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DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING



LECTURE NOTES ON INSTRUMENTS AND MEASUREMENTS (EL 172)

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CHAPTER 1

MEASUREMENT SYSTEM AND TERMINOLOGIES

1.1 Measurement System

Generally, a measurement system consists of three elements, namely:

- i) *The sensing element:* Also called the transducer, the sensing element produces a signal that is related to the quantity being measured. It takes information about the quantity being measured and converts it to a form that can be interpreted by the system by way of assigning a value to
- ii) *The signal converter:* This takes the signal from the sensing element and converts it into a suitable condition worthy of display or appropriate for use in a control system. The signal converter comprises three sub-components:
 - A signal conditioner:* Converts the signal from the sensing element into a physical form suitable for display.
 - A signal processor:* Improves the quality of the signal; eg. amplifier for signal magnification.
 - A signal transmitter:* Conveys the signal some distance to the display.
- iii) *The display element:* The element on which the output from the measuring system is displayed. It takes the information from the signal converter and presents it in a format that can be appreciated by visual contact of the observer.

1.2 Intelligent Measuring System

Are systems that employ the services of a microprocessor or a computer otherwise it is called a *dumb* measuring system. With a dumb instrument, the system gives a measure of the quantity and the observer then processes and interprets the displayed data. An intelligent instrument makes the measurement, processes it and interprets the data. Intelligent instruments can make decisions based on measurements made earlier, carry out calculations on data, manipulate information and initiate action based on the results obtained.

Calibration: This is the process of putting marks on a display or checking a measuring system against a standard when a transducer is in a defined environment.

1.3 Performance Terminology

Terms used to describe the performance of measurement systems or elements are numerous and in some cases confined to the environment of the measurement. Some of these terms include:

a. Accuracy: The extent to which a wrong reading might be obtained. *Static accuracy* is used when the quantity being measured does not change or changes very slowly. *Dynamic accuracy* comes in when the measuring quantities change swiftly. Accuracy may be quoted as plus or minus some value of the variable,

e.g. an ammeter of plus or minus 0.1 A (i.e. ± 0.1 A) at some particular current value or for all its readings. It can also be quoted as a percentage of the *full-scale deflection* (fsd) of the instruments. E.g. an ammeter of 2 % fsd. This means that the accuracy of the reading of the ammeter when used for any reading within the range 0-10 A is plus or minus 2% of 10 A (i.e. ± 0.2 A).

b. Bias: The constant error that exists for the full range of its measurements.

c. Discrimination: The smallest change in the quantity being measured that will produce an observable change in the reading of the instrument.

d. Error: The difference between the result of the measurement and the true value of the quantity being measured. $\text{Error} = \text{Measured value} - \text{True value}$

e. Gain: The output divided by the input.

f. Precision: A measure of the scatter of results obtained from measurements as a result of random errors. It describes the closeness of the agreement occurring between the results obtained for a quantity when it is measured several times under the same conditions.

g. Range: The limits between which readings can be made on an instrument.

h. Reliability: The probability that an instrument will operate to an agreed level of performance under the conditions specified for its use.

i. Resolution: The smallest interval measurable by the instrument.

j. Sensitivity (S): $S = \text{change in instrument scale reading} / \text{change in the quantity being measured}$.

k. Stability: The ability of the instrument to display the same reading when it is used to measure a constant quantity over a period of time or when that quantity is measured on a number of occasions.

1.4 Sources of Errors of Measuring Instruments

Error in a measurement is the difference between the results of the measurement and the true value of the quantity being measured. In effect, error is given by:

$$\text{Error} = \text{Measured value} - \text{True value}$$

Numerous errors associated with measuring instruments, can be classified into random or systematic errors. Random errors are those which can vary between successive readings of the same quantity. Systematic errors are errors which do not vary from one reading to another.

1.4.1 Random Errors

Operation Errors

These are errors associated with the operation of the instrument. There are a variety of causes of operating errors. It consists of the following:

Observational Errors

Errors associated with reading the position of a pointer on a scale due to the scale and pointer not being in the same plane. With this error, the reading obtained depends on the angle at which the pointer is viewed against the scale. This is sometimes called parallax error.

Human Errors

These are errors associated with human imperfections. They include wrong choice of appropriate scale of multi-range instrument. Another form is the mistake in selecting an ohm scale instead of an ampere scale. Other forms of operating errors involve the uncertainty in estimating readings between scale markings on an instrument's display.

Environmental Errors

These are errors which could arise as a result of environmental effects, such as a change in temperature, humidity or electromagnetic interference. Instruments must be shielded or placed far away from equipment or machine that carry strong magnetic field such as motors, generators and transformers (especially in the process of metering).

Stochastic Errors

These are errors that result from stochastic processes such as noise. A stochastic process is one which results in random signals.

Noise

Electrical noise may be defined as any undesired voltages or currents that ultimately end up appearing in the receiver output. To the listener, this electrical noise often manifests itself as *static*. It may only be annoying, such as an occasional burst of static, or continuous and of such amplitude that the desired information is obliterated.

Noise signals at their point of origin are generally very small, for example, at the microvolt level. You may be wondering, therefore, why they create so much trouble. Well, a communications receiver is a very sensitive instrument that is giving a very small signal at its input that must be greatly amplified before it can possibly drive a speaker.

The noise present in a received radio signal is termed *external noise*. The noise introduced by the receiver is termed *internal noise*.

External Noise

Man-made-noise : The most troublesome form of external noise is usually the man-made variety. It is often produced by spark-producing mechanisms such as engine ignition systems, fluorescent lights, and

commutators in electric motors. This noise is actually “radiated” or transmitted from its generating sources through the atmosphere in the same fashion that a transmitting antenna radiates desirable electrical signals to a receiving antenna. This process is called *wave propagation*. If the man-made noise exists in the vicinity of the transmitted radio signal and is within its frequency range, these two signals will “add” together. This is obviously an undesirable phenomenon. Man-made noise occurs randomly at frequencies up to around 500 MHz.

Another common source of man-made noise is contained in the power lines that supply the energy for most electronic systems. In this context the ac ripple in the dc power supply output of a receiver can be classified as noise (an unwanted electrical signal) and must be minimized in receivers that are accepting extremely small intelligence signals. Additionally, ac power lines contain surges of voltage caused by the switching on and off of highly inductive loads such as electrical motors. It is certainly ill advised to operate sensitive electrical equipment in close proximity to an elevator! Man-made noise is weakest in sparsely populated areas, which explains the location of extremely sensitive communications equipment, such as satellite tracking stations, in desert-type locations.

Atmospheric noise: Atmospheric noise is caused by naturally occurring disturbances in the earth's atmosphere, with lightning discharges being the most prominent contributors. The frequency content is spread over the entire radio spectrum, but its *intensity* is inversely related to *frequency*. It is therefore most troublesome at the lower frequencies. It manifests itself in the static noise that you hear on standard AM radio receivers. Its amplitude is greatest from a storm near the receiver but the additive effect of distant disturbances is also a factor. This is often apparent when listening to a distant station at night on an AM receiver. It is not a significant factor for frequencies exceeding about 20 MHz.

Space noise: The other form of external noise arrives from outer space and is called space noise. It is pretty evenly divided in origin between the sun and all the other stars. That originating from our star (the sun) is termed *solar noise*. Solar noise is cyclical and reaches very annoying peaks about every 11 years.

All of the other stars also generate this space noise, and their contribution is termed cosmic noise. Since they are much farther away than the sun, their individual effects are small, but they make up for this by their countless numbers and their additive effects. Space noise occurs at frequencies from about 8 MHz up to 1.5 GHz (1.5×10^9 Hz). While it contains energy at less than 8 MHz, these components are absorbed by the earth's ionosphere before they can reach the atmosphere. The ionosphere is a region above the atmosphere where free ions and electrons exist in sufficient quantity to have an appreciable effect on wave travel. It includes the area from about 60 to several hundred miles above the earth.

Internal Noise

As stated previously, internal noise is introduced by the receiver itself. Thus, the noise already present at the receiving antenna (external noise) has another component added to it before it reaches the output. The receiver's major noise contribution occurs in its very first stage of amplification. It is there that the desired signal is at its lowest level, and noise injected at that point will be at its largest value in proportion to the intelligence signal.

