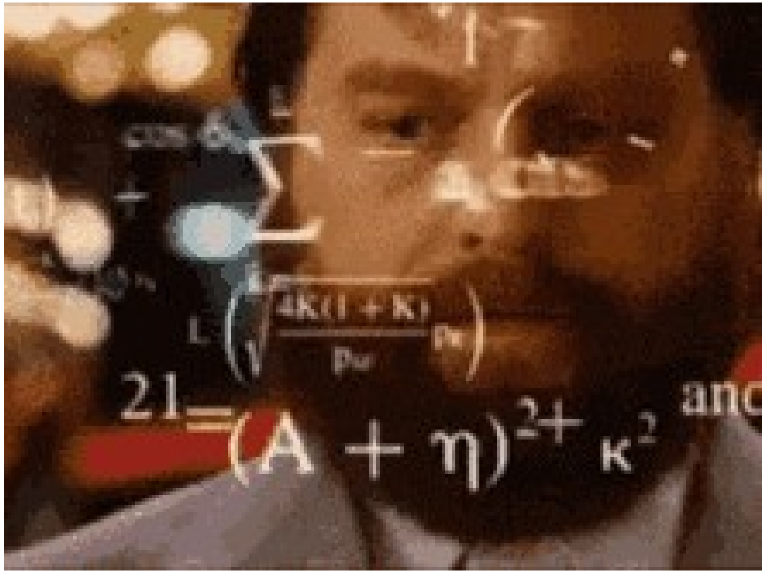
# You know that Ogden hyperelastic formulation right?

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2} \left( \lambda_1^{m_a} + \lambda_2^{m_a} + \lambda_3^{m_a} - 3 \right)$$

# Hmm it is difficult to see how it works as a material law

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2}\left(\lambda_1^{m_a} + \lambda_2^{m_a} + \lambda_3^{m_a} - 3\right)$$

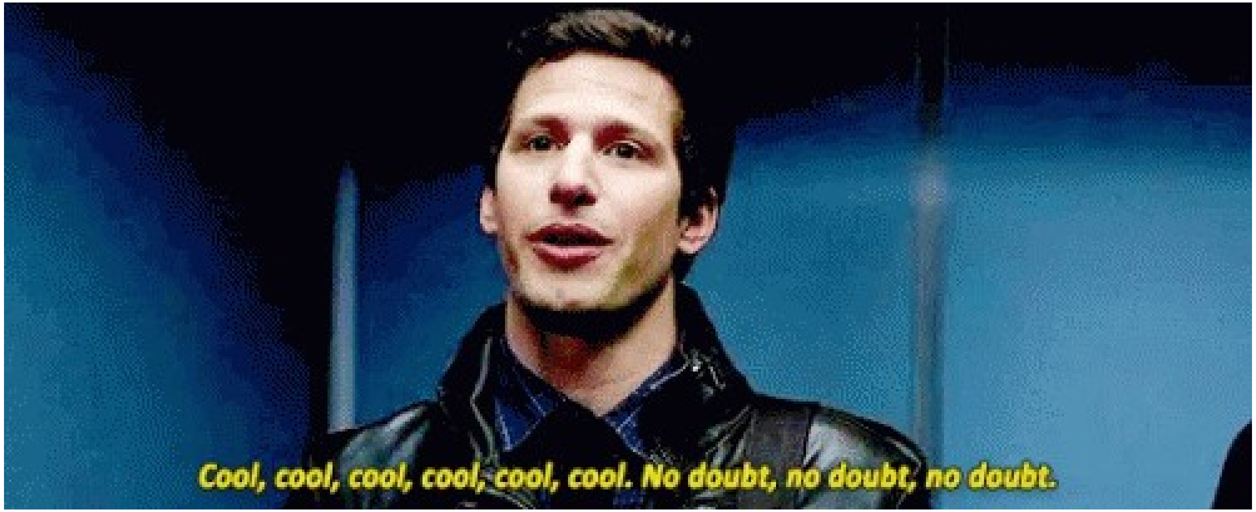


# Let's explore the "anatomy" of the Ogden formulation

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2}\left(\lambda_1^{m_a} + \lambda_2^{m_a} + \lambda_3^{m_a} - 3\right)$$

