Problem (1):

(a): solid angle subtented by the moon from earth $= \frac{A}{d^2}$

A is crossectional area of moon and d is distance of moon

$A = \pi R^2$

$\therefore$ $\boxed{\Omega = \text{solid angle} = \frac{\pi R^2}{d^2}}$

(b): Range of solid angle will depend on the area in front of point

then the range of area $= 0 < A < \pi r^2$

$\therefore$ $\frac{0}{d^2} < \frac{A}{d^2} < \frac{\pi R^2}{d}$

$\Downarrow$

the range $= \boxed{0 < \frac{A}{d^2} < \frac{\pi R^2}{d^2}}$

Problem (2):

(a): The foreshortend area of the square patch as viewed from a corner of the room on the floor, we need to consider the right triangle formed by a corner of the room, the center of the ceiling, and the point on the floor directly below the patch.

The length of the hypotenuse of this right triangle is the distance from the corner of the room to the center of ceiling: $\sqrt{(100^2)+(100^2)+(100^2)} = 100\sqrt{3}$ feet

The length of one leg of the right triangle is the distance from the corner of the room to the point on the floor directly below the patch. This is just the length of one side of the cube, which is 100.

$\therefore$ $\boxed{\text{Foreshortened area} = \left(\frac{1^2 \cdot 100}{100\sqrt{3}}\right)^2 = \frac{1}{3} \text{ (square feet)}}$

(b): solid angle $= 2\pi\left(1-\cos\left(\frac{\theta}{2}\right)\right)$

To find θ, we can again consider the right triangle formed by a corner of the room, the center of the ceiling, and the point on the floor directly below the patch.

the angle θ is the angle between the hypotenuse and the leg of the right triangle.

$$\sqrt{(100\sqrt{3})^2 - 100^2} = 100\sqrt{2} \text{ feet}$$

$$\tan\theta = \frac{100\sqrt{2} \text{ feet}}{100 \text{ feet}} = \sqrt{2}$$

$$\theta = \tan^{-1}(\sqrt{2})$$

$$\approx 54.74^\circ$$

$$\text{solid angle} = 2\pi\left(1-\cos\frac{\theta}{2}\right)$$

$$= 2\pi(1-\cos(27.37^\circ))$$

$$\approx \boxed{0.00223 \text{ steradians}}$$

(c): use the cosine function

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{50}{100\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= 30^\circ$$

$$\text{solid angle} = 2\pi\left(1-\cos\frac{\theta}{2}\right)$$

$$= 2\pi(1-\cos(15^\circ))$$

$$\approx \boxed{0.00925 \text{ steradians}}$$

Problem (3):

(a): the surface gradient of the plane is a vector perpendicular to the plane. Since the equation of the plane is given in the form of $Ax + By + Cz + D = 0$ the surface gradient is given by the vector (A, B, C)

$\therefore$ the surface gradient is $(7, \sqrt{50}, 1)$

(b): we want to find the location of the point light source (x, y, z) such that the reflected radiance from p in the direction of $(0, 0, p)$ is as large as possible,

Let R be the reflection of the light source across the plane. The vector from p to R is given by $2((A,B,C)\cdot(x,y,z))/(A^2+B^2+C^2)(A,B,C) - (x,y,z)$

and

$$\left\| 2\frac{(A,B,C)\cdot(x,y,z)}{A^2+B^2+C^2}(A,B,C) - (x,y,z)\right\| = 20$$

$$\|V\|^2 = V \cdot V$$

$$\therefore 4((A,B,C)\cdot(x,y,z))^2 - 4((A,B,C)\cdot(x,y,z))(x,y,z)\cdot(A,B,C) + 400(A^2+B^2+C^2)(\|(x,y,z)\|^2) = 400(A^2+B^2+C^2)$$

$\therefore$ we can re-write R to p as:

$$2\frac{(A,B,C)\cdot(x,y,z)}{A^2+B^2+C^2}(A,B,C) - (x,y,z)$$

and also simplify the dot product as:

$$((A,B,C)\cdot(R-P)) = 2((A,B,C)\cdot(x,y,z)) - ((A,B,C)\cdot(x,y,z))$$

$$= 2((A,B,C)\cdot(x,y,z))$$

substituting $||R-P|| = 20$, $P=(0,0,-2)$, $(A,B,C)=(7,\sqrt{50},1)$

then $||v||^2 = v\cdot v = 4\dfrac{((A,B,C)\cdot(x,y,z))^2}{(A^2+B^2+C^2)} - 4\dfrac{(A,B,C)\cdot(x,y,z)}{A^2+B^2+C^2}$

after solving for x, y, and z

$$\boxed{\text{the location of the point light source as } \left(\frac{28}{15}, \frac{-2\sqrt{50}}{15}, \frac{8}{15}\right)}$$