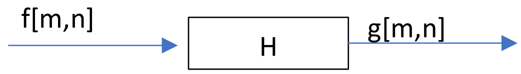
DSC 120: Homework 6

Signal denoising and Image processing

Due 5-18-2022

1) The following system is given:



$$g[m,n] = H\{f[m,n]\} = f[2m-1, 3n+1]$$

Is the system linear? (if yes prove you answer , if not provide an counterexample.)

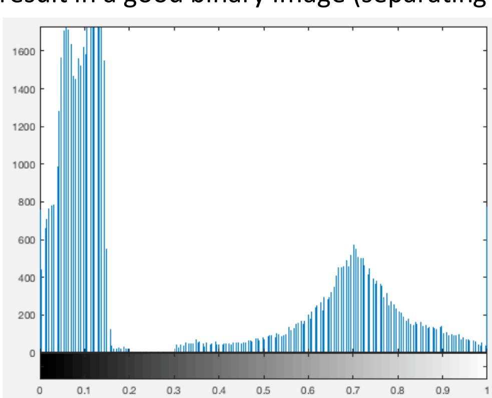
2) Image Processing Pipeline

We want to count the number of coins in the following image

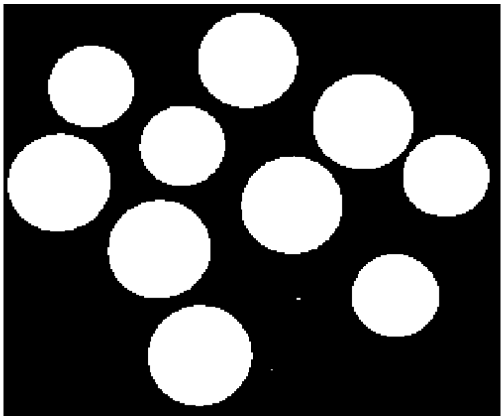
Part A: To do so the first step we want to perform is to convert the image to a **binary** image. The following system takes as input an image with values *f[m,n]*∈[0,1] and makes it binary *g[m,n]*∈{0,1} with respect to threshold τ:

$$g[m,n] = \begin{cases} 0, & f[m,n] < \tau \\ 1, & f[m,n] \geq \tau \end{cases}$$

1. Is the system linear? (if yes prove you answer , if not provide an counterexample.)
2. This is the histogram of the coins image. What is the range of values of threshold τ that will result in a good binary image (separating coins from background)?



Part B: The binarization threshold was chosen such that there are a few residual bright pixels in the background of the image:



To remove these pixels we will use an **erode** operation as follows : For any neighborhood *E*, and input image I_0 we get the output image I_1 by :

$$I_1[m,n] = \min_{i,j \in E}\{I_0[m-i,n-j]\}$$

Where $[m,n]$ is the central pixel of the neighborhood. In this question we will use a 4-adjanceny neighborhood (the four pixels surrounding the central pixel as you can see in the illustration), meaning $E \in \{-1,0,1\}$. The center pixel is the center of the cross and is also defined as a pixel [0,0]:

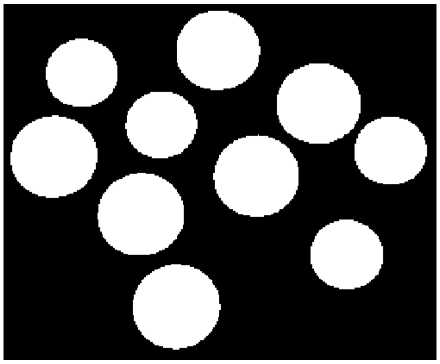
	1	
1	1	1
	1	

1. What will be the output image I_1 for the next input image I_0

$I_0 =$

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
1	1	1	0	0

2. Is the erode operation linear? (If yes prove your answer , If not provide an counterexample)
3. Applying erode to the image has removed the background pixels



Explain what happens to the radius of the coins following the erode operation. Has it changed?

