

EJERCICIO 2

$$y=cx$$

$$dy=c \, dx$$

$$dy/dx = \frac{y}{x}$$

$$dy/dx = -\frac{x}{y}$$

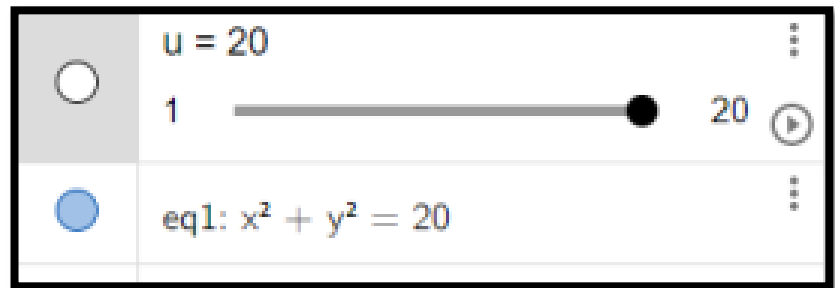
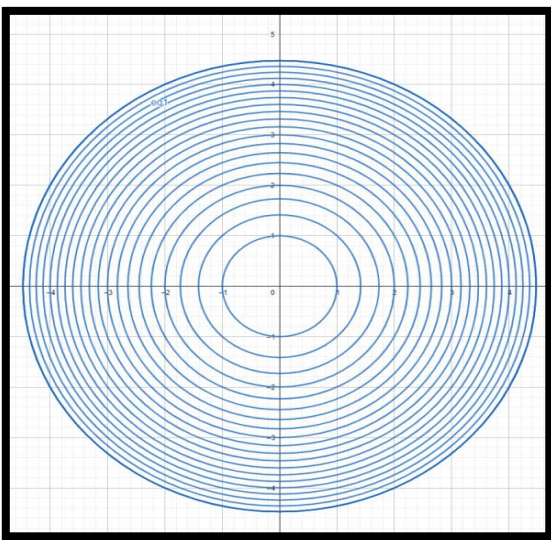
$$dy * y = -dx * x$$

$$\int y \, dy = \int -x \, dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + c$$

$$\frac{y^2}{2} + \frac{x^2}{2} = c$$

$$y^2 + x^2 = k$$



EJERCICIO NO.5

A)