Thermal noise: There are two basic types of noise generated by electronic circuits. The first one to consider is due to thermal interaction between the free electrons and vibrating ions in a conductor. It causes the rate of arrival of electrons at either end of a resistor to vary randomly, and thereby varies the resistor's potential difference. Resistors and the resistance within all electronic devices are constantly producing a noise voltage. This form of noise was first thoroughly studied by *J. B. Johnson* in 1928 and is often termed *Johnson noise*. Since it is dependent on temperature, it is also referred to as *thermal noise*. Its frequency content is spread equally throughout the usable spectrum, which leads to a third designator: *white noise* (from optics, where white light contains all frequencies or colors). The terms *Johnson*, *thermal*, and *white noise* may be used interchangeably. Johnson was able to show that the power of this generated noise is given by:

$$P_n = kT \Delta f$$

where k = Boltzmann's constant ($1.38 \times 10^{-23} \text{ J/K}$)

T = resistor temperature in kelvin (K)

Δf = frequency bandwidth of the system being considered

Since this noise power is directly proportional to the bandwidth involved, it is advisable to limit a receiver to the smallest bandwidth possible.

Transistor noise: The major contributor of transistor noise is called shot noise. It is due to the discrete-particle nature of the current carriers in all forms of semiconductors. These current carriers, even under dc conditions, are not moving in an exactly steady continuous flow since the distance they travel varies due to random paths of motion. The name *shot noise* is derived from the fact that when amplified into a speaker, it sounds like a shower of lead shot falling on a metallic surface. Shot noise and thermal noise are additive. The equation for shot noise in a diode is:

$$i_n = \sqrt{2qI_{dc}\Delta f}$$

where i_n = shot noise (*rms* amperes);
 q = electron charge ($1.6 \times 10^{-19} \text{ C}$);
 I_{dc} = dc current (A); and
 Δf = bandwidth (Hz).

Unfortunately, there is no valid formula to calculate its value for a complete transistor where the sources of shot noise are the currents within the emitter-base and collector-base diodes. Hence, the device user must refer to the manufacturer's data sheet for an indication of shot noise characteristics. Shot noise generally increases proportionally with dc bias currents except in MOSFETS, where shot noise seems to be relatively independent of dc current levels.

1.4.2 Systematic Errors

a. Construction Errors

These errors occur in the manufacturing process of an instrument and arise from such causes as tolerances on the dimensions of components and on the values of electrical components used. The component values to some extent affect the calibration of the instrument and hence form part of *calibration errors*. In other

instances, the frequency at which the instrument was calibrated, if not considered, may lead to errors. These errors, termed *frequency errors*, may be rampant.

b. Approximation Errors

These errors arise from assumptions made regarding relationships between quantities. For example; a linear relationship between two quantities may often be assumed and in practice may only be an approximation to the true relationship.

c. Ageing Errors

These errors result from the instruments' deteriorating components owing to the fact that components decay with the ageing of the instrument. Component deterioration may affect or cause their values to change. There can also be a build-up of deposits on surfaces of components that must have decayed or must have been punctured. This brings about contact resistances and insulation.

d. Insertion Errors

These errors result from the insertion of the instrument into a particular location to measure a quantity. The value of the measured quantity can be affected. For example; inserting an ammeter and voltmeter into a circuit to measure current and voltage respectively creates errors as shown in Fig. 1.1a and 1.1b.

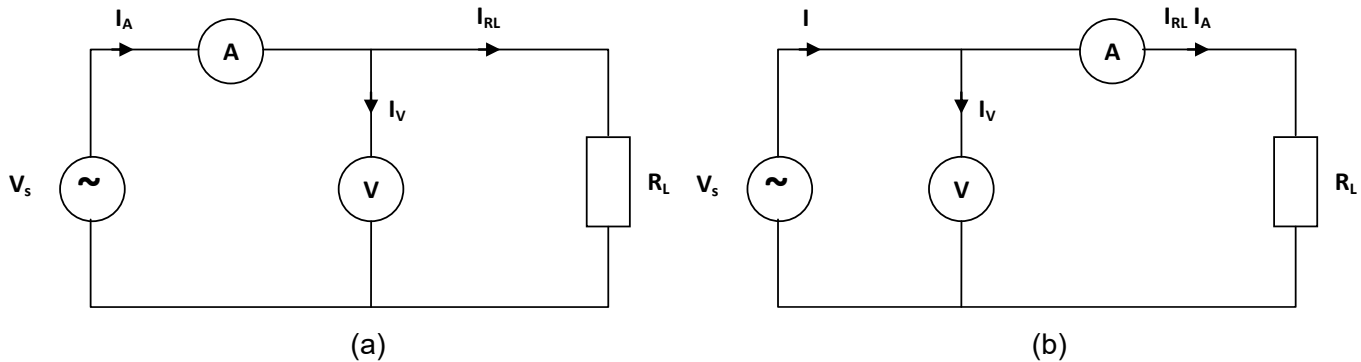


Fig. 1.1 (a) The ammeter A reads current in both voltmeter and load and the voltmeter V reads pd across load, (b) the ammeter A reads exact current in load and the voltmeter V reads pd of both ammeter and load

1.5 Classification of Instruments

Measuring instruments (MIs) could be broadly classified according to the type of signal, whether it is analogue or digital. Analogue instruments are usually based on the motor action, where a force being exerted on a current carrying conductor in a magnetic field produce a relative motion. Digital instruments on the other hand employ analogue to digital converter as the major unit within the measuring instrument. MIs could also be classified into indicating, recording and integrating.

1.5.1 Indicating Instruments

These are the instruments which indicate the instantaneous value of quantity being measured at the time it is being measured. The indication is in the form of pointer deflection (analogue instruments) or digital readout (digital instruments). Ammeters and voltmeters are examples of such instruments.

1.5.2 Recording Instruments

Recording instruments are those instruments which give a continuous record of variations of the electrical quantity being measured over a selected period of time. The moving system of the instrument carries an inked pen which rests tightly on a graph chart e.g. recording voltmeters are used in substations to record the variation of supply voltage. Also recording ammeters are employed in supply stations for registering the amount of current taken from batteries.

1.5.3 Integrating Instruments

These are instruments which measure and register by a set of dials and pointers, either the total quantity of electricity (in ampere – hours) or the total amount of electrical energy (in watt hours or kilowatt hours) supplied to a circuit over a period of time e.g. ampere – hour meters, watt-hour meters, energy meters etc.

1.5.4 Principles of Measurements

The most common analogue instrument or meter is the permanent magnet moving coil (PMMC) instrument and it is used for measuring dc current or voltage of an electric circuit. On the other hand, the indications of alternating current ammeters and voltmeters must represent the RMS values of the current, or voltage, respectively, applied to the instrument. Other types of analogue indicating electromagnetic instruments are:

- i. Moving iron instrument for both DC and AC measurements
- ii. Dynamometer instrument for AC measurement only

Measuring instruments could also be classified according to both the quantity measured by the instrument and the principle of operation. Three general principles of operation available are:

- i. *electromagnetic*, which utilizes the magnetic effects of electric currents;
- ii. *electrostatic*, which utilizes the forces between electrically-charged conductors;
- iii. *electro-thermic*, which utilizes the heating effect.

Electric measuring instruments and meters are used to indicate directly the value of current, voltage, power or energy.

1.6 Basic Parts of Measuring Instruments

Every measuring instrument is composed primarily of three essential parts or devices namely:

- i. Deflecting device;
- ii. Controlling device; and
- iii. Damping device.

1.6.1 Deflecting device

This could be a moving coil or moving iron. An application of electric current, voltage or power into an electromagnet provides a force which deflects a pointer connected to the coil or iron. The value of the deflection of the pointer depends on the magnitude of the current, voltage or power being measured. The deflection of any instrument is determined by the combined effect of the deflecting torque/force, control

torque/force and damping torque/force. The value of the deflecting torque must depend on the electrical signal to be measured; this torque/force causes the instrument to move from its zero position.

1.6.2 Controlling devices

Torque/force exerted by controlling devices must act in the opposite sense to the deflecting torque/force, and the movement will take up an equilibrium or definite position when the deflecting and controlling torques are equal in magnitude. Spiral springs or gravity usually provides the controlling torque. There are two types of these devices.

