahedron w/ side of length $n_{max}$ $\sim \left(\frac{1}{2}hb\right)\cdot h \sim O(n_{max}^3)$

$= C_T n_{max}^3$

$$\varepsilon_F = \frac{ch}{2L}\left(\frac{N}{c_+}\right)^{1/3} = \frac{hc}{2LV}$$

$$= \frac{hc}{c_+^{1/3}}\left(\frac{N}{V}\right)^{1/3} \qquad \text{ift.} \quad c_+ = \left(\frac{3}{8\pi}\right)$$

(b) $$U = 1\sum_{n_x}\sum_{n_y}\sum_{n_z}\varepsilon(\bar{n}) = \sum_{n_x}\sum_{n_y}\sum_{n_z}\frac{ch}{2L}(n_x+n_y+n_z)$$

$$= \int_0^{n_{max}}\int_0^{n_{max}-n_x}\int_0^{n_{max}-n_x-n_y}\frac{ch}{2L}(n_x+n_y+n_z)\,dn_z\,dn_y\,dn_x$$

$$= \cancel{\frac{ch}{2L}\int_0^{n_{max}}\int_0^{n_{max}-n_x}\int_0^{n_{max}-n_x-n_y} n_x n_z + n_y n_z + \frac{n_z^2}{2}}$$

$$= \frac{ch}{2L}\int_0^{n_{max}}\int_0^{n_{max}-n_x}\left(n_x n_z + n_y n_z + \frac{n_z^2}{2}\right|_0^{n_{max}-n_x-n_y} dn_y\,dn_x$$

$$= \frac{ch}{2L}\int_0^{n_{max}}\int_0^{n_{max}-n_x}\Big[n_x(n_{max}-n_x-n_y) + n_y(n_{max}-n_x-n_y)$$

$$+ \frac{1}{2}(n_{max}-n_x-n_y)^2\Big]\,dn_y\,dn_x$$

$$= \frac{ch}{2L}\int_0^{n_{max}}\int_0^{n_{max}-n_x} \cdots$$

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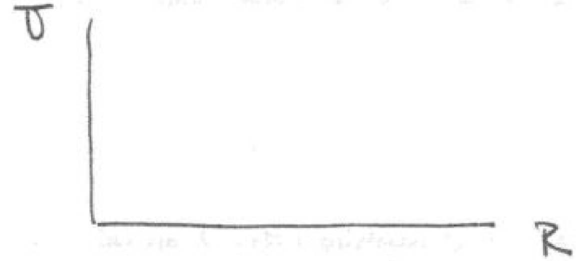
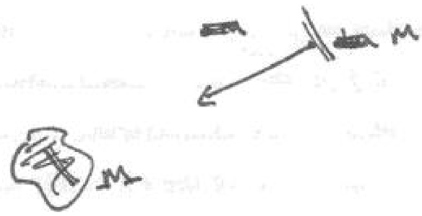
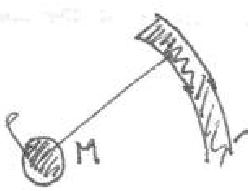
(a) $U_{grav} = -C\frac{GM^2}{R}$


to bring dm to mass M from $\infty$ requires $W = \int F \cdot dr$

$F = GM \cdot dm$ $F = \frac{GMm}{r^2}$




$dM = 4\pi r^2 \rho$

$M = \frac{4}{3}\pi r^3 \rho$

(b) ?

(c) $U_{total} = U_{grav} + U_{k.E} = -C_1\frac{GM^2}{R} + \frac{C_2 h^2 M^{5/3}}{m_e m_p^{5/3} R^2}$

$U_{total}(R)$

$\frac{dU_{total}}{dR} = C_1\frac{GM^2}{R^2} - \frac{2C_2 h^2 M^{5/3}}{m_e m_p^{5/3} R^3} = 0 \rightarrow R = \ldots$

(d) $R = \ldots$

$$\rho = \frac{M}{\frac{4\pi}{3}R^3} = \ldots$$

(e) $$\varepsilon_F = \frac{U}{\left(\frac{3}{5}N\right)}$$

$$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} \qquad kT \ll \varepsilon_F \;?$$

(f) ?

(g) $\langle KE \rangle \cong mc^2$

$\langle KE \rangle =$

?

7.24 one-solar-mass all neutron star.

Follow results of the class

(7.25)

$$C_V = \frac{\pi^2}{2} \frac{N k^2 T}{\epsilon_F}$$

$N = 6.022 \cdot 10^{23}$

$k = \ldots$

$T = 300k$

$$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

$m_{Cu} = (63.54 \ g / 6.022 \cdot 10^{23})$

$V = 7.12 \ cm^3$

What is the contribution due to lattice vibrations?

(7.26) $^3He$

(a) $$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

Assuming 1) $e^-$ has neg mass comp w/ proton 2) proton + neutron have the same mass.

$V = 37 \ cm^3$ $\quad m_{^3He} = \dfrac{4.002 + 1.007}{6.022 \cdot 10^{23}}$

$N = 6.022 \cdot 10^{23}$

$$T_F = \frac{\epsilon_F}{k}$$

(b) $C_V = \frac{\pi^2}{2} \frac{N k^2 T}{\epsilon_F}$

(c) 2 states $\forall$ neutron. $\Omega(N) = \binom{2N}{N}$

$=$

$S = k \ln \Omega(N)$

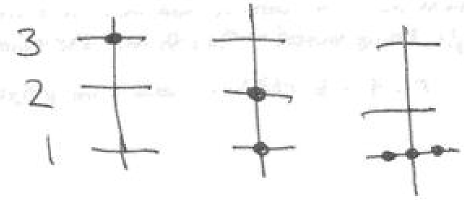
? Not sure

(7.27)

(a) $q = 3$

$= 2+1$

$= 1+1+1$



?

$p(q)$ = unrestricted partitions

$p(1) = 1$ $p(2) = 2$ $p(3) = 3$ $p(4) = 4$ $p(5) = 7$

(b) $p(7) = ?$

$p(n) = 1 + \sum_{k=1}^{n-1} (k + p(k))$

$= 7$

$= 6+1$

$= 5+2,\ 5+1+1$

$= 4+3,\ 4+2+1,\ 4+1+1+1$

$= 3+$

4

3+1

2+1+1

1+1+1+1

5;

5, 4+1, 3+2, 3+1+1

2+2+1, 1+1+1+1+1

2+1+1+1

$p(7) = ?$

7

6+1

5+2, 5+1+1

4+3, 4+2+1, 4+1+1+1

3+4, 3+1+3, 3+2+2, 3+1+1+2, 3+1+1+1+1
x

2+5, 2+1+4, 2+2+3,
x x x

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1. |
| 1 | 0 | 0 | 0 | 0 | 1 | 0. |
| 0 | 1 | 0 | 0 | 1 | 0 | 0. |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 3 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 2 | 1 | 0 | 0 | 0 | 0 |
| 2 | 1 | 1 | 0 | 0 | 0 | 0 |
| 42 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 23 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 2 | 0 | 0 | 0 | 0 | 0 |
| 1 | 3 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 |

$p(7) = 15$.

(c) ?

(d) $$p(q) \approx \frac{e^{\pi\sqrt{2q/3}}}{4\sqrt{3}\,q}$$

$p(10) =$

$$S = k\ln\Omega = k\ln p(q) = \cdots$$

Where does temperature come in?

7.28

(a) 2D:

$$\epsilon = \frac{|p|^2}{2m} \qquad p_x = \frac{h}{\lambda_x} \quad \cancel{p_y} = \frac{h}{\lambda_y}$$

$$\& \ \lambda_x = \frac{2L_x}{n} \quad \lambda_y = \frac{2L_y}{n}$$

$$\therefore \ \epsilon = \frac{(p_x^2 + p_y^2)}{2m}$$

$$= \frac{h^2\left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}\right)}{2m} = \frac{h^2}{2m}\left(\frac{n_x^2}{4L^2} + \frac{n_y^2}{4L^2}\right) = \frac{h^2}{8mL^2}\vec{n}^2$$

$N = 2 \times$ Area of 1/4 circle of radius $n_{max}$ $= 2 \cdot \dfrac{\pi n_{max}^2}{4} = \dfrac{\pi}{2} n_{max}^2$

$$\downarrow$$

$$\& \ \epsilon_F = \frac{h^2}{8mA} n_{max}^2 \qquad n_{max} = \left(\frac{2N}{\pi}\right)^{1/2}$$

$$\Rightarrow \ \epsilon_F = \left(\frac{h^2}{8mA}\right)\left(\frac{2N}{\pi}\right) = \frac{h^2}{4\pi m}\left(\frac{N}{A}\right)$$

~~Avg energy~~

$$U = 2\iint \epsilon(\vec{n})\, dn_x\, dn_y = 2\int_0^{n_{max}}\int_0^{\pi/2}\left(\frac{h^2}{8mA}\right) r^2\, r\, dr\, d\theta$$

$$\epsilon(\vec{n}) = \frac{h^2}{8mA}|\vec{n}|^2 \qquad = 2\left(\frac{h^2}{8mA}\right)\left(\frac{\pi}{2}\right)\int_0^{n_{max}} r^3 dr$$

$$= \pi\left(\frac{h^2}{8mA}\right)\frac{n_{max}^4}{4} = \frac{\pi}{4}\left(\frac{h^2}{8mA}\right) n_{max}^4$$

$$\Rightarrow \cancel{E_{tn}}\ U = \frac{\pi}{4}\underbrace{\left(\frac{h^2}{8mA}\right) n_{max}^2}_{\epsilon_F} \cdot n_{max}^2$$

$$= \frac{\cancel{\pi}}{4}\cdot \epsilon_F \left(\frac{2}{\cancel{\pi}}N\right) = \frac{\epsilon_F \cdot N}{2}$$

$$\Rightarrow \frac{U}{N} = \frac{\epsilon_F}{2} \quad \checkmark.$$

(b) consider

$$U = 2\int_0^{n_{max}}\int_0^{\pi/2} \epsilon(n)\, n\, dn\, d\theta = \pi \int_0^{n_{max}} \epsilon(n)\, n\, dn$$

But $\epsilon(n) = \frac{h^2}{8mA} n^2 \Rightarrow n = \left(\frac{8mA}{h^2}\right)^{1/2} \epsilon^{1/2}$

$$dn = \left(\frac{8mA}{h^2}\right)^{1/2} \frac{1}{2}\epsilon^{-1/2}\, d\epsilon$$

Thus

$$U = \pi \int_0^{\epsilon_F} \epsilon \left(\frac{8mA}{h^2}\right)^{1/2} \cancel{\epsilon^{1/2}} \cdot \left(\frac{8mA}{h^2}\right)^{1/2} \cdot \frac{1}{2} \cdot \cancel{\epsilon^{-1/2}}\, d\epsilon$$

$$= \frac{\pi}{2}\left(\frac{8mA}{h^2}\right)\int_0^{\epsilon_F} \epsilon\, d\epsilon = \frac{\pi}{2}\left(\frac{8mA}{h^2}\right)\frac{\epsilon_F^2}{2}$$

$$U = \frac{\epsilon_F}{2} \cdot \left( \pi \left( \frac{8mA}{h^2} \right) \epsilon_F \right)$$

From $N = \frac{\pi}{2} n_{max}^2 = \frac{\pi}{2} \cdot \left( \frac{8mA}{h^2} \right) \epsilon_F$

$\therefore U = \frac{\epsilon_F}{2} (2N) \cdots$ off by factor of 2?

Now consider

$$U = \pi \int_0^{n_{max}} \epsilon(n) n \, dn = \pi \int_0^{\epsilon_F} \epsilon \cdot \left[ \frac{1}{2} \left( \frac{8mA}{h^2} \right) \right] d\epsilon$$

$g(\epsilon) = \frac{1}{2} \left( \frac{8mA}{h^2} \right)$ ind. of $\epsilon$ ✓.

(c) for non zero T

$$N = \int_0^\infty g(\epsilon) \frac{1}{\left(e^{(\epsilon-\mu)/kT} + 1\right)} d\epsilon = \frac{1}{2} \left( \frac{8mA}{h^2} \right) \int_0^\infty \frac{1}{e^{(\epsilon-\mu)/kT} + 1} d\epsilon$$

$$= \frac{1}{2} \left( \frac{8mA}{h^2} \right) \int_0^\infty \left( \sum_{k \geq 0} (-1)^k e^{k(\epsilon-\mu)/kT} \right) d\epsilon$$ ✗ Not convergent series

$$= \frac{1}{2} \left( \frac{8mA}{h^2} \right) \sum_{k \geq 0} (-1)^k \int_0^\infty e^{k(\epsilon-\mu)/kT} d\epsilon$$

$$N = \frac{1}{2}\left(\frac{8mA}{h^2}\right)\int_0^\infty \frac{e^{-(\varepsilon-\mu)/kT}}{1+e^{-(\varepsilon-\mu)/kT}}\, d\varepsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\int_0^\infty e^{-(\varepsilon-\mu)\beta}\sum_{n\geq 0} e^{-n(\varepsilon-\mu)/kT}\, d\varepsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0}\int_0^\infty e^{-(n+1)(\varepsilon-\mu)\beta}\, d\varepsilon$$

$$\cancel{= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0}\frac{e^{-(n+1)(\varepsilon-\mu)\beta}}{-(n+1)}\Bigg|_0^\infty}$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} e^{+(n+1)\mu\beta}\int_0^\infty e^{-(n+1)\varepsilon\beta}\, d\varepsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} e^{+(n+1)\mu\beta}\,\frac{e^{-(n+1)\varepsilon\beta}}{-(n+1)\beta}\Bigg|_0^\infty$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0}\frac{e^{(n+1)\mu\beta}}{-(n+1)\beta}(0-1)$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} \frac{e^{(n+1)\mu B}}{(n+1)B} \equiv F(\mu)$$

$$\cancel{= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} \int e^{(n+1)\mu B}}$$

Then $$F'(\mu) = \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} e^{(n+1)\mu B}$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right) e^{\mu B}\sum_{n\geq 0} e^{n\mu B} \qquad e^{\mu B} < 1$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right) e^{\mu B}\left(\frac{1}{1-e^{\mu B}}\right) =$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\frac{1}{(e^{-\mu B}-1)}$$

$$\therefore \quad F'(\mu) = \frac{C_1}{(e^{-\mu B}-1)}$$

$$\Rightarrow \quad F(\mu) = C_1\int \frac{d\mu}{e^{-\mu B}-1} = C_1\int \frac{e^{\mu B}}{1-e^{\mu B}}\,d\mu$$

$$= -\frac{C_1}{\beta}\ln(1-e^{\mu B})$$

$$= -\frac{C_1}{\beta}\ln(1-e^{+\mu B})$$

$$N = -\frac{1}{2}\left(\frac{8mA}{h^2}\right) kT \ln\left(1-e^{\mu B}\right)$$

$$\ln\left(1-e^{\mu B}\right) = -2\frac{h^2}{8m}\left(\frac{N}{A}\right)\frac{1}{kT} = -\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}$$

$$\Rightarrow 1-e^{\mu B} = \exp\left\{\frac{-h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}\right\}$$

$$e^{\mu B} = 1 - e^{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}}$$

$$\Rightarrow \mu = \frac{1}{\beta}\ln\left(1-e^{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}}\right) = kT\ln(1- \quad \cdots \quad)$$

(e) $kT \gg \epsilon_F \qquad \epsilon_F = \frac{h^2}{4\pi m}\left(\frac{N}{A}\right)$

$$\therefore \mu = \frac{1}{\beta}\ln\left(1-e^{-\pi\epsilon_F/kT}\right)$$

Assuming $\epsilon_F \ll kT$ $\qquad$ $\cancel{\ln(1-x) \cong x - \frac{x^2}{2} + x}$

$$\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

$$\mu = \frac{1}{\beta}\left[\ln\left(1-\left(1-\frac{\pi\epsilon_F}{kT}\right)\right)\right]$$

$$= \frac{1}{\beta}\ln\left(\frac{\pi\epsilon_F}{kT}\right)$$

$$\mu = -kT \ln\left( \frac{kT}{\pi\left(\frac{h^2}{4\pi m}\left(\frac{N}{A}\right)\right)} \right)$$

$$= -kT \ln\left( \frac{A}{N} \cdot \left(\frac{4mkT}{h^2}\right)^{1} \right)$$

similar to an ideal gas.