# First let’s distribute the - 3 as several - 1’s

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2} \left( \lambda_1^{m_a} + \lambda_2^{m_a} + \lambda_3^{m_a} - 3 \right)$$



# Yep, thats one, let's do another...

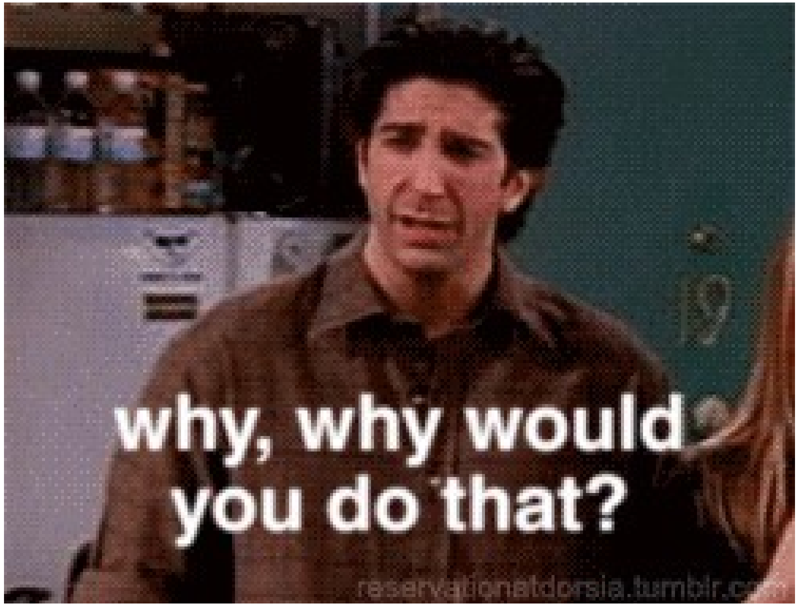
$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2} \left( \lambda_1^{m_a} - 1 + \lambda_2^{m_a} + \lambda_3^{m_a} - 2 \right)$$

# Got it

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2} \left( \lambda_1^{m_a} - 1 + \lambda_2^{m_a} - 1 + \lambda_3^{m_a} - 1 \right)$$

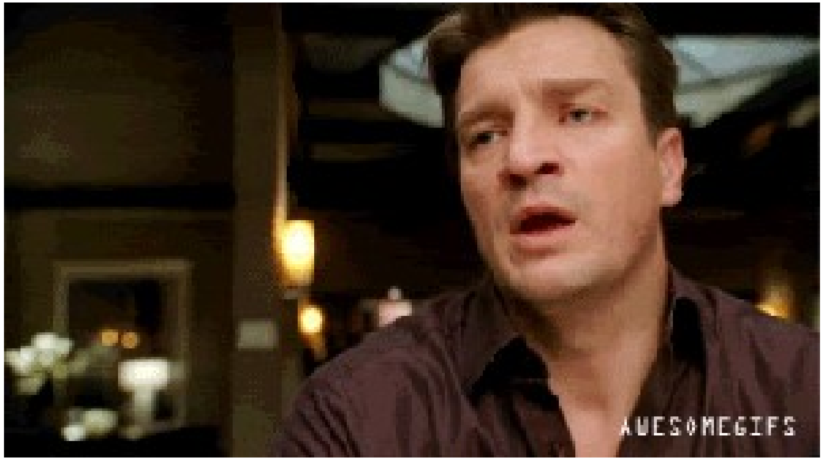
# Okay, great so now we can group a bunch of "stretch minus onezy" bits

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2} \left( (\lambda_1^{m_a} - 1) + (\lambda_2^{m_a} - 1) + (\lambda_3^{m_a} - 1) \right)$$



Hold on. Now lets factor out $\frac{1}{m_a}$

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a^2} \left( (\lambda_1^{m_a} - 1) + (\lambda_2^{m_a} - 1) + (\lambda_3^{m_a} - 1) \right)$$