- i. Gravity control
- ii. Spring control

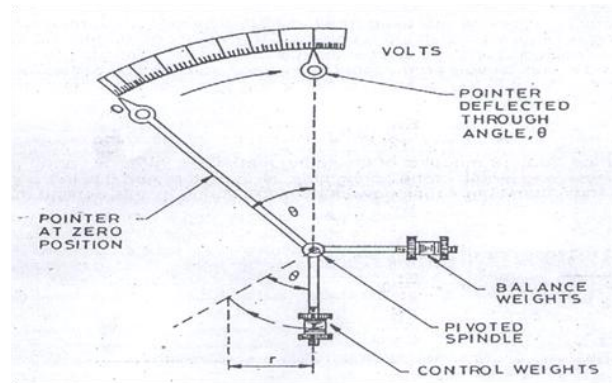
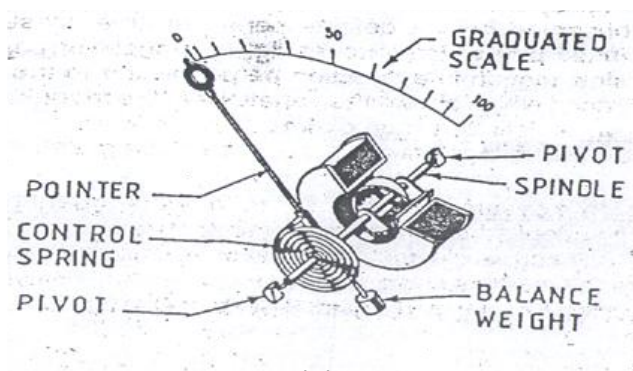


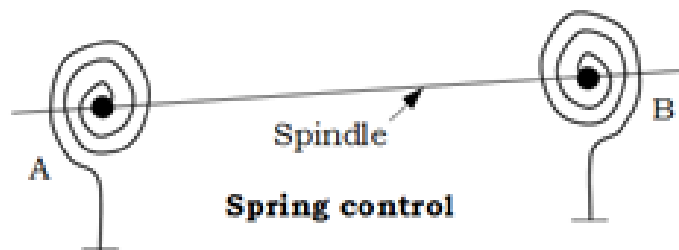
Fig. 1.2 Gravity control type of controlling device

In gravity control (Fig. 1.2), a small weight is attached to the moving system such that the deflecting torque has to act against the action of gravity. Here, two weights are attached to the spindle. A spindle is a rod or pin serving as an axis that revolves or on which something revolves. A counter balance weight of the pointer and another weight used for controlling the movement of the pointer and restore it to its original zero position after the movement of the pointer.

The spring control (Fig. 1.3) is obtained by attaching two counter wound phosphor bronze non-magnetic springs A and B on the spindle. Variations in the pointer for zero setting can be obtained by slacking or tightening on the springs. The outer end of hairspring B is fixed whereas that of A is attached to the end of a lever pivoted at a point thereby enabling zero adjustment to be effected.



(a)



(b)

Fig. 1.3 Spring control type of controlling device

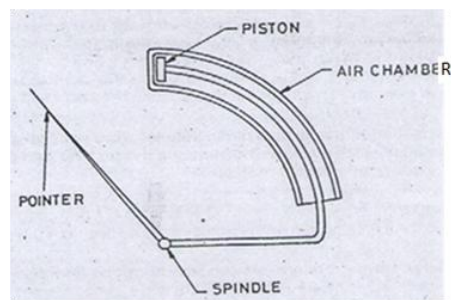
1.6.3 Damping Devices

A damping force is required to act in a direction opposite to the movement of the moving system. A vane is attached to a projecting surface of the spindle, designed to guide the motion of a projectile, e.g. a feather on an arrow. The damping force brings the moving system to rest at the deflected position reasonably quickly without any oscillation or negligible oscillation. This is provided by:

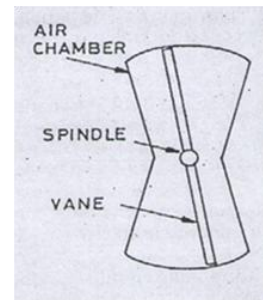
- i) Air friction;
- ii) Fluid friction; and
- iii) Eddy current.

It should be pointed out that any damping force shall not influence the steady state deflection produced by a given deflecting force or torque. Damping force increases with the angular velocity of the moving system, so that its effect is greatest when the rotation is rapid and zero when the system rotation is zero. Details of mathematical expressions for the above torques are considered in the description of the various types of instruments.

1. Air Friction Damping



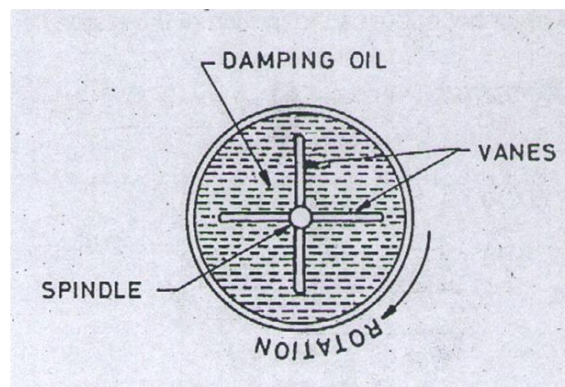
Piston Type



Vane Type

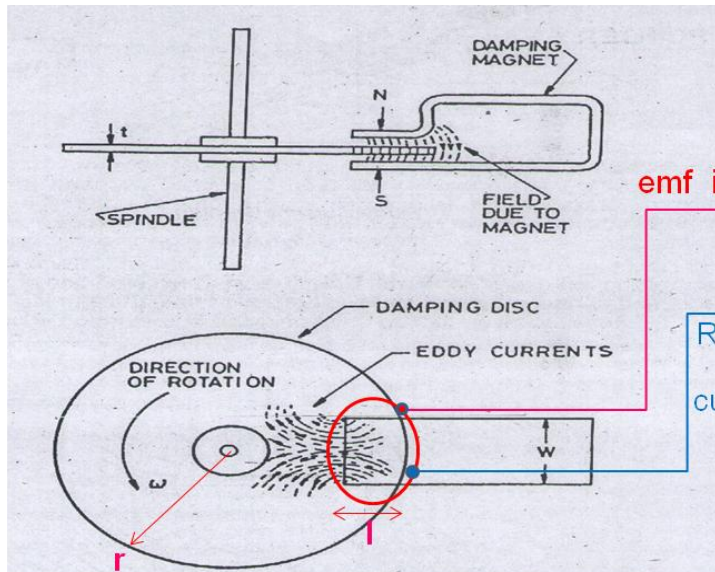
Application: Moving Iron Instrument

2. Fluid Friction Damping



Application: Electrostatic voltmeter

3. Eddy Current Damping : with metal disc



emf induced

$$e = B \times l \times v$$

$$= B \times l \times \omega r$$

Eddy current

$$i_e = \frac{e}{R}$$

Resistance of eddy current path

$$R = \frac{\rho \times l}{w \times t}$$

Damping force

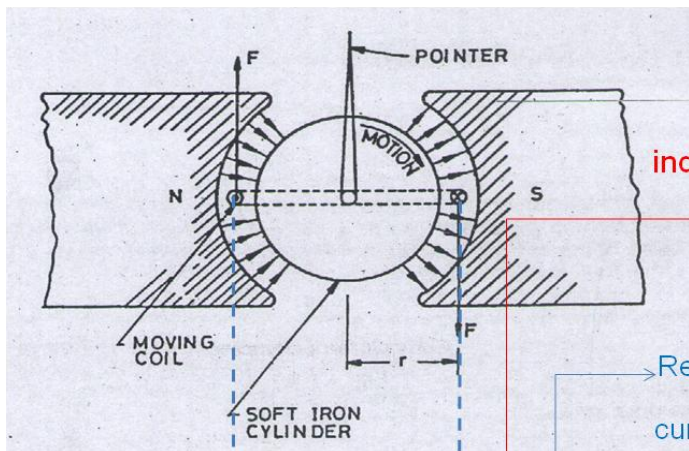
$$F_d = B \times l \times i_e$$

Damping torque

$$T_d = F_d \times r = \left(\frac{B^2 \times r^2 \times A \times t}{k \times \rho} \right) \times \omega = k_d \times \frac{d\theta}{dt}$$

Application: Induction-type wattmeter

4. Eddy Current Damping : with metal former



emf induced

$$2e = 2 \times B \times l \times v$$

$$= 2 \times B \times l \times \frac{\omega d}{2}$$

Eddy current

$$i_e = \frac{e}{R}$$

Resistance of eddy current path

$$R = \frac{\rho \times 2(1+d)}{w \times t}$$

Damping force

$$F_d = B \times l \times i_e$$

Damping torque

$$T_d = F_d \times d$$

$$= k_d \times \omega = k_d \times \frac{d\theta}{dt}$$

Application: PMMC instrument

CHAPTER 2

MOVING COIL/IRON INSTRUMENTS

2.1 Permanent Magnet Moving Coil (PMMC) Instruments

The general theory of moving-coil instruments may be dealt with considering a rectangular coil of turns, free to rotate about a vertical axis.