7.29

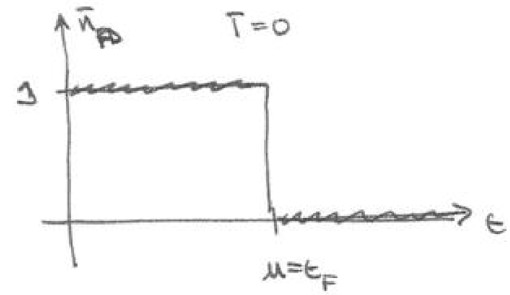
eq 7.68 is

$$U = \frac{3}{5}N\frac{\mu^{5/2}}{\epsilon_F^{3/2}} + \frac{3\pi^2}{8}N\frac{(kT)^2}{\epsilon_F} + \cdots$$

eq 7.54 is

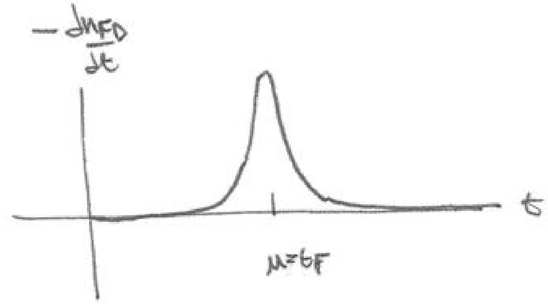
$$U = \int_0^\infty \epsilon\, g(\epsilon)\,\bar{n}_{FD}(\epsilon)\, d\epsilon$$

let $g(\epsilon) = g_0 \epsilon^{1/2}$




$$= g_0 \int_0^\infty \epsilon^{3/2}\, \bar{n}_{FD}(\epsilon)\, d\epsilon$$

$$= g_0 \cancelto{0}{\bar{n}_{FD}(\epsilon)\, \frac{\epsilon^{5/2}}{5/2}\Bigg|_0^\infty}$$




$$- \frac{2}{5} g_0 \int_0^\infty \epsilon^{5/2} \left(\frac{d\bar{n}_{FD}}{d\epsilon}\right) d\epsilon$$

$$= +\frac{2}{5} g_0 \int_0^\infty \epsilon^{5/2} \left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right) d\epsilon$$

$$\approx \frac{2}{5} g_0 \int_{-\infty}^{+\infty} \epsilon^{5/2} \left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right) d\epsilon$$

$$\bar{n}_{FD}(\epsilon) = \frac{1}{e^{+(\epsilon-\mu)/kT} + 1}$$

let $x \equiv \frac{\epsilon - \mu}{kT} \Rightarrow \epsilon = kTx + \mu$

$$\bar{n}_{FD} = \frac{1}{e^x + 1}$$

$$\frac{d\bar{n}_{FD}}{dx} = \frac{-e^x}{(e^x+1)^2}$$

$$= \frac{2}{5} g_0 \int_{-\infty}^{+\infty} (kTx+\mu)^{5/2} \frac{e^{x}}{(e^{x}+1)^2}\, dx\,(kT)$$

$$= \frac{2}{5} g_0\, kT \int_{-\infty}^{\infty} (kT)^{5/2} \left(x+\frac{\mu}{kT}\right)^{5/2} \underbrace{\frac{e^{x}}{(e^{x}+1)^2}}_{\text{peaks only at } x=0.} dx$$

$$\left(x+\frac{\mu}{kT}\right)^{5/2} = \left(\frac{\mu}{kT}\right)^{5/2} + \frac{5}{2}\left(x+\frac{\mu}{kT}\right)^{3/2}\Big|_{x=0}(x) + \frac{5}{2}\cdot\frac{3}{2}\left(x+\frac{\mu}{kT}\right)^{1/2}\Big|_{x=0}\frac{x^2}{2} + O(x^3)$$

$$= \frac{2}{5} g_0 (kT)^{7/2} \int_{-\infty}^{+\infty} \left( \left(\frac{\mu}{kT}\right)^{5/2} + \frac{5}{2}\left(\frac{\mu}{kT}\right)^{3/2} x + \frac{15}{4}\left(\frac{\mu}{kT}\right)^{1/2} \frac{x^2}{2} + O(x^3) \right) \frac{e^{x}}{(e^{x}+1)^2}\, dx$$

$$= \frac{2}{5} g_0 (kT)^{7/2} \left(\frac{\mu}{kT}\right)^{5/2} \underbrace{\int_{-\infty}^{\infty} \frac{e^{x}}{(e^{x}+1)^2}\, dx}_{+1} + 0$$

$$+ \frac{\cancel{2}}{5} g_0 (kT)^{7/2} \cdot \frac{15}{4}\left(\frac{\mu}{kT}\right)^{1/2} \cdot \frac{1}{\cancel{2}} \int_{-\infty}^{+\infty} \frac{x^2 e^{x}}{(e^{x}+1)^2}\, dx$$

$$= \frac{2}{5} g_0 \frac{\mu^{5/2}}{\cancel{kT}} (kT) + \frac{\cancel{2}}{4} g_0\, \mu^{1/2} (kT)^{3} \frac{\pi^2}{\cancel{3}} + \cdots$$

$$\therefore\ U = \frac{2}{5} g_0\, \mu^{5/2} \cdot (kT) + \frac{\pi^2}{4} g_0\, \mu^{1/2} (kT)^{3} + \cdots$$

w/ $g_0 = \dfrac{3N}{2\epsilon_F^{3/2}}$ we get

$$\Rightarrow U = \frac{2}{5}\cdot\frac{3N}{2\epsilon_F^{3/2}}\cdot\mu^{5/2}(kT) + \frac{\pi^2}{4}\cdot\frac{3N}{2\epsilon_F^{3/2}}\mu^{1/2}(kT)^3 + \ldots$$

$$= \frac{3}{5}\frac{N}{\epsilon_F^{3/2}}\mu^{5/2}(kT) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F^{3/2}}\mu^{1/2}(kT)^3 + \ldots \quad \text{eq } 7.67 \checkmark$$

$$\Rightarrow U = \frac{3}{5}\frac{N}{\epsilon_F^{3/2}}\mu^{5/2}(kT) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F^{3/2}}(kT)^{3}$$

$$\left\{ \frac{\mu}{\epsilon_F} = 1 - O((kT)^2) \right.$$

different exponent ...

$$U = \frac{3}{5}N\epsilon_F\left(\frac{\mu}{\epsilon_F}\right)^{5/2} + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^3 + \ldots$$

Assume square

$$= \frac{3}{5}N\epsilon_F\left(1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F}\right)^2\right)^{5/2} + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^2$$

$$= \frac{3}{5}N\epsilon_F\left(1 - \frac{5\pi^2}{24}\left(\frac{kT}{\epsilon_F}\right)^2\right) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^2$$

$$= \frac{3N}{5}\epsilon_F + \underbrace{\left(-\frac{\pi^2}{8} + \frac{3\pi^2}{8}\right)}_{\frac{\pi^2}{4}}\frac{N}{\epsilon_F}(kT)^2 \quad \text{eq } 7.68 \checkmark$$

(Prob 7.30) ~~[illegible]~~ All odd terms would be zero since the integrals are all two from ~~[illegible]~~ $(-\infty, +\infty)$

The $T^4$ term could be evaluated easily enough

(Prob 7.31) From problem 7.28

$$g(\epsilon) = \frac{1}{2}\left(\frac{8mA}{h^2}\right) \quad \underline{\text{independent}} \text{ of } \epsilon.$$

Then

$$U = \int_0^\infty g(\epsilon)\,\epsilon\,\bar{n}_{FD}(\epsilon)\,d\epsilon = \frac{1}{2}\left(\frac{8mA}{h^2}\right)\int_0^\infty \epsilon\,\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\left[\cancelto{0}{\bar{n}_{FD}(\epsilon)\frac{\epsilon^2}{2}\Big|_0^\infty} + \frac{1}{2}\int_0^\infty \epsilon^2\left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right)d\epsilon\right]$$

$$= \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_0^\infty \epsilon^2\left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right)d\epsilon \qquad \bar{n}_{FD} = \frac{1}{\left(e^{(\epsilon-\mu)/kT}+1\right)}$$

$$= \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_0^\infty \epsilon^2\,\frac{e^x}{(e^x+1)^2}\,d\epsilon \qquad x \equiv \frac{(\epsilon-\mu)}{kT} \quad \epsilon = \mu + kTx$$

$$\cong \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_{-\infty}^{+\infty} \epsilon^2\,\frac{e^x}{(e^x+1)^2}\,d\epsilon = \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_{-\infty}^{+\infty}(\mu+kTx)^2\,\frac{e^x}{(e^x+1)^2}\,(kT)\,dx$$

$$= \frac{(kT)}{4}\left(\frac{8mA}{h^2}\right)(kT)^2\int_{-\infty}^{\infty}\left(x+\frac{\mu}{kT}\right)^2\frac{e^x}{(e^x+1)^2}\,dx$$

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$$= \left(\frac{2\pi m A}{h^2}\right)(kT)^3 \int_{-\infty}^{+\infty} \left( \left(\frac{\mu}{kT}\right)^2 + 2\left(\frac{\mu}{kT}\right)x + 2\frac{x^2}{2} \right) \frac{e^x}{(e^x+1)^2}\, dx$$

$$= \left(\frac{2\pi m A}{h^2}\right)(kT)^3 \left[ \left(\frac{\mu}{kT}\right)^2 \cdot 1 + 0 + \overbrace{\int_{-\infty}^{+\infty} x^2 \frac{e^x}{(e^x+1)^2}\, dx}^{\pi^2/3} \right]$$

$$= \frac{2\pi m A}{h^2} \left[ \mu^2 (kT) + \frac{\pi^2}{3}(kT)^3 \right]$$

(I think this is good $\forall$ $kT$)

How Big is the factor of $\epsilon_F$ = ?

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P. 732

(7.53) $$N = \int_0^\infty g(\epsilon) \frac{1}{(e^{(\epsilon-\mu)/kT}+1)} d\epsilon$$

(7.54) $$U = \int_0^\infty g(\epsilon)\epsilon \frac{1}{(e^{(\epsilon-\mu)/kT}+1)} d\epsilon$$

(a) If $kT \equiv \epsilon_F$

(7.53) $$N = \int_0^\infty g_0 \epsilon^{1/2} \frac{1}{(e^{(\epsilon-\mu)/\epsilon_F}+1)} d\epsilon = g_0 \int_0^\infty \epsilon^{1/2} \frac{d\epsilon}{(e^{\epsilon/\epsilon_F}+1)}$$

---

Question is can I evaluate the integrals [illegible] Now

$$N = g_0 \int_0^\infty \epsilon^{1/2} \frac{d\epsilon}{e^{(\epsilon-\mu)/kT}+1} = g_0 \cancel{\int_0^\infty \frac{\epsilon^{1/2}\, d\epsilon}{e^{(x-\mu)/t}+1}}$$

let $x = \frac{\epsilon-\mu}{kT}$ $\epsilon = \mu + kTx$

$$N = g_0 \int_{-\frac{\mu}{kT}}^\infty (\mu+kTx)^{1/2} \frac{(kT)\,dx}{e^x+1} = g_0 (kT)^{3/2} \int_{-\frac{\mu}{kT}} \left(x+\frac{\mu}{kT}\right)^{1/2} \frac{dx}{e^x+1}$$

$$N(kT=\epsilon_F; \mu=0) = g_0 \epsilon_F^{3/2} \int_0^\infty x^{1/2} \frac{dx}{e^x+1}$$

(b) ...?

P. 7.33

(a) $g(\epsilon) = \begin{cases} g_0\sqrt{\epsilon - \epsilon_c} & \epsilon > \epsilon_c \\ g_0\sqrt{\epsilon_v - \epsilon} & \epsilon < \epsilon_v \\ 0 & \text{otherwise } \epsilon_v < \epsilon < \epsilon_c \end{cases}$

$$N = \int_0^\infty g(\epsilon)\,\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$= \int_0^{\epsilon_v} g_0\sqrt{\epsilon_v - \epsilon}\,\bar{n}_{FD}(\epsilon)\,d\epsilon + 0 + \int_{\epsilon_c}^\infty g_0\sqrt{\epsilon - \epsilon_c}\,\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$N = \int_0^{\epsilon_v} \frac{g_0\sqrt{\epsilon_v - \epsilon}\,d\epsilon}{e^{(\epsilon-\mu)/kT} + 1} + \int_{\epsilon_c}^\infty \frac{g_0\sqrt{\epsilon - \epsilon_c}}{e^{(\epsilon-\mu)/kT} + 1}\,d\epsilon$$

Don't know why $\mu$ must be between $\epsilon_v$ + $\epsilon_c$ for a solution to this problem

Assume $\epsilon_c - \epsilon_v \gg kT$

(b) Am I supposed to actually evaluate this integral?

(c) Si

$1 cm^3$ of Si is appox same size as $SiO_2$ which is $22.69\ cm^3$

$\therefore \frac{1\ cm^3}{22.69\ cm^3} = mole^*$

$= \cdot N_A = $ # of Si atoms = has "4" available $e^-$

$=$

For Cu we $\rho$ 7.12 $cm^3$/mole

$\frac{1\ cm^3}{7.12}$ ~ 3 ~~#~~ times larger than Si

But of How many ~~###~~ conduction electrons? $\approx 11.$

~~A~~ $\therefore$ Cu has about 33 times as many conduction $e^-$

(d) Don't know how to get #'s

(e) I would guess $E_C - E_V \gg kT$

say $E_C - E_V = \Delta E \approx 10kT$

$kT = (8.617 \cdot 10^{-5}\ eV/K)(300K) = \cancel{24.617}\ \ 24 \cdot 10^{-3}\ eV$

$= .024\ eV$

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P. 7.34

(a) ?

(b) ~~[illegible]~~

$$N_c = \int_{\epsilon_c}^{\infty} \frac{g_{0c}\sqrt{\epsilon - \epsilon_c}}{e^{(\epsilon-\mu)/kT}+1} \, d\epsilon$$

Assume $\epsilon_c - \mu \gg kT$

$$\frac{\epsilon_c - \mu}{kT} \gg 1$$

$$N_c = g_{0c} \int_{\epsilon_c}^{\infty} \frac{\sqrt{\epsilon - \epsilon_c}\, e^{-(\epsilon-\mu)/kT}}{1 + e^{-(\epsilon-\mu)/kT}} \, d\epsilon$$

$$= g_{0c} \int_{\epsilon_c}^{\infty} \sqrt{\epsilon - \epsilon_c}\, e^{-(\epsilon-\mu)/kT} \left[ 1 - e^{-(\epsilon-\mu)/kT} + \cdots \right] d\epsilon$$

$$= g_{0c} \int_{\epsilon_c}^{\infty} \sqrt{\epsilon - \epsilon_c}\, e^{-(\epsilon-\mu)/kT} \, d\epsilon$$

$$- g_{0c} \int_{\epsilon_c}^{\infty} \sqrt{\epsilon - \epsilon_c}\, e^{-2(\epsilon-\mu)/kT} \, d\epsilon + \cdots$$

Just to get a "feel" for what things look like

lets use the 1st term

$$N_c \approx g_c \int_{\epsilon_c}^{\infty} \sqrt{\epsilon-\epsilon_c}\; e^{-(\epsilon-\mu)/kT}\, d\epsilon$$

let $x = \dfrac{\epsilon-\mu}{kT}$ $\quad dx = \dfrac{1}{kT}\, d\epsilon$

$$\epsilon = kTx + \mu$$

$$N_c \approx g_c \int_{x_c}^{\infty} \sqrt{kTx+\mu-\epsilon_c}\; e^{-x}\, d\epsilon$$

$$= g_c (kT)^{1/2} \int_{x_c}^{\infty} \sqrt{x + \left(\frac{\mu-\epsilon_c}{kT}\right)}\; e^{-x} (kT)\, dx$$

$$= g_c (kT)^{3/2} \int_{x_c}^{\infty} \sqrt{x - x_c}\; e^{-x}\, dx$$

let ~~x~~ $\xi = x - x_c$ Then $x = \xi + x_c$

$$N_c = g_c (kT)^{3/2} \int_{0}^{\infty} \xi^{1/2} e^{-\xi - x_c}\, d\xi$$

$$= g_c (kT)^{3/2} e^{-x_c} \int_{0}^{\infty} \xi^{1/2} e^{-\xi}\, d\xi$$

#

(c) $$N_v = \int_0^{\epsilon_v} g_{0v}\sqrt{\epsilon_v-\epsilon}\;\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$= \int_0^{\epsilon_v} g_{0v}\sqrt{\epsilon_v-\epsilon}\cdot\frac{1}{\left(e^{+(\epsilon-\mu)/kT}+1\right)}\,d\epsilon$$

$$(\mu-\epsilon_v) \gg kT \Rightarrow \frac{-(\epsilon_v-\mu)}{kT} \gg 1$$

$$-(\epsilon_v-\mu) \gg kT \Rightarrow e^{-\frac{(\epsilon_v-\mu)}{kT}} \gg e^{1} = O(1)$$

$$\epsilon_v-\mu \ll -kT$$

$$\frac{\epsilon_v-\mu}{kT} \ll -1 \Rightarrow e^{(\epsilon_v-\mu)/kT} \ll 1$$

$N_v = \#$ of valence $e^-$

$N_{holes} = N_v(T=0) - N_v(T\neq 0)$ $\Big\{\; n_{FD}(T=0) =$ step fn at $\epsilon=\mu$.

$$\cancel{N_v \approx \int_{-\mu/kT}^{\frac{\epsilon_v-\mu}{kT}} g_{0v}\sqrt{\epsilon_v-\epsilon}} \qquad x \equiv \frac{\epsilon-\mu}{kT}$$

$$N_v \approx g_{0v}\int_0^{\epsilon_v}\sqrt{\epsilon_v-\epsilon}\left[1-e^{(\epsilon-\mu)/kT}+\dots\right]d\epsilon$$

1st term is trivial.

2nd term

$$-g_{0V}\int_0^{\epsilon_V}\sqrt{\epsilon_V-\epsilon}\; e^{(\epsilon-\mu)/kT}\, d\epsilon$$

$$x = \frac{\epsilon-\mu}{kT} \qquad dx = \frac{d\epsilon}{kT}$$

$$\epsilon = \mu + kTx$$

$$= -g_{0V}\int_{\frac{0-\mu}{kT}}^{\frac{\epsilon_V-\mu}{kT}}\sqrt{\epsilon_V-\mu-kTx}\; e^{x}\, kT\, dx$$

$$= -g_{0V}(kT) \quad \ldots$$

3(d) $N_{tot} = \cancel{N_V +}\; N_V(\mu) + N_C(\mu)$

invert $\mu = \mu(N)$

(e) ...

P.7.35

(a) ? What should I use for

(b) ?

(c) ?

P.7.36

(a) Due to their thermal motion I would think that the spin states would be in constant flux i.e. always changing and ∴ the total magnetic to moments would tend to be somewhere between $+\mu_B$ & $-\mu_B$ $\approx 0$.

(b) $$P_\uparrow(\ ) = \frac{e^{-\mu B/kT}}{\sum e^{-\mu B/kT} + \sum e^{+\mu B/kT}}$$

(c) ?

