

Problem 1:

$$(a): C = (S + N_A + N_P) A$$

$$\begin{aligned}\therefore \text{VAR}(C) &= \text{VAR}((S + N_A + N_P) A) \\ &= \text{VAR}(SA + N_A A + N_P A)\end{aligned}$$

Assuming that the sources are independent

$$\text{then } = A^2 \text{VAR}(S) + A^2 \text{VAR}(N_A) + A^2 \text{VAR}(N_P)$$

so, S is constant, and $\text{VAR}(N_A) = 2$, $\text{VAR}(N_P) = S$

\Downarrow

$$\begin{aligned}\text{VAR}(C) &= 0 + 2A^2 + S \cdot A^2 \\ &= A^2(2 + S)\end{aligned}$$

$$\therefore \boxed{\text{VAR}(C) = A^2(2 + S)}$$

$$(b): \text{So signal-to-noise (C)}: \frac{E(C)}{\text{standard deviation}(C)}$$

Expected value of C :

$$\begin{aligned}E(C) &= E((S + N_A + N_P) A) \\ &= E(SA + N_A A + N_P A) \\ &= A E(S) + A E(N_A) + A E(N_P)\end{aligned}$$

assuming S is constant, $E(N_A) = 0$, $E(N_P) = 0$

$$E(C) = AS$$

Standard Deviation of C :

$$\text{standard deviation of } (C) : \sqrt{\text{VAR}(C)} = \sqrt{A^2(2+S)}$$

$$\text{Signal to noise } (C) = \frac{AS}{\sqrt{A^2(2+S)}}$$

$$\therefore \boxed{\text{Signal to noise } (C) = \frac{S}{\sqrt{2+S}}}$$

(c):

$$\text{Signal to noise } (C) = \frac{S}{\sqrt{2+S}}$$

if signal to noise $(C) = 50$:

$$50 = \frac{S}{\sqrt{2+S}}$$

$$S = 2502$$

\therefore the minimum value that S can be for the signal to noise to not exceed 50 is being 2502 or less.

Problem 2:

$$f = 4 \text{ cm}$$

$$z' = 6 \text{ cm}$$

$$d = 1 \text{ cm}$$

$$(a): \frac{1}{z'} + \frac{1}{-z} = \frac{1}{f}$$

$$\frac{1}{6\text{cm}} + \frac{1}{-2} = \frac{1}{4\text{cm}}$$

$$z = -12\text{cm}$$

we have to place the image 12cm away from the lens to get the image in focus.

(b): the image blur Equation:

$$b = d \frac{(\bar{z}' - z')}{\bar{z}'}$$

$$\text{since } \frac{2\text{cm}}{100} = \frac{1}{500}\text{cm}$$

$$\therefore \frac{1}{500}\text{cm} = 1\text{cm} \frac{(\bar{z}' - 6\text{cm})}{\bar{z}'}$$

$$\bar{z}' = 6.012\text{cm}$$

$$\frac{1}{6.012\text{cm}} + \frac{1}{-2} = \frac{1}{4\text{cm}}$$

$$\bar{z} = 11.95\text{cm}$$

$$\text{the max distance} = 12\text{cm} - 11.95\text{cm} = 0.05\text{cm}$$

Problem 3:

$$(a): D = (S + N_A + N_P) A + N_Q$$

the expected value of D:

$$\begin{aligned}
 E(D) &= E((S + N_A + N_p)A + N_Q) \\
 &= E(SA + N_A A + N_p A + N_Q) \\
 &= E(SA) + E(N_A A) + E(N_p A) + E(N_Q)
 \end{aligned}$$

$$\begin{aligned}
 E(N_A) &= E(N_p) = E(N_Q) = 0 \\
 \therefore E(D) &= \boxed{H = SA}
 \end{aligned}$$

Variance of D:

$$\begin{aligned}
 \text{VAR}(D) &= \text{VAR}(SA + N_A A + N_p A + N_Q) \\
 &= \text{VAR}(SA) + \text{VAR}(N_A A) + \text{VAR}(N_p A) + \text{VAR}(N_Q)
 \end{aligned}$$

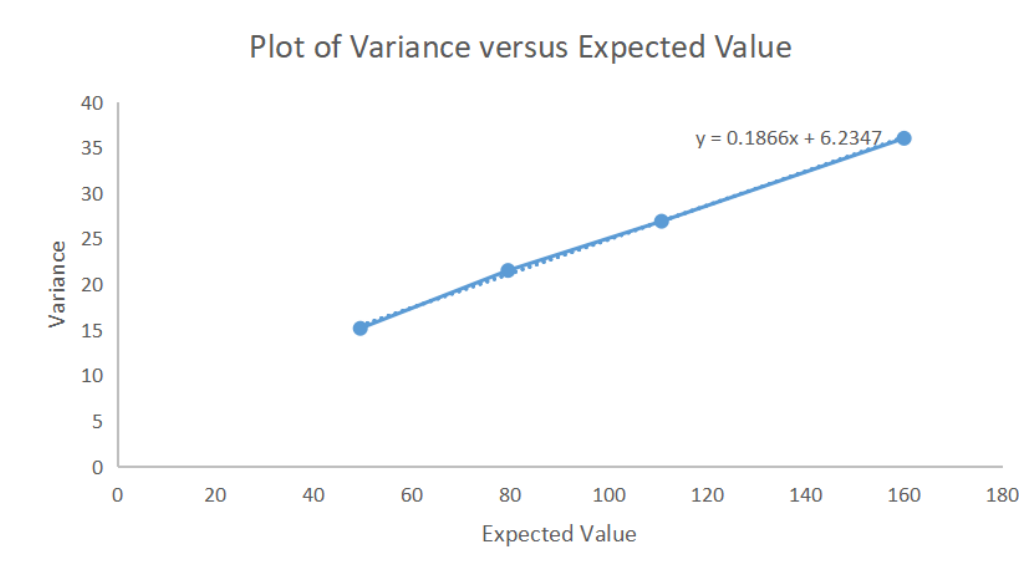
Since $S = \text{constant}$, $\text{VAR}(N_A) = \sigma_A^2$, $\text{VAR}(N_p) = S$,
 $\text{VAR}(N_Q) = \sigma_Q^2$

$$\begin{aligned}
 \therefore \text{VAR}(D) &= 0 + A^2 \sigma_A^2 + A^2 S + \sigma_Q^2 \\
 &= \boxed{\sigma_D^2 = A^2 + \sigma_Q^2}
 \end{aligned}$$

(b) already run my program and show the data to TA.

cC):

image	μ	var of D
image1	49.422600	15.144367
image2	79.483444	21.494219
image3	110.72905	26.888958
image4	160.090271	35.988853



Since $\sigma_D^2 = A\mu + \sigma_c^2$

$$\sigma_D^2 = 0.1866\mu + 6.2347$$

$$\therefore \begin{cases} A = 0.1866 \\ \sigma_c^2 = 6.2347 \end{cases}$$