Problem Chosen	2022	Team Control Number
E	**MCM/ICM Summary Sheet**	**2202616**

The better forests,The better world

Summary

According to the different functional requirements of forests, forests are divided into three types ; for the types of trees in the forest, they were forests are divided into three types

For problem 1, in order to develop a best forest management plan for carbon sequestration, Richards equation was firstly used to fit tree height and age, then logarithmic regression analysis was used to fit the relationship between DBH and tree height, so as to establish the model of single tree carbon sequestration benefit changing with the cutting tree age. After that, this paper describes the age distribution of trees in the forest using the function formula based on Logistic growth curve. For the evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest, the trees with the age of 36 years, 42 years and 53 years have the highest carbon income, and then further formulates the forest management plan with the strongest carbon fixation ability, and obtains the carbon fixation efficiency of evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest in different areas under the implementation of the forest management plan with the strongest carbon fixation ability.

For the second question, this paper established a comprehensive consideration according to the evaluation results of different functions, different types of forests to develop multi-objective balanced management of forest programs, the forest management plan includes deforestation cycle, deforestation species, single deforestation, deforestation age. For tree cutting cycle and the single cutting amount, this paper uses the AHP to weight the nine functional indicators considered, and then takes into account the cost of cutting trees. After that, the score of different single cutting amounts is calculated by the method of distance between good and bad solutions. It is concluded that the optimal single cutting amount for multi-functional forests, wild conservation forests and ecological forests is 2.4%, 2.7% and 6.4% of the total forest area respectively. In view of the optimal cutting age, this paper inherited the idea of the previous question and took the carbon storage capacity as the main reference index. For evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest, the optimal cutting age is 47 years, 58 years and 77 years, respectively. For the species of trees, this paper constructed a nonlinear programming model with the independent variable as the percentage of various trees, and obtained the optimal cutting proportions the three forest respectively.

For the third problem, this paper takes Shenandoah National Park as the research object. The change of carbon fixation with time is analyzed, and the LSTM neural network and Logistic growth curve are used to predict the CO2 fixation of Shenandoah National Park after 100 years. Then, the optimal management plan model of deciduous broad-leaved forest designed in question 2 is substituted into the park, and the relationship between carbon fixation and time is obtained, and the carbon fixation efficiency is compared with that without our business plan : if we adopt our optimal management plan, 100 years later. The carbon fixation of the park will be more than 8.03 thousand tons. At last, we re-planned the management plan for the change of forest managers ’ demand for the use of the park and the extension of deforestation intervals.

Keywords: carbon sequestration Richards equation Optimal management scheme LSTM

Contents

1 Introduction

1.1 Background

Extreme weather such as global warming and sea level rise is mainly caused by the continuous accumulation of greenhouse effects brought by greenhouse gases such as carbon dioxide, which poses a great threat to human life. To mitigate the negative effects of this climate change, we need to take action to reduce the amount of greenhouse gases in the atmosphere.

Forests, as the main body of terrestrial ecosystems, are of great significance in the global carbon cycle, mainly as follows.

- Forests are the largest carbon pool in terrestrial ecosystems, storing 56% of the total carbon pool in terrestrial ecosystems. Trees and wood products in forests, including furniture, wood, plywood and paper, play a significant role in carbon sequestration.
- The carbon storage density per unit area of forest is high, so the change of forest area directly affects the carbon sink function of terrestrial ecosystem.
- The carbon accumulation of forest vegetation is fast.

Therefore, forest plays an important role in mitigating the impact of climate change, and it is particularly critical to formulate scientific and reasonable forest management plans.

Compared with carbon sequestration by trees alone and without cutting at all, the effect of carbon sequestration may be better by cutting mature trees into wood products and replanting them to realize the regeneration of young forests. Therefore, forest managers need to comprehensively consider factors such as tree age, type, geographical location, terrain, benefit and life of wood products, and formulate scientific and reasonable forest management plans to find a balance between whether to cut forests, when to cut and when to rebuild. In addition, forest functions include not only carbon sequestration, but also biodiversity conservation and water conservation. These factors should be taken into account in the formulation of forest management to maximize the social value of forests.

Proposition purposes : It is not scientific to provide unified guidance for forest management around the world due to due to differences in forests, climate, population, interest and values. It is necessary to comprehensively consider the above factors and formulate forest management plans for forest managers around the world to help them scientifically use and manage the forests they are responsible for.

1.2 Statment of the problem

Table 1: Notations

Symbols	Description
DBH	diameter at breast height of the tree
EBL	Evergreen broad-leaved forest/tree
DBL	Deciduous broadleaf forest / tree
CT	Coniferous forest / tree

2 Data Source

The data in this paper are from Our World in Data National Forestry and Grassland Data Center USDA-National Agricultural Statistics Service Homepage National Bureau of Statistics and Forest Resource Inventory Data in Beijing and other places. All source sites are listed in the appendix at the end of the paper. Among them, forest resources inventory data includes vegetation type, dominant tree species, age group, forest volume, area, tree number, tree height and other information.

3 Establishment and Solution of Model in Problem One

For problem 1, the following five models are established : Tree Height With Time Model, DBH and Tree Height Growth Model, Each Cubic Meter Forest Carbon Sequestration Calculation Model, Tree Carbon Sequestration with Forest Harvesting Model, Tree Carbon Sequestration with Forest Density Model, through them to determine the forest and its wood products can store how much carbon dioxide.

The basic idea of the above models is to convert the carbon sequestration per cubic meter into the change of tree quality. The tree quality is calculated by density and volume, and the tree volume is determined by the tree height change model with time and the growth model of the relationship between DBH and tree height. For the carbon sequestration of the forest and its products as a whole, based on the carbon sequestration model of each cubic meter, the influence of forest cutting, tree density and other factors is comprehensively considered, and the correction factor is introduced to measure the influence of regional policies and natural disasters. In addition, based on the above models, this paper developed the most effective forest management plan for carbon dioxide sequestration, namely, a timetable for harvesting and reforestation, and determined the optimal density of trees in reforestation.

3.1 Tree Height With Time Model

3.1.1 Fitting model

In order to describe the growth process of trees and the relationship between tree height and time more accurately, Richard Equation which is the most widely used, adaptable and fitting effect is selected to construct the Tree Height With Time Model.Richard Equation is a kind of growth curve equation based on the famous Berta-lanffy growth theory, which is commonly used to analyze the relationship between animal weight growth rate and the relationship between plant height and time.Its outstanding advantage is flexible curve shape, suitable for many trees allometric growth process, the basic form is as follows :

$$y = A\left(1 \pm Bexp\left(-kt\right)\right)^{\frac{1}{1-m}} \tag{1}$$

3.1.2 Fitting objects

In this paper, the arbor forests located in the United States, Australia and China at the latitude between $23.5°$ and $66.5°$were taken as the research objects. The trees were divided into evergreen broad-leaved trees, deciduous broad-leaved trees and coniferous trees.Based on 123 samples from 78 standard plots of arbor forests as analytical trees, the data of their height changing with time were collected to fit the height growth process of single tree in temperate regions.

