Chapter 1

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The two tables below shows correspondence of question index in 2nd Edition of the textbook and that in 1st Edition of the textbook.

2nd Edition	1.1	1.2	1.3	1.4	1.5	1.6	1.7
1st Edition	1.1	1.2	1.3	1.4	1.5	1.6	1.7

2nd Edition	1.8	1.9	1.10	1.11	1.12	1.13	1.14	1.15
1st Edition	1.8					1.9		

Problem 1.9

Skipped

Problem 1.10

According to Problem 1.3,

$$|+\mathbf{n}\rangle = \cos\frac{\theta}{2}\,|+\mathbf{z}\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|-\mathbf{z}\rangle \tag{1}$$

Therefore,

$$\begin{cases} \cos\frac{\theta}{2} & = \frac{1}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & = \frac{\sqrt{3}}{2}i \end{cases}$$

$$\theta = \frac{2}{3}\pi, \quad \phi = \frac{\pi}{2}$$

$$\boxed{\begin{aligned} \mathbf{n} =& < \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta > \\ =& < 0, \frac{\sqrt{3}}{2}, -\frac{1}{2} > \end{aligned}}$$

Using $P(\lambda_i) = |\langle \lambda_i | \psi \rangle|^2$ and the condition $|\psi\rangle = \frac{1}{2} |+\mathbf{z}\rangle + \frac{\sqrt{3}i}{2} |-\mathbf{z}\rangle$,

$$P(+\mathbf{z}) = \frac{1}{4}, \quad P(-\mathbf{z}) = \frac{3}{4}$$

Using the definition of expectation value $\langle A \rangle = \sum_i P(\lambda_i)\lambda_i$,

$$\boxed{\begin{aligned} \langle S_z \rangle =& \frac{\hbar}{2} P(+\mathbf{z}) + (-\frac{\hbar}{2}) P(-\mathbf{z}) \\ =& -\frac{\hbar}{4} \end{aligned}}$$

Considering $|+\mathbf{x}\rangle = \frac{1}{\sqrt{2}} |+\mathbf{z}\rangle + \frac{1}{\sqrt{2}} |-\mathbf{z}\rangle$ and following a similar procedure,

$$P(+\mathbf{x}) = \frac{1}{2}, \quad P(-\mathbf{x}) = \frac{1}{2}$$

$$\boxed{\langle S_x \rangle = 0}$$

Considering $|+\mathbf{y}\rangle = \frac{1}{\sqrt{2}} |+\mathbf{z}\rangle + \frac{i}{\sqrt{2}} |-\mathbf{z}\rangle$ and following a similar procedure,

$$P(+\mathbf{y}) = \frac{2-\sqrt{3}}{4}, \quad P(-\mathbf{y}) = \frac{2+\sqrt{3}}{4}$$

$$\boxed{\langle S_y \rangle = -\frac{\sqrt{3}}{4}\hbar}$$

Problem 1.11

We can show that these two phases only differ by an overall phase factor by calculating their relative phase difference.
Denote $|\psi_1\rangle = \frac{1}{2} e^{i\delta_+} |+\mathbf{z}\rangle + \frac{\sqrt{3}}{2} e^{i\delta_-} |-\mathbf{z}\rangle$ and $|\psi_2\rangle = \frac{1}{2} e^{i\gamma_+} |+\mathbf{z}\rangle + \frac{\sqrt{3}}{2} e^{i\gamma_-} |-\mathbf{z}\rangle$,

$$\delta_+ = 0, \quad \delta_- = \frac{\pi}{2} \Rightarrow \delta = -\frac{\pi}{2}$$

$$\gamma_+ = -\frac{\pi}{2}, \quad \gamma_- = 0 \Rightarrow \gamma = -\frac{\pi}{2}$$

The relative phase difference $\delta = \gamma$, thus the two states are equivalent, indicating

$$\boxed{\langle S_z \rangle = -\frac{\hbar}{4}, \quad \langle S_x \rangle = 0, \quad \langle S_y \rangle = -\frac{\sqrt{3}\hbar}{4}}$$

