# Métodos de Integração

# Metodos De Integração

① Integração por substituição Simples

② Integração por Partes $\left(\int u\,dv = u \cdot v - \int v\,du\right)$

③ Integração por Substituições Trigonométricas

④ Integração por Frações Parciais

① $\displaystyle\int \frac{x+3}{\underbrace{(x^2+6x)}_{u}^{\frac{1}{3}}}\,dx \therefore \begin{cases} u = x^2+6x\,dx \\ du = 2x+6\,dx \\ du = 2.(x+3)\,dx \\ \frac{du}{2} = x+3\,dx \end{cases}$

$\displaystyle\therefore \int \frac{\frac{du}{2}}{u^{\frac{1}{3}}} = \frac{1}{2}\int \frac{du}{u^{\frac{1}{3}}}$

$\displaystyle\frac{1}{2}\int u^{-\frac{1}{3}}\,du$

$\displaystyle\frac{1}{2} \cdot \frac{u^{\frac{2}{3}}}{\frac{2}{3}} = \frac{1}{2} \cdot \frac{3u^{\frac{2}{3}}}{2}$

$\displaystyle\frac{3}{4}u^{\frac{2}{3}} \therefore \frac{3}{4} \cdot (x^2+6x)^{\frac{2}{3}} + C$

① $\displaystyle\int \frac{dx}{25+16x} \therefore \begin{cases} u = 16x+25\,dx \\ du = 16\,dx \\ \frac{du}{16} = dx \end{cases}$

$\displaystyle\int \frac{\frac{du}{16}}{u} = \frac{1}{16}\int \frac{du}{u} = \frac{1}{16}\int \frac{1}{u}\,du \quad \to \ln|u|$

$\displaystyle\frac{1}{16} \cdot \ln|16x+25| + C$

② $\int \overset{dv}{x}\,\underset{u}{(1+x)}^{\frac{2}{3}}\,dx$ $\quad \left( \int u\,dv = u\,v - \int v\,du \right)$

$u = x+1 \qquad du = dx$

$dv = x \qquad v = \frac{x^2}{2}$

$\frac{x^2}{2} \cdot (x+1) - \int \frac{x^2}{2}\,dx$

$\frac{x^3}{2} + \frac{x^2}{2} - \frac{1}{2}\int x^2\,dx$

$\frac{x^3+x^2}{2} - \frac{1}{2} \cdot \frac{x^3}{3}$

$\frac{x^3+x^2}{2} - \frac{x^3}{6} \therefore \frac{6x^3+6x^2-2x^3}{12}$

$\frac{x^3}{3} + \frac{x^2}{2} + C$ ou $\frac{4x^3+6x^2}{12} + C$

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② $\int \frac{\overset{dv}{x}}{\underset{u}{x^4+3}}\,dx$

$u = x^4+3 \qquad du = 4x^3\,dx$

$dv = x \qquad v = \frac{x^2}{2}$

$\frac{x^2}{2}(x^4+3) - \int \frac{x^2}{2} \cdot 4x^3\,dx$

$\frac{x^6}{2} + \frac{3x^2}{2} - \frac{1}{2}\int x^2 \cdot 4x^3\,dx$

$\frac{x^6}{2} + \frac{3x^2}{2} - 2\int x^5\,dx$

$\frac{x^6}{2} + \frac{3x^2}{2} - 2 \cdot \frac{x^6}{6}$

$\frac{x^6}{2} + \frac{3x^2}{2} - \frac{x^6}{3} \therefore \frac{6x^6+18x^2-4x^6}{12}$

$\frac{2x^6+18x^2}{12} + C$

ou

$\frac{x^6}{6} + \frac{3x^2}{2} + C$

$$\int sec^3(x)\, dx \therefore \int \overset{u}{sec(x)} \,.\, \overset{dv}{sec^2(x)}\, dx \quad \begin{cases} u = sec(x) & du = tg(x) \,.\, sec(x) \\ dv = sec^2(x) & v = tg(x) \end{cases}$$

$$\int sec^3(x)\, dx = tg(x) \,.\, sec(x) - \int tg(x) \,.\, tg(x) \,.\, sec(x)\, dx$$

