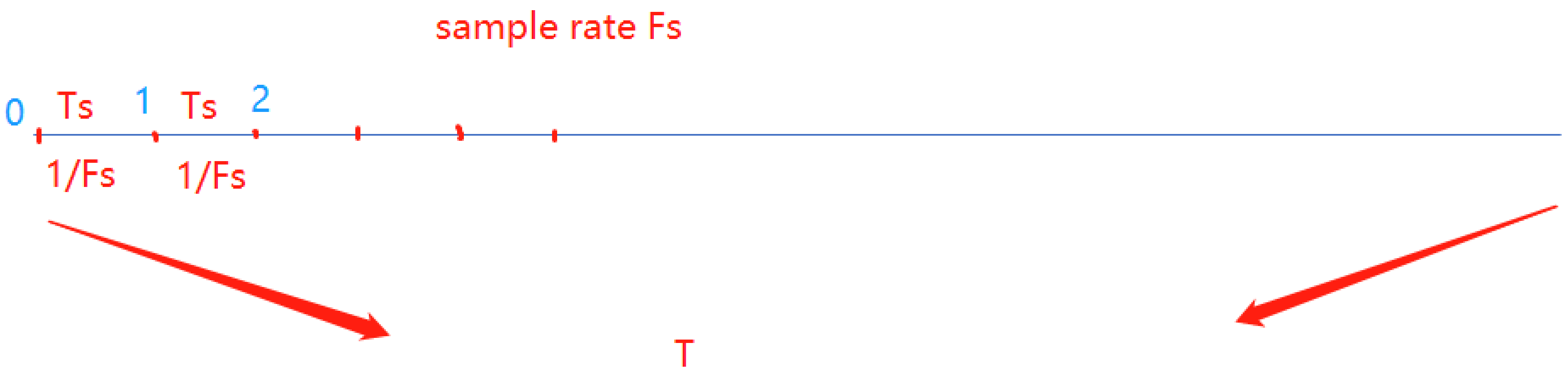
Stochastic Signal Processing

Lesson 7: Experimental Report 2

- Introduction

For continuous-time signals, it can be simulated by discrete sequences with sufficiently small sampling intervals. For example, to generate a continuous signal with a start time zero and a duration of T seconds, the sampling interval can be set to T_S (or sample rate $F_S = 1/T_S$), then sequences length is $N = \frac{T}{T_S} + 1$.



