Defining functions

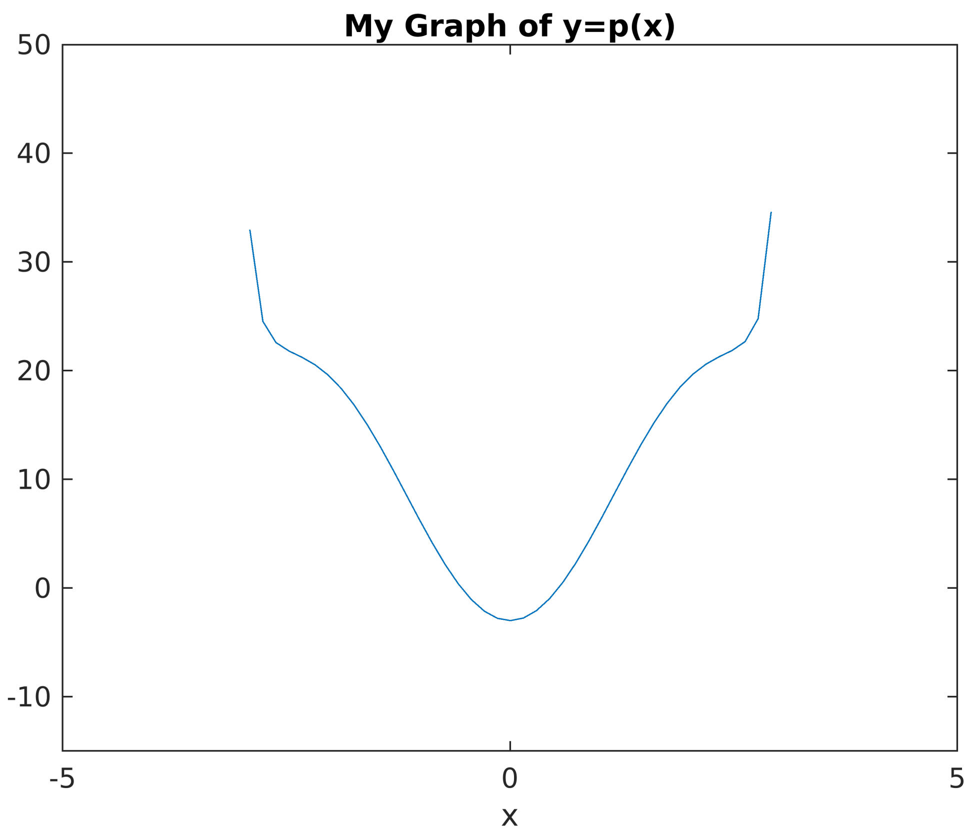
```
syms x
p(x)= -3+50*(sin(.8*x))^2/sqrt(9-x^2)
```

p(x) =

$$\frac{50\sin\left(\frac{4\,x}{5}\right)^2}{\sqrt{9-x^2}}-3$$

Domain - Using the plot and solve commands to determine domain

```
ezplot(p(x),[-5 5 -15 50])
hold on
title("My Graph of y=p(x)")
hold off
```




```
p(2)
```

ans =

$$10\,\sqrt{5}\,\sin\left(\frac{8}{5}\right)^2-3$$

```
vpa(p(2))
```

ans = $19.341614788798846182822034139668$

```
p(5)
```

ans =

$$-3 - \frac{25\sin(4)^2\,i}{2}$$

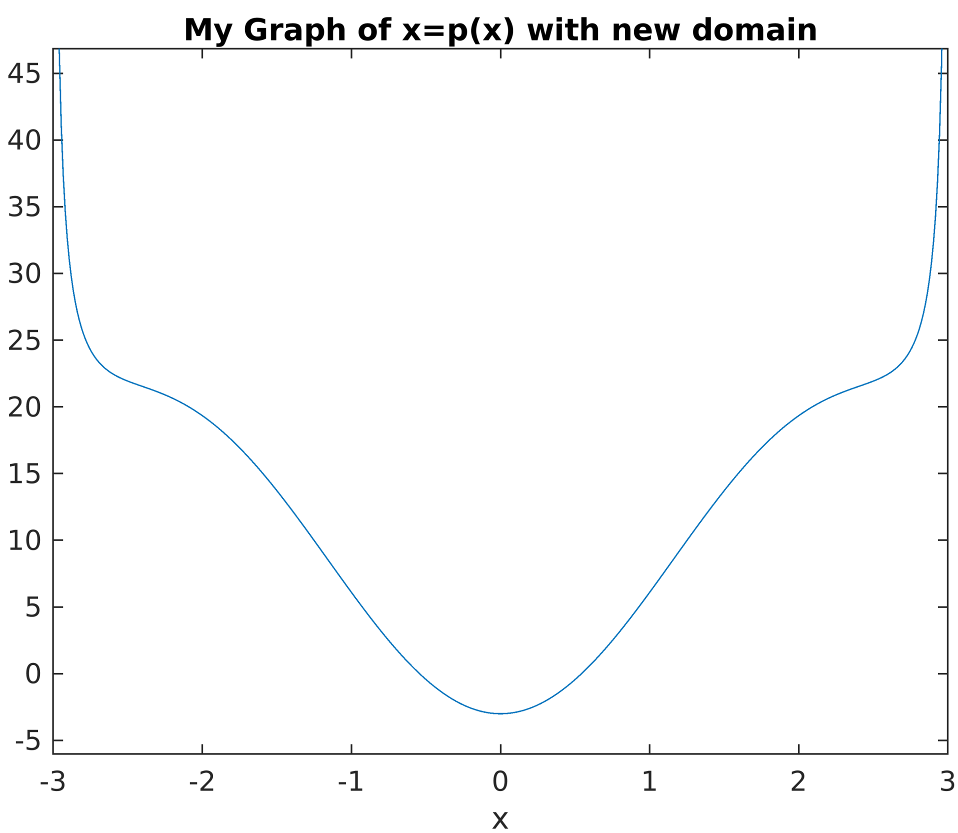
```
vpa(p(5))
```

ans = $-3.0 - 7.1593752113038345366802586364445\,i$

```
syms domain
domain= mysolver(sqrt(9 - x^2)>=0,x);
domain
```

domain = $[-3, 3]$

```
ezplot(p(x),[-3, 3])
hold on
title("My Graph of x=p(x) with new domain")
hold off
```



Comprehension Check # 1

```
syms x
mysolver(sqrt(9 - x^2) == 0,x);

r(x) = -sqrt( 3-x) + 2
```