Part C: We want to detect edges in the binary image. Edge detection along the columns is given as output to the following system:

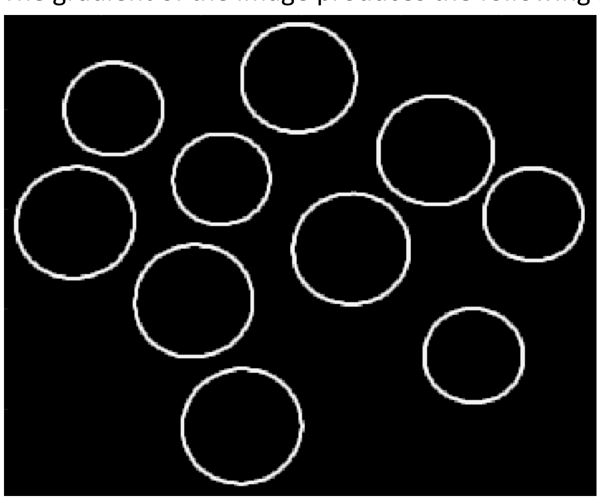
$I_x[m,n] = H_x\{I[m,n]\} = I[m+1,n+1] + I[m,n+1] + I[m-1,n+1] - I[m+1,n-1] - I[m,n-1] - I[m-1,n-1]$

1. Write the system as a convolution with a filter $h_x[m,n]$ and write out $h_x[m,n]$ explictly.

Edge detection along the rows is given by:

$I_y[m,n] = H_y\{I[m,n]\} = I[m+1,n+1] + I[m+1,n] + I[m+1,n-1] - I[m-1,n-1] - I[m-1,n] - I[m-1,n+1]$

2. If $z_1 = H_y\{H_x\{I\}\}$ and $z_2 = H_x\{H_y\{I\}\}$ are z_1 and z_2 equal? Explain.
3. The gradient of the image produces the following results:



and is given by $G[m,n] = \sqrt{\left(I_x\ [m,n]\right)^2 + \left(I_y\ [m,n]\right)^2}$. Is this a linear operation? (If yes prove your answer , If not provide an counterexample)

Bonus: Part D: Given the clean edge image, describe in words

1. How can you determine the number of coins in the image?
2. How can you determine how many different kinds of coins (dime, nickel, quarter,…)are there in the image?

Coding Problems:

1. **Filtering** In the examples we did in class, we were interested in filtering out the high frequency noise in order to find the low dimensional signal. However, there are other scenarios where one wants to remove the low-frequency oscillations and keep the more medium and high frequency data.

   You are given data of daily sales of a major retail chain for one year. The data is found in the 'seasonalData.csv' file located in the homework folder. There are both seasonal oscillations, short term trends, and daily variability (noise). Answer and hand in the following questions:

   (a) Plot the magnitudes of the DFT of this data.

   (b) Determine how seasonality affects sales. Use a low-pass filter and plot the long term periodicity of the sales. Hint: this is on the time scale of seasons (Spring, Summer, Fall, Winter).

   (c) Determine the spikes in sales. Several times during the year, this company released a new product that generated significant sales. Construct a filter that is medium pass (i.e. is only nonzero in medium size frequencies) that removes both the day-to-day noise and the seasonal periodicity. Use this filter to find and plot where the spikes in sales occurred.

2. **Image Filtering** In this problem, we have images with periodic noise that we want to remove. The technique for such denoising is:

   (a) Load the image and make sure it's loaded as a float format (to load an image use *matplotlib.pyplot.imread*)

   (b) Take an FFT2, with fftshift, and visualize the magnitudes of the coefficients (it may help to use a log-color-scale, or display the logs of the DFT magnitudes, to better see spikes)

   (c) Either algorithmically or manually identify and zero out the spikes at high frequency that correspond to the noise

   (d) Covert back to an image with IFFT2 and visualize the new image

   We will do this for three images with different types of noise. You can write one piece of code and apply it to each image, or have different code for each image.

   (a) vertNoise.png

   (b) horizNoise.png

   (c) checkeredNoise.png