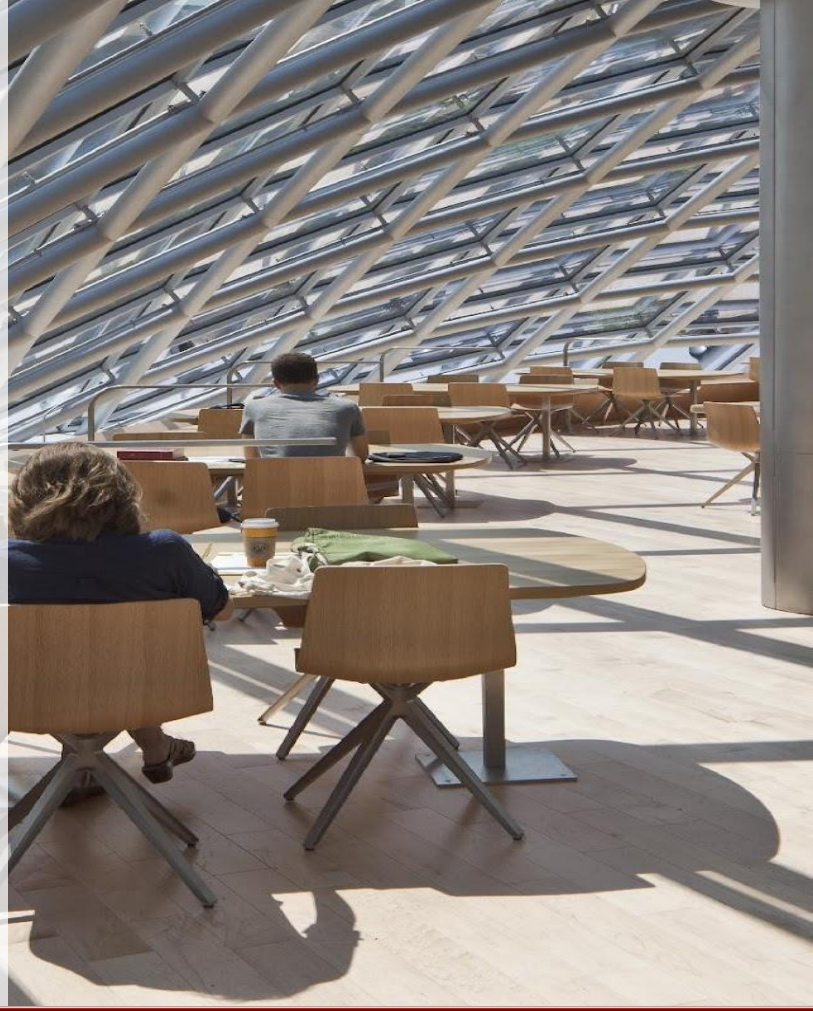
Prime Trading

Project Lab - The University of Chicago
Final Presentation

Futures Basis Model



Presented by

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   - Interest rate
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1. Project Overview

Project Overview

This project aims to analyze and model the relationship between S&P 500 index (SPX), E-mini S&P 500 futures (ES), dividend futures, and related financial costs using high-frequency market data.

The goal is to calculate and visualize the basis — the difference between spot and futures prices, compared with BTIC — while incorporating factors such as interest rates (SOFR), dividends, and financing costs.

Data Usage Overview

1. E-mini S&P 500 (ES) Futures Trade Data

Source: Provided by Prime, ES_1yr_Trade.csv.gz

Purpose: Combined with BTIC and SPX to analyze index-futures relationship

2. Term SOFR (1M, 3M, 6M, 12M)

Source: bloomberg.com, SOFR_partial.csv

Purpose: Used in theoretical price calculations for futures and dividend futures; Helps model financing costs and discounting

3. SPX – 1-Minute Level Data

Source: barchart.com, SPX_1yr_1min.csv

Purpose: Reference data for current index price

4. SPXDIV (S&P 500 Dividend Index), ES_DIV

Source: bloomberg.com, SPXDIV.csv, ES_DIV provided by Prime

Purpose: Reflects expected future dividends of the S&P 500,combined with dividend futures data to calculate expected points and discounts,

5. BTIC Trade Data

Source: Provided by Prime, BTIC_1yr_Trade.csv.gz

Purpose: Compared with our calculated theoretical basis and the market basis

6. Financing spread

Source: SPX_Financing_Spread.parquet

Reflects the cost of financing long SPX positions.

