Time Series Analysis
Winter 2025
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Homework 1 due in class in week 2

Unless otherwise stated, $\varepsilon_t \sim iid\ N\left(0,\sigma^2\right)$.

1. Consider the following series and explain if they are weakly stationary.

   a. $y_t = \begin{cases} \beta_0 + \varepsilon_t \text{ if t<t' } \varepsilon_t \sim iidN\left(0,\sigma_1^2\right) \\ \beta_0 + \eta_t \text{ if } t \geq t' \text{ where } \eta_t \sim iidN\left(0,\sigma_2^2\right) \end{cases} \quad \text{for } \sigma_1^2 \neq \sigma_2^2$

   b. $y_t = \beta_o t + \varepsilon_t$

   c. $y_t^i = \varepsilon_t^i$ where i denotes the i^th realization of the series, $\varepsilon_t^i \sim iidN\left(0,\sigma_i^2\right)$, and $\sigma_i^2 = \eta_i^2$ where $\eta_i \sim iidN(0,1)$

2. Check out the Federal reserve economic data page (FRED) at: https://fred.stlouisfed.org/ . Perform a search for Case-Shiller (in the search window at near the top of the page). Find the Case Shiller 20-city Composite Home Price Index. There are two options (see "select other formats"), the seasonally adjusted and the non-seasonally adjusted. By clicking on each of these series you are taken to a page where you can download the data. Above the graph is a tab to download the data. Click this link and then select "Excel" for the file type. You will now have an Excel spreadsheet with the index values. We will focus on growth rates for this problem. To compute continuously compounded growth rates, you create a new series $r_t = \ln\left(y_t\right) - \ln\left(y_{t-1}\right)$ where ln is the natural logarithm. Create the growth rate series for both the seasonally adjusted series and the unadjusted series. The seasonally adjusted series removes the cyclical variation that repeats itself year after year (i.e. summer tends to have higher prices than the winter).

   a. Examine the ACF of the raw growth rates out through lag 48. Do you see a patter in the autocorrelations? Explain.

   b. Do you think an AR(1) model would work for the raw data? Explain.

   c. Examine the ACF of the adjusted rates. Do you see the same pattern? Explain.

   d. Fit an AR(1) model to this data.

   e. Examine the residual autocorrelations.

f. Does the AR(1) model fit the data?

g. Using residual diagnostics and/or the PACF, find an AR(p) model that fits the data well.

h. Build the one step ahead forecast for every observation in the sample using your fitted AR(p) model. Plot the forecast and the real data on the same plot.

i. What is one step ahead out of sample forecast (i.e. the forecast of T+1) and find the 95% prediction interval.

3. Consider the first order model: $Y_t = .1 + .99Y_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim iidN\left(0,.5^2\right)$.

   a. For the model, find the unconditional mean μ_Y and the unconditional variance σ_Y^2

   b. Simulate 100 observations from the model. Set the initial value Y_0 to a random draw from $N\left(\mu_Y, \sigma_Y^2\right)$. Create 1000 realizations of this process (each time taking a new draw of Y_0. For each realization of the 100 observations, estimate the AR(1) parameter by least squares. Present the histogram of the 1000 estimated slope coefficients.

   c. Compare the mean of the 1000 slope estimates to the true value (.99). What do you learn about any finite sample bias (there is no closed form solution here, so we have to simulate).

4. Consider the model $y_t = .5 + 1.3y_{t-1} - .45y_{t-2} + \varepsilon_t$ where $\varepsilon_t \sim iid\ N\left(0,.5^2\right)$

   a. Find the mean of y.

   b. Find the second moments of the model, the variance and the autocovariances.

   c. Write the companion form representation for this model.

   d. Find the eigenvalues and the eigenvectors of the companion form matrix F. Is this process covariance stationary?

   e. Find the impulse response function using the values from part d.