Based on the above data, Richard Equation was used to fit the relationship between tree height and tree age of three trees.The fitting results of evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest are as follows :

$$H_1 = 20.53 * \left(1 - exp^{-0.08035*t}\right)^{1.73038} \tag{2}$$

$$H_2 = 19.32 * \left(1 - exp^{-0.065*t}\right)^{1.6423} \tag{3}$$

$$H_3 = 21.08 * \left(1 - exp^{-0.0534*t}\right)^{2.1} \tag{4}$$

Where, t is the tree age observation value, H_1, H_2, H_3 are green broad-leaved forest, deciduous broad-leaved forest, coniferous forest height. According to the formula, three tree height growth curves with tree age are as follows :

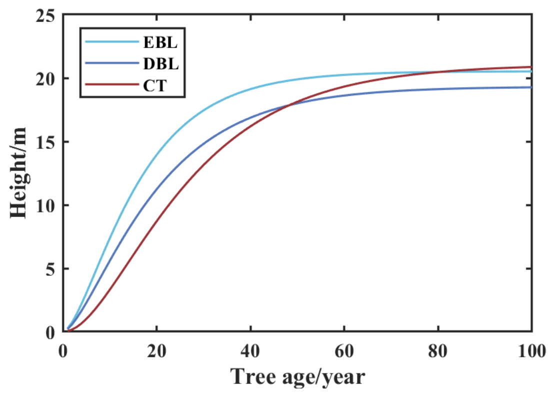

Figure 1: Three Tree Height Growth Curves with Tree Age

It can be seen from the figure that the growth trends of the three kinds of trees are similar. When the tree age is lower than 60 years old, the growth rate of evergreen broad-leaved trees is the fastest. When the tree age is close to 100 years old, the heights of the three trees are all about 20m.The needle-leaf tree has the highest length of 20.635m. and the minimum length of deciduous broad-leaved trees is 18.454m.

3.1.3 Model verification

The age observation value $t_1 = 10$ was substituted into the formula for verification, and the growth heights of the three trees were 3.246 m, 4.625m and 5.221m respectively. The age observation value $t_2 = 15$ was substituted into the formula for verification, and the growth heights of the three trees were 8.202m, 9.616m and 13.162m respectively. This result is highly consistent with the above figure, and the fitting result is good. Therefore, the tree height growth model can be used to simulate the tree height growth process of temperate arbor forest.

3.2 Growth Model of the Relationship between DBH and Tree Height

3.2.1 Fitting model

Regression analysis was used to fit the relationship between DBH and tree height for most scholars, this paper selects samples by random sampling method, studies the Growth Model of the Relationship between DBH and Tree Height through Regression Analysis, and fits the best regression equation to facilitate the calculation of subsequent carbon sequestration.

3.2.2 Fitting objects

In this paper, 184 research quadrats located in the United States, Australia and China were selected as the research objects, and their average DBH and average tree height data were collected. The growth model of the relationship between DBH and tree height was fitted by regression analysis.

3.2.3 Fitting results

According to the above data, the relationship between the overall DBH and tree height of the three plants was obtained by using the method of logarithmic regression analysis.The regression equations of evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest were :

$$ln\left(D_1\right) = 0.0152\left(\frac{H}{3.22}\right) \tag{5}$$

$$ln\left(D_2\right) = 0.016\left(\frac{H}{3.05}\right) \tag{6}$$

$$ln\left(D_3\right) = 0.0165\left(\frac{H}{2.79}\right) \tag{7}$$

Where, D_1, D_2 and D_3 are DBH of evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest respectively. H_1, H_2 and H_3 are corresponding tree height.In Section 1.1.3, we obtained the Richard equation of tree height changing with time. Combined with the DBHtree height regression equation in this section, we can also obtain the quantitative relationship between DBH and time of three kinds of trees.

Draw tree DBH with height curve and tree DBH with tree age curve as follows :

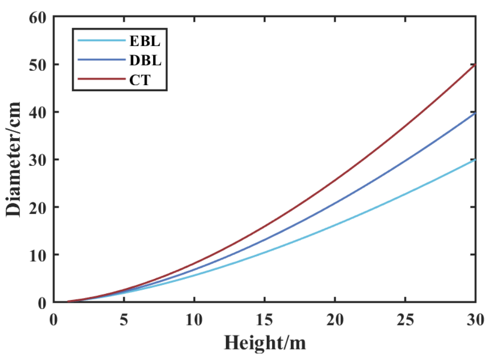

Figure 2: Tree DBH with Height Curve

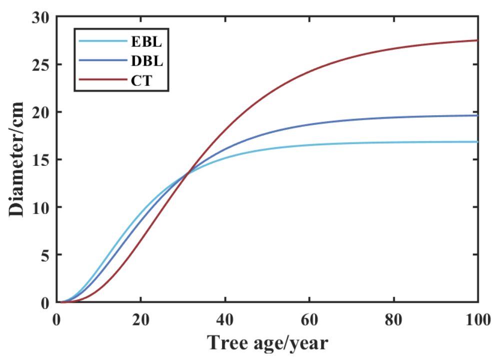

Figure 3: Tree DBH with Tree Age Curve

It can be seen from the figure that the DBH of the three kinds of trees increased with the increase of tree age, among which EBL has the smallest growth rate and CT has the largest growth rate. For the relationship between DBH and tree age, when the tree age is lower than

34 years old, the variation trend of DBH with tree age of the three kinds of trees is relatively similar. However, when the tree age exceeds 34 years old, the DBH of CT still maintains high growth rates and when the tree age is 100 years old, it can reach 26.632cm. In contrast, the DBH growth of DBL and EBL is still relatively stable. When the tree age is 100 years old, the DBHs of DBL and EBL are 19.779cm and 17.436cm respectively.Among them, EBL stands for evergreen broad-leaved forest, DBL stands for deciduous broad-leaved forest, CT stands for coniferous forest.

3.3 Calculation Model of Carbon Sequestration Per Cubic Meter

3.3.1 Model background

The main body of trees is wood, which is the xylem of trees. It is the most carbon storage site in trees. Its carbon sequestration mechanism is to convert carbon dioxide in the atmosphere into organic carbon through photosynthesis. The measurement of tree carbon storage is the basis of the prediction data of tree carbon sequestration.

At present, the measurement methods are mainly based on the calculation of tree rings and physical characteristics of trees, including combustion method, stem analysis method, and growth wheel analysis method. Since both stem analysis method and combustion method destroy trees, the cell wall rate in growth wheel analysis method varies greatly among different trees. For the convenience of calculation, this paper optimized the tree stem analysis method and the growth wheel analysis method to determine the calculation method of carbon sequestration per cubic meter of trees.

3.3.2 Model principle and calculation

The carbon sequestration mechanism of trees is to absorb carbon dioxide through photosynthesis, and then release carbon dioxide through respiration. During photosynthesis, trees convert carbon dioxide into sugar, and finally into trunk branches and roots (The main extracellular substance is cellulose, and the main intracellular substance is protein.) Therefore, the change of tree quality can calculate its carbon storage and then calculate its carbon sequestration.

The carbon element in trees mainly exists in the form of cellulose,whose carbon weight 39.7 % and protein ,whose carbon weight between 50% and 55%. Through consulting the data, it is known that the proportion of carbon in plants is about 53.6 %, and the density of trees is $0.5t/m^3$. It is concluded that the carbon sequestration of trees per cubic meter in the forest is about: $0.268t/m^3$

3.3.3 Model Verification

The conversion factors of wood volume per cubic meter to carbon tons of different tree species are shown in the figure below, which is close to the calculation results in this paper.Therefore, the verification of the model passes.

3.4 Model of Tree Carbon Sequestration Benefit with Tree Age

3.4.1 Basic ideas of the model

Compared with the method of carbon sequestration by trees alone and without cutting at all, the effect of carbon sequestration may be better by cutting mature trees into wood products and replanting them to realize the regeneration of young forests. Therefore, when calculating the carbon sequestration of forest trees, it is necessary to consider the harvesting situation of forests.

In order to obtain the harvesting plan with the optimal fixed efficiency of carbon dioxide for different types of forests, the optimal value of tree age need be calculated, and then the timetable of forest harvesting and reforestation can be determined according to the composition and type of forests.