$$X^2 + 3Y^2 = C$$

$$(X^2 + 3Y^2)' = C'$$

$$2X + 6Y = 0$$

$$X + 3YY' = 0$$

$$x - \frac{3y}{y'} = 0$$

$$xy' - 3y = 0$$

$$x \frac{dy}{dx} - 3y = 0$$

$$xdy - 3ydx = 0$$

$$\frac{1}{xy}(xdy - 3ydx) = \frac{1}{xy} * 0$$

$$\frac{dy}{y} - \frac{3dx}{x} = 0$$

$$\int 1 dy = \int \frac{3}{x} dx$$

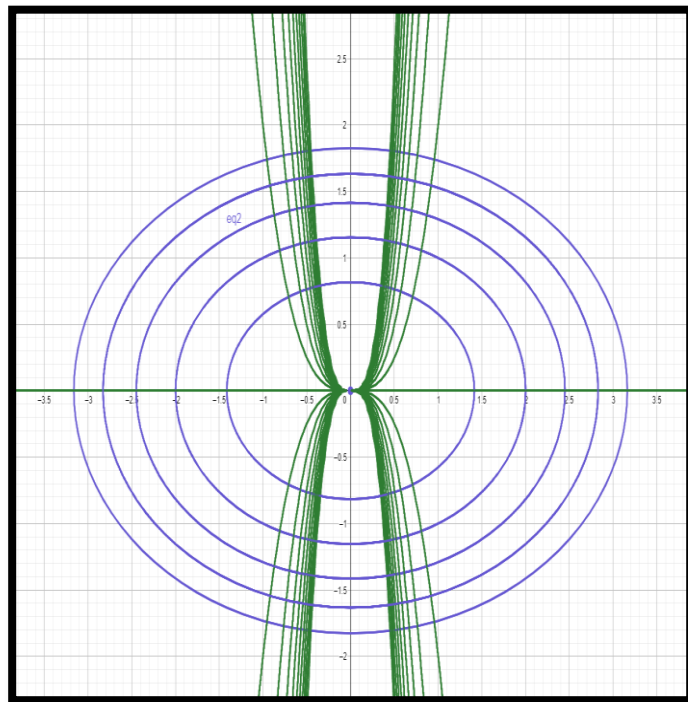
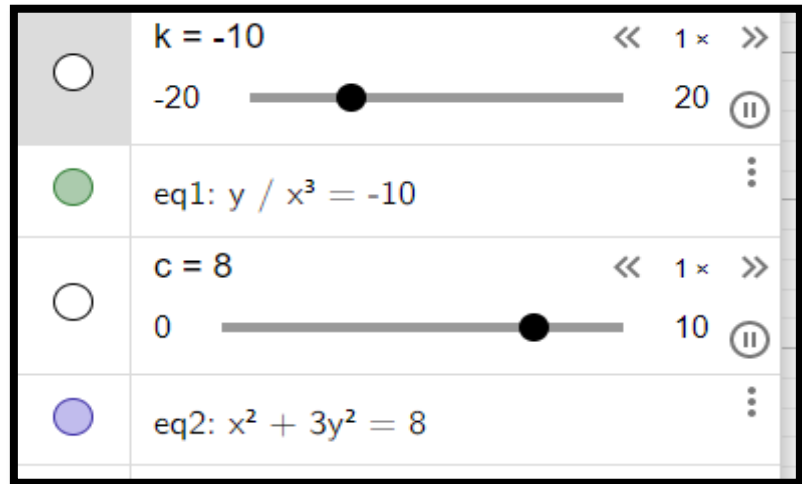
$$\ln(y) = \ln(x^3) + 3$$

$$\ln\left(\frac{y}{x^3}\right) = c$$

$$\frac{y}{x^3} = e^c$$

$$\frac{y}{x^3} = k$$

$$y = kx^3$$



B)

$$y^2 = 2cx$$

$$2yy' = 2c$$

$$y' = \frac{2c}{2y}$$

$$y' = \frac{c}{y}$$

$$y' = \frac{y}{c}$$

$$y' = \frac{y}{\frac{y^2}{2x}}$$

$$y' = \frac{2xy}{y^2}$$

$$y' = \frac{2x}{y}$$

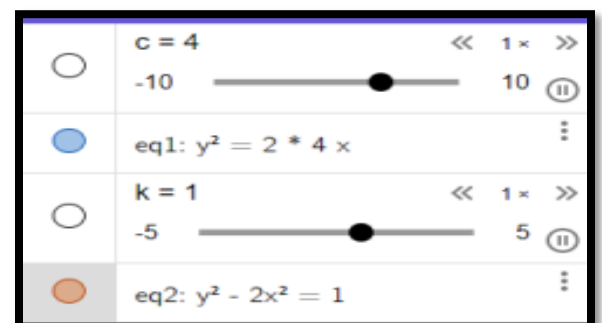
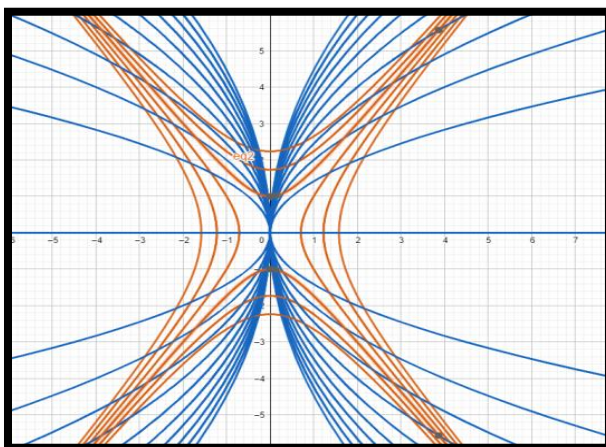
$$\frac{dy}{dx} = \frac{2x}{y}$$

$$\int y \frac{dy}{dx} = \int 2x \frac{dy}{dx}$$

$$\frac{y^2}{2} = \frac{2x^2}{2} + c$$

$$\frac{y^2}{2} - x^2 = c$$

$$y^2 - 2x^2 = k$$



c)