(7.37) integrand in Planck spectrum is

$$f(x) = \frac{x^3}{e^x - 1}$$

$$f'(x) = \frac{3x^2}{e^x - 1} - \frac{x^3 (e^x)}{(e^x-1)^2} = 0$$

set

$x = 0$ $\quad x \neq 0$ $$\frac{3}{(e^x-1)} - \frac{xe^x}{(e^x-1)^2} = 0$$

multiply by $(e^x-1)^2$

$$\Rightarrow \quad 3(e^x - 1) - xe^x = 0$$

$$3e^x - 3 - xe^x = 0$$

$$(3-x)e^x - 3 = 0$$

(7.38) $$u(\epsilon; T) = \frac{8\pi}{(hc)^3} \frac{\epsilon^3}{(e^{\epsilon/kT} - 1)}$$


$k = \cancel{1.381} \cdot \cancel{10}$
$8.617 \cdot 10^{-5}$ eV/K

. . .

7.39
$$\frac{U}{V} = \frac{8\pi}{(hc)^3}\int_0^\infty \frac{\epsilon^3 d\epsilon}{e^{\epsilon/kT}-1}$$

$$f\lambda = c$$

$$\epsilon = h\nu = hf = \frac{hc}{\lambda} \qquad d\epsilon = \frac{-hc}{\lambda^2}\,d\lambda$$

$$\lambda = \frac{hc}{\epsilon}$$

$$\frac{U}{V} = \frac{8\pi}{(hc)^3}\int_\infty^0 \frac{(hc/\lambda)^3(-hc/\lambda^2)\,d\lambda}{e^{hc/\lambda kT}-1} = \frac{8\pi}{(hc)^3}\cdot(hc)^3(hc)\int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda kT}-1)}$$

$$= 8\pi(hc)\int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda kT}-1)}$$

spectrum $\quad u(\lambda) = \dfrac{1}{\lambda^5\left(e^{\frac{hc/kT}{\lambda}}-1\right)}$

Why does the peak not occur at $1/x_c$

Assume $F(x)$ has max at $x_c \Rightarrow F'(x_c) = 0$

$$G(y) \equiv F(x(y)) \qquad x(y) = \frac{1}{y}$$

$$G'(y) = F'(x)$$

$F(x) = F_1(x)F_2(x)$

$F'(x_c) = 0$

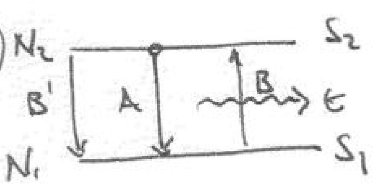
$G(y) \equiv F(x(y)) \qquad y = \frac{c}{x}$

$G'(y) = \frac{dF_1}{dx}\cdot\frac{dx}{dy}F_2 + F_1\frac{dF_2}{dy}\cdot\frac{dx}{dy} \underset{\text{set}}{=} 0$

$\frac{dx}{dx}\left[\frac{dF_1}{dx}\cdot F_2 + F_1\frac{dF_2}{dx}\right] = 0$

7.40 ?

7.41




P

$B = \frac{\text{prob of abs}}{u(f)}$

(a) ~~$\frac{dN_1}{dt} = +AN_1 - BN_1 + B'N_2$~~

$\frac{dN_2}{dt} = \frac{dN_1}{dt} = AN_1 - Bu(f)N_1 + B'u(f)N_2$

$$\frac{N_1}{N_2} \approx C$$

$$\frac{dW_2}{dt} = -\frac{dW_1}{dt}$$

How show related as given?

7.42

(a) $U = V\left(\frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^3}\right)$

(b) $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$ · All

(c) $1 = \int$

$\lambda_{min} \approx 400\,nm \to \epsilon_{max} = \frac{hc}{\lambda} = \cdots$

$\lambda_{max} \cong 700\,nm \to \epsilon_{min} = \frac{hc}{\lambda_{max}} = \cdots$

$$F = \int_{\epsilon_{min}}^{\epsilon_{max}} \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}\,d\epsilon$$

7.43 $T = 5800\,k$

(a) $U = V\cdot\left(\frac{8\pi^5}{15}\right)\frac{(kT)^4}{(hc)^3}$

(b) $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$

(c) - -

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P. 7.44

(a) $N = 2\sum_{n_x}\sum_{n_y}\sum_{n_z} \bar{n}_{PI}(\epsilon) = 2\sum_{n_x,n_y,n_z} \frac{1}{(e^{hcn/2LkT}-1)}$

$$= 2\int_0^\infty dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi \, n^2 \sin\theta \frac{1}{(e^{hcn/2LkT}-1)}$$

$$= 2\left(\frac{\pi}{2}\right) \int_0^{\pi/2} \sin\theta \, d\theta \int_0^\infty \frac{n^2 \, dn}{(e^{hcn/2LkT}-1)}$$

$$= \pi\left[-\cos\theta\right]_0^{\pi/2} \int_0^\infty \frac{n^2 \, dn}{(e^{hcn/2LkT}-1)} \qquad \epsilon = \frac{hcn}{2L}$$

$$d\epsilon = \frac{hc}{2L} dn$$

$$= \pi\left[+1\right] \cdot \int_0^\infty \frac{\frac{4L^2}{h^2c^2}\epsilon^2 \left(\frac{2L}{hc}\right) d\epsilon}{e^{\epsilon/kT}-1}$$

$$= \pi \frac{8L^3}{(hc)^3} \int_0^\infty \frac{\epsilon^2 \, d\epsilon}{e^{\epsilon/kT}-1} \qquad \text{let } x = \frac{\epsilon}{kT}$$

$$dx = \frac{1}{kT} d\epsilon$$

$$= \frac{8\pi V}{(hc)^3}$$

$$= \frac{8\pi V}{(hc)^3} \int_0^\infty \frac{(kT)^2 x^2 \, (kT) \, dx}{e^x - 1}$$

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$$N = \frac{8\pi (kT)^3 V}{(hc)^3} \int_0^\infty \frac{x^2 dx}{e^x - 1} = \frac{8\pi (kT)^3 V}{(hc)^3} I$$

(b) $$S(T) = \frac{32\pi^5}{45} V \left(\frac{kT}{hc}\right)^3 k$$

So $$s = \frac{S}{N} = \frac{\frac{32}{45}\pi^5 V \left(\frac{kT}{hc}\right)^3 k}{\frac{8\pi (kT)^3}{(hc)^3} V \cdot I}$$

$$= \frac{4}{45}\pi^4 \left(\frac{1}{I}\right) \cdot k$$

entropy per photon is constant.

(c) $$\frac{N}{V} = 8\pi \left(\frac{kT}{hc}\right)^3 \cdot I$$

P.7.45

$$P = -\frac{\partial U}{\partial V}\bigg|_{S,N}$$

Since $N = 8\pi V\left(\frac{kT}{hc}\right)^3 I \rightarrow \left(\frac{kT}{hc}\right) = \left(\frac{N}{8\pi V I}\right)^{1/3}$

from P.7.44

$$U = \cancel{\frac{8\pi^5}{18}\frac{1}{(hc)^3}}\; V \cdot \frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^4}\cdot(hc)$$

$$= V \cdot \frac{8\pi^5}{15}(hc)\left(\frac{N}{8\pi V I}\right)^{4/3}$$

$$= V^{1-4/3} \cdot \frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}$$

$$\frac{\partial U}{\partial V}\bigg|_{S,N} = \left(1-\frac{4}{3}\right)V^{-4/3} \cdot \frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}$$

$$= \left(1-\frac{4}{3}\right)\frac{V^{1-4/3}}{V}\cdot\left(\frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}\right)$$

$$= \left(1-\frac{4}{3}\right)\frac{U}{V} = -\frac{1}{3}\frac{U}{V}$$

So $P = -\frac{\partial U}{\partial V}\bigg|_{S,N} = \frac{1}{3}\frac{U}{V}$

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P.7.46

(a)

$$F = U - ST$$

$$= V \cdot \frac{8\pi^5 (kT)^4}{15 (hc)^3} - \frac{32\pi^5}{45} V \left(\frac{kT}{hc}\right)^3 kT$$

$$= \frac{8}{15} V \pi^5 \frac{(kT)^4}{(hc)^3} \left[ \underbrace{1 - \frac{4}{3}}_{-\frac{1}{3}} \right]$$

$$F = -\frac{8}{45} V \pi^5 \frac{(kT)^4}{(hc)^3}$$

(b)

$$S = -\frac{\partial F}{\partial T}\bigg|_V = -\frac{\partial}{\partial T}\left[ \cdots \right]$$

$$= + \frac{32}{45} V \pi^5 \left(\frac{(kT)}{(hc)}\right)^3 k.$$

(c)

$$\frac{\partial F}{\partial V}$$

$$F = U - ST$$

$$dF = dU - TdS - SdT$$

$$dU = \cancel{2}\ TdS - pdV$$

$$dF = T\cancel{dS} - pdV - T\cancel{dS} + -SdT = -pdV - SdT$$

$$\therefore \quad p = -\frac{\partial F}{\partial V}\Big|_S \quad + \quad S = -\frac{\partial F}{\partial T}\Big|_V$$

given $$S = \frac{32}{45} V \pi^5 \left(\frac{kT}{hc}\right)^3 k \Rightarrow \left(\frac{kT}{hc}\right) = \left(\frac{45}{32}\frac{1}{\pi^5}\frac{1}{V}\cdot\frac{1}{k}\cdot S\right)^{1/3}$$

$$p = -\frac{\partial}{\partial V}\Big|_S \left[ -\frac{8}{45} V \pi^5 \frac{(kT)^4}{(hc)^3} \right]$$

$$= -\frac{\partial}{\partial V}\Big|_S \left[ -\frac{8}{45} V \cdot \pi^5 (hc) \left(\frac{45}{32}\cdot\frac{1}{\pi^5}\cdot\frac{S}{V\cdot k}\right)^{4/3} \right]$$

$$= \frac{\partial}{\partial V}\Big|_S \left[ \frac{8}{45} V^{1-\frac{4}{3}} \pi^5 (hc) \left(\frac{45}{32}\frac{1}{\pi^5}\frac{S}{k}\right)^{4/3} \right]$$

$$= \frac{8}{45}\frac{V^{1-\frac{4}{3}}}{V}\cdot\left(1-\frac{4}{3}\right) \ \ldots$$

$$= \frac{\left(1-\frac{4}{3}\right)}{V}\cdot\frac{8}{45} V \pi^5 (hc) \left( \ \ldots \ \right)^{4/3}$$

$$= \frac{1}{3\cdot V}\cdot\frac{8}{45} V \pi^5 (hc)\left(\frac{kT}{hc}\right)^4$$

$$= \frac{1}{3\cdot V}$$ ... check ...

(d) $F = -kT \ln Z$

$$Z = \sum_i e^{-\epsilon_i/kT} \qquad \epsilon_n = \frac{hcn}{2L}$$

$$= \sum_n \exp\left\{-\frac{hcn}{2L\cdot kT}\right\} = \sum_n \left(e^{-\frac{hc}{2LkT}}\right)^n$$

$$= \frac{1}{1 - e^{-\frac{hc}{2LkT}}}$$

$$F \overset{?}{=} kT \ln\left(1 - e^{-\frac{hc}{2LkT}}\right)$$ What's wrong?

P.7.47

~~[illegible]~~ $r_{pH} = 10^9$

? How do?

P.7.48

$\nu, \bar{\nu}$ $T_{eff} = 1.95 K$

(a) $\mu_\nu = \mu_{\bar{\nu}}$ $\nu + \bar{\nu} \leftrightarrow 2\gamma$

By equating chemical potentials for a reaction of this type

$\mu_\nu + \mu_{\bar{\nu}} = \mu_\gamma = 0$ chemical potential of light " for

$\Rightarrow$ w/ fact that $\mu_\nu = \mu_{\bar{\nu}}$ $\Rightarrow$ $\mu_\nu = 0$

(b) Fermion $\bar{n}_{FD}(\epsilon) = \dfrac{1}{e^{\epsilon/kT} - 1}$

$\bar{W} = \cancel{2}^{3} \int \epsilon \bar{n}_{FD}(\epsilon)\, d\epsilon$ How set up?

(c) $N = 3 \int_\epsilon \bar{n}_{FD}(\epsilon) \cdot n(\epsilon)\, d\epsilon$

prob. state w/ energy $\epsilon$ is occupied.

degeneracy of state!

(d) ?

7.49 $\epsilon = \sqrt{(pc)^2 + (mc^2)^2}$ $p = [0, \infty)$

(a) $U = 2^2 \int_0^\infty \epsilon \, n_{F.D}(\epsilon) \, d\epsilon$ $n_{FD}(\epsilon) = \dfrac{1}{e^{\epsilon/kT} + 1}$

$= 4 \int_0^\infty \dfrac{\epsilon \, d\epsilon}{e^{\epsilon/kT} + 1}$

$\epsilon = \sqrt{(pc)^2 + (mc^2)^2}$

$\epsilon^2 = (pc)^2 + (mc^2)^2$

$= 4 \int_0^\infty \cdots$

(b) $u(T) = \int_0^\infty \dfrac{x^2 \sqrt{x^2 + (mc^2/kT)^2}}{e^{\sqrt{x^2 + (mc^2/kT)^2}} + 1} \, dx$

(d) $F = -kT \ln Z$

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P. 7.50 $\frac{kT}{c^2} \gg m$

(a) $V(t)$ $T$ $F = U - ST$

$$\Rightarrow S = \frac{F - U}{T}$$

$$S = -\frac{16\pi (kT)^4}{(hc)^3} f(T) \cdot V - \frac{16\pi (kT)^4}{(hc)^3} u(T) \cdot V$$

(b) ?

(c) ?

7.51 $T = 3000\,K$

$e = 1/3$

(a) $P = 100\,W$.

Using the $\text{power} = \sigma e A T^4$

$$100\,W = \left(5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}\right)\left(\frac{1}{3}\right)(A)(3000\,K)^4$$

$$100\,W = \frac{3^4 \cdot 10^{12}}{3}(A)\ 5.67 \cdot 10^{-8} \frac{W}{m^2}$$

$$100\,W = 3^3 \cdot 10^{12} \cdot A \cdot 5.67 \cdot 10^{-8}\ W/m^2$$

$$A = \frac{100}{3^3 \cdot 10^{12}(5.67 \cdot 10^{-8})}\ m^2 = 6.532 \cdot 10^{-5}\ m^2$$

$$\left\{ (mm)^2 = 10^{-6}\,m \right.$$

$$= .6532 \cdot (mm)^2$$

(b) $\epsilon = 2.82\,kT$

$$= 2.82(1.381 \cdot 10^{-23}\ J/K)(300\,K)$$

$$= 1.1683 \cdot 10^{-20}\ J = 0.0729\ eV$$

$$\epsilon = h\nu = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\epsilon}$$

$$\lambda = \frac{(6.626\cdot 10^{-34}\,J\cdot s)(3\cdot 10^{8}\,m/s)}{1.1683\cdot 10^{-20}\,J}$$

$$= 1.701\cdot 10^{-5}\,m$$

(c) Spectrum of light $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$ energy density per unit photon energy

$u(\epsilon)$ = Now

$$\frac{8\pi}{(hc)^3} = \frac{8\cdot\pi}{[(6.626\cdot 10^{-34}\,J\cdot s)(3\cdot 10^{8}\,m/s)]^3} = 3.1998\cdot 10^{75}$$

$$= \cancel{3.1998\cdot 10^{75}}\ \frac{\cancel{\epsilon^3}}{\phantom{xxxxxx}}$$

$$kT = (1.381\cdot 10^{-23}\,J/k)(3000\,k) = 4.143\cdot 10^{-20}\,J$$

$$\therefore\quad u(\epsilon) = (3.1998\cdot 10^{75})\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$$

let $x \equiv \epsilon/kT$

$$u(x) = \frac{(3.1998\cdot 10^{75})(kT)^3 x^3}{(e^{x}-1)}$$

$$U(x) = (2.2755\cdot 10^{17})\frac{x^3}{(e^x-1)}$$

— — —

Spectrum of light $\quad U(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$

$$\epsilon = h\nu = \frac{hc}{\lambda}$$

$$U(\lambda) = \frac{8\pi}{(hc)^3}\frac{(hc)^3(\lambda^{-3})}{e^{\frac{hc}{kT}\cdot\frac{1}{\lambda}}-1} = \frac{8\pi}{\lambda^3}\frac{1}{\left(e^{\left(\frac{hc}{kT}\right)\cdot\frac{1}{\lambda}}-1\right)}$$

$$\left.\frac{hc}{kT}\right| = \frac{(6.626\cdot 10^{-34}\,J\cdot s)(3\cdot 10^8\,m/s)}{(1.381\cdot 10^{-23}\,J/k)(3000\,k)} = 4.798\cdot 10^{-6}\ m$$

$T=3000k$ $\qquad = 4.798\ \mu m \equiv \lambda_0$

$$U(\lambda) = \frac{8\pi}{\lambda^3}\frac{1}{(e^{\lambda_0/\lambda}-1)}$$ plot in units of

$\lambda \sim 0 \quad \frac{\infty}{\phantom{x}}\cdot\frac{1}{\infty} = \frac{\infty}{\infty}$ undetermined guess $\rightarrow 0.$

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$$I(T) = \int_{400\,nm}^{700\,nm} U(\theta)\, dt = \ldots$$ calculated radiated energy per unit time? in the visible spectrum.

(e)

(f) Find Maximum of $I_{vis}(T)$ intensity of radiated energy per unit time.