This paper defines the average carbon income as follows :

Since we are transplanting seedlings not seeds, the minimum age of new seedlings transplanted after cutting old trees is 2, and the nonlinear programming of the average carbon gain with the cutting age is as follows :

$$Averagecarbongain = (carbonsequestrationofreplantedsaplingsovertime + carbonsequestrationoflog \tag{8}$$

Where, $R_n = t$ is the tree chest radius when the tree age is t , $H_n = t$ is the tree height when the tree age is t , t is the tree age.

For the R and T values at different times in the formula, they can be solved by the Richard equation and regression relationship in 1.1 and 1.2. According to the data, the economic cost of wood transportation and processing cost per cubic meter is converted into CO_2 , which is about $8.024 \times 10^{-4} t/m^3$.

3.4.2 Model solving

We use MATLAB to draw the graph of the average carbon gains of the three kinds of trees with the tree age when cutting trees as follows :

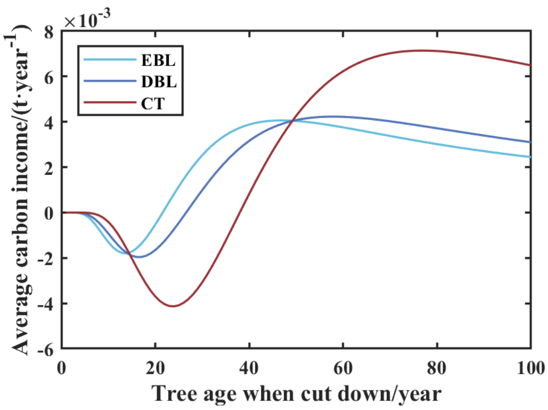

Figure 4: Average Carbon Gains of the Three Kinds of Trees When Cutting Trees

According to the figure and function relationship, the optimal cutting age of evergreen broad-leaved tree was 47 years old, and the corresponding maximum average carbon yield is $4.06 \times 10^{-3} t$ per tree. The optimal cutting age of deciduous broad-leaved trees is 58 years old, and the corresponding maximum average carbon yield is $4.23 \times 10^{-3} t$ per tree ; the optimal cutting age of coniferous trees is 77 years old, and the corresponding maximum average carbon yield is $7.14 \times 10^{-3} t$ per tree.

3.4.3 Model of overall forest carbon sequestration benefits versus tree age

The number of trees in each tree age section of the forest is not uniformly distributed, and there is an inverse correlation between the two. The larger the tree age is, the less the proportion of the total number of trees in the forest is. Therefore, the tree age with the highest carbon sequestration benefit of a single tree is not necessarily the tree age with the highest carbon sequestration benefit of the forest as a whole. After collecting the data of tree age-tree number and conducting regression analysis, this paper, based on the Logistic growth curve, fits the proportion of tree number with tree age t in the total number of trees in the forest as follows :

$$W_t = \frac{1 - 1/\left(1 + 5.8448 * exp\left(-0.0579 * t\right)\right)}{\sum_{i=1}^{i=tmax} 1 - 1/\left(1 + 5.8348 * exp\left(-0.0579 * i\right)\right)} \tag{9}$$

where t is the tree age, and $tmax$ is the maximum value of tree age in the forest.

Using MATLAB to make three kinds of forest carbon income score with the cutting tree age changes as follows :

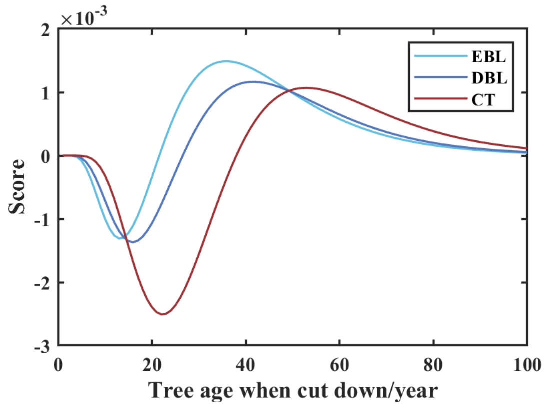

Figure 5: Three Kinds of Forest Carbon Income Score with the Cutting Tree Age Changes

According to the graph and function relationship, the optimal cutting age of evergreen broad-leaved forest is 36 years old, and the corresponding maximum carbon benefit score is $1.49 * 10^{-3}$; the optimal cutting age of deciduous broad-leaved trees is 42 years old, and the corresponding maximum average carbon yield is $1.16 * 10^{-3}$. The optimal cutting age of coniferous trees is 53 years old, and the corresponding maximum average carbon yield is $1.07 * 10^{-3}$.

3.5 Model of Carbon Sequestration Changing with Forest Density

3.5.1 Basic ideas of the model

In the forest, the carbon sequestration capacity of a single tree depends not only on itself, but also on the density of trees in the forest. When the density of trees is low, the carbon sequestration capacity of trees is strong. When the density of trees is high, the distance between trees is too close, which will reduce the carbon sequestration capacity of a single tree. In this paper, Sigmoid function is used as the basis function to consider the relationship between carbon sequestration efficiency of single tree in forest and the ratio of spacing to DBH. When the trees are too dense, the carbon sequestration efficiency of single tree decreases.According to the data,the best spacing and DBH ratio of pine planting is 10.24, the best spacing and DBH ratio of fir planting is 9.94, the best spacing and DBH ratio of eucalyptus planting is 10.16, and the best spacing and DBH ratio of poplar planting is 9.64.In order to make the results scientific and accurate, this paper takes their median value of 10.09. When the ratio of tree spacing to diameter at breast height is greater than 10.09, the carbon fixation efficiency of single tree is almost not affected by the change of stand density. When the ratio of tree spacing to diameter at breast height is less than 10.09, the carbon fixation efficiency of single tree decreases rapidly with the increase of stand density. Therefore, in the relationship based on Sigmoid function constructed by us, when $x = 10.09$, two constraints need to be met :

$$\begin{cases} \frac{df(s)}{dx} = 0.04, (x = 10.09) \\ \frac{df(s)}{d^2x} < 0, (x = 10.09) \end{cases} \tag{10}$$

It is only necessary to find out the points in sigmoid function that meet the corresponding constraints, and these points are symmetrical and translated, so we can determine the relationship between the carbon sequestration efficiency of a single tree and the spacing and DBH ratio. The equation is as follows :

$$\begin{cases} \frac{1}{1+e^{-x}} \cdot \left(1 - \frac{1}{1+e^{-x}}\right) = 0.04 \\ \frac{1}{1+e^{-x}} \cdot \left(1 - \frac{1}{1+e^{-x}}\right)\left(1 - \frac{2}{1+e^{-x}}\right) > 0 \end{cases} \tag{11}$$

By calculating $x = 3.13$, it can be seen that the sigmoid function shift 6.96 units to the right is the relationship between the carbon sequestration efficiency of a single tree and the ratio of spacing to DBH :

$$f(x) = \frac{1}{1 + e^{6.94 - x}} \tag{12}$$

The images of Sigmoid function and Sigmoid function after translation are as follows :

