State Estimation and Control of a Bipedal Robot

Derivation of Kinetic Equations

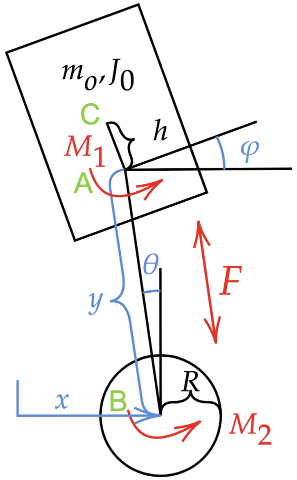
Based on Lagrange equation

$$\frac{(m_0+m_1+m_2+\frac{J_2}{R^2})}{m_0}\ddot{x}-y_0\ddot{\theta}-h\ddot{\varphi}=-\frac{M_2}{m_0R}$$

$$\ddot{y}=\frac{F}{m_0}-g$$

$$-y_0\ddot{x}+(\frac{J_1}{m_0}+y_0^2)\ddot{\theta}+hy_0\ddot{\varphi}=\frac{M_1-M_2}{m_0}+gy_0\theta$$

$$-h\ddot{x}+hy_0\ddot{\theta}+(\frac{J_0}{m_0}+h^2)\ddot{\varphi}=-\frac{M_1}{m_0}+gh\varphi$$



Optimal Control Algorithm

We designed control algorithms based on LQR (implemented on the real-world robot) and MPC (tested in simulation)

Slip Detection and State Estination

Based on physical first principle that the contact points are relatively stationary for pure rolling, we assume that the contact points of the two wheels and the ground are moving relative to each other when slippinp

$$\vec{v}_{\mathrm{WL}}=\vec{v}_0+\vec{\omega}\times\left(-\tfrac{1}{2}\vec{W}+\vec{l}_{\mathrm{L}}+R\hat{t}\right)+\vec{v}_{\mathrm{L}}+\vec{\omega}_{\mathrm{L}}\times R\hat{t}$$

$$\vec{v}_{\mathrm{WR}}=\vec{v}_0+\vec{\omega}\times\left(\tfrac{1}{2}\vec{W}+\vec{l}_{\mathrm{R}}+R\hat{t}\right)+\vec{v}_{\mathrm{R}}+\vec{\omega}_{\mathrm{R}}\times R\hat{t}$$

$$\Rightarrow\vec{v}_{\mathrm{WR}}-\vec{v}_{\mathrm{WL}}=\vec{\omega}\times\left(\vec{W}+\vec{l}_{\mathrm{R}}-\vec{l}_{\mathrm{L}}\right)+\vec{v}_{\mathrm{R}}-\vec{v}_{\mathrm{L}}+(\vec{\omega}_{\mathrm{R}}-\vec{\omega}_{\mathrm{L}})\times R\hat{t}$$

$$|\vec{v}_{\mathrm{WR}}-\vec{v}_{\mathrm{WL}}|>v_{\mathrm{threshold}}\ \text{implies the wheel is slipping}$$

$$s'=As+Bu+g$$

$$P'=APA^{\top}+Q$$

$$s=s'+P'H^{\top}\left(HP'H^{\top}+R\right)^{-1}(z-Hs')$$

$$P=(I-KH)P'$$

When slipping, we switch the Kalman filter's observation noise covariance matrix

Relevant video: https://www.youtube.com/watch?v=DIakTY5WKMU