P.7.52

(a) $P = \sigma e A T^4$ $T \simeq 300\,K$

$A_{Body} \cong 1\,m^2$

$e = 1$

$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

$\Rightarrow P = (5.67 \cdot 10^{-8} \frac{W}{m^2 K^4})(1)(1m^2)(300\,K)^4 = 459.27\,W$

What is surface area of Body?

(b) $E/_{day} = P \cdot (3600)(24) = 3.968092 \cdot 10^7\,J$ radiated in photon's.

$Cal/_{day} = \cancel{8} \cdot 2000$ kilocal $= 8.3736 \cdot 10^6\,J$ ingested as calories.

~~\$~~ to look for humans get $T_H$ exactly + then max return from a black body at $T_H$ is $E \approx 2.8\,kT_H = \cancel{hc}\,\frac{hc}{\lambda} \Rightarrow$ tells one how to build

$f = \frac{c}{\lambda}$ a sensor to look for people.

Dont know?

(c) $\left(\frac{P}{M}\right)_{sun} = \frac{3.9 \cdot 10^{26}\,W}{(2 \cdot 10^{30}\,kg)} = 1.98 \cdot 10^{-4}\,W/kg$

$\left(\frac{P}{M}\right)_{Hum} = \frac{459.27\,W}{81.64\,kg} = 5.62\,W/kg$

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7.53

(a) $T_{BH} =$

$\epsilon_{peak} = 2.8kT_{BH} = 2.8(1.381 \cdot 10^{-23} J/k)(\cancel{\ldots} \cdot h$

$\epsilon_{peak} = 2.8\cancel{k}\dfrac{hc^3}{16\pi^2 \cancel{k} GM} = \dfrac{2.8}{16\pi^2 GM} hc^3$

$= \dfrac{2.8(6.626 \cdot 10^{-34} J \cdot s)(3 \cdot 10^8 m/s)^3}{16\pi^2(6.673 \cdot 10^{-11} N \cdot m^2/kg^2)(2 \cdot 10^{30} kg)}$

$= \cancel{B} 2.3769 \cdot 10^{-30} \dfrac{J \cdot \cancel{s}}{N \cdot \cancel{m^2}} \dfrac{\cancel{m^3}}{\cancel{s^3}} \cancel{kg}$

$=$ " $\dfrac{J\, m \cdot kg}{N \cdot s^2} = J.$ $\quad J = F \cdot L = N \cdot m$

$N = kg \cdot \dfrac{m}{s^2}$

$\lambda = \dfrac{\epsilon}{} \quad \epsilon = h\nu = \dfrac{hc}{\lambda}$

$\lambda = \dfrac{hc}{\epsilon}$

$\lambda_{pk} = \dfrac{hc}{\dfrac{2.8\, hc^3}{16\pi^2 G \cdot M}} = \dfrac{16\pi^2}{2.8} \dfrac{G \cdot M}{c^2} = 8.86 \cdot 10^4\, m$

$SA = 4\pi R^2 \quad R = \left(\dfrac{SA}{4\pi}\right)^{1/2} = 4\pi^{1/2} \dfrac{G M}{c^2}$

$= 1.05 \cdot 10^4\, m$

(b) $P = \sigma e A T^4$

$$= \sigma e \left(\frac{16\pi G^2 M^2}{c^4}\right)\left(\frac{hc^3}{16\pi^2 kGM}\right)$$

$$= \frac{\sigma e}{\pi c}\frac{GM}{k} = 5.8 \cdot 10^{26}\ W$$

(c) $E = Mc^2$

$$\frac{dE}{dt} = c^2\frac{dM}{dt} = P = \left(\frac{\sigma e}{\pi c}\frac{G}{k}\right) \cdot M$$

$$\frac{dM}{dt} = \left(\frac{\sigma e}{\pi c^3}\frac{G}{k}\right) M$$

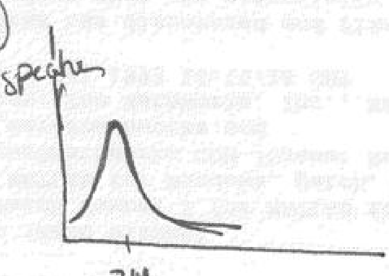
$$M = M_0 \exp\left\{ B\, t \Big/ \frac{\pi c^3 k}{\sigma e G} \right\}$$

$$t_0 = \frac{\pi c^3 k}{\sigma e G} = 3.09 \cdot 10^{20}\ s = 9.7 \cdot 10^{12}\ \text{years}$$

(e) $M_0 = ?$ $\quad M \ldots$

P.7.54

(a)




$$E_{peak} = 2.8kT \Rightarrow T = \frac{2.4\ eV}{2.8(8.617\cdot 10^{-5}\ eV/K)} = 9.94\cdot 10^{3}\ K$$

$$P = \sigma e A T^4 \qquad \cancel{P = \frac{(24\cdot 3.9\cdot 10^{26}\ W)}{(5.67\cdot 10^{-8}\ \frac{W}{m^2K^4})(1)(A)}}$$

$$24\cdot 3.9\cdot 10^{26}\ W = (5.67\cdot 10^{-8}\ \tfrac{W}{m^2K^4})(1)A\,(9.94\cdot 10^{3}\ K)^4$$

$$A = 1.69\cdot 10^{19}$$

$$4\pi R^2 = "$$

$$R = 1.16\cdot 10^{7}\ m = 1.16\cdot 10^{6}\ km$$

$$R_{sun} = 7\cdot 10^{8}\ m \qquad \text{that } 10\times \text{ larger}$$

(b) $P = (.03)(3.9\cdot 10^{26}\ W)$

$$E_{peak} = 2.8kT \underset{\text{set}}{=} 7\ eV$$

$$\Rightarrow T = \frac{7\ eV}{2.8(8.617\cdot 10^{-5}\ eV/K)} = 29{,}000\ K$$

$$P = \sigma e A T^4$$

$$\Rightarrow A = \frac{P}{\sigma e T^4} = \frac{(.03)(3.9\cdot 10^{26}\ W)}{(5.67\cdot 10^{-8}\ \frac{W}{m^2K^4})(1)(29\cdot 10^{3}\ K)^4} =$$

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$A = 2.9\cdot 10^{14}\ m^2$

$\Rightarrow R = 4\cdot 10^6\ m.$ About $\frac{1}{100}$ radius of our sun

(c) $\epsilon = 2.8kT = 0.8eV$

$$T = \frac{0.8\ eV}{(2.8)(8.617\cdot 10^{-5}\ eV/K)} = \cancel{3000K}.\ 3300K.$$

$$\epsilon = 0.8eV = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{\epsilon} = \frac{(4.136\cdot 10^{-15}\ eV\cdot s)(3\cdot 10^8\ m/s)}{.8\ eV} = 1.5\ \mu m$$

visible light $\sim$ 400 nm – 700 nm $\Rightarrow$ .4 $\mu m$ – .7 $\mu m$

$$P = (10^4\cdot 3.9\cdot 10^{26}\ W) = \sigma e A T^4$$

$$\Rightarrow A = \frac{P}{\sigma e T^4} = 5.8\cdot 10^{23}\ m^2$$

$\Rightarrow R = 2.1\cdot 10^{11}\ m$ $\qquad R_{sun} = 7\cdot 10^8$

$\approx$ 1000 times greater than our sun.

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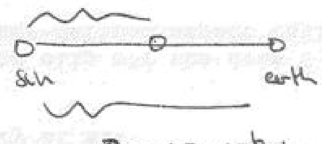
Prob 7.55    7.

P. 7.56  Infrared light responsible for

$$R = .7(150 \cdot 10^6 \text{ km}) = 105 \cdot 10^6 \text{ km}$$

(a)




$$R = 150 \cdot 10^6 \text{ km}$$

$$P = 3.9 \cdot 10^{26} \text{ W}$$

$$\text{Solar constant for Venus} = \frac{P}{4\pi (R^2)} = \frac{3.9 \cdot 10^{26} \text{ W}}{4\pi (105 \cdot 10^6 \cdot 10^3)^2 \text{ m}^2}$$

$$= 2.8 \cdot 10^3 \text{ W/m}^2$$

(a) $\Rightarrow$ (?)

Amt heat absorbed by

$$(\text{solar constant})\pi R^2 = \sigma e \cdot \frac{A}{\pi R^2} \cdot T^4 = \sigma e \, 4\pi R^2 \cdot T^4$$

$$\rightarrow \quad T = \left( \frac{2800 \text{ W/m}^2}{4\pi (5.67 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4})} \right)^{1/4} = 333.3 \text{ k}$$

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(b) Assuming on 30% of incident flux gets through

$$T = \left( \frac{(.3)(2800\ W/m^2)}{\cdots} \right)^{1/4} = \cdots \quad 246.69$$

(c) ?

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P. 7.57 eq 7.112 $U =$

eq 7.108 13

$$U = 3\int_0^{n_{max}} dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi\, n^2 \sin\theta \frac{\epsilon}{e^{\epsilon/kT} - 1}$$

$= \cancel{\ldots}$

$$= 3\left(\frac{\pi}{2}\right)\left(-\cos\theta\right)\Big|_0^{\pi/2} = \frac{3\pi}{2}(1) = \frac{3\pi}{2}$$

$$= \frac{3\pi}{2}\int_0^{n_{max}} dn\, n^2 \left(\frac{hc_s n}{2L}\right)\frac{1}{\left(e^{hc_s n/2LkT} - 1\right)}$$

let $x = \frac{hc_s n}{2LkT} \Rightarrow n = \frac{2LkT}{hc_s}\cdot x$

$$dn = \frac{2LkT}{hc_s}\, dx$$

$$= \frac{3\pi}{2}\int_0^{x_{max}} \left(\frac{2LkT}{hc_s}\right)\cdot dx \left(\frac{2LkT}{hc_s}\right)^2 x^2 (xkT) \frac{1}{\left(e^x - 1\right)}$$

$$= \frac{3\pi}{2}\left(\frac{2LkT}{hc_s}\right)^3 \int_0^{x_{max}} \frac{x^3\, dx}{e^x - 1}\cdot kT$$

$$= \frac{3\pi}{2}\left(\frac{2LkT}{hc_s}\right)^3 kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$x_{max} = \frac{hc_s}{2LkT}\cdot\left(\frac{6N}{\pi}\right)^{1/3} = \frac{hc_s}{2kT}\left(\frac{6N}{\pi V}\right)^{1/3} \equiv \frac{T_D}{T}$$

Then $T_D = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3}$ $\qquad \left(\frac{2k}{hc_s}\right) = \frac{1}{T_D}\left(\frac{6N}{\pi V}\right)^{1/3}$

$$U =$$

$$= \frac{3\pi}{2}\cdot L^3 T^3 \left(\frac{2k}{hc_s}\right)^3 kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$= \frac{3\pi}{2}\,\frac{1}{T_D^3}\left(\frac{6N}{\pi V}\right) k^3 T^3\, kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$= 3^2 \left(\frac{T}{T_D}\right)^3 NkT \int_0^{T_D/T} \frac{x^3\,dx}{e^x-1}$$

$$= \frac{9NkT^4}{T_D^3}\int_0^{T_D/T} \frac{x^3\,dx}{e^x-1} \qquad \text{eq } 7.112 \checkmark$$

$$C_V = \frac{\partial U}{\partial T}$$

$$U = \frac{3\pi}{2}\int_0^{n_{max}} \frac{hc_s}{2L} \frac{n^3}{\left(e^{hc_s n/2LkT} - 1\right)} dn$$

$$\frac{\partial U}{\partial T} = \frac{3\pi}{2}\int_0^{n_{max}} \frac{hc_s}{2L} \frac{n^3 e^{\frac{hc_s n}{2LkT}}\left(\frac{hc_s n}{2LkT^2}\right)(-1)}{\left(e^{hc_s n/2LkT} - 1\right)^2} dn$$

$$\frac{\partial U}{\partial T} = -\frac{3\pi}{2}\frac{hc_s}{2L}\left(\frac{hc_s}{2LkT^2}\right)\int_0^{n_{max}} \frac{n^4 e^{\frac{hc_s n}{2LkT}}}{\left(e^{hc_s n/2LkT} - 1\right)^2} dn$$

let $x = \frac{hc_s n}{2LkT}$ $\quad n = \frac{2LkT}{hc_s} x$ $\quad dn = \left(\frac{2LkT}{hc_s}\right) dx$

$$\frac{\partial U}{\partial T} = -\frac{3\pi}{2}\left(\frac{hc_s}{2L}\right)^2 \frac{1}{kT^2}\int_0^{x_{max}}$$

P.7.58 $c_s = 3560\ \text{m/s}$

$$T_D = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3} \qquad V = 7.12\ \text{cm}^3$$

$$N = N_A = 6.02\cdot 10^{23}$$

$$= \frac{(6.626\cdot10^{-34}\ \text{J}\cdot\text{s})(3560\ \text{m/s})}{2(1.381\cdot10^{-23}\ \text{J/K})}\left(\frac{6\cdot 6.02\cdot10^{23}}{\pi(7.12\cdot10^{-6}\ \text{m}^3)}\right)^{1/3}$$

$$= 465.0686\ \text{K}.$$

Experimentally:

How do I measure $T_D$ from this plot?

$$\text{Slope of curve} = \frac{12\pi^4 Nk}{5T_D^3}$$

$$\Rightarrow \quad = \frac{1.8 - .9}{16 - 2} = \frac{.9}{14} = .0643\ \frac{\text{mJ}}{\text{K}^4}$$

$$= .0643\cdot10^{-3}\ \frac{\text{J}}{\text{K}^4}$$

So that

$$\left(\frac{1}{T_D}\right)^3 = \frac{5}{12\pi^4 N_A k}(.0643\cdot10^{-3}\ \text{J/K}^4)$$

$$T_D = 311.5\ \text{K}$$

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P.7.59 According to the free electron model of specific heat w/ ...

$$\gamma = \frac{\pi^2 N k^2}{2\epsilon_F}$$

If $\epsilon_F$ is approximately the same $\forall$ species then the y-intercept will be also.

P.7.60 $C_V = \gamma T + \frac{12\pi^4 N k}{5 T_D^3} T^3 \quad T \ll T_D$

w/ $\gamma = \frac{\pi^2 N k^2}{2\epsilon_F}$ $\quad T_D = 465.06\,k$

$$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} = \cdots$$

$$\gamma T = \frac{12\pi^4 N k}{5 T_D^3} T^3$$

$$T^2 = (\quad)\gamma$$

P.7.61

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P.7.61

$$^4He = 4.0 + 2\text{ neutrons}$$

$$\approx 6.0\ g/\text{mole} = M \quad \text{atomic weight}$$

$$C_V = Nk\left(\frac{T}{4.67\,K}\right)^3 \qquad T < 0.6\,K$$

$$c_s = 238\ m/s \qquad \rho = 0.145\ g/cm^3 \qquad V = \frac{M}{\rho}$$

$$\frac{N}{V} = \frac{nN_A}{V} =$$

$V =$ (1) take 1 mole of $^4He$

(2) $N = N_A$

(3) $V = \dfrac{M}{\rho} \doteq \dfrac{[M]}{[\rho]} = \dfrac{g}{(g/cm^3)}$

$$\therefore \frac{N}{V} = \frac{N_A \rho}{M}$$

$$n \text{ moles} = nM \text{ grams} = \frac{N}{N_A} M$$

$$C_V = ? \qquad C_V = \frac{12\pi^4}{5}\left(\frac{T}{T_D}\right)^3 Nk$$

$$T_D = \frac{hc_s}{2k_B}\left(\frac{6N}{\pi V}\right)^{1/3}$$

$\frac{N}{V} =$ (1) compute $T_D = \ldots$

(2) compute $C_V$

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P. 7.62 7.112

$$U = \frac{9NkT^4}{T_D^3} \int_0^{T_D/T} \frac{x^3\,dx}{e^x - 1}$$

$$\frac{x^3}{e^x-1} = x^3 \cdot \frac{1}{(e^x-1)}$$

~~Let~~ $f(x) = \dfrac{x^3}{e^x-1}$

$$f(0) = \left.\frac{3x^2}{e^x}\right|_{x=0} = 0 \quad \text{“}\frac{0}{0}\text{”}$$

$$f'(x) = \frac{3x^2}{e^x-1} + \frac{x^3(-1)e^x}{(e^x-1)^2} = \frac{x^2}{(e^x-1)^2}\left[3(e^x-1) + x(-1)e^x\right]$$

$$= \frac{x^2}{(e^x-1)^2}\left(3e^x - 3 - xe^x\right) = \frac{x^2}{(e^x-1)^2} \cdots$$

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P. 7.63 ?

P. 7.64 $f \propto \left(\frac{1}{\lambda}\right)^2$

$E = hf$ + $p = \frac{h}{\lambda}$ for particles

$E \propto p^2$ for magnons

$E = \frac{p^2}{2m^*}$ $m^* = \text{constant} = 1.24 \cdot 10^{-29}\,kg$

$= 14 \cdot m_e$

(a) ?

(b) ?

↓

(P. 7.64)

$\epsilon = hf \qquad p = \frac{h}{\lambda}$

$\epsilon^2 = \frac{p^2}{2m^*} = \frac{h^2}{2m^*}\frac{1}{\lambda^2} \qquad \lambda = \frac{2L}{n}$ sam as for photons

(a) $N = g\sum_n n\, \bar{n}_{PL}(\epsilon) = \int_0^\infty n \ldots$ Not entirely sure

(b) $\frac{M(0) - M(T)}{M(0)} = \frac{2\mu_B N - 2\mu_B N_m}{2\mu_B N}$

$= 1 - \frac{N_m}{N} = \ldots$

c)

P. 7.65 eq 7.124

$$\int_0^\infty \frac{\sqrt{x}\,dx}{e^x-1} = \ldots$$

P. 7.66

(a) $b_0 =$

(b) $kT_c = (0.527)\left(\frac{h^2}{2\pi m}\right)\left(\frac{N}{V}\right)^{2/3}$

?