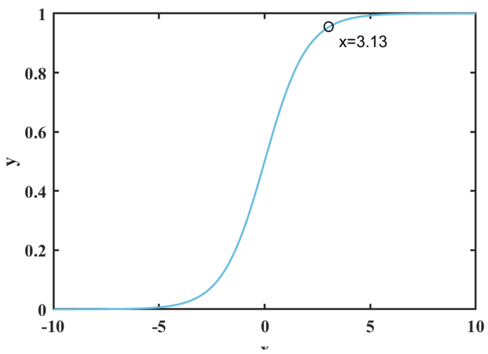

Figure 6: The Images of Sigmoid Function

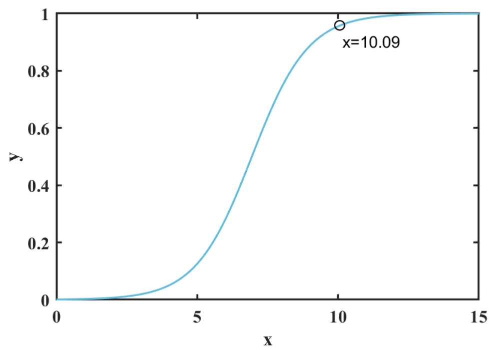

Figure 7: The Images of Sigmoid Function After Translation

3.5.2 Model solution

According to the above formula, the diameter and radius of trees can be calculated by the height of trees, and the relationship between carbon income per unit area of evergreen broad-leaved forest and tree height, spacing-diameter ratio is drawn by MATLAB as follows :

It can be seen from the figure that for evergreen broad-leaved trees of different ages, the ratio of spacing to DBH when the forest has the highest carbon yield is about 7.2. The fitting situation of deciduous broad-leaved forest and coniferous forest is similar to that of evergreen

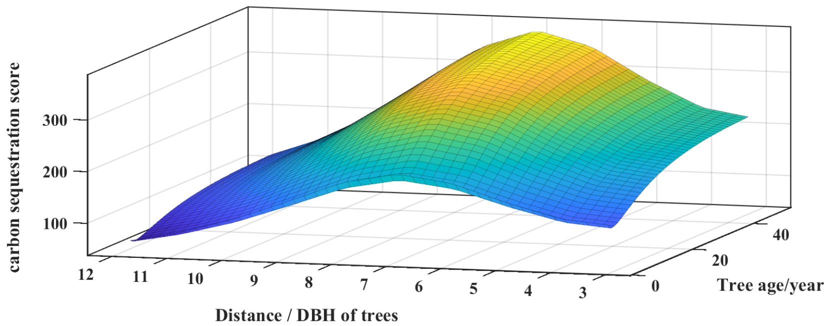

Figure 8: Relationship between Carbon Income of Evergreen Broad-leaved Forest

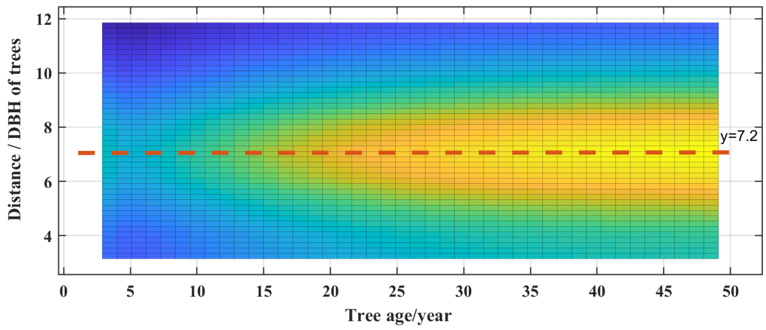

Figure 9: Relationship between Carbon Income of Evergreen Broad-leaved Forest

broad-leaved forest. Although the scores are quite different, the optimal spacing to DBH ratio of the three forests is about 7.2.

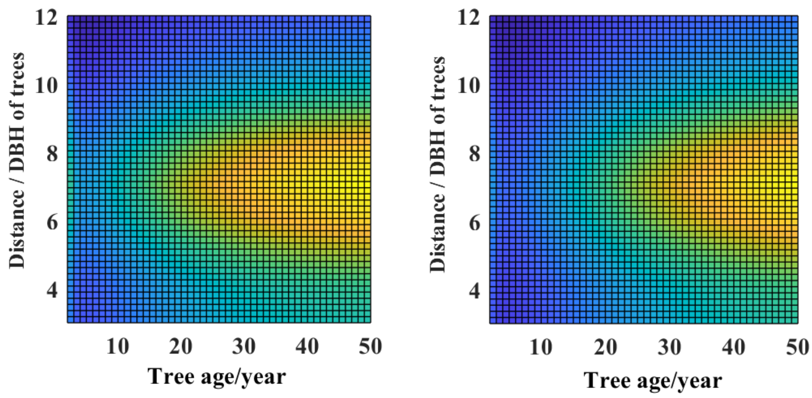

Figure 10: Relationship between DBH and Tree age

To sum up, we can formulate the forest management plan with the strongest carbon sequestration capacity as follows :

Attention should be paid to the following points when replanting the three forests in this paper.

- When replanting seedlings, the ratio of spacing to DBH is guaranteed to be 7.2.

- Ensure that the ratio of tree spacing to DBH in the forest remains at around 7.2, otherwise appropriate deforestation is required.

Besides, management strategy of evergreen broad-leaved forest :

- The cutting cycle of trees should be determined as 36 years. Trees aged 36 and above in the forest should be cut down and seedlings aged 2 should be planted.

Management strategy of deciduous broad-leaved forest :

- The cutting cycle of trees should be determined as 42 years. Trees aged 42 and above in the forest should be cut down and seedlings aged 2 should be planted.

Management strategy of Coniferous forest :

- The cutting cycle of trees should be determined as 53 years. Trees aged 53 and above in the forest should be cut down and seedlings aged 2 should be planted.

We use MATLAB to draw the relationship between forest carbon sequestration rate and area as follows :

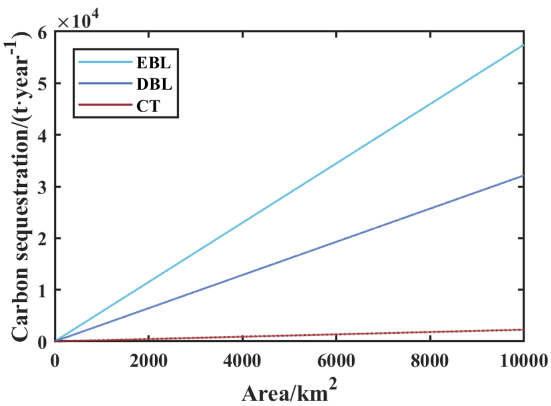

Figure 11: Relationship between Forest Carbon Sequestration Rate and Area

It can be seen that in the case of the same tree species composition, the forest carbon sequestration efficiency is basically proportional to the forest area. For the three forests with an area of 1 000, 5 000 and 10 000 square kilometers in temperate regions, the annual carbon sequestration is as follows after using our carbon sequestration model :

4 Establishment and Solution of Model in Problem Two

For Problem 2, this paper divides forests into three types according to different functions of forests, namely, multi-functional forestry, field protection type and ecological forestry. According to the types of trees in the forest, the forest is divided into evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest, and the comprehensive consideration of forest water conservation, biodiversity maintenance, carbon fixation and oxygen release, purification of air and other functions, in order to ensure that the value of the forest can be evaluated comprehensively. According to the above evaluation results, the most effective forest management plan that can maximize the social benefits of the forest is formulated, that is, according to different functions and different types of forests, the forest scheme design of multi-objective balanced management is formulated, the timetable of different logging and reforestation is formulated, and the indicators such as the amount of cutting trees, the type of cutting trees and the age of cutting trees are determined.

4.1 Establishment of Single Cutting Tree Quantity Model

In this paper, nine indicators such as timber product income, water conservation, carbon fixation and oxygen release, pest control, and pollutant degradation are comprehensively considered to comprehensively evaluate the value of forests. In determining the single tree cutting amount, a two-level comprehensive evaluation model is established in this paper. The income brought by tree cutting amount and the cost required for tree cutting are scored respectively, and the final score is obtained by combining the two scores, so as to determine the optimal proportion of each cutting amount in the total forest area of three types of forests, namely, multi-functional forestry, field protection type, and ecological forestry. The advantage and disadvantage solution distance method is used to calculate the cutting income score with the change of single cutting amount. Due to the different weights of each index in different forest types, this paper uses the analytic hierarchy process to weight each index.