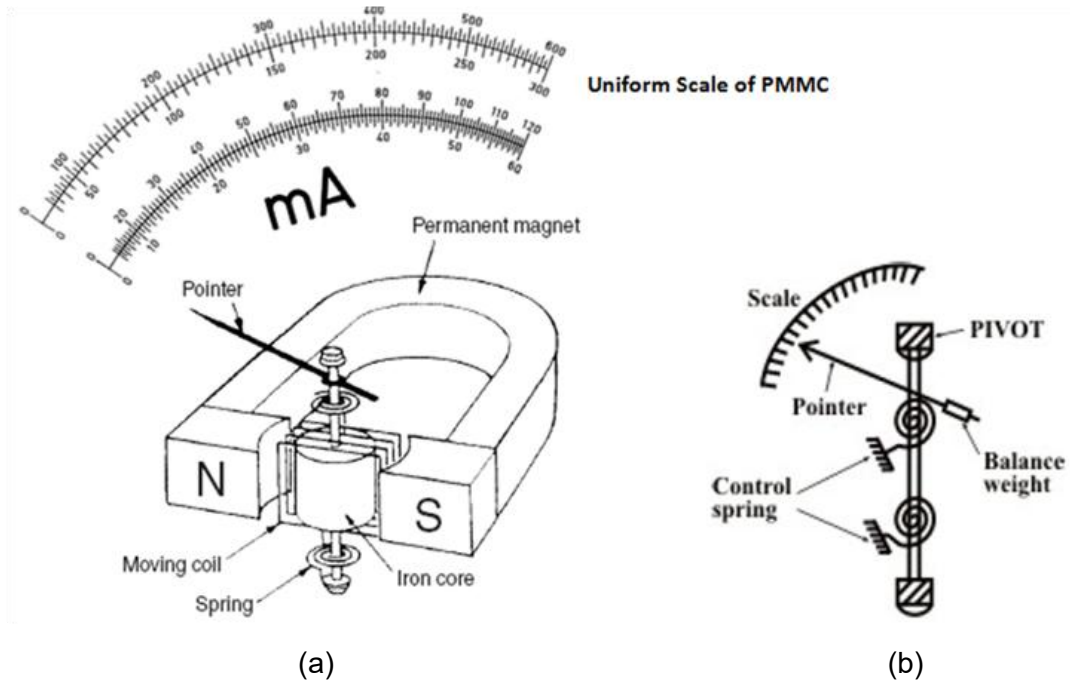


Fig. 2.1 (a) Basic construction of a permanent magnet moving coil (PMMC) instrument and (b) details of all parts attached to the spindle of the instrument

Fig. 2.1a show the basic construction of a permanent magnet moving coil (PMMC) instrument. A moving coil instrument consists basically of a permanent magnet to provide a magnetic field and a small lightweight coil is wound on a rectangular soft iron core that is free to rotate around its vertical axis. When current is passed through the coil windings, torque is developed on the coil by the interaction of the magnetic field and the field set up by the current in the coil. The aluminum pointer attached to the rotating coil and the pointer moves over the calibrated scale indicating the deflection of the coil. Fig. 2.1b shows in detail, all the parts attached to the spindle of the attachment.

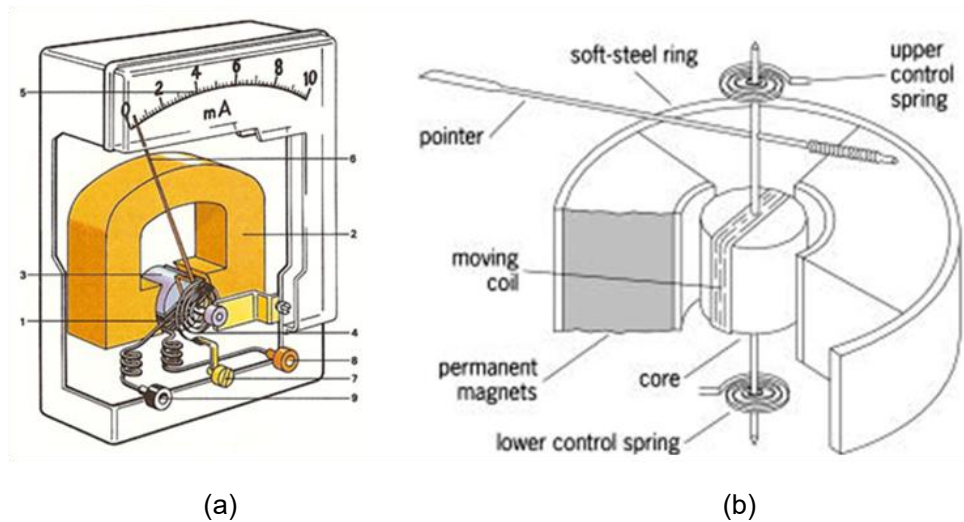


Fig. 2.2 (a) Show the skeleton of the PMMC and (b) detail section of parts of the spindle and pointer of the instrument

To use PMMC device as a meter, two problems must be solved. First, a way must be found to return the coil to its original position when there is no current through the coil. Second, a method is needed to indicate the amount of coil movement. The first problem is solved by the use of hairsprings attached to each end of the coil. These hairsprings are not only supplying a restoring torque but also provide an electric connection to the rotating coil. With the use of hairsprings, the coil will return to its initial position when no current is flowing through the coil. The springs will also resist the movement of coil when there is current passing through the coil. When the developing force between the magnetic fields (from permanent magnet and electro magnet) is exactly equal to the force of the springs, the coil rotation will stop. The coil set up is supported on jeweled bearings in order to achieve free movement. Two other features are considered to increase the accuracy and efficiency of this meter movement. First, an iron core is placed inside the coil to concentrate the magnetic fields. Second, the curved pole faces ensure the turning force on the coil increases as the current increases.

2.1.1 Principle of Operation

It has been mentioned that the interaction between the induced field and the field produced by the permanent magnet causes a deflecting torque, which results in rotation of the coil. It is assumed that the coil sides are situated in a uniform radial magnetic field of flux density B , Wb/m^2 . Let the length of a coil side (within the magnetic field) be l (meter), and the distance from each coil side to the axis be r (meter).

Deflecting Torque

If the coil is carrying a current of i amps, the force on a coil side = $BilN$ (newton, N).

$$\therefore \quad \begin{aligned} \text{Torque due to both coil sides} &= (2r) (BilN) & Nm \\ &= G \times i & (Nm) \end{aligned}$$

where G is the Galvanometer constant and it is expressed as $G = 2rBIN$ (Nm/amp) = NBA (Nm/amp). (Note $A = 2rl$ = area of the coil.)

N = no. of turns of the coil.

B = flux density in Wb/m^2 .
 l = length of the vertical side of the coil, m.
 $2r$ = breadth of the coil, m
 i = current in ampere.
 $A = 2rl$ = area, m^2 .

This equation is valid while the iron core is cylindrical and the air gap between the coil and pole faces of the permanent magnet is uniform.

Controlling torque

The value of control torque depends on the mechanical design of the control device. For spiral springs and strip suspensions, the controlling torque is directly proportional to the angle of deflection of the coil.