$$y = cx^2$$

$$y' = 2cx$$

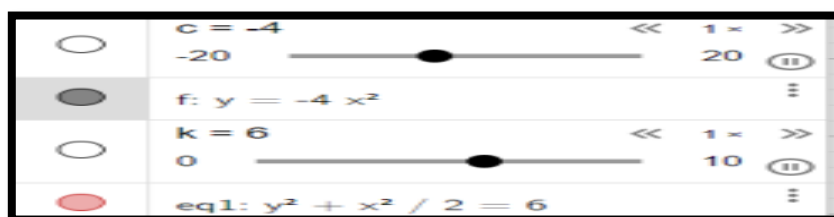
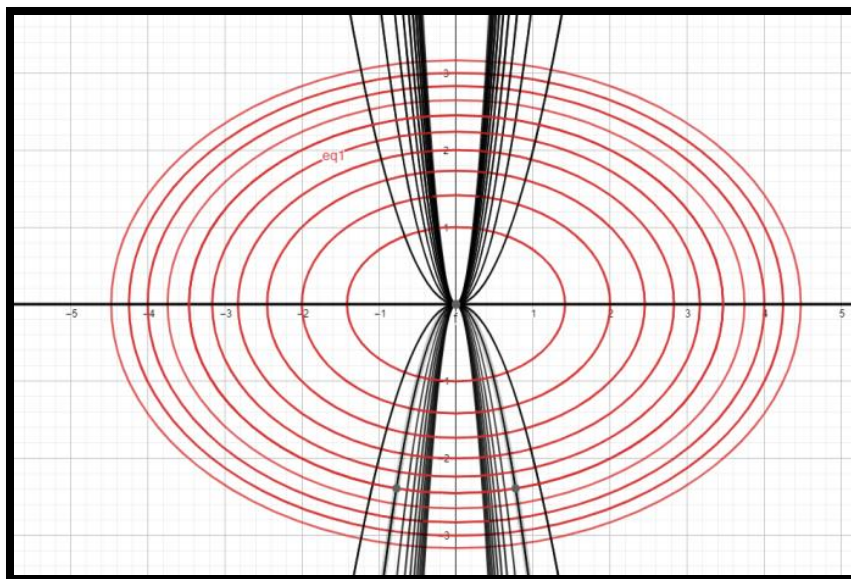
$$\frac{dy}{dx} = -\frac{1}{2cx}$$

$$\frac{dy}{dx} = \frac{-x}{2y}$$

$$\int 2y dy = \int -x dx$$

$$y^2 = \frac{-x^2}{2} + c$$

$$y^2 + \frac{x^2}{2} = c$$



D)

$$xy = c$$

$$y = \frac{c}{x}$$

$$y' = -cx^{-2}$$

$$y' = \frac{-xy}{x^2}$$

$$y' = -\frac{y}{x}$$

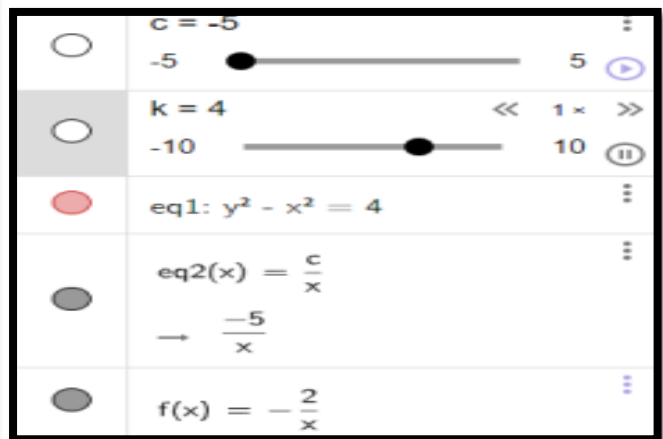
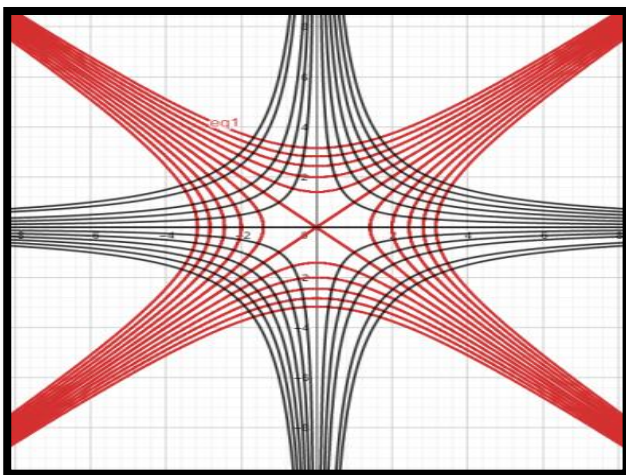
$$\frac{dy}{dx} = \frac{x}{y}$$