4.1.1 Profit Evaluation Method of Wood Products

The income of wood products is the economic value shown by the direct benefits of forests, including the value of trees (wood, furniture, plywood, paper, etc.), economic forest products and forest by-products such as medicinal materials.The calculation formula is as follows :

$$FP = \sum_{i=1}^{n} V_i N_i T P_i \tag{13}$$

4.1.2 Evaluation method of water conservation value

The economic evaluation of the value of forest water conservation is as follows : Due to the forest interception and storage of precipitation, a large number of rainwater is detained in the forest, which reduces the flood peak and reduces the harm of rainstorm and flood to industry, agriculture and people ' s lives and property. The economic evaluation of this part is the value of forest water conservation. The calculation formula is :

$$FWC_t = \sum ps_i \left(1 - I_i\right) \cdot 10 \tag{14}$$

4.1.3 Evaluation Method of Carbon Sequestration and Oxygen Release Value

Based on the net primary production of forest ecosystem and the reflection equation of photosynthesis and respiration, this paper calculates the amount of fixed CO_2 and the amount of released O_2, and then uses the market value method to calculate the value of carbon fixation and oxygen release of plant photosynthesis. The calculation formula is as follows :

$$V_{total} = V_{CO_2} + V_{O_2} \tag{15}$$

$$V_{CO_2} = \sum_{i=1}^{n} B_i C_{CO_2} P_{CO_2} \tag{16}$$

$$V_{O_2} = \sum_{i=1}^{n} B_i C_{O_2} P_{O_2} \tag{17}$$

4.1.4 Evaluation method of pollutant degradation value

Forest plays a certain role in pollutant degradation and dustfall. In this paper, the production cost method was used to calculate the purification effect of forest ecosystem on atmospheric SO2 and the absorption and purification function value of heavy metal elements such as Hg in water. The average value of SO2 absorption per unit area is multiplied by the forest area, and the total amount of SO2 absorption per year is obtained. According to the investment cost of reducing SO2 per unit mass in pollution control projects in recent years, the total value of SO2 absorption is calculated. The calculation formula is as follows :

$$V_1 = S \cdot H \cdot P \tag{18}$$

The value of pollutant degradation is calculated according to the investment cost of 600 yuan per 1 t SO2 reduction in the ' biodiversity study report '. The total value of pollutant degradation is calculated by expert evaluation method. According to expert evaluation method, the total value of pollutant degradation can be obtained by estimating that the value of SO2 degradation in forest ecosystems accounts for 60 % of the total value of pollutant degradation.

4.1.5 Evaluation method of pest control cost

In order to maintain the healthy and normal growth of trees, it is necessary to invest a certain amount of cost for the prevention and control of pests and diseases every year. According to the statistical data of the Ministry of Forestry, if the cost of prevention and control in 2020 is calculated, the average cost of forest land prevention and control is $3.48 yuan/hm^2$. The calculation formula of the cost value of pest control is :

$$V_2 = S \cdot P \tag{19}$$

4.2 Solving of single tree cutting model

4.2.1 Weighting results of each index by AHP

Because the weight of each index is different in different forest types, this paper uses the analytic hierarchy process to empower each index. The empowerment results are as follows :

4.2.2 Calculation results of income fraction of felling by good and bad solution distance method

After the Min-Max standardization of each index to measure the income of single deforestation, the final income score is calculated by using the distance method of superior and inferior solutions. Firstly, the distance between the income and the worst solution of the forest with the cutting area ratio x is obtained.

$$D_N = \sqrt{(w_1 + w_6 + w_{10})\,x^2 + (w_2 + w_3 + w_5 + w_7 + w_8)\,(1 -^2) + w_9\,(m - x)^2 + w_{10}\,(0.3 - x)^2} \tag{20}$$

For forests in different locations,m has different values. When forests are located in the cold zone, temperate zone and tropical zone, the corresponding values of m are 0.8, 1 and 1.2 respectively. The distance between the profit and the optimal solution of the forest with the harvesting area ratio x is :

$$D_P = \sqrt{(w_1 + w_6 + w_{10})\,(x - 1)^2 + (w_2 + w_3 + w_5 + w_7 + w_8 + w_9 + w_{10})\,x^2} \tag{21}$$

Finally, the comprehensive score of the solution distance method is obtainedthe score obtained by the distance method is the final score.

$$Score = D_N / \left(D_N + D_P\right) \tag{22}$$

4.2.3 The results of secondary comprehensive evaluation to determine the single tree cutting amount of different types of forests

When the secondary comprehensive evaluation is used to determine the single tree cutting amount of different types of forests, this paper weights 0.95 for the income score and 0.05 for the cost score.

The cost score is proportional to the cut area, with the lowest score of 0 and the highest score of 1. The income score of the distance between the superior and inferior solutions is substituted into the above equation to obtain the single optimal cut ratio of different forest types as follows :The single cutting amount is 2.4 % of the total trees in the forest ; wilderness conservation was 2.7% and eco-forestry 6.4%.

4.3 Establishment and Solution of Tree Cutting Age Model

For trees of different ages, the biggest difference is their ability to fix carbon and release oxygen. Combined with the growth model of the relationship between DBH and tree height established in question 1, the calculation model of carbon fixation per cubic meter, and the model of tree height changing with time, we take the age of trees with the highest average carbon yield of single tree in the first question as the best cutting tree age in this question. It can be seen from Figs. 1 4 that under the comprehensive consideration of all aspects of forest functions, evergreen broad-leaved forest should cut trees with age of 47 years, deciduous broad-leaved forest should cut trees with age of 58 years, and coniferous forest should cut trees with age of 77.

4.4 Establishment and solution of tree cutting cycle model

In the process of determining the optimal tree cutting cycle, this paper comprehensively considers the three functions of biodiversity, forest recreation and forest income to comprehensively evaluate the value of forests. The above three indicators are all income indicators, namely, the greater the indicator is, the higher the income is. The main steps are as follows :

4.4.1 Min-Max standardization

Because the three indexes considered in this paper have different dimensions, for the convenience of subsequent calculation, we use the following formula to normalize the data to values between 0 and 1.

$$a_{i,jnormalized} = \frac{a_{i,j} - mina_{i,J}}{maxa_{i,J} - mina_{i,J}} \tag{23}$$

4.4.2 Evaluation method of species diversity value

Gao et al. pointed out that tree height and DBH were the decisive factors determining species diversity in forest ecosystems. Based on this, we fit the data in the database , and the calculation formula of species diversity index is as follows :

$$Y = 7.412 - (1.545exp\left(-H/17.631\right) + 1.304exp\left(-W/2.317\right)) \tag{24}$$

4.4.3 Determination of index weight by AHP

The biodiversity, forest recreation and forest tree income considered in this paper vary with forest functions and forest types. AHP is used to determine the weights of the three indicators. The results are as follows :

4.4.4 Solving Tree Cutting Period Model

Based on the above analysis, this paper uses MATLAB to fit the relationship between the comprehensive evaluation scores of different tree species and the cutting cycle under three kinds of forestry as follows :