That is, Control torque = $C\theta$

where, θ = deflection angle in radians and C = spring constant [Nm/rad]

Damping Torque

This is provided by the induced currents in a metal former or core on which the coil is wound or in the circuit of the coil itself. As the coil moves in the field of the permanent magnet, eddy currents are set up in the metal former or core. The magnetic field produced by the eddy currents opposes the motion of the coil. The pointer will therefore swing more slowly to its proper position and come to rest quickly with very little oscillation. Electromagnetic damping is caused by the induced effects in the moving coil as it rotates in the magnetic field, provided the coil forms part of the closed electric circuit.

Let the velocity of the coil be $\omega(t) = \frac{d\theta}{dt}$ rad/sec and let the resistance of the coil circuit be N turns by $R\Omega$.

Then the velocity of a coil side $v(t) = r \frac{d\theta}{dt}$ (m/sec)

\therefore Emf induced in each turn of the coil = $2Blv = 2Blr \frac{d\theta}{dt}$ (volt) = (note that both sides of the coil have same e.m.f but they are additive in nature).

\therefore Induced current across N turns of coil = $\frac{2BLNr}{R} \frac{d\theta}{dt} = \frac{G}{R} \frac{d\theta}{dt}$ (amps) (R = resistance of coil)

By Lenz's Law, torque produced = $Gi = G \frac{G}{R} \frac{d\theta}{dt} = \frac{G^2}{R} \frac{d\theta}{dt} = D \frac{d\theta}{dt}$ (Nm) = opposing torque. Note; $\frac{G^2}{R}$ is the damping constant for the induced currents in the coil due to its motion. This damping torque is active when the coil posses a change in deflection. A metal former or core may be considered as a single-turn coil if its dimensions are l_1 and r_1 and its resistance R_1 . Similarly, damping torque for the former or core can be computed as

Damping torque (for the core or former) = $D_1 \frac{d\theta}{dt}$ (Nm)

where D_1 = damping constant due to induced currents in the core or former. In addition to the induced current damping, there will be a small damping torque due to air friction. Air damping torque $\left(D_2 \frac{d\theta}{dt}\right)$ may be assumed to be proportional to the angular velocity of the coil.

2.1.2 Equation of motion

The resulting torque in a coil or motion of a coil in a magnetic field is due to the combined effect of the deflecting torque (T_d), controlling torque ($C\theta$) and damping torque ($D \frac{d\theta}{dt}$) which is expressed mathematically as:

$$J \frac{d^2\theta}{dt^2} = Gi - C\theta - D \frac{d\theta}{dt} \Rightarrow J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + C\theta = Gi$$

where J is the moment of inertia of the moving parts. One can easily study the dynamic behavior of the above second order system by solving the differential equation.

Remarks: When the moving system reaches a steady state i.e. at final deflected position, *the controlling torque becomes equal and opposite to the deflecting torque*. The deflecting angle is directly proportional to the current in the movable coil. For this reason, the scale of the moving coil instrument is calibrated linearly.

2.2 Multi-Range Ammeters and Voltmeters

An ammeter is required to measure the current in a circuit and it is therefore connected in series with the components carrying the current. If the ammeter resistance is not very much smaller than the load resistance, the load current can be substantially altered by the inclusion of the ammeter in the circuit. To operate a moving coil instrument around a current level of say 5 A is impractical, owing to the bulk and weight of the coil that would be required. So, it is necessary to extend the meter-range shunts (in case of ammeters) and multipliers (in case of volt meters).

For higher range ammeters, a low resistance made up of *manganin* (low temperature coefficient of resistance) is connected in parallel to the moving coil and the instrument is calibrated to read directly the total current. A typical shunt connected moving coil instrument is shown below in Fig. 2.3.

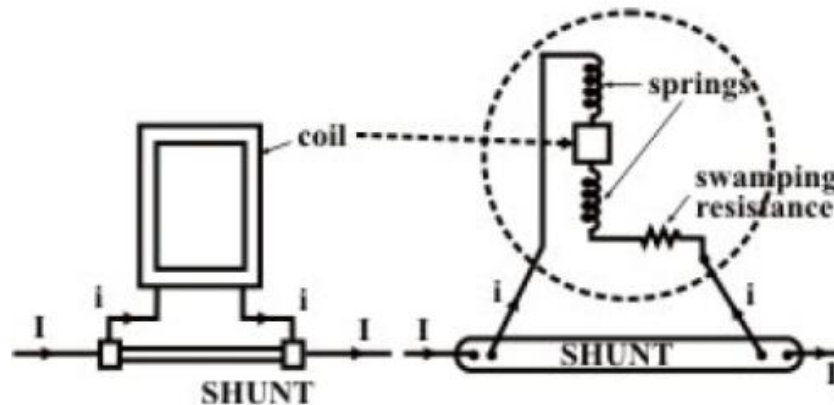
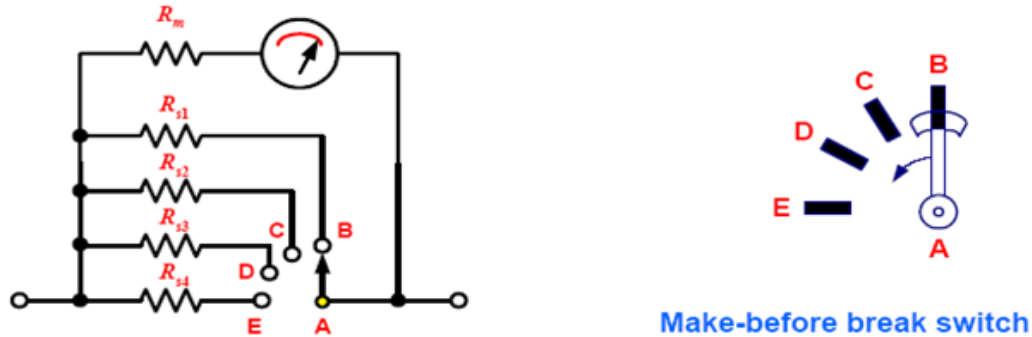


Fig. 2.3 A typical shunt connected moving coil instrument

The movement of PMMC instrument may be temperature-compensated by the appropriate use of series and shunt resistors of copper and manganin. Use of manganin resistance (known as swamping resistance which has a temperature coefficient practically zero) in series with the coil resistance can reduce the error due to the variation of resistance of the moving coil.

2.2.1 Multi-Range Ammeter (MRA)

A multirange ammeter can be constructed simply by employing several values of shunt resistances, with a rotary switch to select the desired range as shown below in Fig. 2.4.



Multirange ammeter using switch shunts

Fig. 2.4 A multirange ammeter with a rotary switch

NB: A make-before-break switch must be used so that the instrument is not left without a shunt in parallel to prevent a large current flow through the ammeter.

When an instrument is used in this fashion, care must be taken to ensure that the shunt does not become open-circuited, even for a very short instant. When the switch is moved from position 'B' to 'C' or moved to any other positions, the shunt resistance will remain open-circuited for a fraction of time, resulting in a very large current flowing through the ammeter and resulting in potential damage to the instrument. To avoid such situation, one may use the make-before-break switch. Another form of MRA arrangement is shown below in Fig. 2.5.

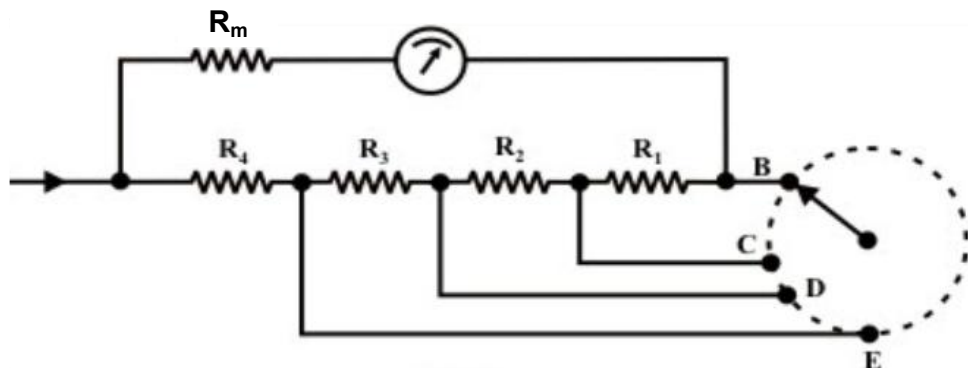
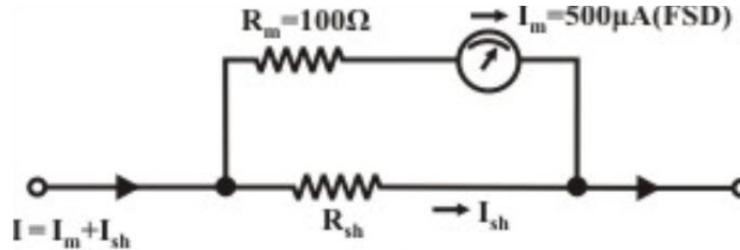


Fig. 2.5 Another multirange ammeter with a different arrangement

The wide-ended make-before-break moving switch gets into contact with the next terminal before it loses contact with the previous terminals. Thus, during the switching time there are two resistances parallel with the instrument before the required shunt finally comes into contact.

