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208 9:00AM 10/15/2021

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Question 1

Family: The Witches

```
syms x c pi
assume(c>0)
witch(x,c)= 8*c/(4 * pi *(4*c^2+(x-pi)^2))
```

```
witch(x, c) =
```

$$\frac{2\,c}{\pi\,((\pi-x)^2+4\,c^2)}$$

Formuals for the Witches

```
witch(x,1)
```

```
ans =
```

$$\frac{2}{\pi\,((\pi-x)^2+4)}$$

```
witch(x,2)
```

```
ans =
```

$$\frac{4}{\pi\,((\pi-x)^2+16)}$$

```
witch(2,3/4)
```

```
ans =
```

$$\frac{3}{2\,\pi\,\left((\pi-2)^2+\frac{9}{4}\right)}$$

Simularitys and differences:

For first half is the same untill you reach the numerator, witchis double what was given, and the last number in the dominator, that is x^4 of the given

Graphs for the Witches

```
hold off
ezplot(witch(x,1))
hold on
```

```
ezplot(witch(x,2))
ezplot(witch(x,3/4), [-3,9,0,0.5])
legend('Witch Curves with c= 1, 2, 3/4')
```

Intecepts for the Witches

```
vpa(witch(0,1))
```

ans =

$$\frac{2.0}{\pi\ (\pi^2 + 4.0)}$$

```
vpa(witch(0,2))
```

ans =

$$\frac{4.0}{\pi\ (\pi^2 + 16.0)}$$

```
vpa(witch(0,3/4))
```

ans =

$$\frac{1.5}{\pi\ (\pi^2 + 2.25)}$$

```
vpa(witch(0,c))
```

ans =

$$\frac{2.0\,c}{\pi\ (4.0\,c^2 + \pi^2)}$$

Asymtopes for the Witches

```
limit(witch(x,1),inf)
```

ans = 0

```
limit(witch(x,1),-inf)
```

ans = 0

```
limit(witch(x,2),inf)
```

ans = 0

```
limit(witch(x,2),-inf)
```

ans = 0

```
limit(witch(x,3/4),inf)
```

ans = 0

```
limit(witch(x,3/4),-inf)
```

ans = 0

Intervuls of increasing and Decressing Local Extreama

```
diff(witch(x, 1),x)
```

ans =

$$\frac{2\,(2\,\pi-2\,x)}{\pi\,((\pi-x)^2+4)^2}$$

```
solve(diff(witch(x, 1),x))
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans = π

```
assume(x,'real')
solve(diff(witch(x, c),x)==0,x)
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans = π

```
assume(x<-pi)
simplify(diff(witch(x,c),x)>=0)
```

ans = $x^2 \le (2\,\pi-x)^2$

```
assume(x,'real')

[pi,witch(pi,3/4)]
```

ans =

$$\left(\pi\quad\frac{2}{3\,\pi}\right)$$

```
vpa([pi,witch(pi,3/4)])
```

ans =

$$\left(\pi\quad\frac{0.66666666666666666666666666666667}{\pi}\right)$$

```
[pi,witch(pi,c)]
```