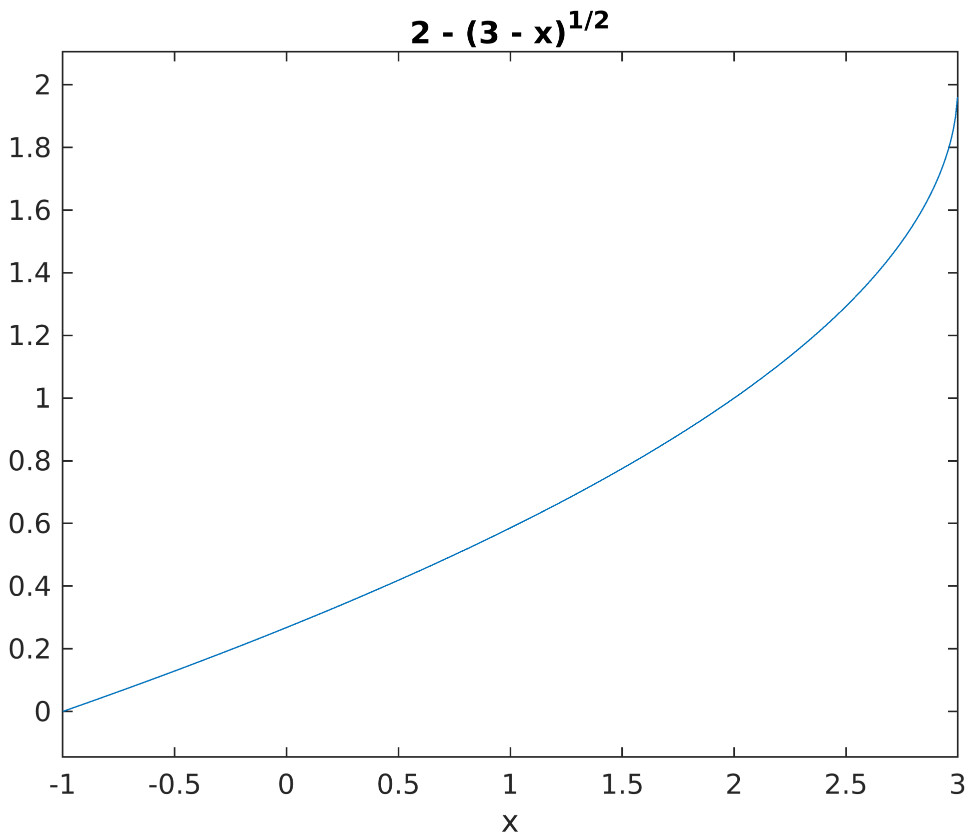
r(x) = $2-\sqrt{3-x}$

```
mysolver(-sqrt( 3-x) + 2 >= 0,x)
```

ans = $[-1,3]$

```


ezplot(r(x),[-1 3])
```



Intercepts

```
syms x
f(x)=(x^3-7*x^2-x+7)/50
```

f(x) =

$$\frac{x^3}{50}-\frac{7\,x^2}{50}-\frac{x}{50}+\frac{7}{50}$$

Y

```
f(0)
```

ans =

$$\frac{7}{50}$$

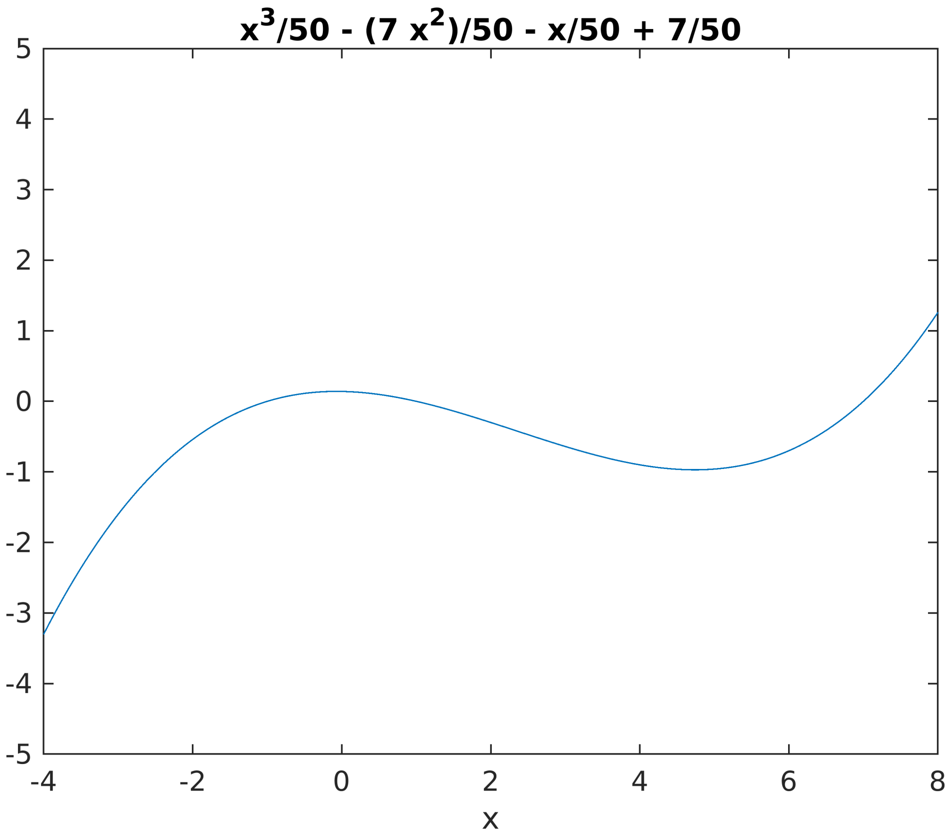
X

```
syms x
mysolver(f(x)==0,x)
```

ans = $\{-1, 1, 7\}$

As a graph

```
ezplot(f(x),[-10 10])
axis([-4 8 -5 5])
```




```
syms x
factor(f(x))
```

ans =

$$\left(\frac{1}{50} \quad x-1 \quad x-7 \quad x+1\right)$$

```
syms x
mysolver(f(x)==0,x)
```

ans = $\{-1, 1, 7\}$

Comprehension Check #2

```
syms x
h(x) = 4*x^4 + 42*x^3 - 3.75*x^2 - 218.75*x + 187.5
```

h(x) =

$$4\,x^4 + 42\,x^3 - \frac{15\,x^2}{4} - \frac{875\,x}{4} + \frac{375}{2}$$

X

```
syms x
factor(h(x))
```

ans =

$$\begin{pmatrix} \frac{1}{4} & x+10 & x+3 & 4\,x-5 & 4\,x-5 \end{pmatrix}$$

```
mysolver(h(x)==0,x)
```