(d) ?

P. 7.67 ?

P. 7.68 ?

P. 7.69

(a) $N = \int_0^\infty g(t)\,\frac{dt}{e^{(t-\mu)/kT}-1}$ let $t = T/T_c$ + $x = \frac{t}{kT_c}$

$c = \frac{\mu}{kT_c}$

$$= \int_0^\infty g(x\,kT_c)\,\frac{(dx/kT_c)}{e^{(xkT_c - ckT_c)/ktT_c}-1} \qquad dt = \frac{dx}{kT_c}$$

$$= \int_0^\infty \frac{g(xkT_c)}{kT_c}\,\frac{dx}{e^{(x-c)/t}-1}$$

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7.122 is

P.7.70 (a) ?

P.7.71 ?

P.7.72 ?

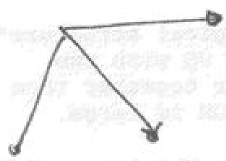
P.7.73 (a) ?

P.7.74 ?

P.7.75 ?

$$Z_{ideal} = \frac{1}{N!}\left(\frac{V Z_{int}}{V_Q}\right)^N \qquad V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$

n(



$$\binom{n}{2} = \frac{n(n-1)}{2}$$ connections for a network of n nodes

$$Z_c = \frac{1}{V^N}\int d^3r_1 \cdots d^3r_N \prod_{pairs} e^{-\beta u_{ij}}$$

$$e^{-\beta u_{ij}} = 1 + f_{ij}$$

$$\prod_{pairs} e^{-\beta u_{ij}} = \prod_{pairs}(1+f_{ij}) = (1+f_{12})(1+f_{13})(1+f_{14})\cdots(1+f_{1N})(1+f_{23})(1+f_{24})\cdots$$

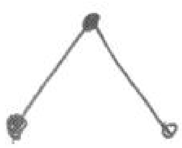
$$= 1 + \sum_{\substack{pairs \\ (connections)}} f_{ij} + \sum_{\substack{distinct \\ pairs}} f_{ij} f_{kl} + \cdots$$

$$\frac{1}{V^N}\int d^3r_1 \cdots d^3r_N\, f_{12} = \frac{1}{V^N}\, V^{N-2}\int d^3r_1\, d^3r_2\, f_{12}$$

$$= \frac{1}{V^2}$$


1 , 2
2 , 1

$$\frac{1}{V^2}\int d^3r_1\, d^3r_2\, f_{12} \qquad \frac{N(N-1)}{2}$$



$$\frac{N(N-1)(N-2)}{\boxed{2}\ \ 2!}$$

$$\cancel{N(N-1)}$$

$$\frac{N(N-1)(N-2)(N-3)}{8}$$

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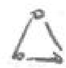
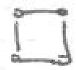
$$Z = Z_{ideal} \cdot Z_c$$

$$F = -kT \ln Z = -kT \ln Z_{ideal} - kT \ln Z_c$$

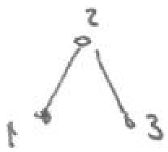
$$= -NkT \ln\left(\frac{V}{N v_Q}\right) - kT\Big($$  $+$  $+$  $+ \cdots \Big)$

$$P = -\left.\frac{\partial F}{\partial V}\right|_{N,T} =$$

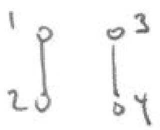
$$F = U - TS$$

P.8.1

 $= \dfrac{N(N-1)}{2V^2} \int d^3r_1\, d^3r_2\, f_{12}$

 $= \dfrac{N(N-1)(N-2)}{2V^3} \int d^3r_1\, d^3r_2\, d^3r_3\, f_{12} f_{23}$

Don't follow the symmetry factor very well

 $= \dfrac{N(N-1)(N-2)(N-3)}{2V^4} \int d^3r_1\, d^3r_2\, d^3r_3\, d^3r_4\, f_{12} f_{34}$

 $= \dfrac{N(N-1)(N-2)}{4\cancel{6}\, V^3} \int d^3r_1\, d^3r_2\, d^3r_3\, f_{12} f_{23} f_{31}$

$2 \cdot 2 = 4$

P.8.2 ...

P.8.3 ..

P.8.4 ...

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P. 8.15

$$U = -\epsilon + \epsilon + \epsilon - \epsilon - \epsilon - \epsilon + \epsilon - \epsilon - \epsilon - \epsilon - \epsilon + \epsilon$$

$$+ \epsilon + \epsilon - \epsilon + \epsilon - \epsilon + \epsilon - \epsilon \mp \epsilon + \epsilon$$

$$+ \epsilon - \epsilon - \epsilon$$

$$= -14\epsilon + 10\epsilon = -4\epsilon$$

P.8.16 100 dipols

$10^9$ terms ~~of~~ per second

$$Z = \sum_{\{s_i\}} e^{-\beta U}$$

# states + # of terms in the sum

$$2^N = 2^{100} \text{ terms}$$

$$\frac{2^{100}}{10^9} \text{ second} = 1.26 \cdot 10^{21} \text{ seconds} = 4.1 \cdot 10^{13} \text{ years}$$

P. 8.17 $\uparrow\uparrow$ ; $\uparrow\downarrow$ ; $\downarrow\uparrow$ ; $\downarrow\downarrow$

$$U = \quad -\epsilon \quad +\epsilon \quad +\epsilon \quad -\epsilon$$

$$Z = \sum_{\{s_i\}} e^{-\beta U} = 2e^{\beta\epsilon} + 2e^{-\beta\epsilon}$$

$$P_{\text{state antiparallel}} = \frac{2e^{-(\beta\epsilon)}}{2e^{\beta\epsilon} + 2e^{-\beta\epsilon}} = \frac{e^{-\beta\epsilon}}{e^{\beta\epsilon} + e^{-\beta\epsilon}}$$

$$P_{\text{state parallel}} = \frac{e^{+\beta\epsilon}}{e^{\beta\epsilon} + e^{-\beta\epsilon}}$$

$$P_{\perp}(kT/\epsilon \equiv x) = \frac{e^{-(1/x)}}{e^{1/x} + e^{-1/x}} =$$

$$P_{\parallel}(kT/\epsilon \equiv x) = \frac{e^{(1/x)}}{e^{1/x} + e^{-1/x}} =$$

I will plot this vs. $\epsilon/kT$ instead.

$$P_{\perp} = \frac{e^{-x}}{e^{x} + e^{-x}} \quad \& \quad P_{\parallel} = \frac{e^{+x}}{e^{x} + e^{-x}}$$

$$= \frac{1}{e^{2x} + 1} \qquad P_{\parallel} = \frac{1}{1 + e^{-2x}}$$

$$\epsilon \doteq O(1\,\text{eV}) \qquad x \equiv \frac{kT}{\epsilon}$$

$$k = O(8.617 \cdot 10^{-5}\,\text{eV}/K) \qquad x = T/(\epsilon/k)$$

$$\frac{\epsilon}{k} = O\left(\frac{1}{8.617 \cdot 10^{-5}\ 1/K}\right)$$

$$= O(1.1 \cdot 10^{4}) \qquad 100 < T < 1000$$

$$\Rightarrow \cancel{x} \qquad \frac{1}{100} < x < \frac{1}{10}$$

$$\sum_{s_N} e^{\beta\epsilon s_{N-1} s_N} = \begin{cases} = e^{\beta\epsilon(1)(1)} + e^{\beta\epsilon(1)(-1)} \\ = e^{\epsilon\beta(-1)(1)} \end{cases} = 2\cosh(\beta\epsilon)$$

$$Z = \underbrace{\sum_{s_1}\sum_{s_2}\cdots\sum_{s_N}}_{N \text{ sums}} \underbrace{e^{\beta\epsilon s_1 s_2} e^{\beta\epsilon s_2 s_3} \cdots e^{\beta\epsilon s_{N-2} s_{N-1}} \underbrace{e^{\beta\epsilon s_{N-1} s_N}}_{2\cosh(\beta\epsilon)}}_{N-1 \text{ terms}}$$

$$= \sum_{s_1} (2\cosh(\beta\epsilon))^{N-1} = 2^N (\cosh(\beta\epsilon))^{N-1}$$

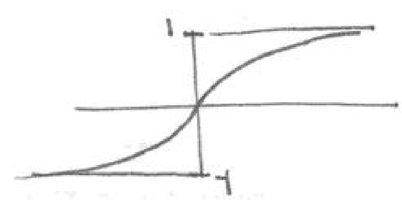
$$\approx 2^N \cosh(\beta\epsilon)^N$$

$$\bar{U} = -\frac{\partial}{\partial\beta}\ln Z = -\frac{\partial}{\partial\beta}\left(N \ln\cosh(\beta\epsilon)\right)$$

$$= \frac{-N}{\cosh(\beta\epsilon)}\sinh(\beta\epsilon)\cdot\epsilon = -N\epsilon\tanh(\beta\epsilon) \quad \text{eq } 8.44$$

$T \to \infty \Rightarrow \beta \to 0$

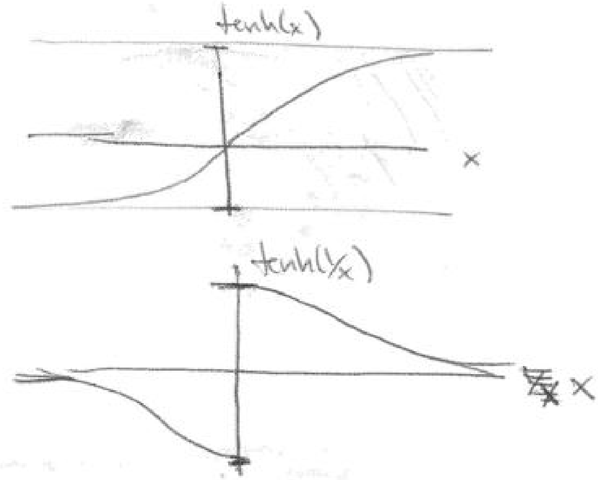
$T \to 0 \Rightarrow \beta \to \infty$



(3.13)

$$\bar{U} = -\frac{\partial}{\partial\beta}\ln Z$$ see notes pg 342

$$U(T) = -N\epsilon\tanh(\beta\epsilon)$$



(P. 8.19) $kT_c = n\epsilon$

$$\epsilon = \frac{kT_c}{n}$$

$$\epsilon = \frac{(8.617\cdot10^{-5}\,eV/k)(1043k)}{n} = \cdots$$

Think iron is BCC. $\Rightarrow n = 8$

(P.8.20) $\bar{s} = \tanh(\beta\epsilon n\bar{s})$ implicitly this is a fn of

$$s = s(\beta\epsilon) = s\left(\frac{\epsilon}{kT}\right)$$

From $kT_c = n\epsilon$

$\frac{kT_c}{\epsilon} \sim n \Rightarrow$ plot Above for $\frac{kT}{\epsilon} \sim 1-20.$

(P.8.21) $\bar{s} = \tanh(\beta\epsilon n\bar{s}) = \frac{\sinh(\beta\epsilon n\bar{s})}{\cosh(\beta\epsilon n\bar{s})}$

$T = 0 \Rightarrow \beta = +\infty$

$$= \frac{e^{\beta\epsilon n\bar{s}} - e^{-\beta\epsilon n\bar{s}}}{e^{\beta\epsilon n\bar{s}} + e^{-\beta\epsilon n\bar{s}}} \cdot \frac{e^{-\beta\epsilon n\bar{s}}}{e^{-\beta\epsilon n\bar{s}}}$$

$=$

$\bar{s} = \cancel{f(x_0) + f'(x_0)}($

Expand about $T = 0$ $\qquad \operatorname{sech}^2(\beta \epsilon n \bar{s}) \cdot \frac{(\epsilon n \bar{s})(-1)}{kT^2}$ $\qquad$ A, M, J, J|

$\bar{s} = 1 +$

let $\bar{s} = \tanh(\frac{1}{x})$ $\qquad x \to \infty$

$\bar{s} = \tanh(x)$ $\qquad x \ll 1$

Dammit how do you take the limit as $x \to \infty$ + do a taylor series type expansion?

$\bar{s} = \tanh(\beta \epsilon n \bar{s})$

$\to \beta \epsilon n \bar{s} = \tanh^{-1}(\bar{s})$

$\frac{\epsilon n \bar{s}}{kT} = \tanh^{-1}(\bar{s})$

$\epsilon n \bar{s} = kT \tanh^{-1}(\bar{s})$

$\frac{\epsilon n \bar{s}}{\tanh^{-1}(\bar{s})} = kT$

P.B.22 $\pm \mu_B B$

$$E_\uparrow = -\epsilon \sum_{\text{neighbors}} s_{\text{neighbor}} - \mu_B B$$

$$= -\epsilon n \bar{s} - \mu_B B$$

$$E_\downarrow = +\epsilon n \bar{s} + \mu_B B$$

$$Z_i = e^{-\beta E_\downarrow} + e^{-\beta E_\uparrow} = e^{-\beta(\epsilon n \bar{s} + \mu_B B)} + e^{-\beta(-\epsilon n \bar{s} - \mu_B B)}$$

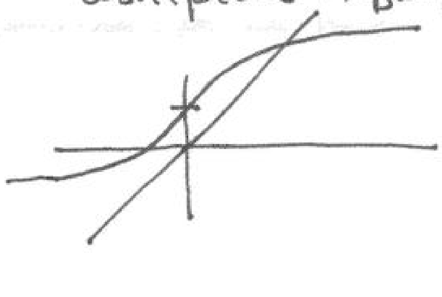
$$= e^{-\beta(\epsilon n \bar{s} + \mu_B B)} + e^{+\beta(\epsilon n \bar{s} + \mu_B B)}$$

$$= 2\cosh(\beta(\epsilon n \bar{s} + \mu_B B))$$

Avg

$$\bar{s}_i = \frac{1}{Z_i}\left[+1 e^{+\beta(\epsilon n \bar{s} + \mu_B B)} - 1 e^{-\beta(\epsilon n \bar{s} + \mu_B B)}\right]$$

$$\bar{s} = \frac{\sinh(\beta(\epsilon n \bar{s} + \mu_B B))}{\cosh(\beta(\epsilon n \bar{s} + \mu_B B))} = \tanh(\beta \epsilon n \bar{s} + \beta \mu_B B)$$



2 variables

$\beta \epsilon n + \beta \mu_B B$

P. 8.23 (a) ?

(b) ?

(c) ?

P. 8.24 (a) $\tanh x \approx x - \frac{1}{3}x^3$

(b) $\bar{s} \cong \beta \epsilon n \bar{s} - \frac{1}{3}(\beta \epsilon n \bar{s})^3$

?

(c) $\chi = \left.\frac{\partial M}{\partial B}\right|_T$

Prob A.1

(a) $hc = (6.63 \cdot 10^{-34}\ \text{J}\cdot\text{s})(3 \cdot 10^{8}\ \text{m/s})$

$= 1.98 \cdot 10^{-25}\ \text{J}\cdot\text{m}$

$1\ \text{m} = 10^{9}\ \text{nm}$

$1\ \text{J} = \frac{1}{1.602 \cdot 10^{-19}}\ \text{eV}$

$\therefore\ hc = \frac{1.98 \cdot 10^{-25} \cdot 10^{9}}{1.602 \cdot 10^{-19}}\ \text{eV nm}$

$= 1241.\ \text{eV nm}$

(b) $E_{photon} = hf = \frac{hc}{\lambda} \cong \frac{1240\ \text{eV}\cdot\text{nm}}{\lambda}$

$\therefore\ E(\text{red light}) = \frac{1240\ \text{eV}\cdot\text{nm}}{680\ \text{nm}} = 1.9\ \text{eV}$

$E(\text{blue light}) = \frac{1240\ \text{eV}\cdot\text{nm}}{450\ \text{nm}} = 2.7\ \text{eV}$

$E(\text{x-ray}) = \frac{1240\ \text{eV}\cdot\text{nm}}{.1\ \text{nm}} = 1.24 \cdot 10^{4}\ \text{eV}$

$E(\text{cosmic radiation}) = \frac{1240\ \text{eV}\cdot\text{nm}}{10^{6}\ \text{nm}} = 1.24 \cdot 10^{-3}\ \text{eV}$

$1\ \text{mm} = 10^{-3}\ \text{m}$
$= 10^{-3} \cdot 10^{9}\ \text{nm}$
$= 10^{6}\ \text{nm}$

07-24-02 &

(c) $1\cdot 10^{-3}$ W at red laser

$E(\text{red}) \cong 2$ eV. per photon

$$1\text{ W} = 1\text{ J/s} = \frac{1}{1.602\cdot 10^{-19}}\text{ eV/s} = 6.24\cdot 10^{18}\text{ eV/s}$$

$$\therefore\ \#\text{photons} = (3.12\cdot 10^{18}\text{ photons/s})(10^{-3})$$

since it is a 1 milliwatt laser

$$= 3.12\cdot 10^{+16}\text{ photons/sec.}$$

Prob. A.2 $K_{max} = Ve = (.8)e = .8$ eV.

(a) $K_{max} = \frac{hc}{\lambda} - \phi$

$$\Downarrow$$

$$.8\text{ eV} = \frac{1240\text{ eV}\cdot\text{nm}}{400\text{ nm}} - \phi$$

$$\phi = 2.3\text{ eV.}$$

(b) $\phi$ does not change

$$K_{max} = \frac{1240\text{ eV}\cdot\text{nm}}{300\text{ nm}} - 2.3\text{ eV} = 1.8\text{ eV}$$

$\Rightarrow V = 1.8$ eV $\quad V(300\text{ nm}) = 1.8$ V

$V(500\text{ nm}) = .18$ V

$V(600\text{ nm}) = -.23$ V.