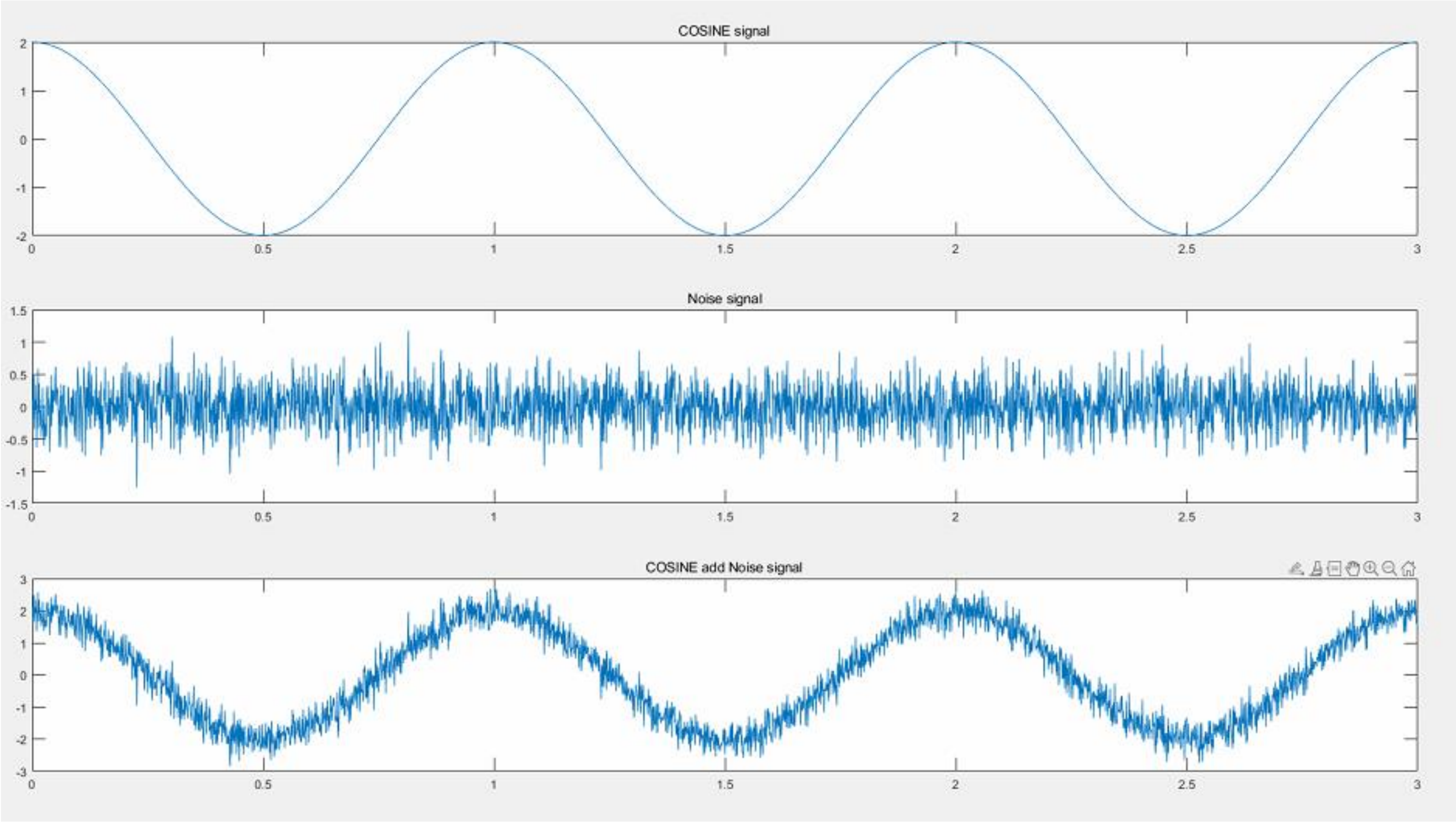
- Practice 1

The simulation generates a sinusoidal signal with frequency 1Hz and Gaussian noise. The Signal amplitude is 2 and noise variance $\sigma^2 = 0.1$. Generate a signal sampled at F_S= 1 kHz for T = 3 seconds. Plot a sinusoidal signal with Gaussian noise.

In order to ensure that the signal is not distorted, the sampling rate should be greater than twice the highest frequency of the signal.

- Practice 1

```
T = 3;                    % signal time_len(s)
Fs = 1000;                % sample rate(Hz)
t = 0:1/Fs:T-1/Fs; % sample point
s = 2*cos(2*pi*t); % signal cos
subplot(311);
plot(t,s);
title('\fontname{}COSINE signal');
n = sqrt(0.1)*randn(size(t)) %noise
subplot(312);
plot(t,n);
title('\fontname{}Noise signal');
x=s+n;                    % signal cos add noise
subplot(313);
plot(t,x);
title('\fontname{}COSINE add Noise signal');
```



- Review Lesson 6

Cross-Correlation, Cross-Covariance and Jointly WSS

Properties of jointly WSS:

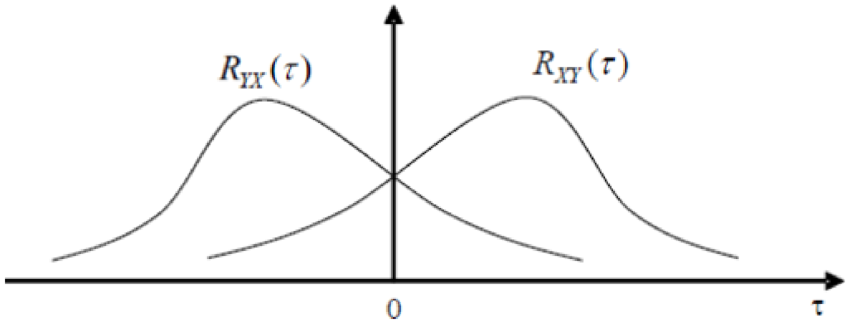
- Property 1: for real valued process

$$R_{XY}(-\tau) = R_{YX}(\tau)$$
$$C_{XY}(-\tau) = C_{YX}(\tau)$$

$$\begin{aligned} R_{XY}(\tau) &= E[X(t)Y(t-\tau)] \\ R_{XY}(-\tau) &= E[X(t)Y(t+\tau)] \end{aligned} \rightarrow R_{XY}(\tau) \neq R_{XY}(-\tau) \quad R_{XY}(\tau) \neq -R_{XY}(-\tau)$$

