Kevin White

11/1/2021

208 9:00AM 10/15/2021

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Question 2

Family: The Hills

```
syms x c pi
assume(c>0)
hill(x,c) = (c*exp(-(x-1/10)*c))/((1+exp(-c*(x-1/10)))^2)
```

```
hill(x, c) =
```

$$\frac{c\,e^{-c\left(x-\frac{1}{10}\right)}}{\left(e^{-c\left(x-\frac{1}{10}\right)}+1\right)^2}$$

Formuals for the hills

```
hill(x,1.1)
```

```
ans =
```

$$\frac{11\,e^{\frac{11}{100}-\frac{11\,x}{10}}}{10\left(e^{\frac{11}{100}-\frac{11\,x}{10}}+1\right)^2}$$

```
hill(x,2.2)
```

```
ans =
```

$$\frac{11\,e^{\frac{11}{50}-\frac{11\,x}{5}}}{5\left(e^{\frac{11}{50}-\frac{11\,x}{5}}+1\right)^2}$$

```
hill(2,0.1)
```

```
ans =
```

$$\frac{e^{-\frac{19}{100}}}{10\left(e^{-\frac{19}{100}}+1\right)^2}$$

Simularitys and differences:

The equetions are mostly the same exept for the numerator, the leftovers of the numerator are givin to the denominator.

Graphs for the hills

```
hold off
ezplot(hill(x,1.1))
hold on
ezplot(hill(x,2.1))
ezplot(hill(x,0.1), [-6,6,0,1])
legend('hill Curves with c= 1.1, 2.1, 0.1')
```

Intecepts for the hills

```
vpa(hill(0,1.1))
```

ans = $0.27416979974339781925289278150886$

```
vpa(witch(0,2.1))
```

ans =

$$\frac{4.2}{\pi\ (\pi^2 + 17.64)}$$

```
vpa(witch(0,0.1))
```

ans =

$$\frac{0.2}{\pi\ (\pi^2 + 0.04)}$$

```
vpa(hill(0,c))
```

ans =

$$\frac{c\,e^{0.1\,c}}{(e^{0.1\,c} + 1.0)^2}$$

Asymtopes for the hills

```
limit(hill(x,1.1),inf)
```

ans = 0

```
limit(hill(x,1.1),-inf)
```

ans = 0

```
limit(hill(x,2.1),inf)
```

ans = 0

```
limit(hill(x,2.1),-inf)
```

ans = 0

```
limit(hill(x,0.1),inf)
```

ans = 0

```
limit(hill(x,0.1),-inf)
```

ans = 0

Intervuls of increasing and Decressing Local Extreama

```
diff(hill(x, 1.1),x)
```

ans =

$$\frac{121\,e^{\frac{11}{50}-\frac{11\,x}{5}}}{50\,\left(e^{\frac{11}{100}-\frac{11\,x}{10}}+1\right)^3}-\frac{121\,e^{\frac{11}{100}-\frac{11\,x}{10}}}{100\,\left(e^{\frac{11}{100}-\frac{11\,x}{10}}+1\right)^2}$$

```
solve(diff(hill(x, 1.1),x))
```

ans =

$$\frac{1}{10}$$

```
assume(x,'real')
solve(diff(hill(x, c),x)==0,x)
```

ans =

$$\frac{1}{10}$$

```
assume(x<-pi)
simplify(diff(hill(x,c),x)>=0)
```

ans =

$$e^{-\frac{c\,(10\,x-1)}{10}} \le e^{-\frac{c\,(10\,x-1)}{5}}$$

```
assume(x,'real')

[pi,hill(pi,0.1)]
```

ans =

$$\left(\pi \quad \frac{e^{\frac{1}{100}-\frac{\pi}{10}}}{10\,\left(e^{\frac{1}{100}-\frac{\pi}{10}}+1\right)^2}\right)$$

```
vpa([pi,hill(pi,0.1)])
```