ans =

$$\left\{-10, -3, \frac{5}{4}\right\}$$

Y

```
syms x
h(0)
```

ans =

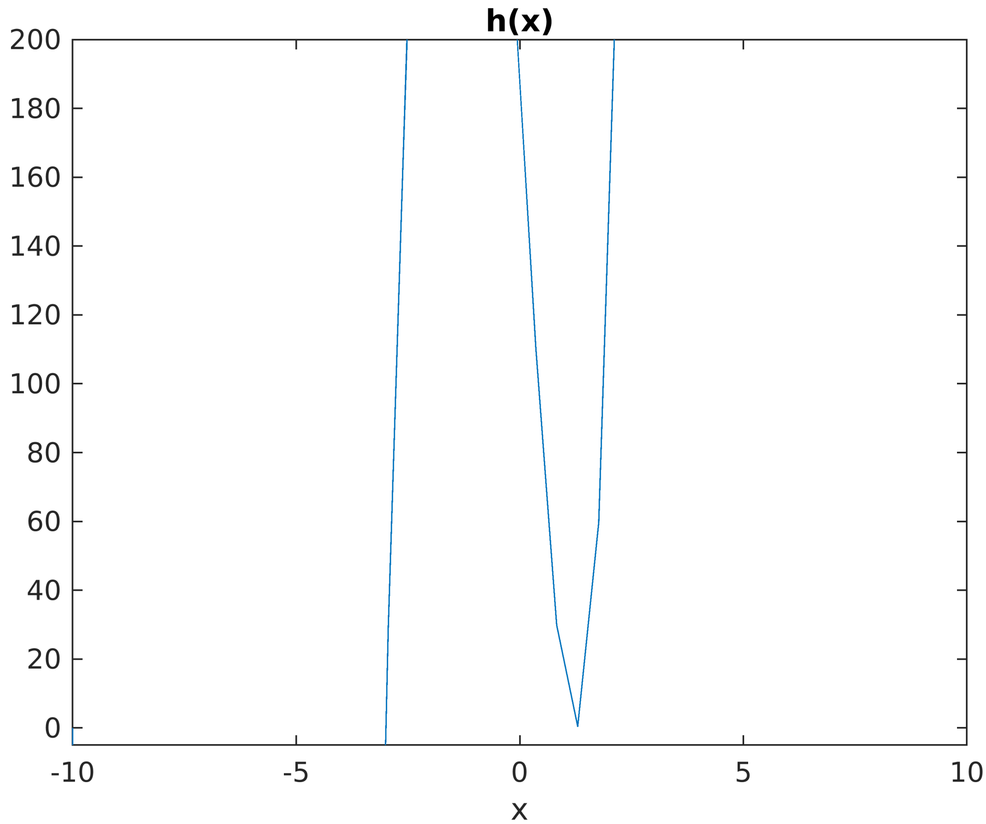
$$\frac{375}{2}$$

```
vpa(h(0))
```

ans = 187.5

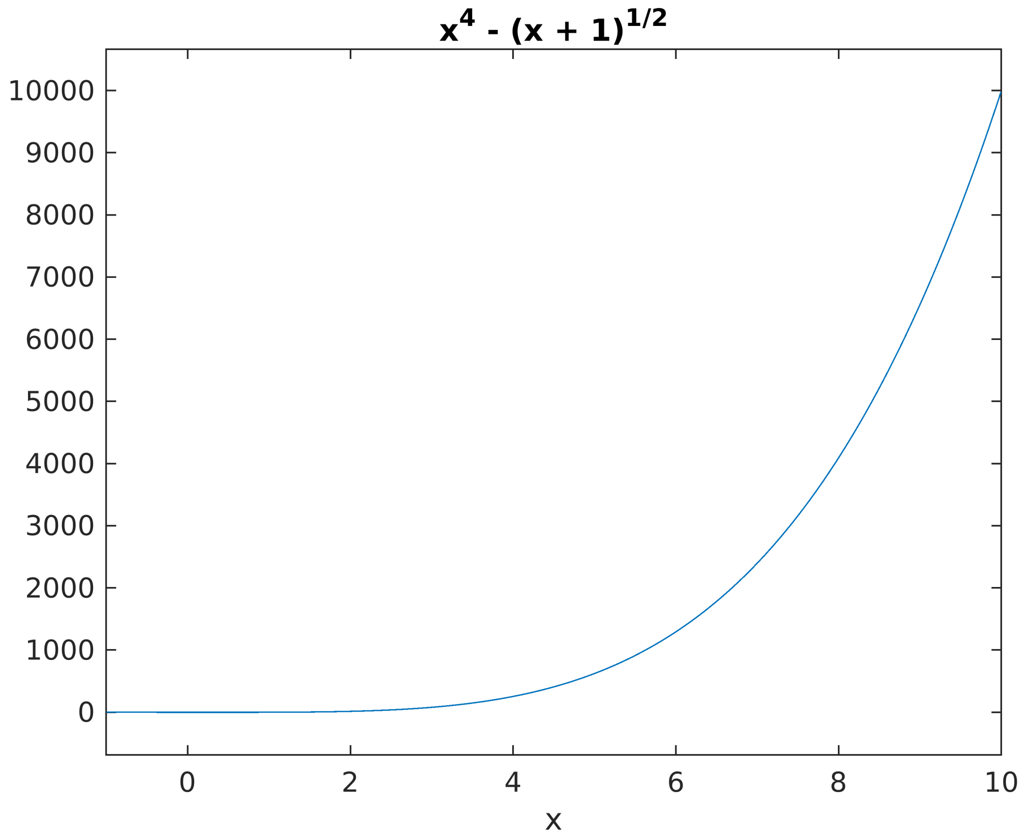
Graph

```
syms x
h(x) = 4*x^4 + 42*x^3 - 3.75*x^2 - 218.75*x + 187.5;
ezplot(h(x),[-10 10 -5 200])
hold on
title("h(x)")
hold off
```