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Prob A.3 $E = hf$ $E = pc$

$f \cdot \lambda = c$

$hf = pc$

$h\frac{f}{c} = p \rightarrow \frac{h}{\lambda} = p. \rightarrow \lambda = \frac{h}{p}$

Prob A.4

$$E = \frac{mc^2}{\sqrt{1+(\frac{v}{c})^2}} \qquad p = \frac{mv}{\sqrt{1+(\frac{v}{c})^2}}$$

If $v=c$.

$E =$

?

Prob A.5

$$E = pc$$

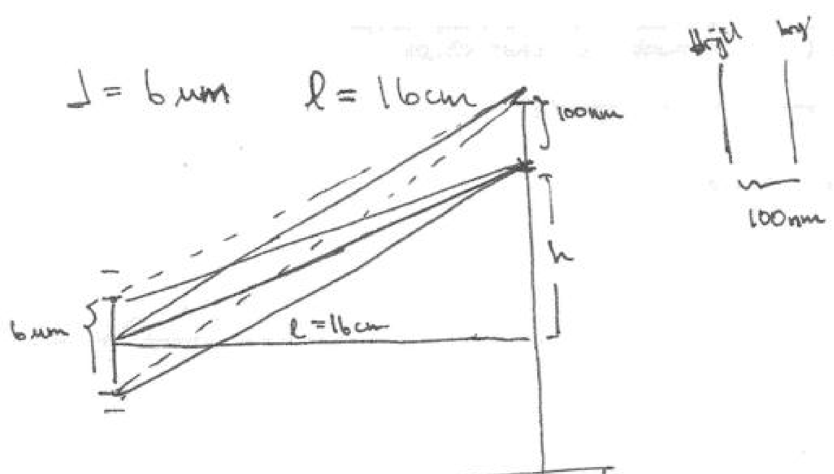
$$10^5\,eV = pc \Rightarrow p = \frac{10^5\,eV}{3 \cdot 10^8\,m/s}$$

$1\,eV = 1.602 \cdot 10^{-19}\,J$

$p = 5.34 \cdot 10^{-23}\,kg\,m/s$ $\qquad 1J = kg \cdot m^2/s^2$

$$\lambda = \frac{h}{p} = \frac{6.626 \cdot 10^{-34}\,J \cdot s}{5.34 \cdot 10^{-23}\,kg\,m/s} = 1.24 \cdot 10^{-11}\,\frac{kg \cdot m^2/s^2 \cdot s}{kg\,m/s} = m.$$

Prob A.6 $\quad d = 6\,\mu m \quad \ell = 16\,cm$




path length for 1st maximum $\hat{=} \sqrt{h^2+\ell^2}$

path " " 2nd max. $\hat{=} \sqrt{(h+\Delta)^2+\ell^2}$

$$\lambda \cdot \lambda = \sqrt{(h+\Delta)^2 + \ell^2} - \sqrt{h^2+\ell^2}$$

$$=$$

How Do?

But given $\lambda$ one has $E = eV = pc = \frac{h}{\lambda} c$ one ~~&~~ can calculate the voltage

Prob A.7

(a) $\lambda = \frac{h}{p}$ $\quad \frac{1}{2}\frac{p^2}{m} = K.E.$

$$p = \sqrt{2mk} =$$

$m_{neutron} = ? \simeq m_{proton} = 1.673 \cdot 10^{-27}$ kg

$k = 1eV = 1.602 \cdot 10^{-19} J.$

$\therefore p = 2.3 \cdot 10^{-23}$ kgm/s

(b) $m \cong 1lb = \frac{1}{2.2}$ kg

$1kg = 2.2lbs$
$1lb = \frac{1}{2.2}$ kg

$v \cong 90 \, mi/hr = 40 \, m/s$

$p = 18.1$ kgm/s

$\lambda = \frac{h}{p} = 3.6 \cdot 10^{-35}$ m

Prob A.8

$$\psi(x) = A(\cos(kx) + i\sin(kx))$$

(a) $k = \frac{2\pi}{\lambda}$ w/ $\lambda$ the deBroglie wave length

$$p = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p}$$

$$k = \frac{2\pi}{h} \cdot p = \frac{2\pi p}{h}$$

How to justify?

(b) $|\psi(x)|^2 = |A|^2(\cos(kx) + i\sin(kx))(\cos(kx) - i\sin(kx))$

$$= |A|^2(\cos^2(kx) + \sin^2(kx)) = |A|^2$$

$$P_{ab} = \int_a^b |\psi|^2 dx = |A|^2(b-a)$$

$P_{-\infty,+\infty} = \int_{-\infty}^{+\infty} |A|^2 dx = 1$ if $A \neq$ infinitesimal ~~te~~

(d) $\frac{d\psi}{dx} = A(-\sin(kx) + i\cos(kx))k$

$$= i(\psi)k$$

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A9

(a) $\psi(x)\psi^* = A^2 e^{-2ax^2} = |\psi|^2$

(b)
$$\int_{-\infty}^{+\infty} |\psi|^2 dx = 1$$

(c)
$$\overline{x^2} = \int_{-\infty}^{+\infty} x^2 |\psi|^2 dx = \ldots$$

$$\Delta x = \sqrt{\overline{x^2} - \overline{x}^2}$$

(d)
$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-ikx} A e^{ik_0 x - ax^2} dx$$

(e)
$$\Delta k = \sqrt{\overline{k^2} - \overline{k}^2} = \int_{-\infty}^{+\infty} k^2 |\tilde{f}(k)|^2 dk$$

(f) $\Delta p_x =$

$k = \frac{2\pi}{\lambda}$ $\quad p = \frac{h}{\lambda} \Rightarrow p = \frac{h \cdot k}{2\pi}$

$\Delta p = \frac{h \Delta k}{2\pi} = \hbar \Delta k$

$\lambda = \frac{2\pi}{k}$

$$\Delta x \cdot \Delta p = \frac{1}{2\sqrt{a}} \cdot \hbar \cdot \sqrt{a} = \frac{\hbar}{2}$$

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A.11

$$\sin(x) - \sin(2x)$$

$$\sin(x) + \sin(2x)$$

$$\sin(x) \times (2x)$$

A.12

$$l = 10^{-15}\ \text{m}$$

$$E = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$E_{min} = \frac{h^2}{8m(L)^2}(1+1+1)$$

Prob A,12

$$E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2)$$

Prob A,13

$$E = \frac{p^2}{2m} \quad \text{but} \quad E = pc$$

(a) $p_n$ is still quantized by quantizing the wave length ...

$$\therefore E_n = p_n c = c\left(\frac{h}{\lambda_n}\right)$$

$$\lambda_n = \frac{2L}{n}$$

$$\therefore E_n = ch\left(\frac{n}{2L}\right) = \frac{ch}{2L}n$$

(b) $$E_1 = \frac{(3\cdot10^8\ m/s)(6.626\cdot10^{-34}\ J\cdot s)}{2(10^{-15}\ m)} = 9.939\cdot10^{-11}\ J$$

$$= 6.02\cdot10^8\ eV$$

They would have to have a huge amount of energy ... prob. to much.

(c) $$E_1 = 6.02\cdot10^8\ eV \qquad 1J = F\cdot L = M\frac{L^2}{s^2}$$

$$\frac{E_1}{c^2} = 1.104\cdot10^{-27}\ J/(m/s)^2$$

$$= \text{''} \quad (kg)$$

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A.14 $$E = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$n_x^2 + n_y^2 + n_z^2 \le 15$$

$\Rightarrow$ ~~vecto~~ 4 $\vec{n} = (1,0,0),\ (0,1,0),\ (0,0,1)$

$(1,1,0),\ (1,0,1),\ ($

Avg # of states per unit energy increases.

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Prob. A.15 $f = 6.4 \cdot 10^{13}\ s^{-1}$

(a) $E = \frac{1}{2}hf,\ \frac{3}{2}hf,\ \frac{5}{2}hf,\ \frac{7}{2}hf,\ \frac{9}{2}hf$

$= \cancel{\frac{1}{2}(6.626 \cdot 10^{-34}\ J \cdot s)(6.4 \cdot 10^{13}\ s^{-1})},$

Now $hf = (4.136 \cdot 10^{-15}\ eV \cdot s)(6.4 \cdot 10^{13}\ s^{-1})$

$= \cancel{.2647}\ .2647\ eV$

$.1323\ eV,\ .397\ eV,\ .6617\ eV,\ \cancel{.926\ eV}\ .926\ eV,$

$1.19\ eV$

(b) $E = h\nu_L = \frac{hc}{\lambda}$

$\frac{hc}{\lambda} = \frac{1}{2}hf \Rightarrow \nu_L = \frac{f}{2} = 3.2 \cdot 10^{13}\ s^{-1}$

$\lambda = \frac{c}{\nu_L} = 9.3 \cdot 10^{-6}\ m = 9.3\ \mu m$

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Prob A16

(a) ~~???~~ ? Don't know what is being asked?

(b) ?

(c) ?

(d) ?

(e) ?

Prob A17 $E = \frac{(2n_1+1)}{2} hf + \frac{(2n_2+1)}{2} hf$

$= (n_1 + n_2) hf + hf$

$= (n_1+n_2+1) hf$

if E is fixed how many ways can we distribute the energy

$\underbrace{|| \ | \ ||||| \ |}_{n_1+n_2 \text{ sticks}} \quad |$

$n_1+n_2$ sticks + ~~2~~ we are asked to select

1 location from the $n_1+n_2+1$ spots

$\Rightarrow \binom{n_1+n_2+1}{1}$ is the degeneracy

$= n_1+n_2+1$

Prob A718

$$E = \frac{(2n_1+1)fh}{2} + \frac{(2n_2+1)fh}{2} + \frac{(2n_3+1)fh}{2}$$

$$= (n_1+n_2+n_3)hf + \frac{3}{2}fh$$

$$\binom{n_1+n_2+n_3+1}{2} = \text{degeneracy for a given energy.}$$

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A,19

~~B~~ $\Delta E = E_2 - E_1$ $\qquad\qquad$ $E_n = -\dfrac{13.6\,eV}{n^2}$

$= \dfrac{-13.6}{4} + 13.6 =$

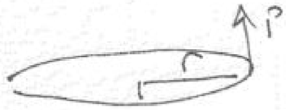
$E_2 > E_1$

n=2

n=1

$\Delta E = h\nu = \dfrac{hc}{\lambda}$

(A.20)



$\pm rp$

$f = f(s) = A\sin(\lambda s)$ ? Don't follow

(A.21)

$n = 1, 2, 3, \ldots$

$\ell = 0, 1, 2, 3, \ldots, n-1$

$m = -\ell, -\ell+1, -\ell+2, \ldots \ell-1, \ell$

$\underline{n=1}$:

$\ell = 0$ only
$m = 0$ only $\Big\}$ # independent states $= 1^2 = 1$ ✓

$\underline{n=2}$: Then independent states are

$\ell = 0, 1$ $(n, \ell, m)$:

$m=0$, $m = -1, 0, +1$ $(2,0,0), (2,1,-1), (2,1,0), (2,1,+1)$

# independent states $= 4 = 2^2$ ✓

$\underline{n=3}$: Then independent states are

$\ell = 0, 1, 2$ $(n, \ell, m)$

$m=0$, $m = -1, 0, +1$, $m = -2, -1, 0, +1, +2$ $(3,0,0)$; ~~[illegible]~~

$(3,1,-1)$; $(3,1,0)$; $(3,1,+1)$

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$(3,2,-2); (3,2,-1); (3,2,0); (3,2,+1); (3,2,+2)$

# independent states

$= 5 + 3 + 1 = 9 = 3^2$. ✓

Prob A.22

$$\epsilon = \frac{\hbar^2}{2I}$$

(a) $$\frac{\hbar^2}{2I} = .00024\,eV$$

$$E_{rot} = j(j+1)\cdot\epsilon$$

$$\Delta E = E(j=1) - E(j=0) = \epsilon = h\nu = \frac{hc}{\lambda}$$

$$= \frac{(4.136\cdot10^{-15}\,eV\cdot s)(3\cdot10^{8}\,m/s)}{\lambda} = 2.4\cdot10^{-5}\,eV$$

(b) $$I = \frac{\hbar^2}{2\epsilon} = \frac{(4.136\cdot10^{-15}\,eV\cdot s)^2}{4\pi^2}\,\frac{1}{2(2.4\cdot10^{-5}\,eV)}$$

$$= \left(\frac{1}{8\pi^2}\right)(\quad)^2\,eV\cdot s^2$$

$$1\,eV = 1.602\cdot10^{-19}\,J$$

$$1\,eV\cdot s^2 = J\cdot s^2$$

$$[I] = kg\,m^2$$

$$[E] = [F\cdot d] = \left\{kg\frac{m}{s^2}\cdot m\right.$$

$$= kg\frac{m^2}{s^2}$$

$$= \frac{[I]}{s^2}$$

$$\Rightarrow [I] = [E]\cdot s^2 \quad \checkmark$$

(c) $m_O \cong \dfrac{16\,g}{6.022\cdot 10^{23}} \approx \dfrac{8}{3}\cdot 10^{-23} < 2.6\cdot 10^{-23}\,g$

$m_C \cong \dfrac{12\,g}{6.022\cdot 10^{23}} \approx 2\cdot 10^{-23}\,g$

$I = m_O r_{1/2}^2 + m_C r_{1/2}^2$

$= r_{1/2}^2 (m_O + m_C)$

$\Rightarrow r = 2 r_{1/2} = 2\sqrt{\dfrac{I}{(m_O+m_C)}} \approx 2\sqrt{\dfrac{I}{5\cdot 10^{-23}\,g}}$

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A.23 $|J| = \sqrt{s(s+1)}\,\hbar$

$s = 1/2$

$$|J| = \sqrt{\tfrac{1}{2}\cdot\tfrac{3}{2}}\,\hbar$$

$$J_z \in \{ s\hbar, (s-1)\hbar, \ldots, (-s+1)\hbar, -s\hbar \}$$

$$J_z = \tfrac{1}{2}\hbar, -\tfrac{1}{2}\hbar.$$

$s = 3/2$

$$|J| = \sqrt{\tfrac{3}{2}\cdot\tfrac{5}{2}}\,\hbar$$

$$J_z = \tfrac{3}{2}\hbar, \tfrac{1}{2}\hbar, -\tfrac{1}{2}\hbar, -\tfrac{3}{2}\hbar.$$

(A 24)

$E_0 = \frac{1}{2}kT$ $\quad \lambda > \frac{\ell}{2} \therefore \exists$ a smallest $\lambda$

$\lambda_{min} \geq \frac{\ell}{2}$ w/ $\ell$ = lattice spacing of 1D lattice

(a) $E_{\vec{n}} = G(n_x^2 + n_y^2 + n_z^2)$

w/ $n_i = p\frac{2\pi}{L}$

$E_{(\ell,m,n)} = G\frac{4\pi^2}{L^2}(\ell^2 + m^2 + n^2)$ ?

(b) How get $\sqrt{\frac{G\hbar}{c^3}}$

$[G] = \frac{N \cdot m^2}{kg^2}$ $\quad [\hbar] = J \cdot s = kg\frac{m^2}{s^2}s = kg\frac{m^2}{s}$

$[c] = m/s$

$\therefore \left[\frac{G\hbar}{c^3}\right] = \frac{\left(\frac{N \cdot m^2}{kg^2}\right)\left(\frac{kg\, m^2}{s}\right)}{\frac{m^3}{s^3}} = \frac{N \cdot m^4 s^2}{kg\, m^3} = \frac{kg\frac{m}{s^2}s^2}{kg} = m^2$ ✓

$\sqrt{\frac{(6.673 \cdot 10^{-11}\, N \cdot m^2/kg^2)(6.626 \cdot 10^{-34}\, J \cdot s)}{(3 \cdot 10^8)^3 \quad (2\pi)}}$

$$= \left( \frac{6.6^2}{3^3} \frac{10^{-11-34-24}}{2\pi} \right)^{1/2}$$

$$\begin{array}{r} 45 \\ 24 \\ \hline 69 \end{array}$$

$$= \left( \frac{6.6^2}{3^3 \cdot 2\pi} \right)^{1/2} \left( 10^{-69} \right)^{1/2}$$

$$= \left( \frac{6.6^2}{3^3\, 2\pi} \right)^{1/2} 10^{-34.5}$$

(c) $\int_{e_p}$

$\bar{E}$

$$\frac{J/m^3}{m/s} = \frac{\frac{N \cdot m}{m^3}}{m/s} = \frac{\frac{N}{m^2}}{m/s} = \frac{N \cdot s}{m}$$

$$= \frac{kg\,\frac{m}{s}}{m} = \frac{kg}{s}$$

B.1

$$F(x) = \int_0^x e^{-\xi^2}\,d\xi = \frac{\sqrt{\pi}}{2}\,\mathrm{erf}(x) \quad \text{plot.}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-\xi^2}\,d\xi$$

B.2

$$\int_0^{\infty} x^2 e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$\frac{d}{da} \rightarrow$

$$-\int_0^{\infty} x^4 e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\pi}\,\frac{d(a^{-3/2})}{da} = \frac{\sqrt{\pi}}{4}\left(-\frac{3}{2}\right)a^{-5/2}$$

$$\Rightarrow \int_0^{\infty} x^4 e^{-ax^2}\,dx = \frac{3\sqrt{\pi}}{8\,a^{5/2}}$$

B.3 (a) $= 0$

(b) $F(x) = \int_0^x v e^{-av^2}\,dv =$

$$u = av^2$$

$$du = 2av\,dv \Rightarrow v\,dv = \frac{du}{2a}$$

$$F(x) = \int_0^{ax^2} \frac{du}{2a} e^{-u} = -\frac{1}{2a}\left(e^{-u}\right|_0^{ax^2}$$

$$= \cancel{\ } -\frac{1}{2a}\left(e^{-ax^2} - 1\right)$$

$$\therefore -\frac{e^{-ax^2}}{2a} + \frac{1}{2a} = F(x)$$

(c) $\displaystyle\int_0^\infty x e^{-ax^2}\,dx = F(\infty) = +\frac{1}{2a}$ ? Can't be correct.