ans =

$$\left(\pi\quad\frac{1}{2\,c\,\pi}\right)$$

```
vpa([pi,witch(pi,c)])
```

ans =

$$\left(\pi \quad \frac{0.5}{c\,\pi}\right)$$

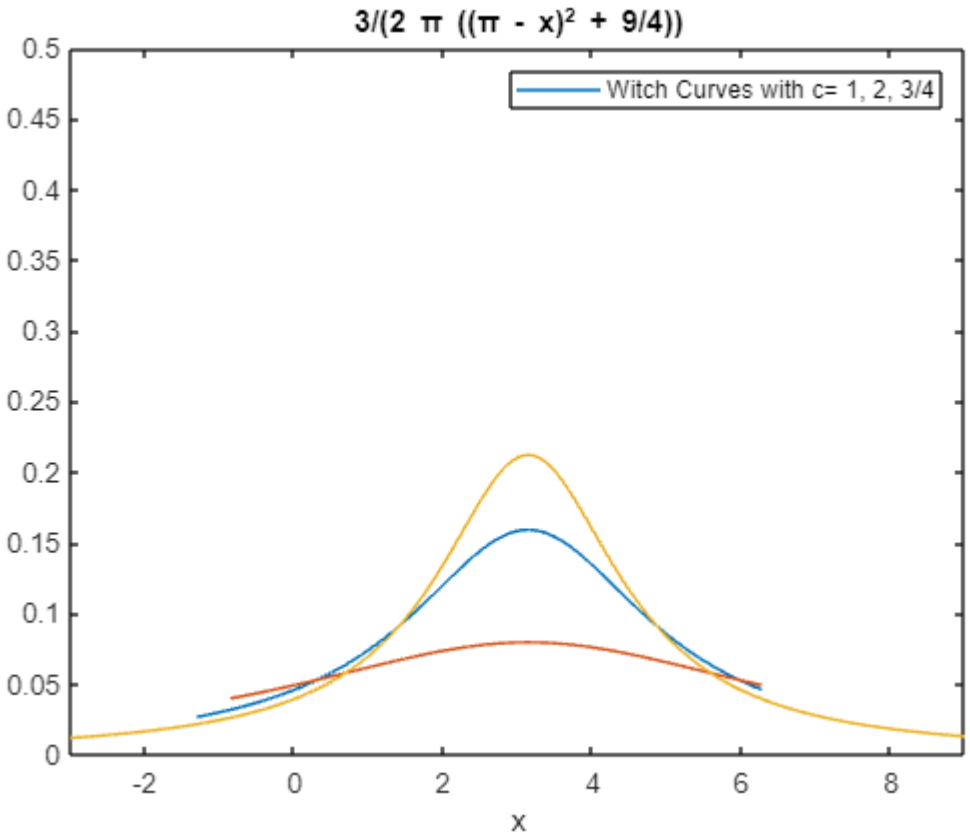
Intervals of concavity And inflection points for the Witches

```
diff(witch(x,c),x,2)
```

```
ans =
```

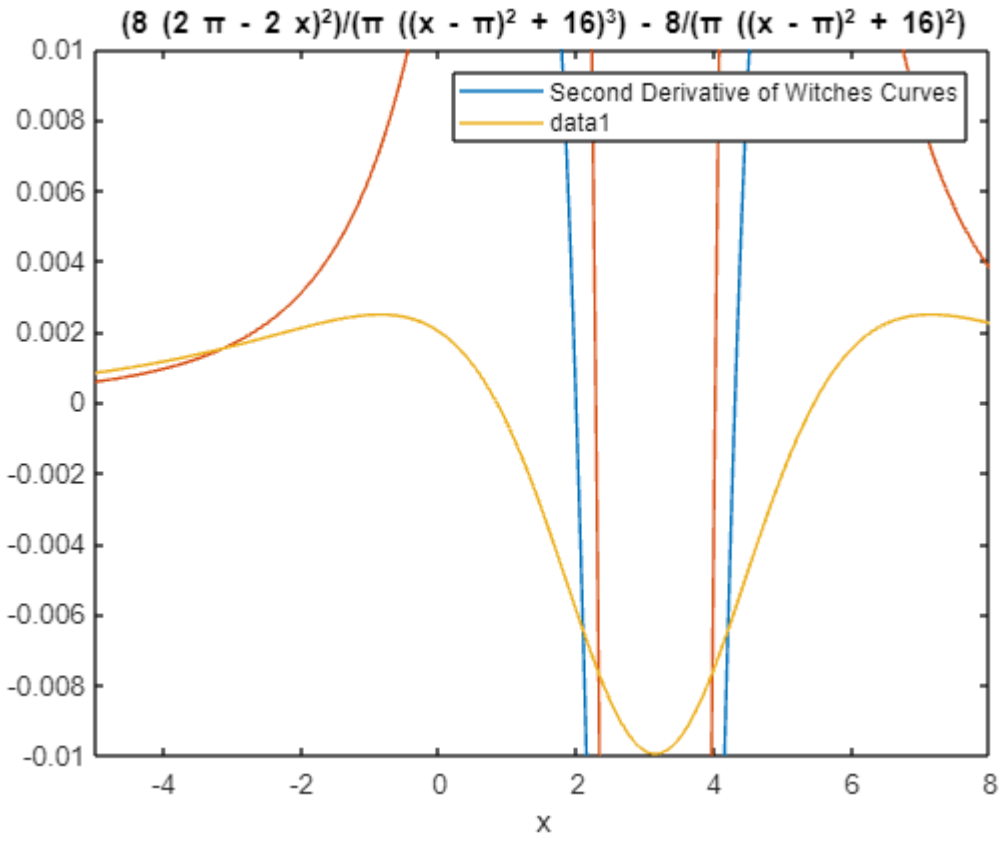
$$\frac{4\,c\,(2\,\pi-2\,x)^2}{\pi\,((\pi-x)^2+4\,c^2)^3}-\frac{4\,c}{\pi\,((\pi-x)^2+4\,c^2)^2}$$

```
hold off
```




```
ezplot(diff(witch(x,1),x,2))
hold on
ezplot(diff(witch(x,3/4),x,2),[-5,8,-2,1])
legend('Second Derivative of Witches Curves')

ezplot(diff(witch(x,2),x,2),[-5,8,-.01,0.01])
```