# Cool now work that in front of each of the"stretch minus onezy bits"

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \frac{1}{m_a} \left( \left(\lambda_1^{m_a} - 1\right) + \left(\lambda_2^{m_a} - 1\right) + \left(\lambda_3^{m_a} - 1\right) \right)$$

# Got it!

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \frac{1}{m_a} (\lambda_1^{m_a} - 1) + \frac{1}{m_a} (\lambda_2^{m_a} - 1) + \frac{1}{m_a} (\lambda_3^{m_a} - 1) \right)$$

# Wait those look strangely like strains!

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a}\left(\frac{1}{m_a}\left(\lambda_1^{m_a} - 1\right) + \frac{1}{m_a}\left(\lambda_2^{m_a} - 1\right) + \frac{1}{m_a}\left(\lambda_3^{m_a} - 1\right)\right)$$

# Yep! The Seth-Hill class of strains

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \frac{1}{m_a}(\lambda_1^{m_a} - 1) + \frac{1}{m_a}(\lambda_2^{m_a} - 1) + \frac{1}{m_a}(\lambda_3^{m_a} - 1) \right)$$

$$E_i^{(m_a)} = \frac{1}{m_a}(\lambda_i^{m_a} - 1)$$

# Which allows you to pick a strain flavour

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \frac{1}{m_a}(\lambda_1^{m_a} - 1) + \frac{1}{m_a}(\lambda_2^{m_a} - 1) + \frac{1}{m_a}(\lambda_3^{m_a} - 1) \right)$$

$$E_i^{(m_a)} = \frac{1}{m_a}(\lambda_i^{m_a} - 1)$$

# Like chocolate or strawberry or Green-Lagrange strain if you use m=2 (Neo-Hookean!)

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a}\left(\frac{1}{m_a}(\lambda_1^{m_a} - 1) + \frac{1}{m_a}(\lambda_2^{m_a} - 1) + \frac{1}{m_a}(\lambda_3^{m_a} - 1)\right)$$