$$\neq R_{YX}(\tau)$$

Topical curve of Cross-Correlation of Jointly WSS processes



- Cross-Correlation

Suppose $\{x_n\}_{n=0}^{N-1}$ and $\{y_n\}_{n=0}^{N-1}$ are samples of real random signals X_n and Y_n, respectively. For real valued process. The formula of the cross-correlation function:

$$\hat{R}_{XY}[m] = \begin{cases} \alpha \sum_{n=0}^{N-m-1} x_{n+m} y_n , & 0 \leq m \leq N-1 \\ \hat{R}_{YX}[-m], & -(N-1) \leq m \leq 0 \end{cases}$$

The formula of the autocorrelation function:

$$\hat{R}_X[m] = \alpha \sum_{n=0}^{N-|m|-1} x_{n+|m|} x_n , \; 0 \leq |m| \leq N-1$$

- xcorr

Cross-correlation r = xcorr(___,scaleopt)

scaleopt	归一化系数α
none	1
biased	1/N
unbiased	1/(N-|m|)
coeff	$1/\sqrt{\hat{R}_X(0)\hat{R}_Y(0)}$

`scaleopt` — Normalization option
`'none'` (default) | `'biased'` | `'unbiased'` | `'normalized'` | `'coeff'`

Normalization option, specified as one of the following.

- `'none'` — Raw, unscaled cross-correlation. `'none'` is the only valid option when x and y have different lengths.
- `'biased'` — Biased estimate of the cross-correlation:

$$\hat{R}_{xy,\mathrm{biased}}(m) = \frac{1}{N}\hat{R}_{xy}(m).$$

- `'unbiased'` — Unbiased estimate of the cross-correlation:

$$\hat{R}_{xy,\mathrm{unbiased}}(m) = \frac{1}{N-|m|}\hat{R}_{xy}(m).$$

- `'normalized'` or `'coeff'` — Normalizes the sequence so that the autocorrelations at zero lag equal 1:

$$\hat{R}_{xy,\mathrm{coeff}}(m) = \frac{1}{\sqrt{\hat{R}_{xx}(0)\hat{R}_{yy}(0)}}\hat{R}_{xy}(m).$$

- Practice 2

Simulate Gaussian noise sequence with mean 0 and variance 5, and estimate its mean, variance and autocorrelation

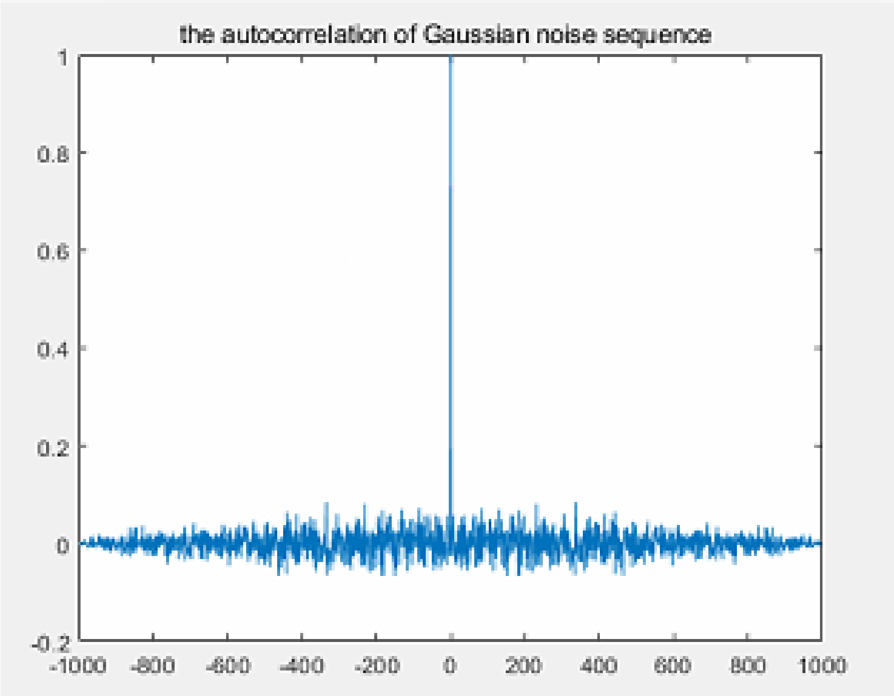
```
N = 1000;                    % sequence_len
sigma = sqrt(5);             % std
x = randn(1,N)*sigma;        % Simulate Gaussian sequence with
m = mean(x);
v = var(x);
r = xcorr(x,'coeff');        % Normalizes the sequence so that the
                             autocorrelations at zero lag equal 1
plot(1-N:N-1,r)
title('\fontname{}the  autocorrelation of Gaussian noise sequence ')
```