**Teorema Fundamental**

$$\boxed{sen^2 + cos^2 = 1}$$

**da Trigonometria**

$$- \int tg^2(x) \,.\, sec(x)\, dx$$

$$\frac{sen^2}{cos^2} + \frac{\cancel{cos^2}}{\cancel{cos^2}} = \frac{1}{cos^2}$$

$$- \left( \int (sec^2 - 1) \,.\, sec\, dx \right)$$

$$tg^2 + 1 = sec^2$$

$$- \int sec^3(x)\, dx + \int sec(x)\, dx$$

$$tg^2 = sec^2 - 1$$

$$\to \ln \left| sec(x) + tg(x) \right| + C$$

$$\int sec^3(x)\, dx = tg\, x \,.\, sec(x) - \int sec^3(x)\, dx + \ln \left| sec(x) + tg(x) \right|$$

$$2 \int sec^3(x)\, dx = tg(x) \,.\, sec(x) + \ln \left| sec(x) + tg(x) \right|$$

$$\int sec^3(x)\, dx = \frac{1}{2} \cdot tg(x) \,.\, sec(x) + \ln \left| sec(x) + tg(x) \right|$$

③ $\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$

$$\begin{cases} a = 4 \\ x = 4\,\text{sen}\,\theta \therefore \text{sen}\,\theta = \frac{x}{4} \\ dx = 4\cos\theta\,d\theta \end{cases}$$

$\text{sen}^2 + \cos^2 =$

$\cos^2 = 1 - \text{sen}^2$

$$\int \frac{4\cos\theta}{(4\,\text{sen}\,\theta)^2 \cdot \sqrt{16-(4\,\text{sen}\theta)^2}}\,d\theta \therefore \int \frac{4\cos\theta}{16\,\text{sen}\,\theta \cdot \sqrt{16-16\,\text{sen}^2\theta}} \therefore \int \frac{4\cos\theta}{16\,\text{sen}^2\theta\,\sqrt{16.(1-\text{sen}^2\theta)}}$$

evidência

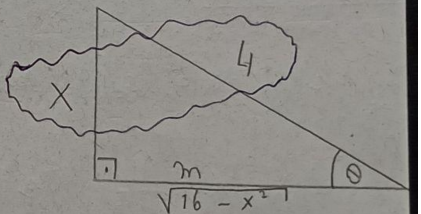
substituição trigonométrica

$$\int \frac{4\cos\theta}{16\,\text{sen}^2\theta\,\sqrt{16\cos^2}}\,d\theta \therefore \int \frac{4\cancel{\cos}\theta}{16\,\text{sen}^2\theta \cdot 4\cancel{\cos}\theta}\,d\theta \therefore \int \frac{1}{16\,\text{sen}^2\theta}\,d\theta$$

$$\frac{1}{16}\int \frac{1}{\text{sen}^2\theta}\,d\theta \quad \left\{ \frac{\text{sen}^2}{\text{sen}^2} + \frac{\cos^2}{\text{sen}^2} = \frac{1}{\text{sen}^2} \therefore 1 + \text{cotg}^2 = \text{cossec}^2 \right.$$

$$\frac{1}{16}\int \text{cossec}^2\,d\theta \therefore \frac{1}{16}\cdot(-\text{cotg}\,\theta) + C \therefore -\frac{1}{16}\cdot\frac{\cos}{\text{sen}} + C$$

$$-\frac{\cos}{16\,\text{sen}} + C \therefore -\frac{\frac{\sqrt{16-x^2}}{4}}{16\cdot\frac{x}{4}}$$