(d) $\displaystyle\int_0^\infty x e^{-ax^2}\,dx = \frac{1}{2a}$

$\frac{d}{da} =$

$$+\int_0^\infty x(-x^2)e^{-ax^2}\,dx = \frac{-1}{2a^2}$$

$$\int_0^\infty x^3 e^{-ax^2}\,dx = \frac{1}{2a^2}$$

B.4 

$$\int_x^\infty e^{-t^2} dt = ?$$

let $s = t^2$

$ds = 2t\,dt \Rightarrow$ ~~dtt~~ $dt = \frac{ds}{2t} = \frac{ds}{2\sqrt{s}}$

$$= \int_{x^2}^\infty e^{-s} \cdot \frac{ds}{2\sqrt{s}}$$

$$= \frac{1}{2} \int_{x^2}^\infty s^{-1/2} e^{-s}\, ds$$

expand $s^{-1/2}$ in taylor series abt $x^2$

$$s^{-1/2} \cong x^{-1} + \frac{d}{ds}(s^{-1/2})\Big|_{s=x^2} (s - x^2) + \frac{1}{2!}\frac{d^2}{ds^2}(s^{-1/2})\Big|_{s=x^2} (s - x^2)^2 + \cdots$$

$\cong x^{-1}$

$$\frac{d}{ds}(s^{-1/2}) = -\frac{1}{2} s^{-3/2}\Big|_{x^2} = -\frac{1}{2} x^{-3}$$

$$\frac{d^2}{ds^2}(s^{-1/2}) = \frac{3}{4} s^{-5/2}\Big|_{x^2} = \frac{3}{4} x^{-5}$$

$$s^{-1/2} = \frac{1}{x} - \frac{1}{2x^3}(s-x^2) + \frac{3}{24x^5}(s-x^2)^2 + O((s-x^2)^3)$$

$$\frac{1}{2}\int_{x^2}^{\infty} s^{-1/2}e^{-s}\,ds = \frac{1}{2}\int_{x^2}^{\infty}\frac{1}{x}e^{-s}\,ds - \frac{1}{2\cdot 4x^3}\int_{x^2}^{\infty}(s-x^2)e^{-s}\,ds$$

$$+ \frac{3}{8x^5}\int_{x^2}^{\infty}(s-x^2)^2e^{-s}\,ds + \cdots$$

$$= \frac{1}{2x}\left[-e^{-s}\right|_{x^2}^{\infty} - \frac{1}{2\cdot 4x^3}\left[\int_{x^2}^{\infty}se^{-s}\,ds - x^2\int_{x^2}^{\infty}e^{-s}\,ds\right]$$

$$+ \frac{3}{8x^5}\left[\int_{x^2}^{\infty}(s^2-2sx^2+x^4)e^{-s}\,ds\right]$$

$$= \frac{1}{2x}\left[0+e^{-x^2}\right] \cancel{- \frac{1}{2\cdot 4x^3}\left[-se^{-s}\right|_{x^2}^{\infty}} + \int_{x^2}^{\infty}e^{-s}\,ds\Bigg]$$

$$+ \frac{1}{4x}\left[-e^{-s}\right|_{x^2}^{\infty} + \frac{3}{8x^5}\left[-s^2e^{-}\right.$$

$$\therefore \int_x^\infty e^{-t^2}dt = \frac{1}{2}\int_{x^2}^\infty \frac{1}{x} e^{-s}ds - \frac{1}{4x^3}\int_{x^2}^\infty (s-x^2)e^{-s}ds$$

$$v = s - x^2$$

$$dv = ds$$

$$+ \frac{3}{2\cdot 8x^5}\int_{x^2}^\infty (s-x^2)^2 e^{-s}ds + \dots$$

$$s = v + x^2$$

$$= \frac{1}{2x}\left(-e^{-s}\Big|_{x^2}^\infty\right) - \frac{1}{4x^3}\int_0^\infty v e^{-v-x^2}dv$$

$$+ \frac{3}{2\cdot 8x^5}\int_0^\infty v^2 e^{-v-x^2}dv$$

$$= \frac{1}{2x}e^{-x^2} - \frac{e^{-x^2}}{4x^3}\left[\cancelto{0}{-ve^{-v}}\Big|_0^\infty + \int_0^\infty e^{-v}dv\right]$$

$$+ \frac{3}{2\cdot 8x^5}e^{-x^2}\left[-v^2e^{-v}\Big|_0^\infty + 2\int_0^\infty ve^{-v}dv\right]$$

$$= \frac{e^{-x^2}}{2x} - \frac{e^{-x^2}}{4x^3} + \frac{3}{2\cdot 8x^5}e^{-x^2}\,2\left[\cancelto{0}{-ve^{-v}}\Big|_0^\infty + \int_0^\infty e^{-v}dv\right]$$

$$= e^{-x^2}\left(\frac{1}{2x} - \frac{1}{4x^3} + \frac{3}{8x^5} + \dots\right)$$

$\xi$

B.5

$$I(x) = \int_x^\infty t^2 e^{-t^2}\, dt$$

let $s = t^2$

$$ds = 2t\,dt \rightarrow dt = \frac{ds}{2t} = \frac{ds}{2s^{1/2}}$$

$$= \int_{x^2}^\infty s e^{-s} \frac{ds}{2s^{1/2}} = \frac{1}{2}\int_{x^2}^\infty s^{1/2} e^{-s}\, ds$$

$$s^{1/2} = x.$$

$$\frac{ds^{1/2}}{ds} = \frac{1}{2} s^{-1/2} \Big|_{x^2} = \frac{1}{2x}$$

$$\frac{d^2 s^{1/2}}{ds^2} = \frac{-1}{4} s^{-3/2} \Big|_{x^2} = \frac{-1}{4x^3}$$

$\vdots$

$$I(x) = \frac{1}{2}\int_{x^2}^\infty x e^{-s}\, ds + \frac{1}{2}\cdot\frac{1}{2x}\int_{x^2}^\infty (s-x^2) e^{-s}\, ds - \frac{1}{2\cdot 2}\frac{1}{4x^3}\int_{x^2}^\infty (s-x^2)^2 e^{-s}\, ds$$

$$+ \cdots$$

$$I(x) = \frac{x}{2} e^{-s}(-1)\Big|_{x^2}^{\infty}$$

$$+ \frac{1}{4x}\int_0^{\infty} v e^{-v-x^2}\,dv$$

$$- \frac{1}{16x^3}\int_0^{\infty} v^2 e^{-v-x^2}\,dv$$

$$= \frac{x}{2} e^{-x^2} + \frac{e^{-x^2}}{4x}\left[\cancelto{0}{-ve^{-v}\Big|_0^{\infty}} + \int_0^{\infty} e^{-v}\,dv\right]$$

$$- \frac{e^{-x^2}}{16x^3}\left[-v^2 e^{-v}\Big|_0^{\infty} + 2\cancelto{1}{\int_0^{\infty} v e^{-v}\,dv}\right]$$

$$= \frac{x}{2} e^{-x^2} + \frac{e^{-x^2}}{4x} - \frac{e^{-x^2}}{8x^3}$$

$$= e^{-x^2}\left[\frac{x}{2} + \frac{1}{4x} - \frac{1}{8x^3} + \cdots\right]$$

3.b (a)

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

(a) $\mathrm{erf}(-\infty) = \frac{2}{\sqrt{\pi}} \int_0^{-\infty} e^{-t^2}\,dt \qquad v = -t$

$$\cancel{= \frac{2}{\sqrt{\pi}} \int_0^{+\infty} e^{-v^2}(-dv)}$$

$$= -\frac{2}{\sqrt{\pi}} \int_{-\infty}^{0} e^{-t^2}\,dt = -\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-t^2}\,dt = -1 \checkmark$$

(b) $\int_0^x t^2 e^{-t^2}\,dt = \int_0^x t\left(t e^{-t^2}\right) dt$

$$= t\frac{(-1)}{2} e^{-t^2} \Big|_0^x + \int_0^x \frac{1}{2} e^{-t^2}\,dt$$

$$= -\frac{x}{2} e^{-x^2} + \frac{\sqrt{\pi}}{4} \mathrm{erf}(x)$$

(c) $\frac{\sqrt{\pi}}{2} = \int_0^x e^{-t^2}\,dt + \int_x^{\infty} e^{-t^2}\,dt$

$\frac{\sqrt{\pi}}{2}\mathrm{erf}(x)$

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$$1 = \underbrace{\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt}_{\mathrm{erf}(x)} + \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt$$

$$\therefore\ \mathrm{erf}(x) = 1 - \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt$$

$$\approx 1 - \frac{2}{\sqrt{\pi}}\left[e^{-x^2}\left(\frac{1}{2x} - \frac{1}{4x^3}\cdots\right)\right]$$

(B.7) Pr: $\Gamma(n+1) = n\Gamma(n)$

$$\Gamma(n+1) = \int_0^\infty x^n e^{-x}\,dx = \cancel{\int_0^\infty x\,(x^{n-1}e^{-x})\,dx}$$

$\cancel{v = x}$

$\cancel{dv = dx} \quad \cancel{du = x^{n-1}e^{-x}}$

let $v = x^n \qquad u = -e^{-x}$

$dv = nx^{n-1} \qquad du = e^{-x}\,dx$

$$= -x^n e^{-x}\Big|_0^\infty + \int_0^\infty nx^{n-1}e^{-x}\,dx = n\int_0^\infty x^{n-1}e^{-x}\,dx = n\Gamma(n)$$

(B.8) $\Gamma(1/2) = ?$

$$\Gamma(1/2) = \int_0^\infty x^{-1/2}e^{-x}\,dx$$

let $v = x^{+1/2} \Rightarrow x = v^2$

$dv = \frac{1}{2}x^{-1/2}dx \Rightarrow dx = 2x^{1/2}dv$

$= 2v\,dv$

$$= \int_0^\infty v^{-1}e^{-v^2}2v\,dv = 2\int_0^\infty e^{-v^2}dv$$

$$= 2\cdot\frac{1}{2}\sqrt{\frac{\pi}{1}} = \sqrt{\pi}.$$

$$\Gamma(3/2) = 1/2\,\Gamma(1/2) = \frac{\sqrt{\pi}}{2}$$

$\Gamma(-1/2) = ?$ $\quad (-\tfrac{1}{2})\Gamma(-1/2) = \Gamma(1/2)$

so $\Gamma(-1/2) = -2\Gamma(1/2) = -2\sqrt{\pi}$

(B9) $\Gamma(n+1) = \int_0^\infty x^n e^{-x}\,dx$

$\Gamma(1/3) = ?$

$$\Gamma(1/3) = \int_0^\infty x^{-2/3} e^{-x}\,dx = \cdots$$

$\Gamma(\tfrac{2}{3}) = ?$

$$\Gamma(2/3) = \int_0^\infty x^{-1/3} e^{-x} = \cdots$$

Check:

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin(n\pi)}$$

$$n = 1/3 \Rightarrow \Gamma(1/3)\Gamma(2/3) = \frac{\pi}{\sin(\frac{\pi}{3})} = \frac{\pi}{\frac{\sqrt{3}}{2}} = \frac{2\pi}{\sqrt{3}}$$

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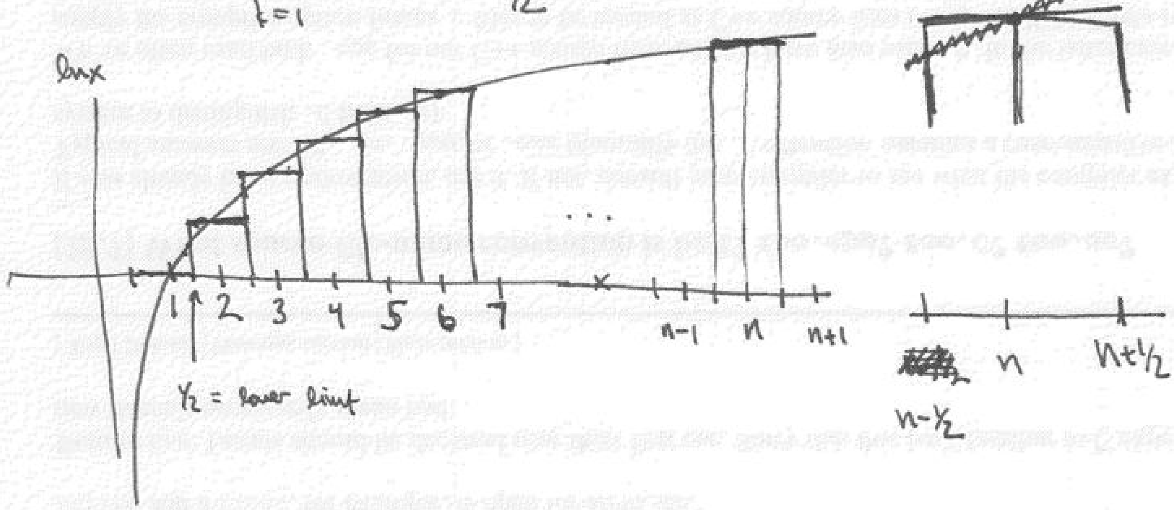
B.10

$$\ln n! = \ln n + \ln(n-1) + \ln(n-2) + \cdots + \ln 2 + \ln 1$$

$$= \sum_{k=1}^{n} \ln k \cong \int_{1/2}^{n+1/2} \ln x \, dx$$




$$= x\ln x - x \Big|_{1/2}^{n+1/2} = \left[(n+\tfrac12)\ln(n+\tfrac12) - (n+\tfrac12)\right] - \left[\tfrac12\ln(\tfrac12) - \tfrac12\right]$$

$$= (n+\tfrac12)\ln\left(n\left(1+\tfrac{1}{2n}\right)\right) - (n+\tfrac12) - \tfrac12\ln\tfrac12 + \tfrac12$$

large n approx

$$= (n+\tfrac12)\ln\left(n\left(1+\tfrac{1}{2n}\right)\right) - n - \cancel{\tfrac12} - \tfrac12\ln\tfrac12 + \cancel{\tfrac12}$$

$$= \cancel{(n+\tfrac12)\ln(n\tfrac{}{})}$$

$$= (n+\tfrac12)\left[\ln(n) + \ln\left(1+\tfrac{1}{2n}\right)\right] - n - \tfrac12\ln\tfrac12$$

$$\ln(1+x) = \sum_{k=0}^{\infty}\frac{x^k}{(k+1)} \qquad \frac{1}{1-x} = \sum_{k\geq0} x^k \qquad -\ln(1-x) = \sum_{k\geq0}\frac{x^{k+1}}{k+1}$$

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$$\therefore \ln(1+x) = \sum_{k\geq 0} \frac{(-1)^k x^{k+1}}{k+1}$$

$$\therefore \ln\left(1+\frac{1}{2n}\right) \simeq \sum_{k\geq 0} \frac{(-1)^k}{(k+1)} \frac{1}{(2n)^{k+1}}$$

$\therefore$

$$\ln(n!) \cong (n+\tfrac{1}{2})\left(\ln(n) + \sum_{k\geq 0} \frac{(-1)^k}{(k+1)} \frac{1}{(2n)^{k+1}}\right) - n - \frac{1}{2}\ln\frac{1}{2}$$

$$\simeq (n+\tfrac{1}{2})\left(\ln(n) + \frac{1}{2n}\right) - n - \frac{1}{2}\ln\frac{1}{2}$$

$$= (n+\tfrac{1}{2})\ln(n) + \overset{\sim 0}{\cancel{\frac{1}{2}}} + \overset{\sim 0}{\cancel{\frac{1}{4n}}} - n - \overset{\sim 0}{\cancel{\frac{1}{2}\ln\frac{1}{2}}}$$ constants become insignificant for large $n$.

$$\therefore n! \cong n^{(n+\frac{1}{2})} \cdot e^{-n} \cdot \cancel{e^{-\frac{1}{2}\ln\frac{1}{2}}} =$$

$$n^n \cdot n^{\frac{1}{2}} e^{-n} \qquad \checkmark \qquad n! \approx \left(\frac{n}{e}\right)^n \sqrt{n} \cdot C_1.$$

Check

$\cancel{\ln(0!)} = \ln(1!) = \cancel{0} = \frac{3}{2}\cancel{0} + \frac{1}{2} - 1$ Can't expect to be close to excellent at 1.