Range

```
syms x
g(x)=x^4-sqrt(1+x);
ezplot (g(x),[-1 10])
```



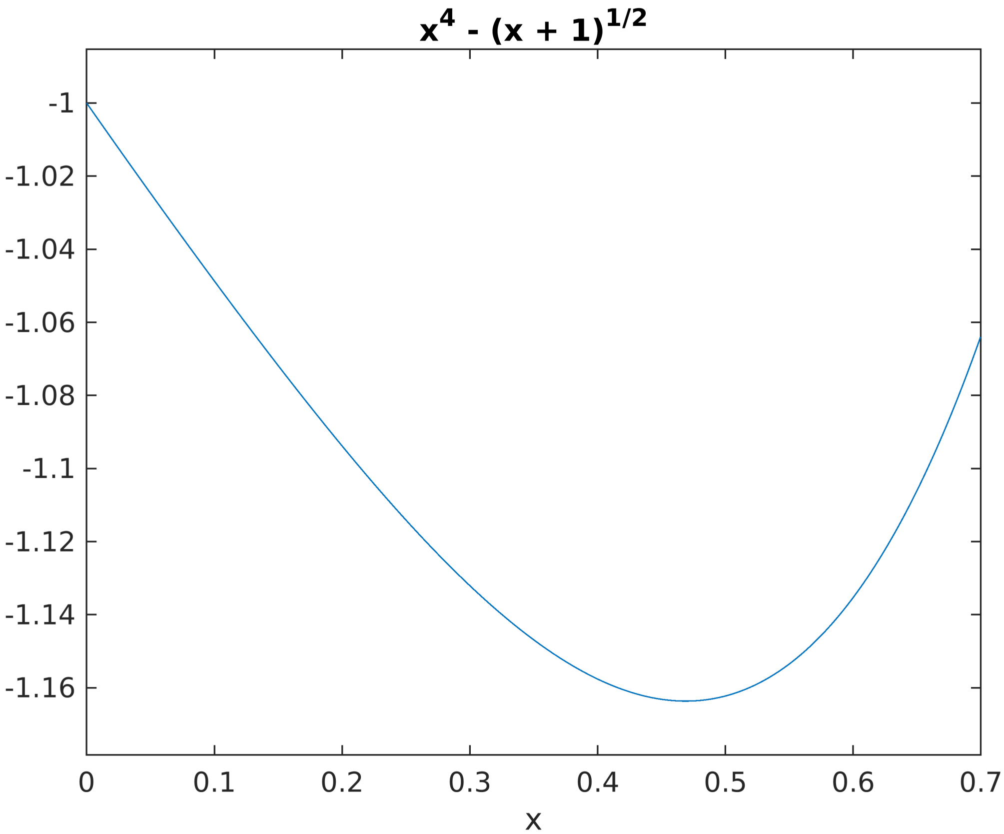

```
g(10)
```

ans = $10000 - \sqrt{11}$

```
vpa(g(10)) %to get a decimal value
```

ans = $9996.6833752096446001508850672633$

```
ezplot(g(x),[0 0.7])
```



Inequalities

```
syms x
s(x)= x^2
```

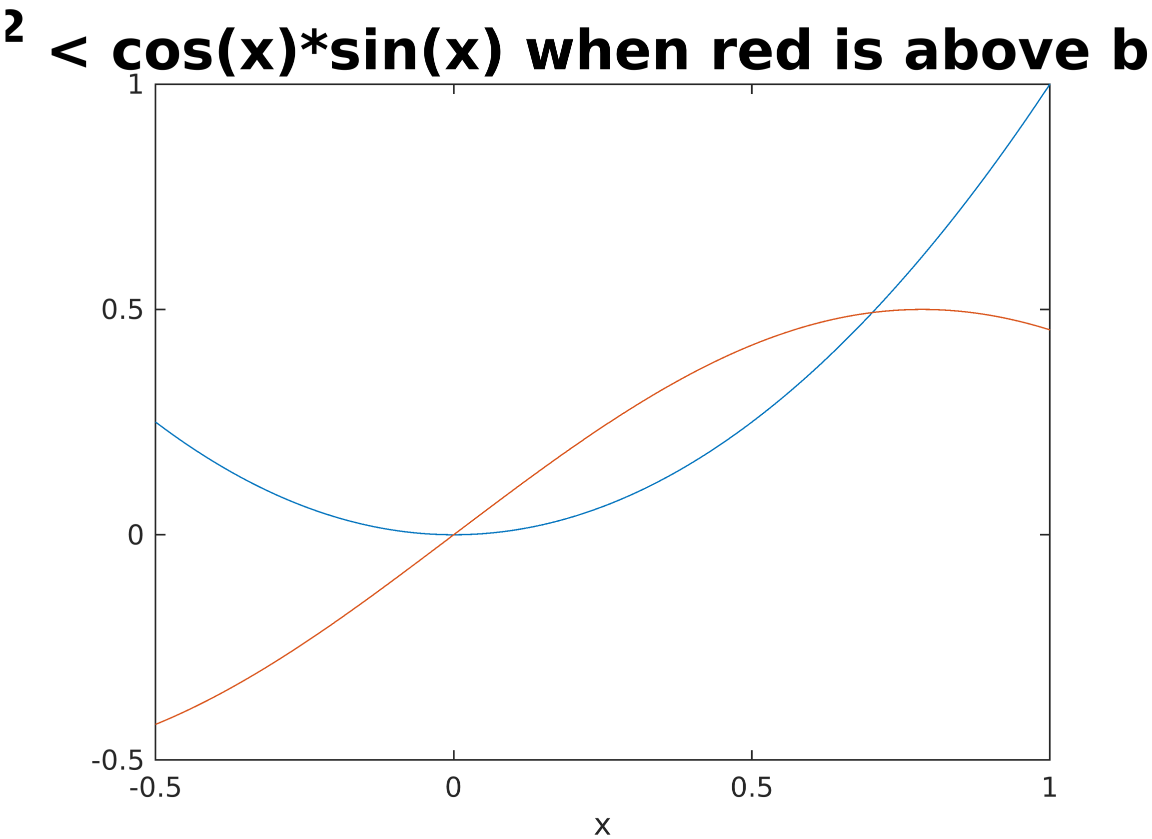
s(x) = x^2

```
t(x)=sin(x)*cos(x)
```

t(x) = $\cos(x)\sin(x)$

```
ezplot(s(x),[-2 2])
hold on %when you need two curves on the same plot hold on
ezplot(t(x),[-2 2])

axis([-0.5 1 -0.5 1])
title('x^2 < cos(x)*sin(x) when red is above blue','FontSize',20)
```




```
syms x
solve(s(x)==t(x),x) %not helpful
```

Warning: Unable to solve symbolically. Returning a numeric solution using vpasolve.

ans = 0

```
syms x
vpasolve(s(x)==t(x),x) %still not helpful. Only found solution we knew!
```

ans = 0

```
vpasolve(s(x)==t(x),x, 0.8) %tells vpasolve to look close to 0.8
```