Example: A PMMC instrument has a coil of resistance $100\ \Omega$ and gives a full-scale deflection (FSD) for a current of $500\ \mu\text{A}$. Determine the value of the shunt resistance required if the instrument is to be employed as an ammeter with a FSD of $5\ \text{A}$.

Solution: Let current flowing through the shunt be I_{sh} . From the circuit shown below, the following expression using Kirchhoff's laws could be derived.



$$I_m = \frac{R_{sh}}{R_{sh} + R_m} I$$

where R_{sh} = shunt resistance, R_m = ammeter resistance = $100\ \Omega$, I_m = ammeter full scale deflection current = $500\ \mu\text{A}$ and I the desired range of ammeter = $5\ \text{A}$. From the above expression, we get;

$$500 \times 10^{-6} = \frac{R_{sh}}{R_{sh} + 100} \times 5 \Rightarrow R_{sh} = 0.01\ \Omega$$

2.2.2 Multi-Range Voltmeter (MRV)

A dc voltmeter is constructed by connecting a resistor in series with a PMMC instrument in Fig. 2.6. Unlike an ammeter, a voltmeter should have a very high resistance R_{se} and it is normally connected in parallel with the circuit where the voltage is to be measured. To minimize voltmeter loading, the voltmeter operating current should be very small i.e., the resistance connected in series with the coil should be high.

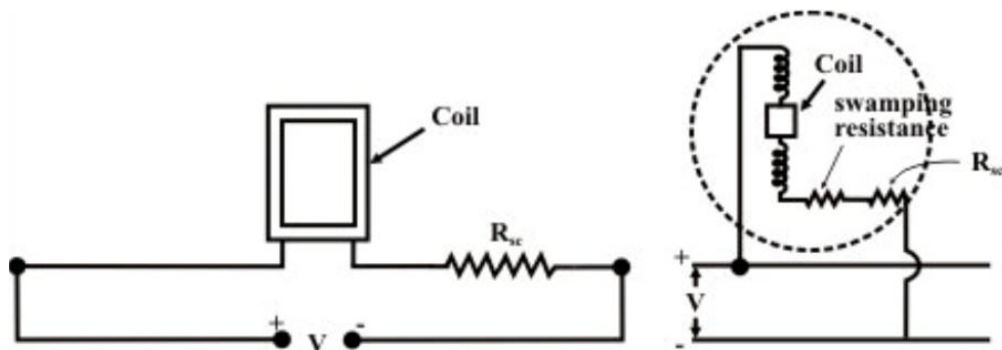


Fig. 2.6 A dc voltmeter constructed by connecting a resistor in series with a PMMC instrument

The moving coil instrument can be suitably modified to act either as an ammeter or as a voltmeter. For multi-range voltmeters, the following arrangement applies: In the figure below, any one of several multiplier resistors is selected by means of a rotary switch. Unlike the case of the ammeter, the rotary switch used with the voltmeter should be a break-before make type, that is, the moving contact should disconnect from one terminal before connecting to the next terminal.

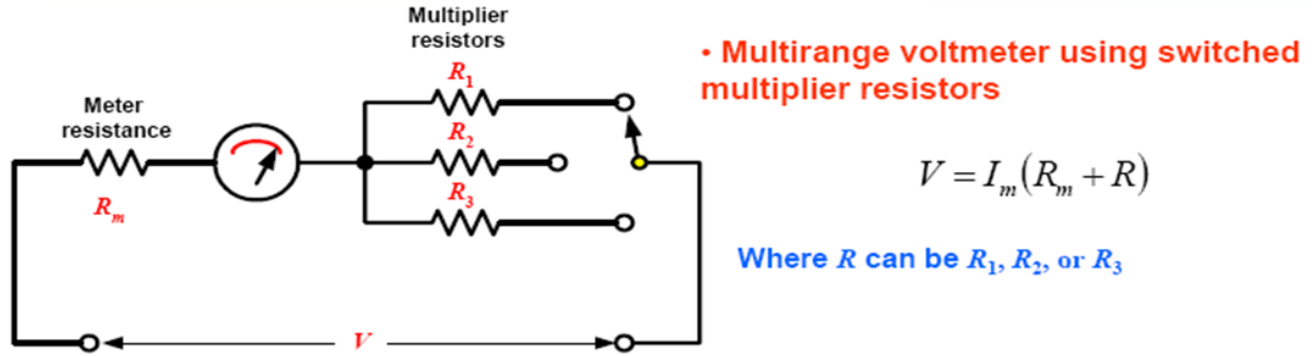


Fig. 2.7 A multirange voltmeter constructed using a switched multiplier resistors

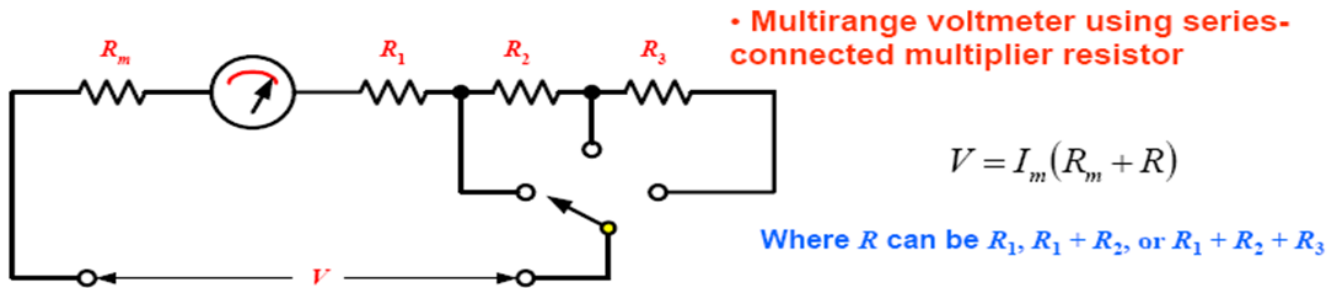
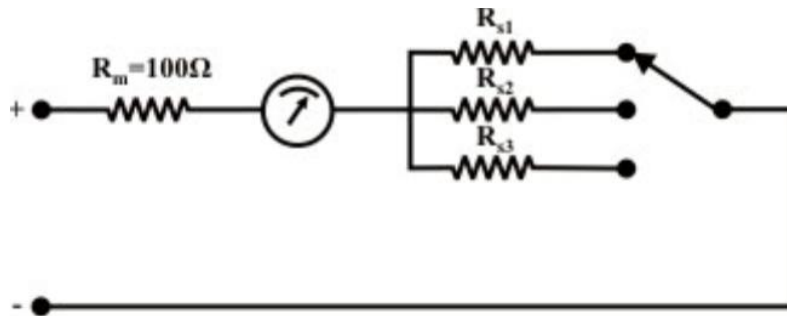


Fig. 2.8 A multirange voltmeter constructed using a series-connected multiplier resistors

Example: A PMMC meter with a coil resistance $100\ \Omega$ and a full scale deflection current of $100\ \mu\text{A}$ is to be used in the voltmeter circuit as shown below. The voltmeter ranges are to be $50\ \text{V}$, $100\ \text{V}$ and $150\ \text{V}$. Determine the required value of resistances for each range.