ans =

$$\begin{pmatrix} \pi & \frac{0.1\,\mathrm{e}^{0.01-0.1\,\pi}}{(\mathrm{e}^{0.01-0.1\,\pi}+1.0)^2} \end{pmatrix}$$

```
[pi,hill(pi,c)]
```

ans =

$$\begin{pmatrix} \pi & \frac{c\,\mathrm{e}^{-c\,\left(\pi-\frac{1}{10}\right)}}{\left(\mathrm{e}^{-c\,\left(\pi-\frac{1}{10}\right)}+1\right)^2} \end{pmatrix}$$

```
vpa([pi,hill(pi,c)])
```

ans =

$$\begin{pmatrix} \pi & \frac{c\,\mathrm{e}^{-1.0\,c\,(\pi-0.1)}}{(\mathrm{e}^{-1.0\,c\,(\pi-0.1)}+1.0)^2} \end{pmatrix}$$

Intervals of concavity And inflection points for the hills

```
diff(hill(x,c),x,2)
```

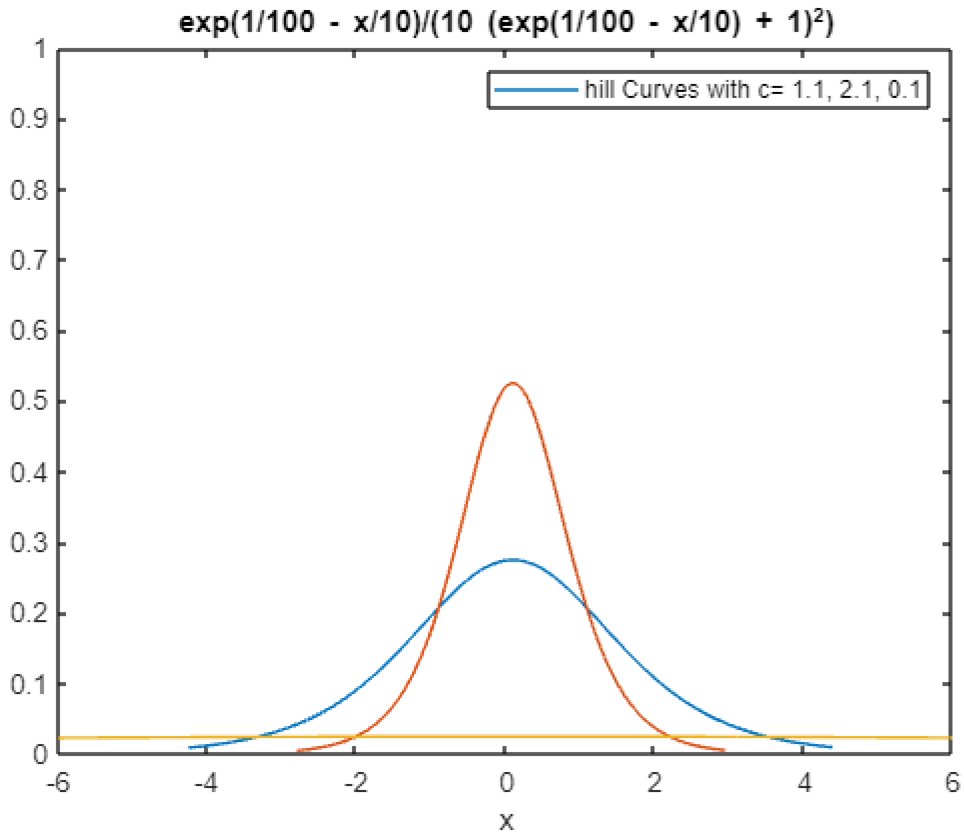
ans =

$$\frac{c^3\,\mathrm{e}^{-c\,\left(x-\frac{1}{10}\right)}}{{\sigma_1}^2}-\frac{6\,c^3\,\mathrm{e}^{-2\,c\,\left(x-\frac{1}{10}\right)}}{{\sigma_1}^3}+\frac{6\,c^3\,\mathrm{e}^{-3\,c\,\left(x-\frac{1}{10}\right)}}{{\sigma_1}^4}$$