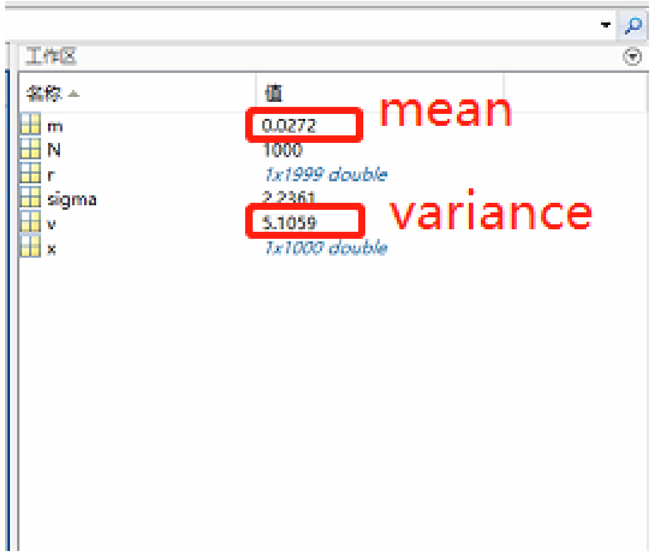
- Practice 2

For real valued Stationary Stochastic Processes:

$R_X(0) \geq |R_X(\tau)|$

From the figure, we can see that 0-point is maximum and autocorrelation is an even function. Except for 0, other real values are basically stable around zero. It is called White Gaussian Noise.





- Experimental Report 2
- Basic 1 (20ponits)： A stochastic signal

$$X(t) = \sin(2\pi f_1 t) + 2cos(2\pi f_2 t) + N(t)$$

where f_1=50Hz, f_2=200Hz, $N(t)$ is White Gaussian Noise, and σ^2=0.1. Generate a signal sampled at 1 kHz for 3 seconds. Plot the signal and estimate autocorrelation function $R_X(\tau)$, Cross-Correlation $R_{XN}(\tau)$.

Hint : see practice 1, 2.

- Basic knowledge of Basic 2

Characteristics of Autocorrelation of Stationary Stochastic Processes

Periodic Signal / Processes: $X(t)=X(t+T)$

- If $X(t)=X(t+T)$, and it is Stationary, then:

$$R_X(\tau)=E[X(t+\tau)X(t)]=E[X(t+T+\tau)X(t)]=R_X(\tau+T)$$

If the stochastic process has a periodic component, the autocorrelation function also has a periodic component.

- Experimental Report 2
- Basic 2 (40 points): construct a sinusoidal signal with Gaussian noise, where σ^2= 0.1 and other parameters are designed for you.
  - Use characteristics of autocorrelation to estimate your signal frequence and verify that it is the same as you defined. Hint: $X(t)$=$X(t+T)$ $\Rightarrow$ $R_X(\tau) = R_X(\tau + T)$
  - When σ^2=0.5, 1, 5 (you can try more), plot the figure of Autocorrelation and analyse the situation in different noise.
- Requirement

1. Your code must be runnable (no error, warning accepted), otherwise, 0 point.
2. Explain the phenomenon of your experiment figure.

- Experimental Report 2

- Basic 2 (40 points): construct a sinusoidal signal with Gaussian noise, where σ^2= 0.1 and other parameters are designed for you.（sample rate =1 kHz）
  - For example, you define the signal frequence f=2Hz, and plot the figure of Autocorrelation. In the figure, count the N between two adjacent top point. It is the period for $R_X(\tau)$ and $X(t)$ at the same time. At last, estimated value is whether approximately equal to f .

Hint: $X(t)$=$X(t+T)$ $\Rightarrow$ $R_X(\tau) = R_X(\tau + T)$

1. S=cos(2 pi f t)+N(t)

f = 2Hz, sample rate Fs= 1k Hz ,Ts = 0.001s

From the figure2, N ≈ 500, T =500*0.001=0.5s

T is not only the period of $R_X(\tau)$, but also the period of $X(t)$, so estimate the frequence of X(t) :f=1/T=2Hz