Solution: From the circuit shown above, one can write the expression for the $50\ \text{V}$ range as

$$I = \frac{V}{R_{in} + R_s} \Rightarrow R_s = \frac{V}{I} - R_{in}$$

- Now for
- (i) $V = 50\ \text{V}, I = 100\ \mu\text{A}$ and $R_{in} = 100\ \Omega$, the value of series resistance $R_{s1} = 0.4999\ \text{M}\Omega$
 - (ii) $V = 100\ \text{V}, I = 100\ \mu\text{A}$ and $R_{in} = 100\ \Omega$, the value of series resistance $R_{s2} = 0.9999\ \text{M}\Omega$
 - (iii) $V = 150\ \text{V}, I = 100\ \mu\text{A}$ and $R_{in} = 100\ \Omega$, the value of series resistance $R_{s3} = 1.4999\ \text{M}\Omega$

2.2.3 Advantages, Limitations and Sources of Errors of MRV

Advantages

- i) The scale is uniformly divided (at steady state, $\theta = \frac{G}{C} I_s$).
- ii) The power consumption can be made very low.
- iii) The torque-weight ratio can be made high with a view to achieve high accuracy.
- iv) A single instrument can be used for multi range ammeters and voltmeters.
- v) Error due to stray magnetic field is very small.

Limitations

- i) They are suitable for direct current only.
- ii) The instrument cost is high.
- iii) Variation of magnetic strength with time.

The Errors include:

- i) Frictional error;
- ii) Magnetic decay;
- iii) Thermo electric error; and
- iv) Temperature error.

Errors can be reduced by following the steps given below:

- i) Proper pivoting and balancing weight may reduce the frictional error;
- ii) Suitable aging can reduce the magnetic decay;
- iii) Use of manganin resistance in series (swamping resistance) can nullify the effect of variation of resistance of the instrument circuit due to temperature variation; and
- iv) The stiffness of spring, permeability of magnetic core (Magnetic core is the core of electromagnet or inductor which is typically made by winding a coil of wire around a ferromagnetic material) decreases with increases in temperature.

2.2.4 Ammeter Sensitivity

Ammeter sensitivity is determined by the amount of current required by the meter coil to produce full-scale deflection of the pointer. The smaller the amount of current required producing this deflection, the greater the sensitivity of the meter. A meter movement that requires only 100 microamperes for full-scale deflection has a greater sensitivity than a meter movement that requires 1 mA for the same deflection.

2.2.5 Voltmeter Sensitivity

The sensitivity of a voltmeter is given in ohms per volt. It is determined by dividing the sum of the resistance of the meter (R_m), plus the series resistance (R_s), by the full-scale reading in volts. In equation form, sensitivity is expressed as follows:

$$\text{Sensitivity} = \frac{R_m + R_s}{V}$$

This is the same as saying the sensitivity is equal to the reciprocal of the full-scale deflection current. In equation form, this is expressed as follows:

$$\text{Sensitivity} = \frac{\text{ohms}}{\text{volt}} = \frac{1}{\text{volt/ohms}} = \frac{1}{\text{ampere}}$$

Therefore, the sensitivity of a 100-microampere movement is the reciprocal of 0.0001 ampere, or 10,000 ohms per volt.

2.3 Moving-Iron Instruments

In moving –iron instruments, the movable system consists of one or more pieces of specially-shaped soft iron, which are so pivoted as to be acted upon by the magnetic field produced by the current in the coil. There are two general types of moving-iron instruments namely:

- i) Repulsion (or double iron) type and
- ii) Attraction (or single-iron) type.

The different components of a moving-iron instrument could be briefly described as follows:

- i) *Moving element*: a small piece of soft iron in the form of a vane or rod;
- ii) *Coil*: conductor wrapped round an iron core to produce the magnetic field due to current flowing through it and also to magnetize the iron pieces;
- iii) In *repulsion type*, a *fixed* vane or rod is also used and magnetized with the same polarity;
- iv) *Control torque* is provided by spring or weight (gravity);
- v) *Damping torque* is normally pneumatic; the damping device consisting of an air chamber and a moving vane attached to the instrument spindle; and
- vi) *Deflecting torque* produces a movement on an aluminium pointer over a graduated scale.

2.3.1 Construction of Moving-iron Instruments

Repulsive type

The deflecting torque in any moving-iron instrument is due to forces on a small piece of magnetically ‘soft’ iron that is magnetized by a coil carrying the operating current. The repulsion type moving–iron instrument (Fig. 2.9) consists of two cylindrical soft iron vanes mounted within a fixed current-carrying coil. One iron vane is held fixed to the coil frame and other is free to rotate, carrying with it the pointer shaft. Two irons lie in the magnetic field produced by the coil that consists of only few turns if the instrument is an ammeter or of many turns if the instrument is a voltmeter. Current in the coil induces both vanes to become magnetized and repulsion between the similarly magnetized vanes produces a proportional rotation.

The deflecting torque is proportional to the square of the current in the coil, making the instrument possible for reading true ‘RMS’ quantity. Rotation is opposed by a hairspring that produces the restoring torque. Only the fixed coil carries load current, and it is constructed so as to withstand high transient current.

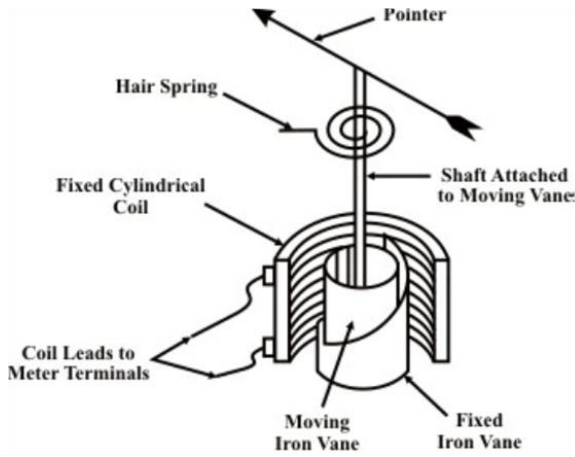


Fig. 2.9 Repulsion Type of a Moving-iron Instrument

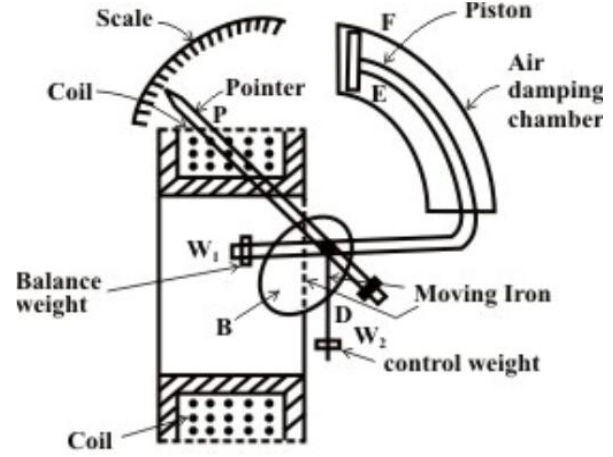


Fig. 2.10 Attractive Type of a Moving-iron Instrument

Attractive type

The attractive type of this instrument (Fig. 2.10) consists of a few soft iron discs (B) that are fixed to the spindle (D), pivoted in jeweled bearings. The spindle (D) also carries a pointer (P), a balance weight (W_1), a controlling weight (W_2) and a damping piston (E), which moves in a curved fixed cylinder (F). The special shape of the moving-iron discs is for obtaining a scale of suitable form.

The instruments may be effectively shielded from the influence of external magnetic fields by enclosing the working parts, except the pointer, in a laminated iron cylinder with laminated iron end covers.

Moving iron instruments have scales that are nonlinear (Fig. 2.11) and somewhat crowded in the lower range of calibration as shown below.