ans = $0.70220741204621718207416397187288$

Comprehenstion Check #3

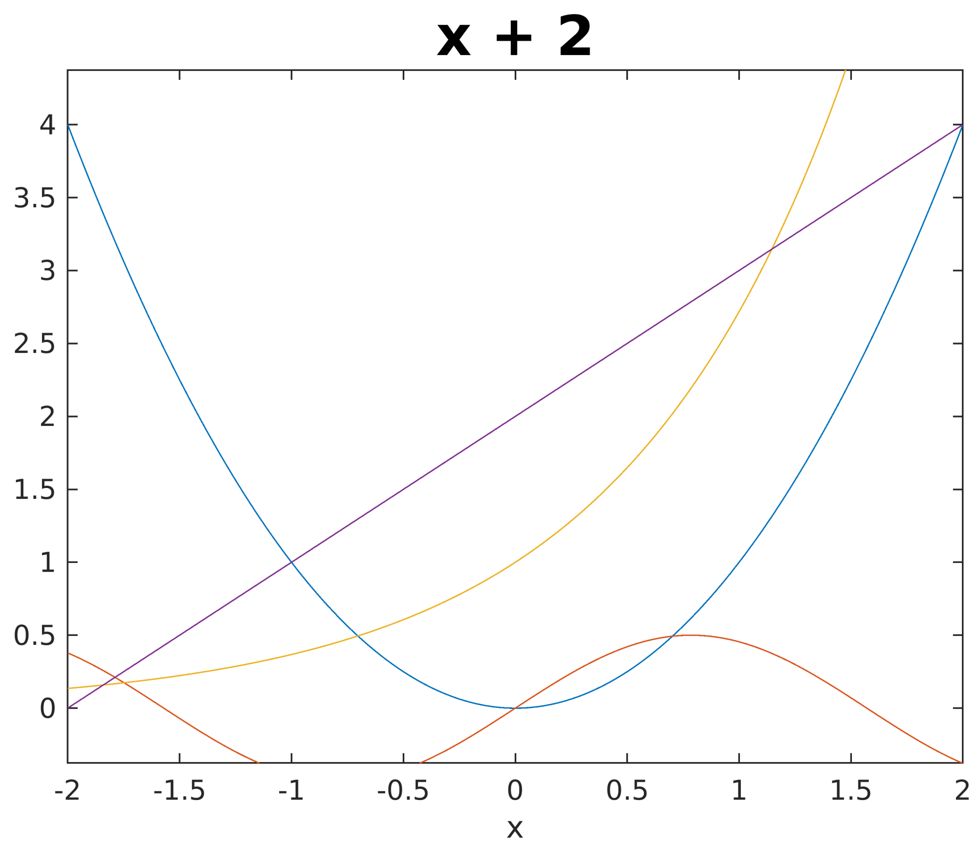
```
syms x
f(x)= exp(x)
```

f(x) = e^x

```
g(x)=x+2
```

g(x) = $x+2$

```
ezplot(f(x),[-2 2])
hold on
ezplot(g(x),[-2 2])
```



Table

```
myxvalues = [-2, -1, 0, 1, 2];
syms x
f(x)= x^2
```

f(x) = x^2

```
[myxvalues', f(myxvalues)']
```

ans =

$$\begin{pmatrix} -2 & 4 \\ -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 4 \end{pmatrix}$$

Limits

```
syms x
f(x)= (x^4-1)/(x-1)
```

f(x) =

$$\frac{x^4 - 1}{x - 1}$$

```
myx=2
```

myx = 2

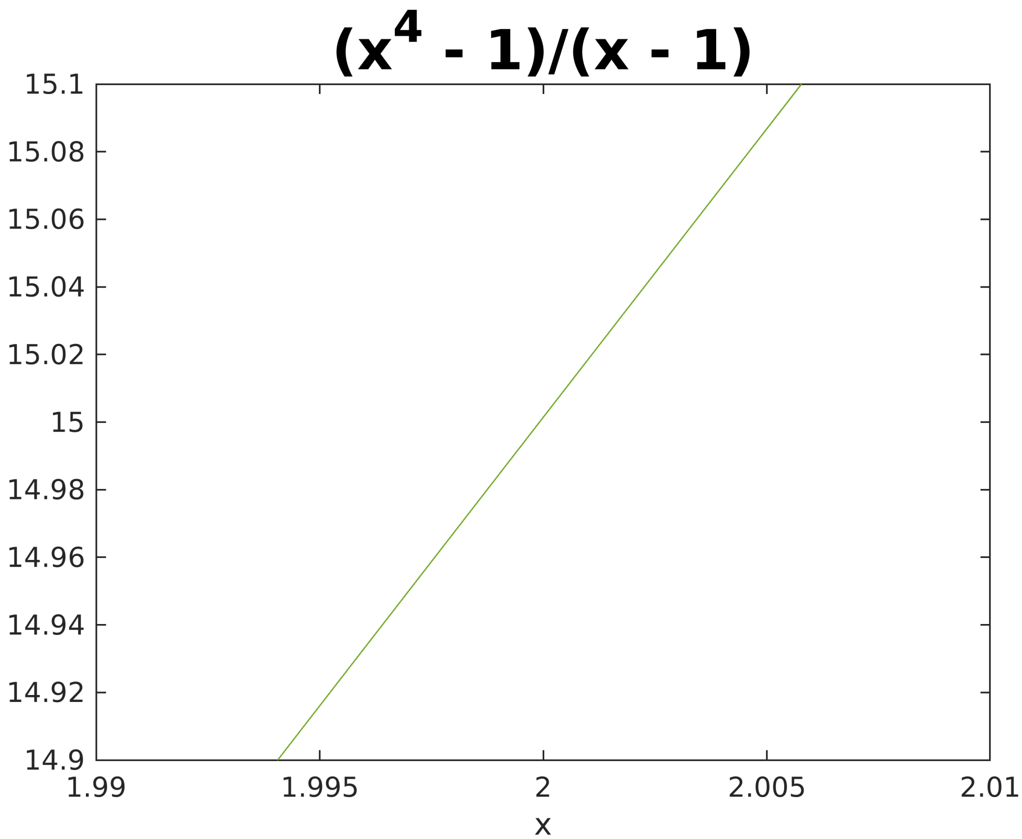
```
xnear_myx = [myx-0.1 myx-0.01 myx-0.001 myx+0.001 myx+0.01 myx+0.1];
vpa([xnear_myx', f(xnear_myx)'])
```

ans =

$$\begin{pmatrix} 1.9 & 13.369 \\ 1.99 & 14.830699 \\ 1.999 & 14.983006999 \\ 2.001 & 15.017007001 \\ 2.01 & 15.170701 \\ 2.1 & 16.771 \end{pmatrix}$$

```
ezplot(f(x),[2-0.01 2+0.01 15-0.5 15+0.5])

axis([2-0.01 2+0.01 15-0.1 15+0.1])
```




```
limit(f(x),x,2)
```