where

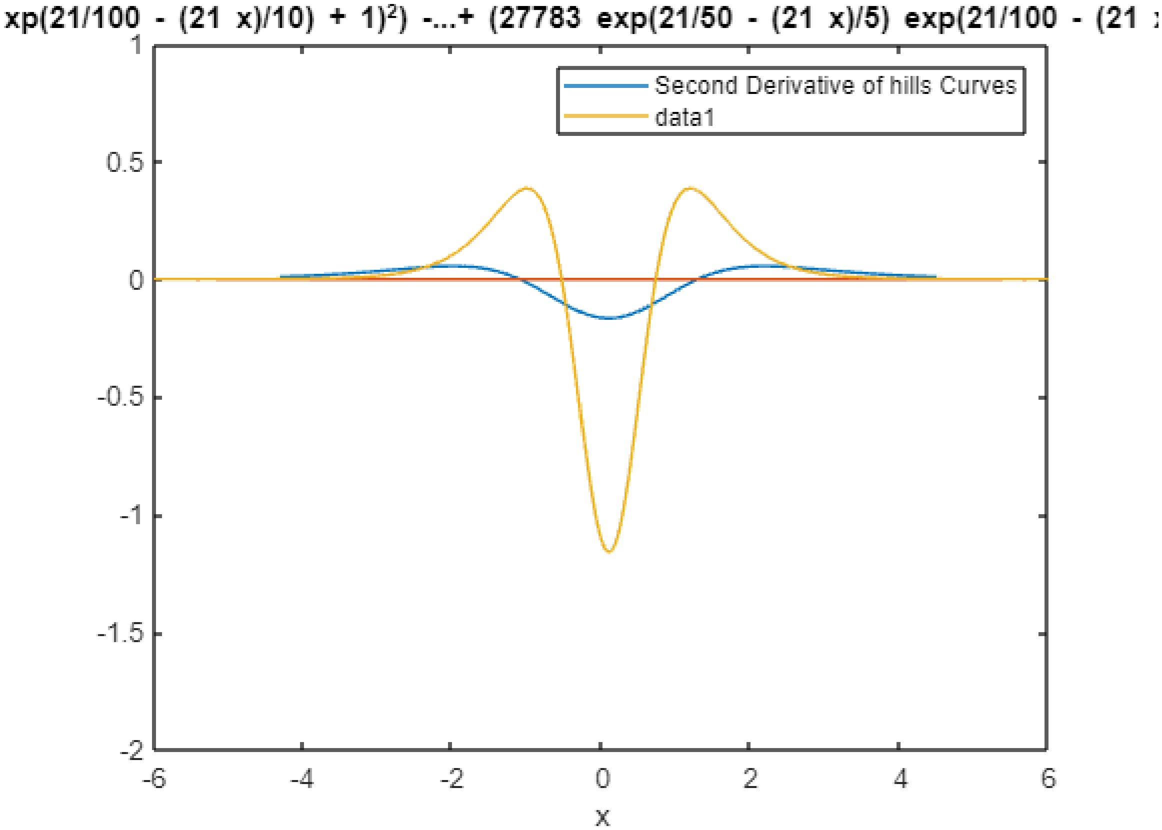
$$\sigma_1=\mathrm{e}^{-c\,\left(x-\frac{1}{10}\right)}+1$$

```
hold off
```




```
ezplot(diff(hill(x,1.1),x,2))
hold on
ezplot(diff(hill(x,0.1),x,2),[-12,12,-5,5])
legend('Second Derivative of hills Curves')

ezplot(diff(hill(x,2.1),x,2),[-6,6,-2,1])
```