$$-\frac{\sqrt{16-x^2}}{\cancel{4}}\cdot\frac{\cancel{4}}{16x}$$

$$\begin{cases} x^2 + m^2 = 4^2 \\ m^2 = 16 - x^2 \\ m = \sqrt{16-x^2} \end{cases}$$

$$-\frac{\sqrt{16-x^2}}{16x} + C$$

$$\int \frac{\sqrt{x^2-9}}{x}\,dx \qquad \begin{cases} a = 3 \\ x = 3 \,.\, \sec\theta \;\therefore\; \sec\theta = \dfrac{x}{3} \\ dx = \dfrac{3 \operatorname{sen}\theta}{\cos^2\theta}\, d\theta \end{cases} \qquad \begin{cases} \dfrac{\operatorname{sen}^2}{\cos^2} + \dfrac{\cos^2}{\cos^2} = \dfrac{1}{\cos^2} \\ tg^2 + 1 = \sec^2 \\ \boxed{tg^2 = (\sec^2 - 1)} \end{cases}$$

$3 \,.\, tg \,.\, \sec$

$\frac{3}{1} \cdot \frac{\operatorname{sen}}{\cos} \cdot \frac{1}{\cos}$

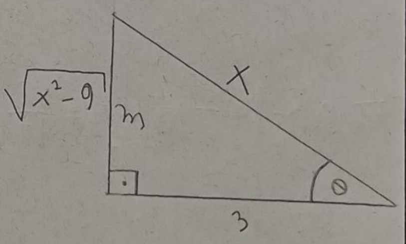
$$\int \frac{\sqrt{(3\sec^2\theta) - 9}}{3\sec\theta}\,dx \;\therefore \int \frac{\sqrt{(9\sec^2 - 9)}}{3\sec\theta}\,dx \;\therefore \int \frac{\sqrt{9.(\sec^2-1)}}{3\sec\theta}\,dx$$

$$\int \frac{\sqrt{9\,tg^2}}{3\sec\theta}\,dx \;\therefore \int \frac{\cancel{3}\,tg\,\theta}{\cancel{3}\sec\theta} \cdot \frac{3\operatorname{sen}\theta}{\cos^2\theta}\,d\theta \;\therefore 3\int tg^2\,d\theta$$

$$3\,tg\,\theta - 3\theta + C$$

$$3\,\frac{\sqrt{x^2-9}}{3} - 3\theta \;\therefore\; \sqrt{x^2-9} - 3\theta$$

$$\frac{1}{\cos} = \frac{x}{3} \;\therefore\; \frac{1}{\frac{\text{cat A}}{\text{Hip}}} = \frac{x}{3} \;\therefore\; \frac{\text{Hip}}{\text{cat A}} = \frac{x}{3}$$




$$m^2 + 3^2 = x^2$$

$$m^2 = x^2 - 9$$

$$m = \sqrt{x^2-9}$$

$$④ \int \frac{x^2+4}{x^4-x^3-7x^2+x+6}\,dx$$

$\pm1 \quad \pm2 \quad \pm3 \quad \pm6$

(Raízes possíveis → Divisores do termo independente)

$1-1-7+1+6=0$

$\boxed{\text{Raiz }(1)}$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | -1 | -7 | 1 | 6 |
| | 1 | 0 | -7 | -6 | 0 |

$[x^3 + 0x^2 - 7x - 6]$

$-1+7-6=0$

$\boxed{\text{Raiz }(-1)}$

| | | | | |
|---|---|---|---|---|
| -1 | 1 | 0 | -7 | -6 |
| | 1 | -1 | -6 | 0 |

$[x^2 - x - 6]$

$S = -\frac{b}{a} \qquad P = \frac{c}{a}$

$S = \frac{1}{1} = 1$

$P = \frac{-6}{1} = -6$

$[3] \quad [-2]$

OU

$(-1)^2 - 4 \cdot 1 \cdot -6$

$1 + 24 = 25$

$\frac{1 \pm 5}{2} \to \frac{6}{2} = 3$

$\to -\frac{4}{2} = -2$

$\boxed{\text{Raiz }(3)}$

$\boxed{\text{Raiz }(-2)}$

$$\int \frac{x^2+4}{(x-1)\cdot(x+1)\cdot(x-3)\cdot(x+2)} = \int \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x-3)} + \frac{D}{(x+2)}$$