$$dy y = x dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c$$

$$\frac{y^2}{2} - \frac{x^2}{2} = c$$

$$y^2 - x^2 = k$$



E)

$$x + 2y = c$$

$$\frac{c - x}{2} = y$$

$$y' = \frac{1}{2}$$

$$y'_2 = \frac{-1}{-\frac{1}{2}}$$

$$y'_2 = 2$$

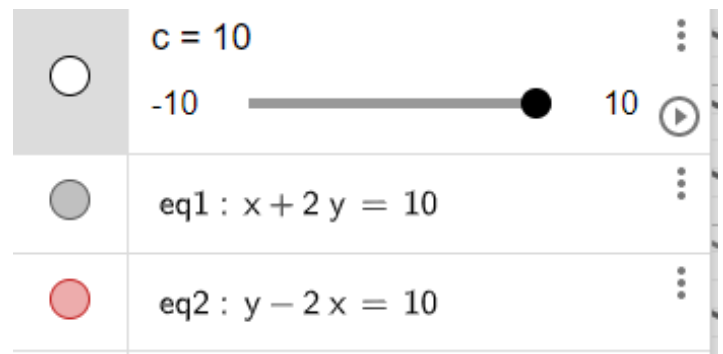
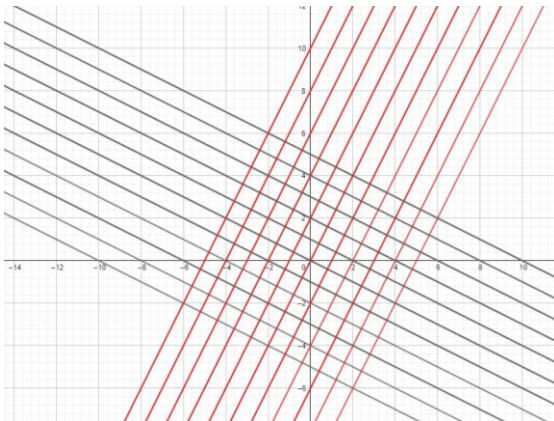
$$\frac{dy}{dx} = 2$$

$$dy = 2dx$$

$$\int 1dy = \int 2dx$$

$$y = 2x + c$$

$$y - 2x = c$$



EJERCICIO 2

$$y' = f(x, y) = (x + y + 1)^2, y(0) = 2$$

$$y_{n+1} = y_n + h(w_1 k_1 + w_2 k_2)$$

$$w_2 = \frac{3}{4}, \quad y_0 = 0.5, \quad x_0 = 0$$

$$w_1 = 1 - w_2 = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\alpha = \frac{1}{2w_2} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$\beta = \frac{1}{2w_2} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$k_1 = f(x_0, y_0)$$

$$k_1 = (x_0 + y_0 - 1)^2$$

$$k_1 = (0 + 2 - 1)^2$$

$$k_1 = 1$$

$$k_2 = f(x_0 + \alpha h, y_0 + \beta h k_1)$$

$$k_2 = [(x_0 + \frac{2}{3}h) + (y_0 + \beta \frac{2}{3}k_1) - 1]^2$$

$$k_2 = [(0 + \frac{2}{3} * 0.1) + (2 + 0.1 * \frac{2}{3} * 1) - 1]^2$$

$$k_2 = [0.0677 + 2.0677 - 1]^2$$

$$k_2 = 1.2844$$

$$y_1 = y_0 + h(\frac{1}{4}k_1 + \frac{3}{4}k_2)$$

$$y_1 = y_0 + \frac{h}{4}(k_1 + 3k_2)$$

$$y_1 = 2 + \frac{0.1}{4}(1 + 3 * 1.2844)$$

$$y_1 = 2.1213$$

| x_n | $k_1 + 3k_2$ | y_n | valor |
|-------|--------------|--------|--------|
| 0.0 | 4.5832 | 2 | 2 |
| 0.1 | 7.2663 | 2.1213 | 2.1230 |
| 0.2 | 11.1370 | 2.3030 | 2.3085 |
| 0.3 | 16.8500 | 2.5814 | 2.5958 |
| 0.4 | 30.2106 | 3.003 | 3.0650 |
| 0.5 | | 3.7579 | 3.9082 |