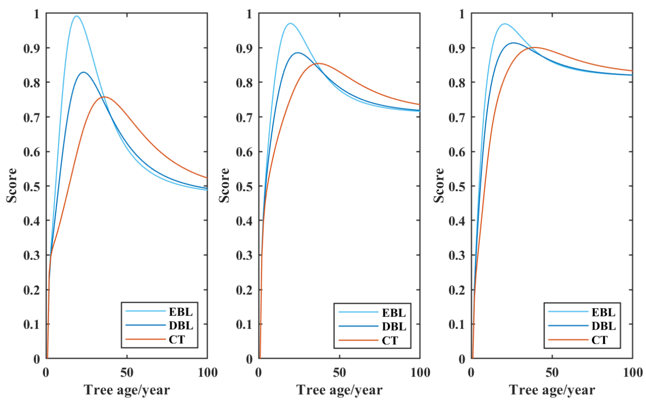

Figure 12: Scores of Different Tree Species and Cutting Cycle under Three Kinds of Forestry

Taking the first point with the first derivative less than 0, namely the point where the final score begins to decline, as the optimal solution, the optimal tree cutting cycle can be obtained by analyzing the image as follows :

4.5 Prohibition of forest exploitation

Forest ecosystem plays an irreplaceable role in maintaining balance, regulating regional climate and preventing natural disasters. In Problem 2, this paper constructs models for three main trees in temperate regions, and analyzes the decision-making scheme for maximizing forest benefits. However, the service functions of forests should be fully considered in assessing the use of forest ecosystems. This section takes the ability of forest windbreak and sand fixation as an example to analyze the forests that should be prohibited from mining due to the role of sand storm prevention.

As shown in figure, sand storms are concentrated in North America, Central East Asia, Oceania and northern Africa. Therefore, the US ' Roosevelt Grassland Project ', China ' s ' Three North ' shelter forest, Morocco ' Green Dam ' construction and other shelter forests play a crucial role in the windbreak and sand fixation link. If it is cut down, resulting in insufficient protection capacity, sand storms will likely invade surrounding cities and bring huge economic losses. Therefore, for the protective forest belt with strategic significance, the amount of carbon fixation and oxygen release will be much smaller than the economic loss caused by natural disasters due to exploitation, and exploitation should be prohibited.

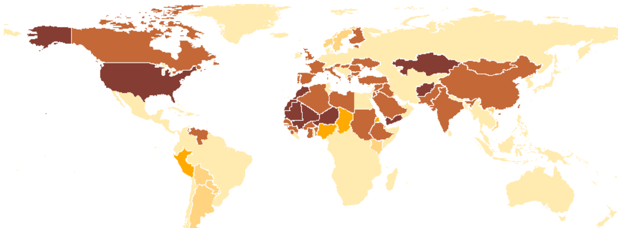

Figure 13: Sand Storms

4.6 Change in management plans

Although this paper constructs a universal forest benefit model, and according to this model, the decision-making plan corresponding to the maximum forest benefit is obtained. However, it is undeniable that for different forest characteristics, it is difficult to have a certain mathematical expression that can perfectly fit all forest income models. Therefore, this section comprehensively considers the temperature, precipitation and ecological function of the forest location, and constructs a more targeted income model.

Spearman's rank correlation coefficient is used to measure the correlation between the two variables. Compared with the Pearson correlation coefficient, its application scope is wider:

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n\left(n^2-1\right)} \tag{25}$$

Spearman's rank correlation coefficient between the new indicators and the original indicators are as follows :

Table 2: Spearman's Rank 1

	Forest return	water conservation	carbon sequestration
x_1	0.542	0.337	0.214
x_2	0.638	0.674	0.336
x_3	0.113	0.431	0.712
x_4	0.197	0.311	0.571

Table 3: Spearman's Rank 2

	forest recreation	soil maintain	biodiversity	pollutant degradation
x_1	0.498	0.183	0.256	0.194
x_2	0.214	0.577	0.229	0.316
x_3	0.196	0.479	0.337	0.455
x_4	0.217	0.334	0.679	0.332

According to the analysis of the table, when the correlation coefficient is greater than 0.4, it is defined as ' correlation '. The score relationship between the two indicators with correlation changes, and the two indicators without correlation do not affect each other :Construction of influence factor $k = exp(sp)$

The new income function expression is generalized to:

$$G_I = Z_i \times k_i \times H_i \tag{26}$$

New index income calculation formula construction :Annual average temperature ,Precipitation ,Environmental protection needs,Protection needs of endangered species .

According to the newly constructed evaluation system, the original optimal solution of Problem 2 is evaluated. 37 forests worldwide are selected as examples to evaluate the economic benefits of the original optimal solution, and the results are shown in the figure.

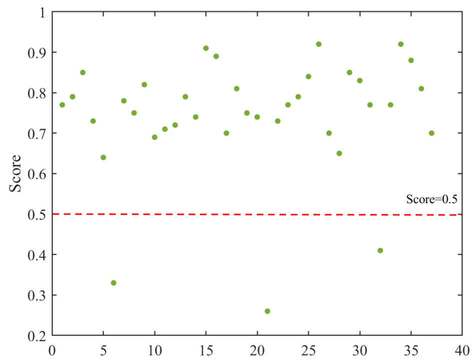

Figure 14: Comprehensive Scores of Nature Reserves under New Indicators

According to the observation figure, the scores of nature reserves No. 6, 21 and 32 were significantly lower. Control database available, the three are : China Saihanba Nature Reserve, Tanzania Serengeti National Park, China Sanjiangyuan Nature Reserve. The reason why the scores of the three are significantly lower is that they play an important role in wind prevention and sand fixation, the protection of endangered animals and plants, and ecological regulation, respectively. For example, Saihanba Nature Reserve in China has the effect of preventing dust storms in Beijing and its neighboring cities. Therefore, when forests have special functions, the special function of forests is the transition point. This part of the model should be used to comprehensively consider latitude and precipitation, and re-evaluate its economic value.

5 Establishment and Solution of Model in Problem Three

For the same forest, the average height of trees in the forest can be regarded as no significant change at different times. Therefore, we can calculate the standing tree density of the forest based on the tree volume and the covered area of the forest, and then establish a relationship with the carbon dioxide absorption per unit area of the forest, so as to obtain the change of carbon sequestration capacity per unit volume of trees in the forest with the standing tree density.

For the third problem, this paper takes the American Shenandoer Forest Park as the research object. Firstly, the change of carbon fixation with time is analyzed, and the LSTM neural network and Logistic growth curve are used to predict the CO_2 fixation after 100 years. Then, the optimal management plan model of deciduous broad-leaved forest designed in the second problem is substituted into the Shenandoer Forest Park, and the change of carbon fixation with time is obtained, and the carbon fixation efficiency is compared with that without our business plan. Finally, in view of the change of forest managers ' demand for the use of Shenandoer Forest Park and the extension of deforestation interval, the business plan was redesigned.

5.1 Prediction of carbon sequestration in Shenandoer Forest Park

Xienandoer Forest Park began to be built in 1926, where the park was set up. Most of the park land is eroded slopes, unreusable farmland and sparse second and third generation forests. Today, the Shenandoeu Forest Park has an area of more than 800 square kilometers, becoming a famous tourist attraction and forest oxygen bar in Virginia. The changes of CO_2 fixation and forest stock over time in the forest park from 1941 to 2021 are as follows. The data are from the website of Swan Valley Idaho Forest Service Campgrounds and RV Parks.

