Electrodynamics and Optics

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1 Preliminary knowledge

Classical electrodynamics is governed by the four Maxwell's equations

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\dot{\mathbf{B}} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \dot{\mathbf{D}}, \end{aligned} \tag{1}$$

where $\mathbf{D}$ and $\mathbf{B}$ are the electric/magnetic flux density while $\mathbf{E}$ and $\mathbf{H}$ are the corresponding fields. These are related via $\mathbf{D} = \epsilon_0 \underline{\underline{\epsilon}} \cdot \mathbf{E}, \quad \mathbf{B} = \mu_0 \underline{\underline{\mu}} \cdot \mathbf{H}$, where $\underline{\underline{\epsilon}}$ and $\underline{\underline{\mu}}$ are Hermitian tensors (thus diagonalisable) of the second rank.

In the absence of charge and current, the EM field can propagate as harmonic waves

$$\mathbf{E} = \mathbf{E_0} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \quad \mathbf{B} = \mathbf{B_0} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)},$$

satisfying $\mathbf{k} \cdot D = 0, \quad \mathbf{k} \cdot \mathbf{B} = 0, \quad \mathbf{k} \times \mathbf{E} = \omega \mathbf{B}, \quad \mathbf{k} \times \mathbf{H} = -\omega \mathbf{D}$. The wave also carries an energy density $u = \frac{1}{2}\mathbf{E} \cdot \mathbf{D} + \frac{1}{2}\mathbf{B} \cdot \mathbf{H}$ and the energy flux is $\mathbf{N} = \mathbf{E} \times \mathbf{H}$.

2 Optics

The Maxwell's equations are linear, making it possible to investigate polarised orthogonal components of fields separately.

2.1 Jones Notation