Problem 1.12

Following (1),

$$\theta = \frac{3\pi}{2}, \quad \phi = 0$$

thus

$$\mathbf{n} = < \frac{\sqrt{3}}{2}, 0, -\frac{1}{2} >$$

As $|\psi\rangle = \frac{1}{2}|+\mathbf{z}\rangle + \frac{\sqrt{3}}{2}|-\mathbf{z}\rangle$,

$$P(+z) = \frac{1}{4}, \quad P(-z) = \frac{3}{4}$$

$$\boxed{\langle S_z\rangle = -\frac{\hbar}{4}}$$

As $|+\mathbf{x}\rangle = \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle + \frac{1}{\sqrt{2}}|-\mathbf{z}\rangle$,

$$P(+x) = \frac{2+\sqrt{3}}{4}, \quad P(-x) = \frac{2-\sqrt{3}}{4}$$

$$\boxed{\langle S_x\rangle = \frac{\sqrt{3}}{4}\hbar}$$

As $|+\mathbf{z}\rangle = \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle + \frac{i}{\sqrt{2}}|-\mathbf{z}\rangle$,

$$P(+y) = \frac{1}{2}, \quad P(+y) = \frac{1}{2}$$

$$\boxed{\langle S_y\rangle = 0}$$

Problem 1.14

Because $P(\lambda_i) = |\langle\lambda_i|\psi\rangle|^2$ and in light of the overall phase irrelevency, we can write $|\psi\rangle$ as

$$|\psi\rangle = e^{i\theta}(\frac{3}{5}|+\mathbf{z}\rangle + e^{i\phi}\frac{4}{5}|-\mathbf{z}\rangle)$$

As $|+\mathbf{x}\rangle = \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle + \frac{1}{\sqrt{2}}|-\mathbf{z}\rangle$, the probability of measuring $+x$ would be

$$\begin{aligned}
P(+x) =& \left|\frac{3}{5\sqrt{2}} + e^{i\phi}\frac{4}{5\sqrt{2}}\right|^2 \\
=& (\frac{3}{5\sqrt{2}} + e^{i\phi}\frac{4}{5\sqrt{2}})(\frac{3}{5\sqrt{2}} + e^{-i\phi}\frac{4}{5\sqrt{2}}) \\
=& \frac{1}{50}(25 + 12e^{i\phi} + 12e^{-i\phi}) \\
=& \frac{1}{2} + \frac{24}{50}\cos\phi
\end{aligned}$$

It is given that $P(+x) = \frac{1}{2}$, so

$$\cos\phi = 0 \Rightarrow \phi = \frac{\pi}{2} \Rightarrow e^{i\phi} = i$$

Therefore,

$$\boxed{|\psi\rangle = e^{i\theta}(\frac{3}{5}|+\mathbf{z}\rangle + \frac{4i}{5}|-\mathbf{z}\rangle) \quad \theta \in \mathbb{R}}$$

Problem 1.15

Similar to Problem 1.14, we can express

$$|\psi\rangle = e^{i\theta}()\frac{3}{\sqrt{10}}|+\mathbf{z}\rangle + \frac{1}{\sqrt{10}}e^{i\phi}|-\mathbf{z}\rangle)$$

As $|+\mathbf{z}\rangle = \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle + \frac{i}{\sqrt{2}}|-\mathbf{z}\rangle$, the probability of measuring $+y$ would be

$$\begin{aligned} P(+y) &= \left|\frac{3}{\sqrt{20}} + \frac{1}{\sqrt{20}}ie^{-i\phi}\right|^2 \\ &= \frac{1}{20}(9 + 1 - 3i(e^{i\phi} - e^{-i\phi})) \\ &= \frac{1}{2} + \frac{3}{10}\sin\phi \end{aligned}$$

It is given that $P(+y) = \frac{1}{2}$, so

$$\sin\phi = -1 \Rightarrow \phi = -\frac{\pi}{2} \Rightarrow e^{i\phi} = -i$$

Therefore,

$$\boxed{|\psi\rangle = e^{i\theta}(\frac{3}{\sqrt{10}}|+\mathbf{z}\rangle - \frac{i}{\sqrt{10}}|-\mathbf{z}\rangle) \quad \theta \in \mathbb{R}}$$

As $|+\mathbf{x}\rangle = \frac{1}{\sqrt{2}}|+\mathbf{z}\rangle + \frac{1}{\sqrt{2}}|-\mathbf{z}\rangle$,

$$\boxed{P(+x) = \left|\frac{3}{\sqrt{20}} - \frac{1}{\sqrt{20}}i\right|^2 = \frac{1}{2}}$$