ans = 15

```
limit(f(x),x,2,'left')
```

ans = 15

```
limit(f(x),x,2,'right')
```

ans = 15

```
f(2)
```

ans = 15

Comprehention Check #4

```
syms x
f(x)= ((x^2)-4)/(x+2)
```

f(x) =

$$\frac{x^2-4}{x+2}$$

```
myx= -2
```

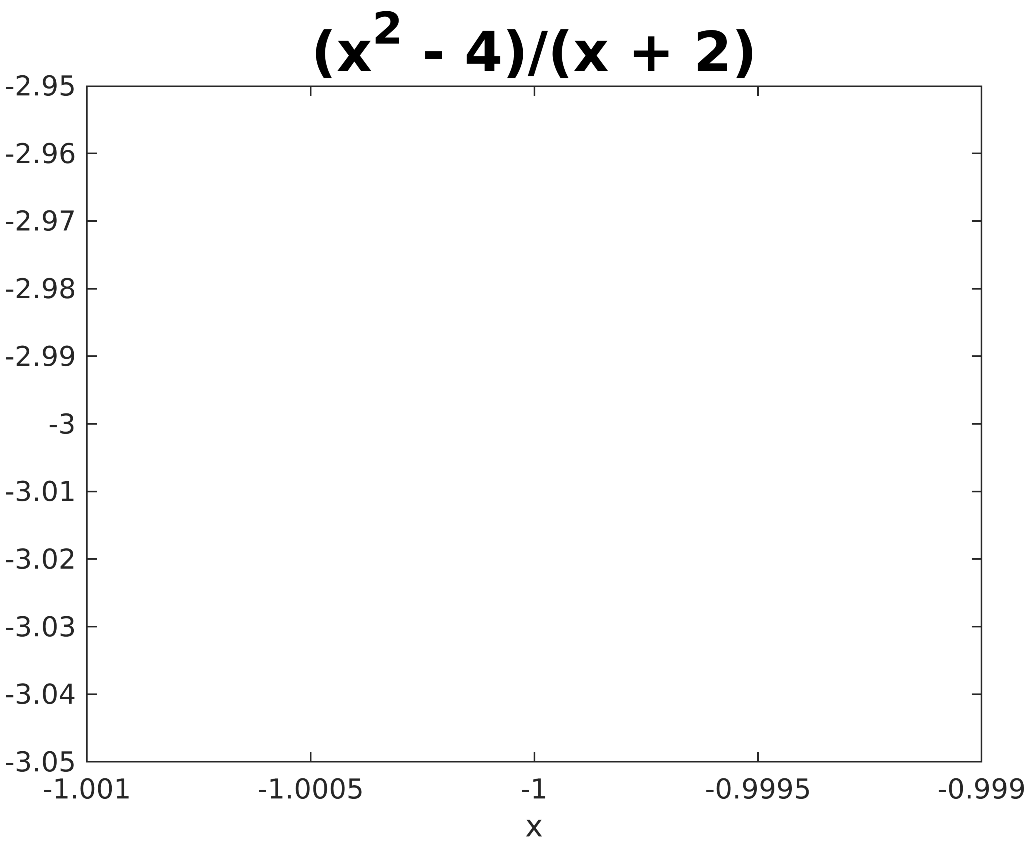
myx = -2

```
xnear_myx = [myx-0.1 myx-0.01 myx-0.001 myx+0.001 myx+0.01 myx+0.1];
vpa([xnear_myx',f(xnear_myx)'])
```

ans =

$$\begin{pmatrix} -2.1 & -4.1 \\ -2.01 & -4.01 \\ -2.001 & -4.001 \\ -1.999 & -3.999 \\ -1.99 & -3.99 \\ -1.9 & -3.9 \end{pmatrix}$$

```
ezplot(f(x),[-2-0.01 -2+0.01 -3-0.5 -3+0.5])
axis([-1-0.001 -1+0.001 -3-0.05 -3+0.05])
```




```
limit(f(x),x,-2)
```

ans = -4

```
limit(f(x),x,-2,'r')
```

ans = -4

```
limit(f(x),x,-2,'l')
```

ans = -4

```
syms x
f(x)= (3*x^2)/(1-cos(x))
```

f(x) =

$$-\frac{3\,x^2}{\cos(x)-1}$$

```
myx= 0
```

myx = 0

```
xnear_myx= [myx-0.1 myx-0.01 myx-0.001 myx+0.001 myx+0.01 myx+0.1];
vpa([xnear_myx',f(xnear_myx)'])
```

ans =

$$\begin{pmatrix} -0.1 & 6.0050025009924108270550881165474 \\ -0.01 & 6.0000500002500009920669642969878 \\ -0.001 & 6.0000005000000250000009920635268 \\ 0.001 & 6.0000005000000250000009920635268 \\ 0.01 & 6.0000500002500009920669642969878 \\ 0.1 & 6.0050025009924108270550881165474 \end{pmatrix}$$

```
limit(f(x),x,0)
```

ans = 6

```
limit(f(x),x,0,'left')
```

ans = 6

```
limit(f(x),x,0,'right')
```

ans = 6

Putting it all together

For the functions listed below, complete the following (i) create a table of values for x near the given value. If there appears to be a limit L for x near “a”, (ii) find a “b” and make a plot showing that if x is b-close to a, then the function values are 0.01-close to L, and (iii) use the "limit" command to confirm the limit. If the limit doesn’t seem to exist, consider whether one-sided limits exist.

(a)

```
syms x
f(x)= (x^3-1)/(x-1)
```

f(x) =

$$\frac{x^3 - 1}{x - 1}$$

```
myx= 1
```

```
myx = 1
```

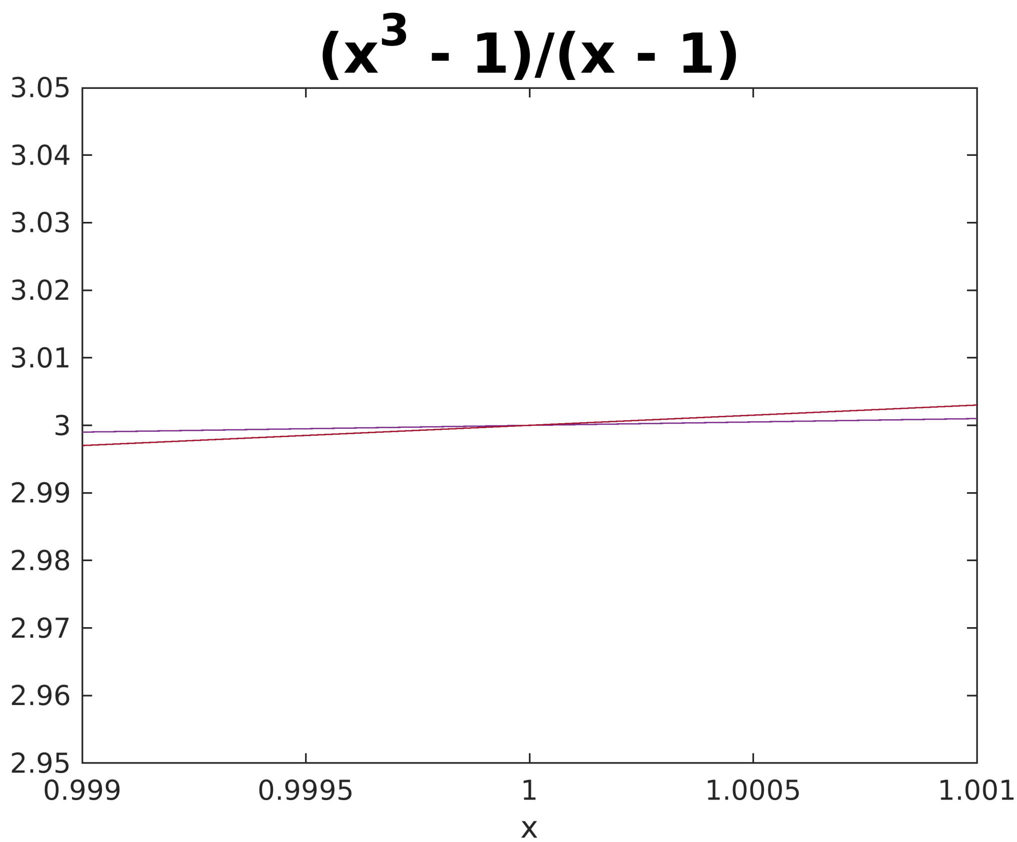
```
xnear_myx= [myx-0.1 myx-0.01 myx-0.001 myx+0.001 myx+0.01 myx+0.1];
vpa([xnear_myx',f(xnear_myx)'])
```

```
ans =
```

$$\begin{pmatrix} 0.9 & 2.71 \\ 0.99 & 2.9701 \\ 0.999 & 2.997001 \\ 1.001 & 3.003001 \\ 1.01 & 3.0301 \\ 1.1 & 3.31 \end{pmatrix}$$

```
ezplot(f(x),[1-0.01 1+0.01 3-0.5 3+0.5])

axis([1-0.001 1+0.001 3-0.05 3+0.05])
```




```
limit(f(x),x,1)
```

```
ans = 3
```

```
limit(f(x),x,1,'left')
```

```
ans = 3
```

```
limit(f(x),x,1,'right')
```

```
ans = 3
```