Inflection Points or the Witches

```
solve(diff(witch(x,1),x,2),x)
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans =

$$\begin{pmatrix} \pi - \frac{2\sqrt{3}}{3} \\ \pi + \frac{2\sqrt{3}}{3} \end{pmatrix}$$

```
vpa(solve(diff(witch(x,1),x,2),x))
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans =

$$\begin{pmatrix} \pi - 1.1547005383792515290182975610039 \\ \pi + 1.1547005383792515290182975610039 \end{pmatrix}$$

```
solve(diff(witch(x,2),x,2),x)
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans =

$$\begin{pmatrix} \pi - \frac{4\sqrt{3}}{3} \\ \pi + \frac{4\sqrt{3}}{3} \end{pmatrix}$$

```
vpa(solve(diff(witch(x,2),x,2),x))
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans =

$$\begin{pmatrix} \pi - 2.309401076758503058036595122007 8 \\ \pi + 2.309401076758503058036595122007 8 \end{pmatrix}$$

```
solve(diff(witch(x,3/4),x,2),x)
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans =

$$\begin{pmatrix} \pi - \frac{\sqrt{3}}{2} \\ \pi + \frac{\sqrt{3}}{2} \end{pmatrix}$$

```
vpa(solve(diff(witch(x,3/4),x,2),x))
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

ans =

$$\begin{pmatrix} \pi - 0.866025403784438646763723170752 94 \\ \pi + 0.866025403784438646763723170752 94 \end{pmatrix}$$

Using Precise Location of the Inflection Points

```
[pi+1,witch(pi+1,1)]
```

ans =

$$\begin{pmatrix} \pi + 1 & \frac{2}{5\,\pi} \end{pmatrix}$$

```
vpa([pi+1,witch(pi+1,1)])
```

ans =

$$\begin{pmatrix} \pi + 1.0 & \frac{0.4}{\pi} \end{pmatrix}$$

```
[pi-1,witch(pi-1,1)]
```

ans =

$$\begin{pmatrix} \pi - 1 & \frac{2}{5\,\pi} \end{pmatrix}$$

```
vpa([pi-1,witch(pi-1,1)])
```

```
ans =
```

$$\left(\pi - 1.0 \quad \frac{0.4}{\pi}\right)$$

For an Arbitrary Value of c

```
[pi+c,witch(pi+c,1)]
```

```
ans =
```

$$\left(c + \pi \quad \frac{2}{\pi\ (c^2 + 4)}\right)$$

```
vpa([pi+c,witch(pi+c,1)])
```

```
ans =
```

$$\left(c + \pi \quad \frac{2.0}{\pi\ (c^2 + 4.0)}\right)$$

```
[pi-c,witch(pi-c,1)]
```

```
ans =
```

$$\left(\pi - c \quad \frac{2}{\pi\ (c^2 + 4)}\right)$$

```
vpa([pi-c,witch(pi-c,1)])
```

```
ans =
```

$$\left(\pi - 1.0\,c \quad \frac{2.0}{\pi\ (c^2 + 4.0)}\right)$$

Role of the Paramater