EJERCICIO 7

$$y' = f(x, y) = e^x, y(0) = 0$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_0, y_0) = e^{y_0} = 1$$

$$k_2 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hk_1\right) = e^{-(0+\frac{1}{2}*0.1*1)} = 0.9512$$

$$k_3 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hk_2\right) = e^{-(0+\frac{1}{2}*0.1*0.9512)} = 0.9536$$

$$k_4 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hk_3\right) = e^{-(0+\frac{1}{2}*0.1*0.9536)} = 0.9534$$

$$y_1 = y_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = 0 + \frac{0.1}{6}(1 + 2 * 0.9512 + 0.9536 + 0.9534)$$

$$y_1 = 0.0961$$

| x_n | $k_1 + 2k_2 + 2k_3 + k_4$ | y_n | $y_n = \ln(x + 1)$ | Error absoluto |
|-------|---------------------------|--------|--------------------|----------------|
| 0.0 | 5.763 | 0.00 | 0.00 | 0.0000 |
| 0.1 | 5.217 | 0.0961 | 0.0953 | 0.0008 |
| 0.2 | 4.800 | 0.1830 | 0.1823 | 0.0007 |
| 0.3 | 4.444 | 0.2630 | 0.2624 | 0.0006 |
| 0.4 | 4.137 | 0.3371 | 0.3365 | 0.0006 |
| 0.5 | | 0.4055 | 0.4055 | 0.0006 |

EJERCICIO 10

$$y' = f(x, y) = xy + \sqrt{y}, y(0) = 1$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_0, y_0) = x_0 * y_0 + \sqrt{y_0} = 1$$

$$k_2 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hk_1\right)$$

$$k_2 = \left(x_0 + \frac{1}{2}h\right)\left(y_0 + \frac{1}{2}hk_1\right) + \sqrt{y_0 + \frac{1}{2}hk_1}$$

$$k_2 = \left(0 + \frac{1}{2}0.1\right)\left(1 + \frac{1}{2}0.1 * 1\right) + \sqrt{1 + \frac{1}{2} * 0.1 * 1}$$

$$k_2 = 1.0772$$

$$k_3 = \left(x_0 + \frac{1}{2}h\right)\left(y_0 + \frac{1}{2}hk_2\right) + \sqrt{y_0 + \frac{1}{2}hk_2}$$

$$k_3 = \left(0 + \frac{1}{2} * 0.1\right)\left(1 + \frac{1}{2}0.1 * 1.0772\right) + \sqrt{1 + \frac{1}{2}0.1 * 1.0772}$$

$$k_3 = 1.0793$$

$$k_4 = \left(x_0 + \frac{1}{2}h\right)\left(y_0 + \frac{1}{2}hk_3\right) + \sqrt{y_0 + \frac{1}{2}hk_3}$$

$$k_4 = \left(0 + \frac{1}{2}0.1\right)\left(1 + \frac{1}{2}0.1 * 1.0793\right) + \sqrt{1 + \frac{1}{2} * 0.1 * 1.0793}$$

$$k_4 = 1.108$$

$$y_1 = y_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = 0 + \frac{0.1}{6}(1 + 2 * 1.0772 + 2 * 1.0793 + 1.108)$$

$$y_1 = 1.107$$

| x_n | $k_1 + 2k_2 + 2k_3 + k_4$ | y_n |
|-------|---------------------------|--------|
| 0.0 | 6.4210 | 0.00 |
| 0.1 | 7.5420 | 1.1070 |
| 0.2 | 8.8150 | 1.2327 |
| 0.3 | 10.3391 | 1.3796 |
| 0.4 | 12.1740 | 1.5519 |
| 0.5 | | 1.7548 |