$$x^2+4 = A(x+1)\cdot(x-3)\cdot(x+2) + B(x-1)\cdot(x-3)\cdot(x+2) + C(x-1)\cdot(x+1)\cdot(x+2) + D(x-1)\cdot(x+1)\cdot(x-3)$$

$\boxed{\text{Para } x=1}$

$x^2+4 = -12A$

$A = -\frac{5}{12}$

$\boxed{\text{Para } x=-1}$

$x^2+4 = 8B$

$B = \frac{5}{8}$

$\boxed{\text{Para } x=3}$

$x^2+4 = 40C$

$C = \frac{13}{40}$

$\boxed{\text{Para } x=-2}$

$x^2+4 = -15D$

$D = -\frac{8}{15}$

$$\int \frac{-\frac{5}{12}}{(x-1)} + \frac{\frac{5}{8}}{(x+1)} + \frac{\frac{13}{40}}{(x-3)} + \frac{-\frac{8}{15}}{(x+2)}$$

$$-\frac{5}{12}\int \frac{1}{(x-1)}dx + \frac{5}{8}\int \frac{1}{(x+1)}dx + \frac{13}{40}\int \frac{1}{(x-3)}dx - \frac{8}{15}\int \frac{1}{(x+2)}dx$$

$$-\frac{5}{12}\ln|x-1| + \frac{5}{8}\ln|x+1| + \frac{13}{40}\ln|x-3| - \frac{8}{15}\ln|x+2| + C$$

# Fórmulas

(Trigonométricas)

① $\int \operatorname{sen}(kx)\,dx = -\frac{1}{k}\cos(kx) + C$

② $\int \cos(kx)\,dx = \frac{1}{k}\operatorname{sen}(kx) + C$

③ $\int \sec^2(kx)\,dx = \frac{1}{k}\operatorname{tg}(kx) + C$

④ $\int \operatorname{cossec}^2(kx)\,dx = -\frac{1}{k}\operatorname{cotg}(kx) + C$

⑤ $\int \operatorname{sen}(kx)\cdot\operatorname{tg}(kx)\,dx = \frac{1}{k}\sec(kx) + C$

⑥ $\int \operatorname{cossec}(kx)\cdot\operatorname{cotg}(kx)\,dx = -\frac{1}{k}\cdot\operatorname{cossec}(kx) + C$

⑦ $\int \operatorname{tg}(x)\,dx = -\ln|\cos(x)| + C$

⑧ $\int \operatorname{cotg}(x)\,dx = \ln|\operatorname{sen}(x)| + C$

⑨ $\int \sec(x)\,dx = \ln|\sec(x) + \operatorname{tg}(x)| + C$

⑩ $\int \operatorname{cossec}(x)\,dx = \ln|\operatorname{cossec}(x) - \operatorname{cotg}(x)| + C$

(Por Partes)

$\int u\,dv = uv - \int v\,du$

(Exponenciais e Logarítmicas)

① $\int e^x\,dx = e^x + C$

② $\int a^x\,dx = \frac{1}{\ln|a|}\cdot a^x + C$

③ $\int \ln|x|\,dx = x(\ln|x| - 1) + C$

④ $\int \log_a x\,dx = \frac{1}{\ln|a|}\int \ln|x|\,dx$

(Trigonométricas Inversas)

① $\int \frac{1}{\sqrt{a^2 - u^2}}\,du = \operatorname{arc\,sen}\frac{u}{a} + C$

② $\int \frac{1}{a^2 + u^2}\,du = \frac{1}{a}\operatorname{arc\,tg}\frac{u}{a} + C$