Inflection Points or the Witches

```
solve(diff(hill(x,1.1),x,2),x)
```

ans =

$$\begin{pmatrix} \frac{10\log(2-\sqrt{3})}{11}+\frac{1}{10} \\ \frac{10\log(\sqrt{3}+2)}{11}+\frac{1}{10} \end{pmatrix}$$

```
vpa(solve(diff(hill(x,1.1),x,2),x))
```

ans =

$$\begin{pmatrix} -1.0972344517498333714773148611891 \\ 1.2972344517498333714773148611891 \end{pmatrix}$$

```
solve(diff(hill(x,2.1),x,2),x)
```

ans =

$$\begin{pmatrix} \frac{10\log(2-\sqrt{3})}{21}+\frac{1}{10} \\ \frac{10\log(\sqrt{3}+2)}{21}+\frac{1}{10} \end{pmatrix}$$

```
vpa(solve(diff(hill(x,2.1),x,2),x))
```

ans =

$$\begin{pmatrix} -0.52712280805943652791668873681332 \\ 0.72712280805943652791668873681332 \end{pmatrix}$$

```
solve(diff(hill(x,0.1),x,2),x)
```

ans =

$$\begin{pmatrix} 10\log(2-\sqrt{3})+\frac{1}{10} \\ 10\log(\sqrt{3}+2)+\frac{1}{10} \end{pmatrix}$$

```
vpa(solve(diff(hill(x,0.1),x,2),x))
```

ans =

$$\begin{pmatrix} -13.069578969248167086250463473308 \\ 13.269578969248167086250463473308 \end{pmatrix}$$

Using Precise Location of the Inflection Points

```
[pi+1,hill(pi+1,1)]
```

ans =

$$\begin{pmatrix} \pi+1 & \frac{e^{-\pi-\frac{9}{10}}}{\left(e^{-\pi-\frac{9}{10}}+1\right)^2} \end{pmatrix}$$

```
vpa([pi+1,hill(pi+1,1)])
```

ans =

$$\begin{pmatrix} \pi+1.0 & \frac{e^{-1.0\,\pi-0.9}}{(e^{-1.0\,\pi-0.9}+1.0)^2} \end{pmatrix}$$

```
[pi-1,hill(pi-1,1)]
```

ans =

$$\begin{pmatrix} \pi-1 & \frac{e^{\frac{11}{10}-\pi}}{\left(e^{\frac{11}{10}-\pi}+1\right)^2} \end{pmatrix}$$

```
vpa([pi-1,hill(pi-1,1)])
```

ans =

$$\begin{pmatrix} \pi-1.0 & \frac{e^{1.1-1.0\,\pi}}{(e^{1.1-1.0\,\pi}+1.0)^2} \end{pmatrix}$$

For an Arbitrary Value of c

```
[pi+c,hill(pi+c,1)]
```

ans =

$$\left(\begin{array}{cc} c+\pi & \frac{e^{\frac{1}{10}-\pi-c}}{{\left(e^{\frac{1}{10}-\pi-c}+1\right)}^2} \end{array}\right)$$

```
vpa([pi+c,hill(pi+c,1)])
```

ans =

$$\left(\begin{array}{cc} c+\pi & \frac{e^{0.1-1.0\,\pi-1.0\,c}}{{\left(e^{0.1-1.0\,\pi-1.0\,c}+1.0\right)}^2} \end{array}\right)$$

```
[pi-c,hill(pi-c,1)]
```

ans =

$$\left(\begin{array}{cc} \pi-c & \frac{e^{c-\pi+\frac{1}{10}}}{{\left(e^{c-\pi+\frac{1}{10}}+1\right)}^2} \end{array}\right)$$

```
vpa([pi-c,hill(pi-c,1)])
```

ans =

$$\left(\begin{array}{cc} \pi-1.0\,c & \frac{e^{c-1.0\,\pi+0.1}}{{\left(e^{c-1.0\,\pi+0.1}+1.0\right)}^2} \end{array}\right)$$

Role of the Paramater