2. Theoretical Value for Basis

Cost Carrying Model: Equations

(1) Definition of Basis

$$\boxed{B_t = F_t^{\text{mkt}} - S_t}$$

(2) Cost-of-Carry Model (Fair-Value Futures Price)

$$\boxed{F_t^{\text{fair}} = S_t\, e^{(r-q)(T-t)}}$$

(3) Definition of Fair-Value Basis

$$\boxed{B_t^{\text{fair}} = F_t^{\text{fair}} - S_t = S_t\left(e^{(r-q)(T-t)} - 1\right)}$$

(4) Net Carry

$$\boxed{\text{Carry} = q - r}$$

Interest Rate Methodology

- We use SOFR as a benchmark funding rate
- Secured Overnight Financing Rate (SOFR) is a risk free, one day collateralized rate
- SOFR is a 1-day interest rate
- Obtained Term SOFR data (1M, 3M, 6M, 12M)
- Convert ACT/360 simple rates → ACT/365 → continuously-compounded rate

$$r_{\text{cc}} = \frac{1}{T} \ln\left(1 + r_{\text{simple}}\, T\right)$$

- Instantaneous annual rates are used interpolate a curve

Dividend Methodology

- **What is the potential candidate method for dividend yield points?**
- Used Dividend index and futures relationship
- Dividend futures is a expectation of dividend in points at the expiration, whereas dividend index only accrues on the ex-dates
- Thus, difference between the two is the projection of the dividends.
- However, this method doesn't account for special dividends. Only regular cash payouts.
- Although, special dividend is not so common it is announced from time to time
- Costco 12/2023 etc. $15 per share

Cost Carrying Model: Equations

(1) Definition of Basis

$$B_t = F_t^{\text{mkt}} - S_t$$

(2) Fair-Value Futures Price (with Point Dividends)

$$F_t^{\text{fair}} = \left(S_t - D_t \cdot \frac{1 - e^{-r(T-t)}}{r(T-t)} \right) \cdot e^{r(T-t)}$$

(3) Definition of Fair-Value Basis

$$B_t^{\text{fair}} = F_t^{\text{fair}} - S_t = S_t \left(e^{r(T-t)} - 1 \right) - D_t \cdot \frac{1 - e^{-r(T-t)}}{r(T-t)} \cdot e^{r(T-t)}$$

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where

- D_t: expected dividend in index points over $[t, T]$
- r: continuously compounded risk-free rate

Financing Cost

Reference: D. E. Shaw Group, *Imbalance Sheet: Supply, Demand, and S&P 500® Financing* (2025).

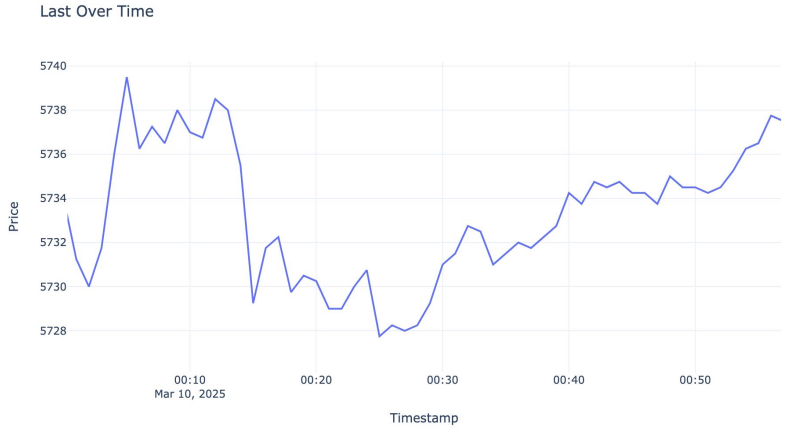
- Our initial model produced values consistently below market basis, which motivated us to consider financing costs.
- Financing cost is the **spread** over the risk-free rate required to borrow again the index.
- We observe a surge in financing cost starting in early 2024, likely due to **high demand** from asset managers for S&P 500 futures and **limited supply** of dealer balance sheets, as banks face capital constraints and regulatory pressures that restrict their ability to lend.
- Implied S&P 500 financing cost was extracted from the following plot using ChatGPT and incorporated into our cost-of-carry model.