(b)

```
syms x
f(x)= (sin(10*x))/(3*x)
```

f(x) =

$$\frac{\sin(10\,x)}{3\,x}$$

```
myx= 0
```

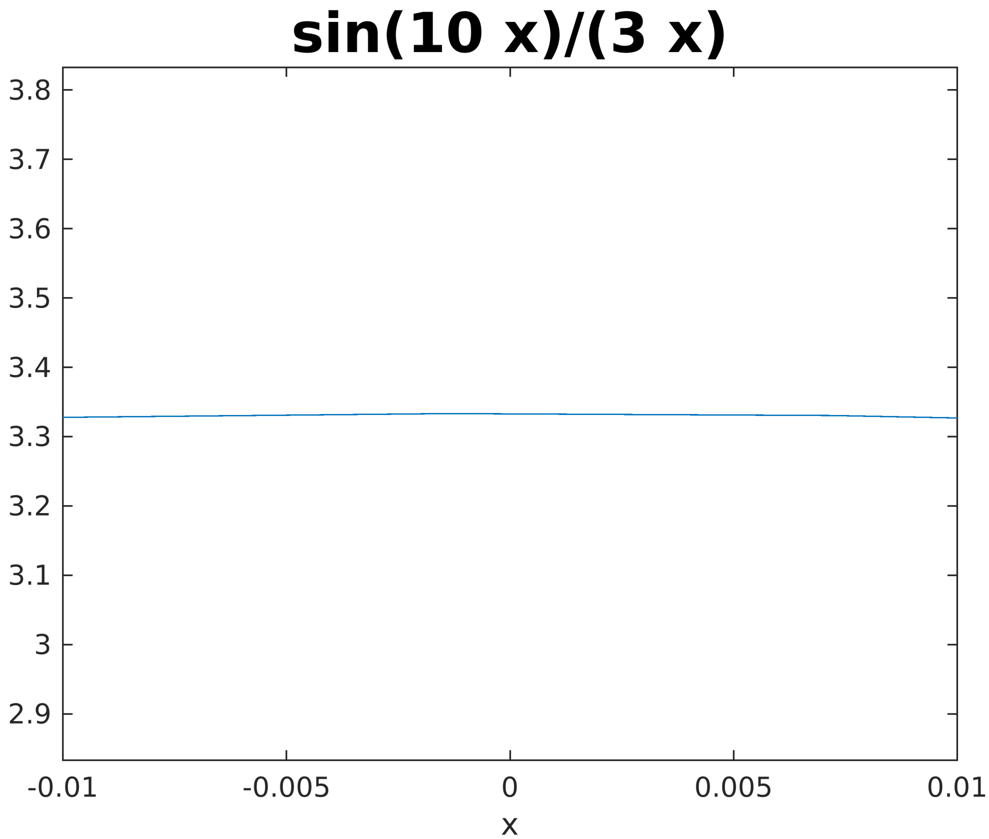
```
myx = 0
```

```
xnear_myx= [myx-0.1 myx-0.01 myx-0.001 myx+0.001 myx+0.01 myx+0.1];
vpa([xnear_myx',f(xnear_myx)'])
```

ans =

$$\begin{pmatrix} -0.1 & 2.8049032826929883555083410721 01 \\ -0.01 & 3.3277805548942717435604732803541 \\ -0.001 & 3.3332777780555548941808127563666 \\ 0.001 & 3.3332777780555548941808127563666 \\ 0.01 & 3.3277805548942717435604732803541 \\ 0.1 & 2.804903282692988355508341072101 \end{pmatrix}$$

```
ezplot(f(x),[0-0.01 0+0.01 (10/3)-0.5 (10/3)+0.5])
```




```
limit(f(x),x,0)
```

ans =

$$\frac{10}{3}$$

```
limit(f(x),x,0,'left')
```

ans =

$$\frac{10}{3}$$

```
limit(f(x),x,0,'right')
```

ans =

$$\frac{10}{3}$$

(c)

```
syms f(x)
f(x)= abs(x+1)/(x+1)
```

f(x) =

$$\frac{|x+1|}{x+1}$$

```
myx= -1
```

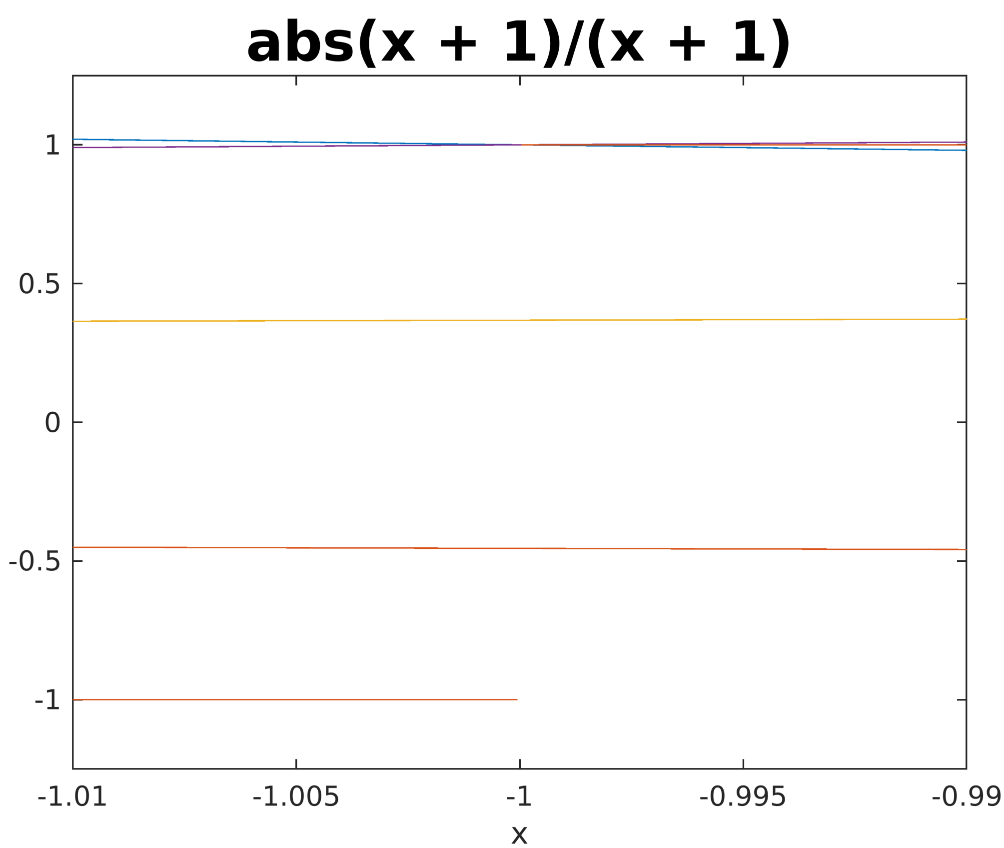
```
myx = -1
```

```
xnear_myx= [myx-0.1 myx-0.01 myx-0.001 myx+0.001 myx+0.01 myx+0.1];
vpa([xnear_myx',f(xnear_myx)'])
```