$$E_i = \frac{1}{2}(\lambda_i^2 - 1)$$

# Postive/negative m's favour tension/compression

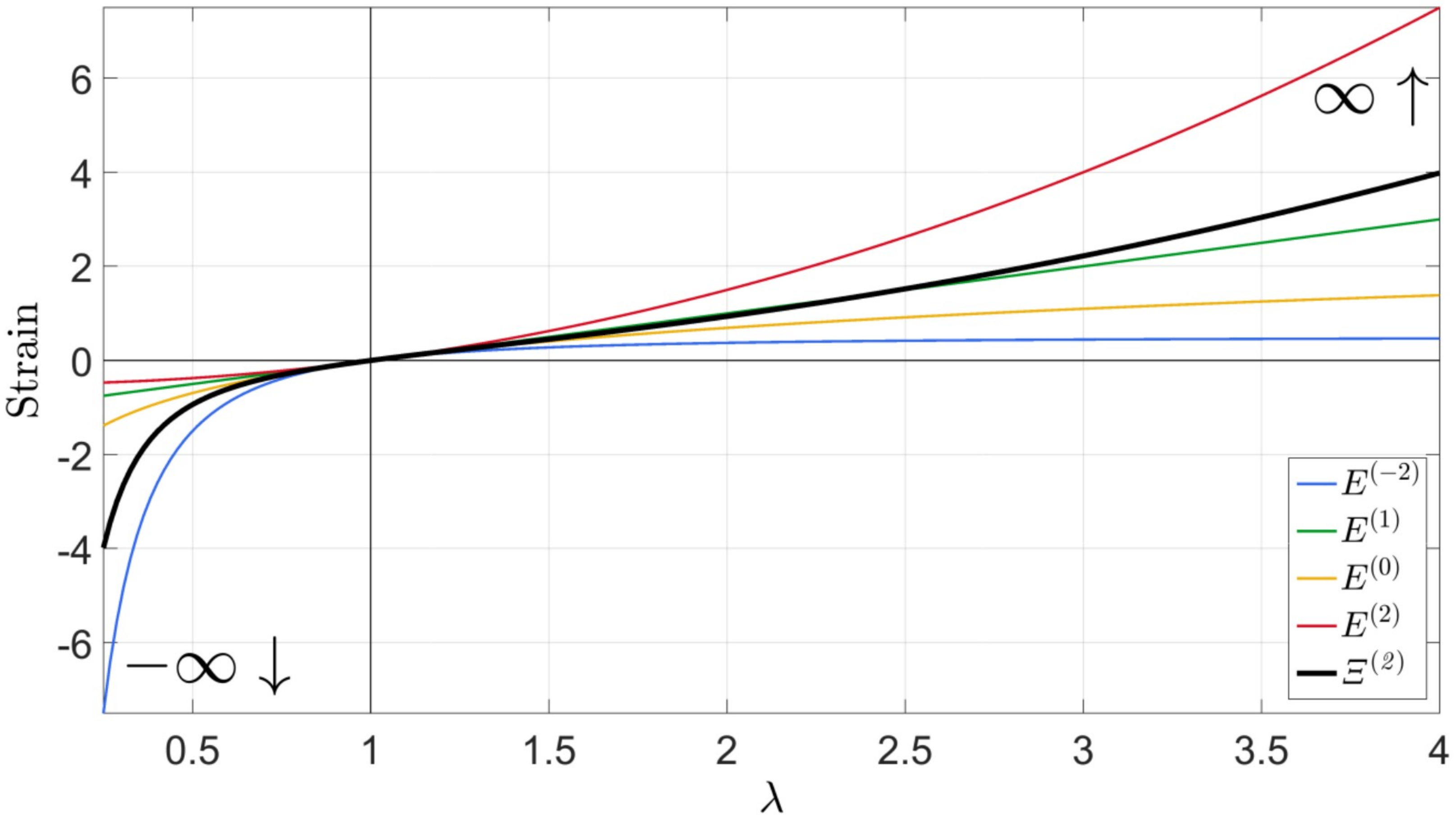



*Figure 1 Various strain measures as a function of stretch (uniaxial). The first 4 coloured curves represent Seth-Hill strains*

But we can clean this up a bit again. Let's use summation for those three strains.

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \frac{1}{m_a}(\lambda_1^{m_a} - 1) + \frac{1}{m_a}(\lambda_2^{m_a} - 1) + \frac{1}{m_a}(\lambda_3^{m_a} - 1) \right)$$

$$E_i^{(m_a)} = \frac{1}{m_a}(\lambda_i^{m_a} - 1)$$

# Ahhh... that is so much better

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \sum_{i=1}^{3} \frac{1}{m_a} (\lambda_i^{m_a} - 1) \right)$$

$$E_i^{(m_a)} = \frac{1}{m_a} (\lambda_i^{m_a} - 1)$$

# Wait.. we're using principal stretches.... so those are principal strains....

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \sum_{i=1}^{3} \frac{1}{m_a} (\lambda_i^{m_a} - 1) \right)$$

$$E_i^{(m_a)} = \frac{1}{m_a} (\lambda_i^{m_a} - 1)$$

So we are summing all three principal strains of $\mathbf{E}^{(m_a)}$

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \sum_{i=1}^{3} \frac{1}{m_a} (\lambda_i^{m_a} - 1) \right)$$

$$E_i^{(m_a)} = \frac{1}{m_a} (\lambda_i^{m_a} - 1)$$

# But that is the trace of that strain tensor!

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \left( \sum_{i=1}^{3} \frac{1}{m_a} (\lambda_i^{m_a} - 1) \right)$$

$$\sum_{i=1}^{3} \frac{1}{m_a} (\lambda_i^{m_a} - 1) = \mathrm{tr}(\mathbf{E}^{(m_a)})$$

Wow so Ogden features:

**"a bunch of traces of desired strain flavours"**

$$\Psi = \sum_{a=1}^{N} \frac{c_a}{m_a} \mathrm{tr}(\mathbf{E}^{(m_a)})$$