Task 1

Let: $\begin{cases} f(x_0-h) \approx f(x_0) - hf'(x_0) + \frac{h^2}{2}f''(x_0) - \frac{h^3}{6}f'''(x_0) + \frac{h^4}{24}f''''(x_0) \\ f(x_0+h) \approx f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \frac{h^3}{6}f'''(x_0) + \frac{h^4}{24}f''''(x_0) \\ f(x_0) = f(x_0) \end{cases}$

Then, $f''(x_0) \approx \dfrac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$ becomes:

$$\frac{\left(f(x_0) - hf'(x_0) + \frac{h^2}{2}f''(x_0) - \frac{h^3}{6}f'''(x_0) + \frac{h^4}{24}f''''(x_0) + \dots\right) - 2f(x_0) + \left(f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \frac{h^3}{6}f'''(x_0) + \frac{h^4}{24}f''''(x_0)\right.}{h^2}$$

$$= \frac{\cancel{2f(x_0)} - \cancel{hf'(x_0)} + \cancel{hf'(x_0)} + h^2f''(x_0) - \cancel{\frac{h^3}{6}f'''(x_0)} + \cancel{\frac{h^3}{6}f'''(x_0)} + \frac{h^4}{12}f''''(x_0) - \cancel{2f(x_0)}}{h^2}$$

$$= \frac{h^2f''(x_0) + \frac{h^4}{12}f''''(x_0)}{h^2} = f''(x_0) + \frac{h^2}{12}f''''(x_0)$$

Since we found $f''(x_0) \approx f''(x_0) + \frac{h^2}{12}f''''(x_0)$, we can conclude

that $f''(x_0)$ is of order h^2 and that

$$\left| f''(x_0) - \left(f''(x_0) + \frac{h^2}{12}f''''(x_0)\right)\right| \leq Ch^2 \Rightarrow$$

$$\left| -\frac{h^2}{12}f''''(x_0)\right| \leq Ch^2 \Rightarrow \underline{\underline{\frac{1}{12}h^2f''''(x_0) \leq Ch^2}}$$

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