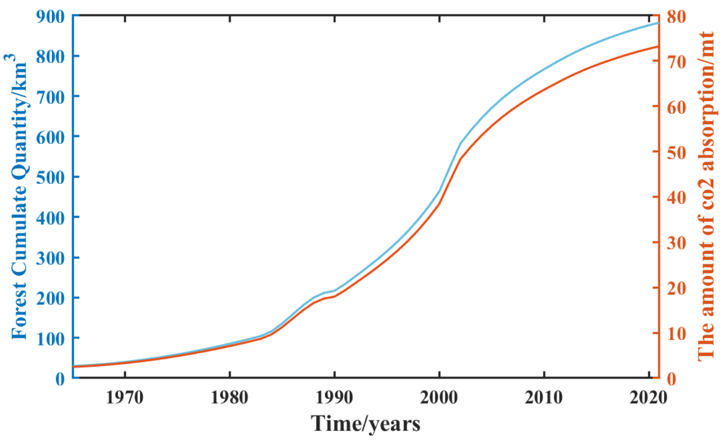

Figure 15: Chart of CO_2 Fixation and Forest Volume Over Time

It can be seen that there is a strong correlation between them, and the Pearson correlation coefficient is 0.92, indicating a strong linear correlation between them.

In order to compare the carbon sequestration efficiency of Xienandoer Forest Park before and after adopting our forest management scheme, this paper uses Logistic Growth Curve and LSTM Neural Network to predict the change of forest carbon sequestration with time when maintaining the existing forest management scheme. According to the change data of CO_2 fixation with time from 1965 to 2021, this paper constructs the logistic growth curve function as follows :

$$f = 3/\left(c \cdot exp\left(b \cdot t^2 - d\right) + a\right) \tag{27}$$

$$\begin{cases} a = 0.03897, b = -0.1384 \\ c = 0.58450, d = -2.5390 \end{cases} \tag{28}$$

The logistic growth curve's $RMSE = 1.754$, LSTM neural network's $RMSE = 1.538$, the mean square error is small, and the fitting effect is good. The two models are applied to the prediction of forest carbon sequestration in Xienan Doer Forest Park. The prediction map is as follows:

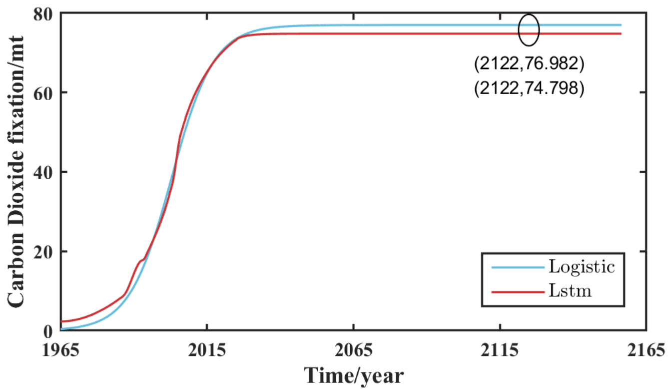

Figure 16: Forest Carbon Sequestration Prediction Map of Xienandoer Forest Park

It can be seen that the increase rate of forest carbon sequestration in Xienandoer Forest Park increases first and then decreases without changing the existing management scheme of forests. Around 2035, forest carbon sequestration began to remain stable and no significant increase was observed. After 100 years (i.e. 2122), the carbon sequestration predicted by Logistic growth curve and LSTM neural network was 76.982 million tons and 74.798 million tons respectively. The average value obtained by model fusion is as follows :

Without changing the existing management mode of Shenandoer Forest Park, the total carbon sequestration of Shenandoer Forest Park forest 100 years later was 7589,000 tons and 383,000 tons between 2022 and 2122.

If forest management is carried out according to the model established in the second question of this paper, as a typical multi-functional forest, the single cutting amount of Xienanduoer Forest Park should be 2.4 %. At the same time, the dominant plants of Xienanduoer Forest Park at the junction of temperate and subtropical regions are deciduous broad-leaved trees such as oak and pecan, so the optimal cutting cycle is 23 years, namely, cutting in 2022,2045,2068,2091 and 2114 respectively.

In terms of the determination of the age of cutting trees, since the forest park in Xienandoe was constructed on the eroded slopes and unusable farmlands in 1926, we believe that the oldest tree in the reserve is 94 years. Substituting the functional relationship between tree age and tree quantity based on logistic growth curve into the first question, we can obtain the ratio relationship between tree age and the total amount of trees :

$$W_t = \frac{1 - 1/\left(1 + 5.8448 * exp\left(-0.0579 * t\right)\right)}{\sum_{i=1}^{i=94} 1 - 1/\left(1 + 5.8348 * exp\left(-0.0579 * i\right)\right)} \tag{29}$$

The corresponding curve relation is as follows:

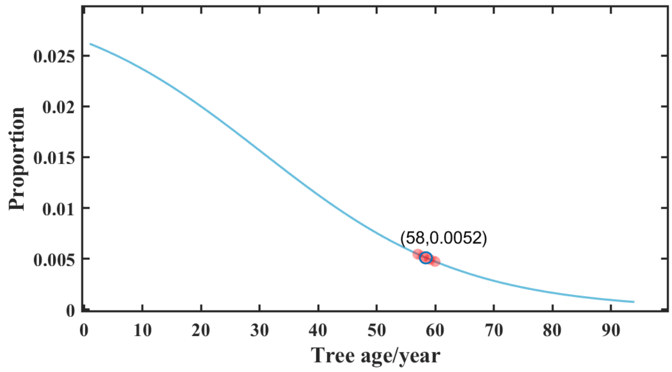

Figure 17: Ratio of Tree Age to Total Trees

According to the figure, the cutting age strategy is to cut all trees between 56 and 59, and then less than 2.4 % of the trees are filled with trees aged 60.

In conclusion, as the Shenandoeu Forest Park is a multi-functional forest and dominated by deciduous broad-leaved forests, the management strategies are as follows :

- The logging cycle is 23 years and will be conducted in 2022, 2045, 2068, 2091 and 2114 respectively.
- 2.4% of total forest volume should be cutted per felling
- The best logging age is 58 years old, when the tree age is between 56 and 59 years old clear cutting ; when the tree age is 60 years old, the insufficient part of the total volume is 2.4 %, accounting for about 55.3% of the total number of trees aged 43 years old.

5.2 Prediction of CO_2 Absorption After Adoption of This Scheme

From the analysis results in Fig above, it can be seen that there is an obvious linear correlation between the forest volume and the co2 absorption in the forest park of Xienandoe. Therefore, it can be used to predict the change of forest volume with time in the forest park of Xienandoe by fitting the logistic growth curve similar to the change of co2 with time. After fitting by using the cftool toolbox in matlab, the functional relationship between forest volume and time is as follows :

$$f\left(x\right) = 3/\left(0.5845 * exp\left(bx + c\right) + a\right) \tag{30}$$

$$a = 0.003137, b = -0.1267, c = -0.3467 \tag{31}$$

The projected forest volume in 2022, 2045, 2068, 2091 and 2114 is 8.973, 9.529 , 9.561 , 9.563 and 9.563 million cubic metres respectively. A total of 113.25 million cubic meters can be harvested five times.

According to the fitted relationship function between carbon yield per tree and tree age, the total carbon yield of five times felling was 0.803 million tons.

Therefore, if we adopt our management mode, the total carbon sequestration of Xienandoer Forest Park forest is 766.93 million tons, during 2022 and 2122 the carbon sequestration is 463.33million tons, compared with the proposal not adopted increased 0.803 million tons.

5.3 The model of ten-year extension of cutting cycle

In our best management plan, when the interval between the two fellings is 23 years and the extension is 10 years, the tree felling cycle is 33 years. The extension of time may be caused by the change of forest managers demand for forest use and management purposes. From the modeling results of the problem, we can see that the process from multi-functional forestry to wild protection forestry and ecological forestry will lead to the change of the felling cycle. The factors are discussed as follows :

5.3.1 Demand becomes wild protection forestry :

The main demand of wild conservation forestry is to protect species diversity, conserve water and so on, so we should limit the single felling amount and avoid its ecosystem as far as possible. By solving the single felling amount in the second question, the single felling amount can be changed to 2.7 %. At this time, the total amount of trees between 56 and 60 cannot meet the needs of felling, and the age of felling trees can be changed to between 55 and 60.