③ $\int \frac{1}{u\sqrt{u^2 - a^2}}\,du = \frac{1}{a}\operatorname{arc\,sen}\frac{u}{a} + C$

(Substituição Trigonométricas Possíveis)

① $\sqrt{a^2 - x^2} \therefore x = a\cdot\operatorname{sen}\theta$

② $\sqrt{a^2 + x^2} \therefore x = a\cdot\operatorname{tg}\theta$

③ $\sqrt{x^2 - a^2} \therefore x = a\cdot\sec\theta$

Regra do quociente (Derivadas)

$$F(x) = \frac{f(x)}{g(x)} \qquad \left[\frac{u'v - uv'}{v^2}\right]$$

$$F'(x) = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{g^2(x)}$$

Regra do Produto

$$F(x) = g(x) \cdot h(x) \quad [u'v + uv']$$

$$F'(x) = g(x) \cdot h'(x) + h(x) \cdot g'(x)$$

$\frac{1}{sen} \rightarrow$ cossecante

$\frac{1}{cos} \rightarrow$ secante

$\frac{cos}{sen} \rightarrow$ cotangente

$$\int e^{2x}\,dx = \frac{e^{2x}}{2} + C$$

$$\int e^{3x}\,dx = \frac{e^{3x}}{3} + C$$

$$\int e^{4x}\,dx = \frac{e^{4x}}{4} + C$$

$$sen^2\theta + cos^2\theta = 1$$

(Volume)

$$V = \pi \int_a^b f^2(x)\,dx$$

$$V_{anel} = \pi \int_a^b f^2(x) - g^2(x)\,dx$$

(comprimento de curva)

$$\int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

$u$: $sec(x)$ — $du$: $tg(x) \cdot sec(x)$

$u$: $sen(x)$ — $du$: $cos(x)$

$u$: $cos(x)$ — $du$: $-sen(x)$

$u$: $tg(x)$ — $du$: $sec^2(x)$

$u$: $ln|x|$ — $du$: $\frac{1}{x}\,dx$

$u$: $e^{2x}$ — $du$: $2e^{2x}$

$u$: $e^{3x}$ — $du$: $3e^{3x}$

$u$: $e^{4x}$ — $du$: $4e^{4x}$

$u$: $e^{5x}$ — $du$: $5e^{5x}$

$u$: $e^{100x}$ — $du$: $100e^{100x}$

(Integral Impropria)

$$\int_1^{+\infty} \frac{1}{x}\,dx$$

$$\lim_{t \to +\infty} \int_1^{t} \frac{1}{x}\,dx \quad \left[\text{Divergente ou Convergente}\right]$$

# Derivadas

**Regra do quociente**

$$\left[\frac{u'v - uv'}{v^2}\right]$$

**Regra do Produto**

$$\left[u'v + uv'\right]$$

$u$: $sen(x)$    $du$: $cos(x)$

$u$: $cos(x)$    $du$: $-sen(x)$

$u$: $tg(x)$    $du$: $sec^2(x)$

$u$: $ln(x)$    $du$: $\frac{1}{x}\,dx$

$u$: $e^{x}$    $du$: $1 \cdot e^{x}$

$u$: $e^{2x}$    $du$: $2e^{2x}$

$u$: $e^{5x}$    $du$: $5e^{5x}$

$u$: $e^{100x}$    $du$: $100e^{100x}$

$u$: $\log_b(x)$    $du$: $\frac{1}{x\,ln(b)}$

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$f(x) = x^3 - 9x + 3$

$f'(x) = 3x^2 - 9$

$f''(x)$ $6x$

$f(x) = xe^x - 1$

$f'(x) = e^x + xe^x$

$f''(x) = e^x + e^x + xe^x = 2e^x + xe^x$

$f(x) = cos(5x)$

$f'(x) = -5 \cdot sen(5x)$

$f''(x) = -25 \cdot cos(5x)$