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B,11

$x^n e^{-x}$ $x^n \nearrow$ $x \gg 1$

$e^{-x} \searrow$ $x \gg 1$

$$\frac{d}{dx} f = n x^{n-1} e^{-x} - x^n e^{-x} \underset{\text{set}}{=} 0$$

$$\frac{n}{x} - 1 = 0 \to n = x \checkmark$$

B,12 $f_n(x) = x^n e^{-x} = e^{n \ln x - x}$

let $y = x - n$

$$x^n = e^{\log x^n} = e^{n \ln x} \qquad x = y + n$$

$$= e^{n \ln(n+y) - (y+n)}$$

B,13

$$\ln\left(1 + \frac{y}{n}\right) \cong \frac{y}{n} - \frac{1}{2}\left(\frac{y}{n}\right)^2 + \frac{1}{3}\left(\frac{y}{n}\right)^3 \text{ keep.}$$

$$y \sim \sqrt{n} \qquad \sim \frac{1}{\sqrt{n}}\,;\ \frac{1}{n}\,;\ \frac{1}{n^{3/2}}$$

$$n! \cong \int_0^\infty x^n e^{-x}\, dx \cong \int_{-\infty}^{\infty} e^{n \ln n - n + n \ln(1 + \frac{y}{n}) - y}\, dy$$

$$n! \cong e^{n\ln n-n}\int_{-\infty}^{+\infty}\exp\left\{y-\frac{1}{2}\frac{y^2}{n}+\frac{1}{3}\frac{y^3}{n^2}-y\right\}dy$$

$$= e^{n\ln n-n}\int_{-\infty}^{\infty}e^{\frac{1}{3}\frac{y^3}{n^2}}\,e^{-\frac{1}{2}\frac{y^2}{n}}\,dy$$

$$\approx n^n e^{-n}\int_{-\infty}^{\infty}\left(1+\frac{y^3}{3n^2}\right)e^{-\frac{1}{2}\frac{y^2}{n}}\,dy$$

let $\frac{1}{2n}$ $\beta$ $y=\sqrt{2n}\cdot v \Rightarrow v=\frac{y}{\sqrt{2n}}$

$$dy=\sqrt{2n}\cdot dv$$

$$\cong n^n e^{-n}\int_{-\infty}^{\infty}\Big(1+\underbrace{\frac{1}{3n^2}\cdot(2n)^{3/2}v^3}_{\text{this term will vanish}}\Big)e^{-v^2}$$

... should have kept a higher order ~~term~~ term. (dropping $O(y^3)$ term...)

$$n! \cong \left(\frac{n}{e}\right)^n\int_{-\infty}^{\infty}\exp\left\{-\frac{y^2}{2n}-\frac{y^4}{4n^3}\right\}dy$$

$$=\left(\frac{n}{e}\right)^n\int_{-\infty}^{\infty}e^{-\frac{y^4}{4n^3}}\,e^{-\frac{y^2}{2n}}\,dy=\left(\frac{n}{e}\right)^n\int_{-\infty}^{\infty}\left(1-\frac{y^4}{4n^3}\right)e^{-\frac{y^2}{2n}}\,dy$$

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let $\cancel{v = \sqrt{2n}\cdot y}$ $\qquad \cancel{v = \frac{y}{\sqrt{2n}}}$ $\quad y = \sqrt{2n}\cdot v$

$\cancel{dv = \sqrt{2n}\,dy}$ $\qquad dy = \sqrt{2n}\cdot dv$

so

$$n! \cong \left(\frac{n}{e}\right)^n \int_{-\infty}^{\infty} \left(1 - \frac{v^4 (2n)^2}{4n^3}\right) e^{-v^2} \sqrt{2n}\cdot dv$$

$$= \left(\frac{n}{e}\right)^n \cdot \sqrt{2n} \int_{-\infty}^{\infty} \left(1 - \frac{v^4}{n}\right) e^{-v^2}\, dv$$

$$= \left(\frac{n}{e}\right)^n \sqrt{2n} \cdot \left[ \int_{-\infty}^{+\infty} e^{-v^2} dv - \frac{1}{n} \int_{-\infty}^{+\infty} v^4 e^{-v^2} dv \right]$$

$$\int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\frac{d}{da} \rightarrow -\int_0^{\infty} x^4 e^{-ax^2} dx = \cancel{\frac{1}{4}\sqrt{\frac{\pi}{a^3}}} - \frac{1}{4}\sqrt{\pi}\,\frac{3}{2}\cdot a^{-5/2}$$

$$\rightarrow \int_0^{\infty} x^4 e^{-ax^2} dx = \frac{3}{8}\sqrt{\pi}\,.$$

$\therefore$

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2n} \left[ \sqrt{\pi} - \frac{1}{n}\cdot\frac{3}{4}\sqrt{\pi} \right]$$

$$\approx n^n e^{-n} \sqrt{2\pi n} \left[ 1 - \frac{3}{4n} \right]$$

error. 3 should be in denom...

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(3.14) (a) B.28 $$\int_0^\pi (\sin\theta)^n\,d\theta = \frac{\sqrt{\pi}\,\Gamma(\frac{n}{2}+\frac{1}{2})}{\Gamma(\frac{n}{2}+1)}$$

$n=0$ $$\pi \stackrel{?}{=} \frac{\sqrt{\pi}\cdot\Gamma(1/2)}{\Gamma(1)} \checkmark \qquad \Gamma(1)=1 \qquad \Gamma(1/2)=\sqrt{\pi}$$

$n=1$ $$-\cos\theta\Big|_0^\pi \stackrel{?}{=} \frac{\sqrt{\pi}\,\Gamma(1)}{\Gamma(3/2)} \qquad \Gamma(n+1)=n\Gamma(n)$$

$$= \cancel{+4-} -(-1-1) = 2 \stackrel{?}{=} \frac{\sqrt{\pi}}{\frac{1}{2}\cdot\sqrt{\pi}} = 2 \checkmark$$

(b) Pr: $$\int_0^\pi (\sin\theta)^n\,d\theta = \left(\frac{n-1}{n}\right)\int_0^\pi (\sin\theta)^{n-2}\,d\theta \qquad n\geq 2$$

$$\parallel$$

$$\int_0^\pi (\sin\theta)^{n-2}(1-\cos^2\theta)\,d\theta = \int_0^\pi (\sin\theta)^{n-2}\,d\theta - \int_0^\pi (\sin\theta)^{n-2}\cos^2\theta\,d\theta$$

$$u=\cos\theta \qquad v=\frac{(\sin\theta)^{n-1}}{n-1}$$

$$du=-\sin\theta \qquad dv=(\sin\theta)^{n-2}\cos\theta\,d\theta$$

$$= \int_0^\pi (\sin\theta)^{n-2}\,d\theta - \left[\overbrace{\cancel{\frac{(\cos\theta)(\sin\theta)^{n-1}}{n-1}\Big|_0^\pi}}^{0} + \frac{1}{n-1}\int_0^\pi (\sin\theta)^n\,d\theta\right]$$

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$$\therefore \int_0^{\pi} (\sin\theta)^n d\theta = \int_0^{\pi} (\sin\theta)^{n-2} d\theta - \frac{1}{n-1}\int_0^{\pi} (\sin\theta)^n d\theta$$

$$= \left(1+\frac{1}{n-1}\right)\int_0^{\pi} (\quad)^n d\theta =$$

$$\int_0^{\pi} (\sin\theta)^n d\theta = \left(\frac{n-1}{n}\right)\int_0^{\pi} (\sin\theta)^{n-2} d\theta$$

(c) $$\int_0^{\pi} (\sin\theta)^n d\theta = \left(\frac{n-1}{n}\right)\int_0^{\pi} (\sin\theta)^{n-2} d\theta = \left(\frac{n-1}{n}\right) \frac{\sqrt{\pi}\,\Gamma\left(\frac{n-2}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{n-2}{2}+1\right)}$$

By induction

$$= \left(\frac{n-1}{n}\right) \frac{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}-\frac{1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \qquad \Gamma(n+1) = n\Gamma(n)$$

$$= \frac{\left(\frac{n}{2}-\frac{1}{2}\right)\sqrt{\pi}\,\Gamma\left(\frac{n}{2}-\frac{1}{2}\right)}{\frac{n}{2}\,\Gamma\left(\frac{n}{2}\right)} = \frac{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)}$$

eq

B.28 ✓

B.15

$$A_d(r) = \frac{2\pi^{d/2}}{\Gamma(d/2)} r^{d-1}$$

(a) $$\int_{-\infty}^{+\infty} e^{-r^2} dr \equiv \prod_{i=1}^{d} \left( \int e^{-x_i^2} dx_i \right) = \left(\sqrt{\pi}\right)^d = \pi^{d/2}$$

area tends to $r$.

(b) $$\int_{\mathbb{R}^d} e^{-\vec{r}^2} d\vec{r} = \int_0^\infty e^{-|r|^2} (dr) \cdot A_d(1) \cdot r^{d-1}$$

All angular integrals give surface of sphere in d dimensions

$$= A_d(1) \int_0^\infty e^{-r^2} dr\, r^{d-1} = \cancel{\ldots}\ \cancel{A_d(1)}$$

$$= A_d(1) \int_0^\infty r^{d-1} e^{-r^2} dr$$

let $r^2 = v$

$r = \sqrt{v}$

$dr = \frac{1}{2} \frac{1}{\sqrt{v}} dv$

$$= A_d(1) \int_0^\infty v^{\left(\frac{d-1}{2}\right)} e^{-v} \frac{1}{2} \frac{1}{\sqrt{v}} dv$$

$$= \frac{A_d(1)}{2} \int_0^\infty v^{\frac{d}{2}-\frac{1}{2}} v^{-\frac{1}{2}} e^{-v} dv = \frac{A_d(1)}{2} \int_0^\infty v^{\frac{d}{2}-1} e^{-v} dv = \frac{A_d(1)}{2} \Gamma\left(\frac{d}{2}\right)$$

$$\Gamma(x) \equiv \int_0^\infty \xi^{x-1} e^{-\xi} d\xi$$

$$A_d(r) = \frac{A_d(1)}{2}\Gamma(d/2)$$

w/ $A_d(1) = \dfrac{2\pi^{d/2}}{\Gamma(d/2)}$

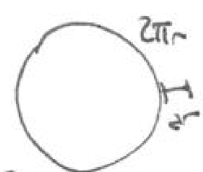
$\therefore A_d(r)$ . . . something wrong . . .

B.1b

$$V_d(r) = \int_0^R A_d(r)\cdot dr$$

$$= \frac{2\pi^{d/2}}{\Gamma(d/2)}\int_0^R r^{d-1}\,dr$$

$$= \frac{2\pi^{d/2}}{\Gamma(d/2)}\frac{R^d}{d}$$




$$\int_0^R 2\pi r\,dr = 2\pi\frac{R^2}{2}$$

$$\int_0^R 4\pi r^2\,dr = \frac{4\pi}{3}r^3\Big|_0^R$$

$$= \frac{4\pi}{3}R^3$$

$$\therefore\ V_d(r) = \frac{2\pi^{d/2}}{\Gamma(d/2)}\frac{r^d}{d}$$

Check $V_2(r) = \dfrac{2\pi}{\Gamma(1)}\dfrac{r^2}{2} = \pi r^2$

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$$V_3(r) = \frac{2\pi^{3/2} r^3}{\Gamma(3/2)\, 3} = \frac{2\pi^{3/2} r^3}{\frac{1}{2}\Gamma(1/2)\cdot 3}$$

$$= \frac{4}{3}\pi r^3 \ \checkmark.$$

B,17

Show B.36

$$\int_0^\infty \frac{x^n}{e^x-1}\,dx = \Gamma(n+1)\,\zeta(n+1)$$

&

$$\int_0^\infty \frac{x^n}{e^x+1}\,dx = \left(1-\frac{1}{2^n}\right)\Gamma(n+1)\,\zeta(n+1)$$

~~odd integer~~

consider

$$\int_0^\infty \frac{x^n}{e^x-1}\,dx = \int_0^\infty \frac{x^n e^{-x}}{1-e^{-x}}\,dx$$

$$= \int_0^\infty x^n e^{-x}\sum_{k=0}^\infty e^{-kx}\,dx = \sum_{k\geq 0}\int_0^\infty x^n e^{-(k+1)x}\,dx$$

Now

$$\int_0^\infty x^n e^{-(k+1)x}\,dx =$$

$$= \left.\frac{x^n e^{-(k+1)x}}{-(k+1)}\right|_0^\infty + \frac{n}{(k+1)}\int_0^\infty x^{n-1} e^{-(k+1)x}\,dx$$

$$\therefore \int_0^\infty x^n e^{-(k+1)x}\,dx = \frac{n}{k+1}\int_0^\infty x^{n-1} e^{-(k+1)x}\,dx = \frac{n(n-1)}{(k+1)^2}\int_0^\infty x^{n-2} e^{-(k+1)x}\,dx$$

$$= \cdots = \frac{n(n-1)(n-2)\cdots 1}{(k+1)^n}\int_0^\infty e^{-(k+1)x}\,dx$$

$$= \frac{n(n-1)(n-2)\cdots(n-(n-1))}{(k+1)^n}\left(\frac{-1}{(k+1)}e^{-(k+1)x}\right|_0^\infty$$

$$= \frac{n!}{(k+1)^{n+1}}$$

$$\therefore \int_0^\infty \frac{x^n}{e^x-1}\,dx = \sum_{k\ge 0} n!\,\frac{1}{(k+1)^{n+1}} = n!\sum_{k\ge 0}\frac{1}{(k+1)^{n+1}}$$

$$= n!\sum_{k\ge 1}\frac{1}{k^{n+1}} = \Gamma(n+1)\zeta(n+1) \qquad \text{B.36} \quad \checkmark$$

consider

$$\int_0^\infty \frac{x^n}{e^x+1}\,dx = \int_0^\infty \frac{x^n e^{-x}}{1+e^{-x}}\,dx = \int_0^\infty x^n e^{-x}\sum_{k\ge 0}(-1)^k e^{-kx}\,dx$$

$$= \cancel{\int_0^\infty x^n e^{-x}} = \cancel{\int_0^\infty x^n} - \sum_{k\ge 0}(-1)^k\int_0^\infty x^n e^{-(k+1)x}\,dx$$

From before $\displaystyle\int_0^\infty x^n e^{-(k+1)x}\,dx = \frac{n!}{(k+1)^{n+1}}$

So $$\int_0^\infty \frac{x^n}{e^x+1}\,dx = \sum_{k\geq 0} \frac{n!\,(-1)^k}{(k+1)^{n+1}}$$

$$= n!\left[\sum_{k=0,2,4,6,\dots} \frac{1}{(k+1)^{n+1}} + \sum_{k\ \text{odd}} \frac{-1}{(k+1)^{n+1}}\right]$$

$$= n!\left[\sum_{\substack{\cancel{k=0,1,2,\dots}\\ k\geq 0}} \frac{1}{(2k+1)^{n+1}} \cancel{+} - \sum_{k\geq 0} \frac{1}{(2k+2)^{n+1}}\right] \qquad \cancel{2k+1}$$

$$= n!\left[\sum_{k\geq 0} \frac{1}{(2k+1)^{n+1}} - \sum_{k\geq 0} \frac{1}{2^{n+1}(k+1)^{n+1}}\right]$$

$$= n!\left[\underbrace{\sum_{k\geq 0} \frac{1}{(2k+1)^{n+1}} + \sum_{k\geq 1} \frac{1}{(2k)^{n+1}}}_{\zeta(n+1)} - \sum_{k\geq 1} \frac{1}{(2k)^{n+1}} - \sum_{k\geq 0} \frac{1}{2^{n+1}(k+1)^{n+1}}\right]$$

$$- \frac{1}{2^{n+1}}\left[\underbrace{\sum_{k\geq 1} \frac{1}{k^{n+1}} + \sum_{k\geq 0} \frac{1}{(k+1)^{n+1}}}_{2\zeta(n+1)}\right]$$

$\cancel{\zeta(n+1)}$

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$$= n! \left[ L(n+1) - \frac{1}{2^{n+1}} \left[ L(n+1) + L(n+1) \right] \right]$$

$$= n! \, L(n+1) \left[ 1 - \frac{1}{2^n} \right]$$

B.36

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(B,18) $$\frac{\pi}{4} = \sum_{k \text{ odd}} \frac{\sin(kx)}{k} \quad \cdots$$

(B,19) B,42

$$\frac{\pi x}{4} = \sum_{k \text{ odd}} \frac{1}{k^2}(1-\cos kx)$$

$$\int_0^x f(x')\,dx' \rightarrow \quad \frac{\pi x^2}{8} = \sum_{k \text{ odd}} \frac{1}{k^2}\left(x - \frac{\sin kx}{k}\Big|_0^x\right)$$

$$= \sum_{k \text{ odd}} \frac{1}{k^2}\left(x - \frac{\sin kx}{k}\right) = \sum_{k \text{ odd}} \frac{1}{k^3}(kx - \sin(kx))$$

evalu' at $x = \frac{\pi}{2}$ to get ... will not get a simple sum

$$\sum_{k \text{ odd}} \frac{1}{k^3}\left(k\frac{\pi}{2} - \sin\left(k\frac{\pi}{2}\right)\right) \quad \cdots$$

integrate again

$$\frac{\pi x^3}{24} = \sum_{k \text{ odd}} \frac{1}{k^3}\left(\frac{kx^2}{2} + \frac{\cos(kx)}{k}\Big|_0^x\right.$$

$$\frac{\pi x^3}{24} = \sum_{k\text{ odd}} \frac{1}{k^2}\left(\frac{kx^2}{2} + \frac{\cos(kx)}{k} - \frac{1}{k}\right)\Bigg|_{x=\frac{\pi}{2}}$$

$$= \sum_{k\text{ odd}} \frac{1}{k^2}\cdot\frac{x^2}{2} + \sum_{k\text{ odd}} \frac{\cos(k\pi/2)}{k^4} - \sum_{k\text{ odd}} \frac{1}{k^4}$$

gives powers of 4.

(B,20) $$\frac{\pi}{4} = \sum_{\text{odd } k} \frac{\sin(kx)}{k}\Bigg|_{x=\pi/2} = \sum_{k\text{ odd}} \frac{\sin(k\pi/2)}{k} = \sum_{k\text{ odd}} \frac{(-1)^{k+1}}{k}$$

(B,21) replacing $x$ w/ $-x$ gives

$$\frac{x^2 e^{-x}}{(e^{-x}+1)^2} = \frac{x^2 e^x}{(1+e^x)^2} \quad \checkmark$$

$$\int_0^\infty \frac{x^2 e^x}{(e^x+1)^2}\,dx = \frac{-x^2}{(1+e^x)}\Bigg|_0^\infty + \int_0^\infty \frac{2x\,dx}{(1+e^x)} \quad \ldots$$