ans =

$$\begin{pmatrix} -1.1 & -1.0 \\ -1.01 & -1.0 \\ -1.001 & -1.0 \\ -0.999 & 1.0 \\ -0.99 & 1.0 \\ -0.9 & 1.0 \end{pmatrix}$$

```
ezplot(f(x),[-1-0.01 -1+0.01])
```




```
limit(f(x),x,-1)
```

ans = NaN

```
limit(f(x),x,-1,'left')
```

ans = -1

```
limit(f(x),x,-1,'right')
```

ans = 1

2. Limits as t "goes to infinity". The number, P, of animals as a function of time t in a certain population is given by the formula () = 400 2 + 8 (0.75F) Define this function in MATLAB Plot the function for t ranging from 0 to various positive values (try at least 10, 15, 20, 25). As t gets bigger, P never seems to get above 200. When values of a function like P are close to some number and stay close as regardless of how large t gets, we call that the limit of P(t) as t approaches infinity. We write this as lim FG () Use MATLAB's limit function to calculate the limit of P(t) as t approaches infinity. Use "inf" (without quotes) instead of a given number in the limit command.

```
syms t
P(t)=400/(2+8*(0.75^t))
```

P(t) =

$$\frac{400}{8\left(\frac{3}{4}\right)^t+2}$$

```
myx=inf
```

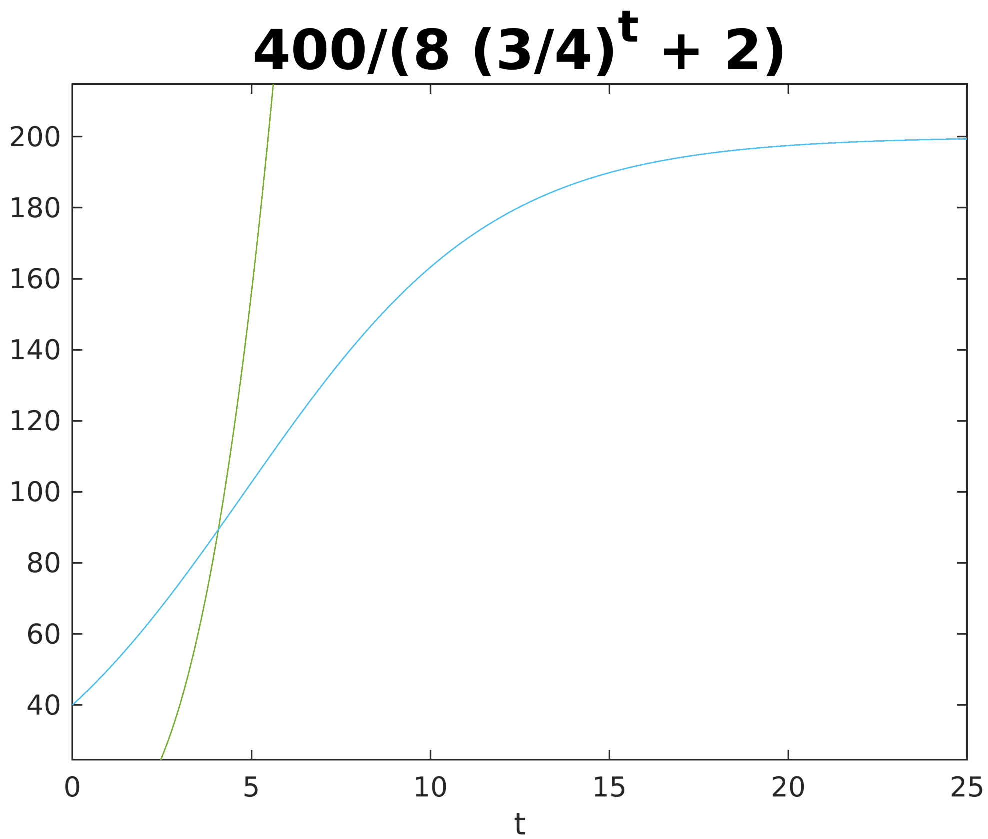
myx = Inf

```
xnear_myx= [myx-0.1 myx-0.01 myx-0.001 myx+0.001 myx+0.01 myx+0.1];
vpa([xnear_myx',f(xnear_myx)'])
```

ans =

$$\begin{pmatrix} \infty & \mathrm{NaN} \\ \infty & \mathrm{NaN} \\ \infty & \mathrm{NaN} \\ \infty & \mathrm{NaN} \\ \infty & \mathrm{NaN} \\ \infty & \mathrm{NaN} \end{pmatrix}$$

```
ezplot(P(t),[0,10])
ezplot(P(t),[0,15])
ezplot(P(t),[0,20])
ezplot(P(t),[0,25])
```




```
limit(P(t),t,inf)
```

ans = 200

3. Here are two models for the population of the earth. We will do some basic analysis of these two models. One of them is called the doomsday model. Use the limit as t goes to infinity to explain why.

```
syms t
P(t)=30/(3+7*exp(-0.03*0))
```

P(t) = 3

```
Q(t)=(20*10^6)/(1+(2027-t)^(4/3))
```

Q(t) =

$$\frac{20000000}{(2027-t)^{4/3}+1}$$

a) Use each model to estimate the world population in 1960. Be sure to account for the different meanings of t in the two different models. Do they agree?

```
vpa((30)/(3+7*exp(-0.03*0)), 3)
```

ans = 3.0

```
vpa((20*10^6)/(1+(2027-1960)^(4/3)), 3)
```

ans = 7.32e+4

They do not agree on the population

b) Use each model to estimate the world population now. How well does each approximate the current world population of 7.5 billion people?

```
vpa((30)/(3+7*exp(-0.03*61)))
```

ans = 7.2764405074798572670147223107051

```
vpa((20*10^6)/(1+(2027-2021)^(4/3)))
```

ans = 1680287.7008976675570011138916016

P(x) is the best prepresentation of today

c) One of them is called the “doomsday” model. The other is a “logistic” model that levels off at the “carrying capacity.” Use the limit as t goes to infinity with each of these two functions to find out the long term population of the earth. Which function (P or Q) gives which model (doomsday or logistic), and why you would say so?

```
limit(P(t),t,inf)
```

ans = 3

```
limit(Q(t),t,inf)
```

ans = 0

P(t) Is the doomsday modle while Q(t) is the carrying Capacity model