Taking into account the conservation of species diversity, the indicator of deforestation in the second question could be incorporated into the management system, with trees aged between 56 and 60 accounting for 2.61% of the total forest. In order to avoid the excessive impact of total deforestation on forest biodiversity, 0.09% of coniferous forests such as fir and pine could be considered for remaining deforestation.

Thus, it is determined that if the manager ’s demand for Shenandoer Forest Park changes to wilderness protection, then :

- Logging cycle is 33 years and single felling amount changes to 2.7 %.
- The age of trees felled changes from 56-60 to 55-60.
- If the demand for biodiversity conservation is relatively strong, 2.61 % of the 2.7 % of the cutting demand is for deciduous broad-leaved trees aged 56-60, and 0.09% is for coniferous trees aged 77.

5.3.2 Demand becomes ecological forestry :

In the process of building ecological forestry, not only the protection of forests, carbon dioxide fixation and other indicators need to be considered, but also the economic benefits of forests need to be considered. Therefore, in the case of a long logging cycle, it can be considered to increase the amount of single logging. By solving the single cutting amount in the second question, the single cutting amount can be changed to 6.4%. At this time, the total amount of tree age 56-60 cannot meet the cutting needs, and the cutting age is 53-63 years old.

Considering the economic value brought by the management plan, the wood made of coniferous trees such as pines and firs is sold at a high price in the market. At the same time, it can be seen from the carbon income-age curve of a single tree fitted in question 1 that coniferous trees with a tree age of 77 years have the highest carbon income of single tree cutting. Therefore, the proportion of coniferous trees in the cutting plan can be appropriately increased to improve the economic benefits of forests. However, due to the dominant deciduous broad-leaved forest in the Forest Park of Xienandoe, the number of trees cut in the coniferous forest is not easy to be too many, otherwise it will lead to the danger of further reducing or even endangering the number of coniferous forests that are originally inferior species.

Thus it is determined that if the manager 's demand for Shenandoer Forest Park changes to ecological forestry, then :

- Logging cycle is 33 years and single felling amount changes to 6.4 %.
- The age of trees felled changes from 56-60 to 53-63.
- If the economic benefits of forests are to be considered, an appropriate increase in the proportion of coniferous forests in harvested trees may be considered, but should not exceed the proportion of coniferous broad-leaved forests in forests (about 19%).

6 Sensitivity Analysis

In this section, we test the sensitivity of different trees to the geographical location in the relevant models in our work by changing the parameters and comparing the differences between the original results and the change results. In the model of Problem 1, this paper established Richard equation for trees in temperate regions. In this section, we test the sensitivity of different types of trees to temperature by analyzing the tree height changes between cold and tropical trees:

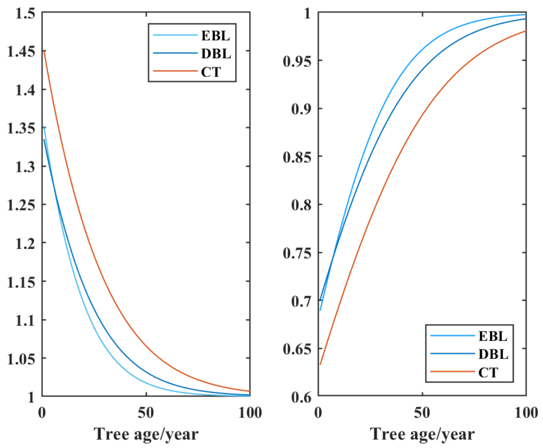

Figure 18: Tree DBH with Height Curve

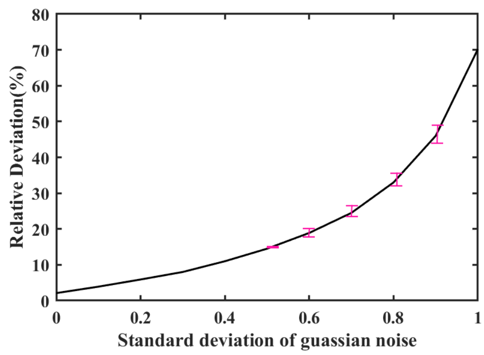

Figure 19: Tree DBH with Tree Age Curve

The left figure is the sensitivity index of tropical trees, and the right figure is the sensitivity index of cold trees. According to the image, coniferous forest is the most sensitive to temperature, evergreen broad-leaved forest is the least sensitive. The sensitivity of the three trees decreased with the increase of planting time.

In constructing transition point selection models based on different forest features, we used the geographical features of 37 nature reserves. However, data from different sources are slightly different, which may lead to inaccurate input data. This may bring some deviations to the results of our model. Therefore, in order to verify the stability of our global garbage prediction model, we add some Gaussian noise to the input data to see how the relative deviation of the results changes with the increase of standard deviation. According to Figure, when the standard deviation of Gaussian noise is less than 0.5, the relative deviation of the results is less than 10 %, which shows the robustness of our model.

7 Establishment and Solution of Model in Problem Four

Dear ICM, there are millions of planets,but only one earth belongs to us. Countless stars flying in the sky every day, but only the earth is our hometown. We love this planet more than anything else. Thus, for a better ecological environment, we present our research on forest management to you.

First, let ' s start deforestation ! Persisting in not cutting down trees in forests is not always pleasant.When trees grow older, their growth slows or even slows down, and no longer fixes carbon dioxide efficiently. Our study found that for most trees, when the tree age reaches 50 years old, the growth rate will be greatly slowed down. For the elderly trees entering the overripe stage, cutting them will not bring disaster to the ecological environment. Whats more ,the benefits of the new tree seedlings planted together with the wood products processed by the old trees to the ecological environment are far beyond that of an old tree with low carbon fixation efficiency.

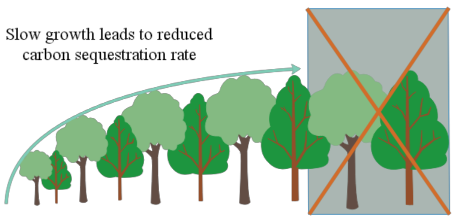

Figure 20: Ratio of Tree Age to Total Trees

Secondly, please don ' t look only focus on one point ! Mitigation of greenhouse gases is our eternal pursuit of forest management, but water conservation and biodiversity maintenance are also treasure of forests. We studied the management of forests and concluded that for different types of forests, the highest comprehensive benefit management is as follows : For multi-functional forest, wild-type protection forest and ecological forest, the optimal single cutting amount was 2.4%, 2.7% and 6.4 % of the total forest area, respectively. For evergreen broad-leaved forest, deciduous broad-leaved forest and coniferous forest, the most suitable cutting ages are 47, 58 and 77 years, respectively.

For different forests, we get the logging cycles

Finally, it is an honor to tell you. We tried to apply the forest management method with the highest comprehensive benefit designed by us to the Shenandoah National Park, and obtained encouraging results. If we adopted the management method designed by us to manage the

Table 4: Optimal Cutting Cycle for Different Forests

	broad-leaved evergreen forest	deciduous broadleaved forest	coniferous forest
x_1	19	23	32
x_2	22	24	33
x_3	20	27	37

Shenandoah National Park, then after 100 years, 80.3 thousand tons of carbon would be extra fixed, which has important application value for the current earth with excessive greenhouse gases. At the same time, we also re-planned the business plan for the changes in the possible use demand of forest managers for the Shenandoah National Park and the extension of cutting interval. Without destruction, there is no construction.No breaking, there is no creating.Please believe that sometimes, appropriate logging will bring greater benefits.

Without destruction, there is no construction. Without breaking, there is no making.