Figure 1: Implied S&P 500® Financing Spread (Rolling 20-Day Average)
Jul 2012 – Dec 2024

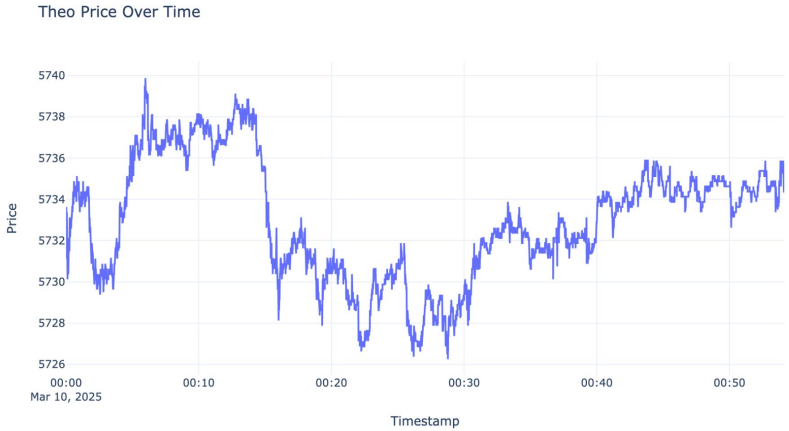
Sources: Bank of America Global Research; the D. E. Shaw group.

Market Implied Pricing

- Attempted plotting market price with quote data vs trade data
- Due to the high liquidity condition of ES, conclude that it is not necessary to include filtering
- Trade data shows much less noise
- We use the market trade data to get price for ES futures and Btic



Trade data plot



Mid price from quote data plot

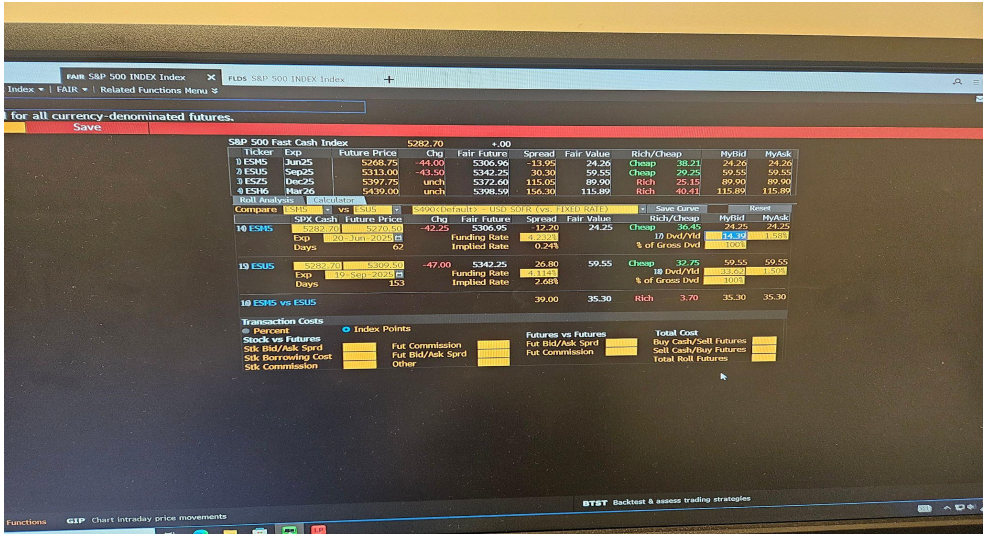
3. Results and Possible Enhancements

Possible Enhancements

- **Interest rate**
- Can try to calculate interest rate more dynamically by replicating how Term SOFR is constructed.
- Currently supports linear interpolation and cubic spline method but can extend to more sophisticated method like N-S
- https://www.cmegroup.com/market-data/files/cme-term-sofr-reference-rates-benchmark-methodology.pdf
- It has its own methodology to calculate Term SOFR rates
- See page 13 - 16

Possible Enhancements

- **Dividend Points / Rate**
- We can use future projected points using Bloomberg FAIR screen
- Note: Currently Bloomberg doesn't support historical data for it, so should stack it my ourselves.



Possible Enhancements

- **Financing cost**

  Reference: *The Implied Futures Financing Rate*, Gunther et al., 2021.

- Problems with current approach:
  - The plot from D.E.Shaw used a 20-day rolling average.
  - Estimates from GPT may be less accurate.

- FIR (Futures-Implied Financing Rate) is calculated by:
  - Comparing prices of two sequential S&P 500 futures contracts (e.g., near-term and next-term).
  - Adjusting for dividends and, if needed, a small convexity correction.

Visualization

- We will build our plots based on:
  - Btic
  - Theoretical future price - index price
  - Market future price - index price

- We will plot the above three time series':
  - Value itself
  - The percentage change
  - Twinx
  - Twinx on specific chosen date
- See html file for the plots (in plotly)

Thank You!