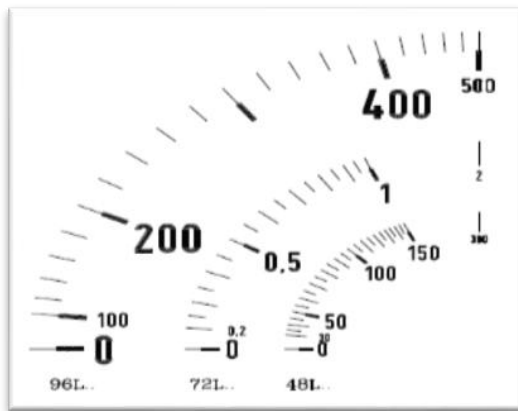


Fig. 2.11 Typical Non-linear Scale of Moving Iron Instrument

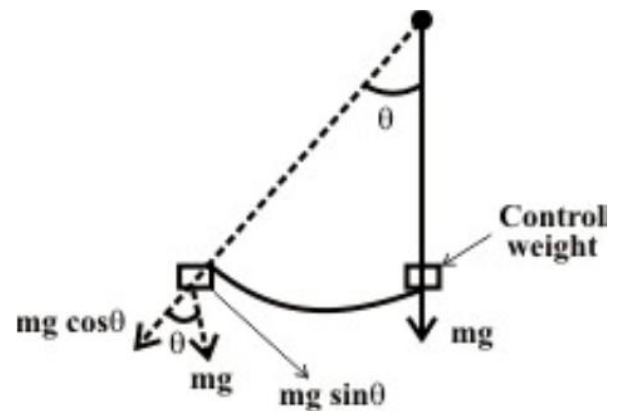


Fig. 2.12 Resolution of forces in gravity control case

2.3.2 Torque Expressions

Torque expression may be obtained in terms of the inductance of the instrument. Suppose the initial current is I , the instrument inductance L and the deflection θ . Then let I change to $I+dI$, being a small change of current; as a result let θ change to $(\theta+d\theta)$, and L to $(L+dL)$. In order to get an incremental change in current dI there must be an increase in the applied voltage across the coil.

Applied voltage
$$v = \frac{d(LI)}{dt} = I \frac{dL}{dt} + L \frac{dI}{dt}$$

The electric energy supplied to the coil in dt is

$$vI dt = I^2 dL + IL dI$$

Increase in energy stored in the magnetic field:

$$= \frac{1}{2} (I + dI)^2 (L + dL) - \frac{1}{2} I^2 L \cong IL dI + \frac{1}{2} I^2 dL$$

(neglecting second and higher terms in small quantities).

If T is the value of the control torque corresponding to deflection θ , the extra energy stored in the control due to the change $d\theta$ is $Td\theta$. Then, the stored energy:

$$\cong IL dI + \frac{1}{2} I^2 dL + T d\theta$$

From the principle of the conservation of energy, it could be written that:

Electric energy drawn from the supply = increase in stored energy + mechanical work done

$$\cong I^2 dL + IL dI = IL dI + \frac{1}{2} I^2 dL + T d\theta$$

$$T(\text{torque}) = \frac{1}{2} I^2 \frac{dL}{d\theta} \quad (\text{Nm})$$

2.3.3 Controlling torque

- i Spring control: $T_s = K_s \theta$, where K_s is the spring constant
- ii Gravity control: $T_g = K_g \sin \theta$, where $K_g = mgl$

At equilibrium i.e. for steady deflection:

$$\text{Deflecting torque} = \text{Controlling torque}$$

If the instrument is gravity controlled, then

$$T_D = T_C$$

$$KI = K_g \sin \theta \Rightarrow \theta = \sin^{-1} \left(\frac{K}{K_g} I \right)$$

2.3.4 Ranges of Ammeters and Voltmeters

For a given moving-iron instrument, the ampere-turns necessary to produce full-scale deflection is constant. One can alter the range of ammeters by providing a shunt coil with the moving coil.

2.3.5 Shunts and Multipliers for MI instruments

For moving-iron ammeters: For the circuit shown below (Fig. 2.13), let R_m and L_m respectively be the resistance and inductance of the coil and R_{sh} and L_{sh} the corresponding values for shunt.

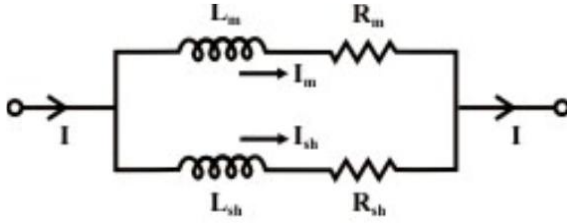


Fig. 2.13 Shunts and Multipliers for MI instruments

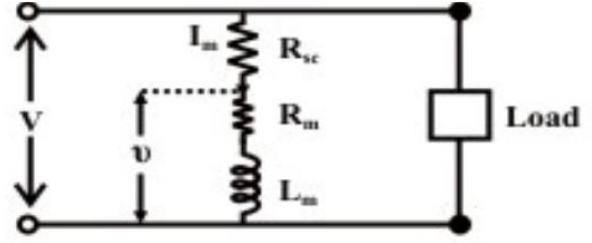


Fig. 2.14 Altering a Voltmeter range by connecting a resistance in series with the coil in moving-iron voltmeters

The ratio of currents in two parallel branches is:

$$\frac{I_{sh}}{I_m} = \frac{\sqrt{R_m^2 + (\omega L_m)^2}}{\sqrt{R_{sh}^2 + (\omega L_{sh})^2}} = \frac{R_m \sqrt{1 + \left(\frac{\omega L_m}{R_m}\right)^2}}{R_{sh} \sqrt{1 + \left(\frac{\omega L_{sh}}{R_{sh}}\right)^2}}$$

The above ratio will be independent of frequency ω provided that the time constants of the two parallel branches are the same i.e.

$$\frac{L_m}{R_m} = \frac{L_{sh}}{R_{sh}}$$

In other words,

$$\frac{I_{sh}}{I_m} = \frac{R_m}{R_{sh}} \quad \text{if} \quad \frac{L_{sh}}{R_{sh}} = \frac{L_m}{R_m}$$

Now,

$$I = I_{sh} + I_m = I_m \frac{R_m}{R_{sh}} + I_m = I_m \left(1 + \frac{R_m}{R_{sh}}\right)$$

Multipliers for the shunt = $\left(1 + \frac{R_m}{R_{sh}}\right)$. It is difficult to design a shunt with the appropriate inductance, and shunts are rarely incorporated in moving iron ammeters. Thus, the multiple ranges can effectively be obtained by winding the instrument coil in sections which may be connected in series, parallel or series-parallel combination which in turn changes the total ampere-turns in the magnetizing coil.

For moving-iron voltmeters: Voltmeter range may be altered connecting a resistance in series with the coil. Hence the same coil winding specification may be employed for a number of ranges. Let us consider a high resistance R_{sc} (almost purely non inductive resistance) connected in series with the moving coil.

$$v = I_m \sqrt{R_m^2 + (\omega L_m)^2}$$

$$V = I_m \sqrt{(R_{se} + R_m)^2 + (\omega L_m)^2}$$

$$\text{Multiplier} = m = \frac{V}{v} = \frac{\sqrt{(R_{se} + R_m)^2 + (\omega L_m)^2}}{\sqrt{R_m^2 + (\omega L_m)^2}}$$

Note: An ordinary arrangement with a non-inductive resistance in series with the fixed coil – results in error that increases as the frequency increases. The change of impedance of the instrument with change of frequency introduces error in signal measurements. In order to compensate the frequency error, the multiplier may be easily shunted by the capacitor.



Fig. 2.15 Connection for method of compensating frequency error in moving-iron voltmeters

2.3.6 Advantages of Moving Iron Instruments

- i) The instruments are suitable for use in ac and dc circuits.
- ii) The instruments are robust, owing to the simple construction of the moving parts.
- iii) The stationary parts of the instruments are also simple.
- iv) Instrument is low cost compared to moving coil instrument.
- v) Torque/weight ration is high, thus less frictional error.

2.3.7 Errors

Errors associated with moving iron instruments are as follows:

- i) Errors due to temperature variation;
- ii) Errors due to friction are quite small as torque-weight ratio is high in moving-iron instruments;
- iii) Stray fields cause relatively low values of magnetizing force produced by the coil. Efficient magnetic screening is essential to reduce this effect;
- iv) Error due to variation of frequency causes change of reactance of the coil and also changes the eddy currents induced in neighboring metal; and
- v) Deflecting torque is not exactly proportional to the square of the current due to non-linear characteristics of iron material.

CHAPTER 3

BRIDGE MEASUREMENTS

3.1 Introduction

A bridge circuit employs the null method and operates on the principle of comparison; i.e. a known (standard) value is adjusted until it is equal to the unknown value.

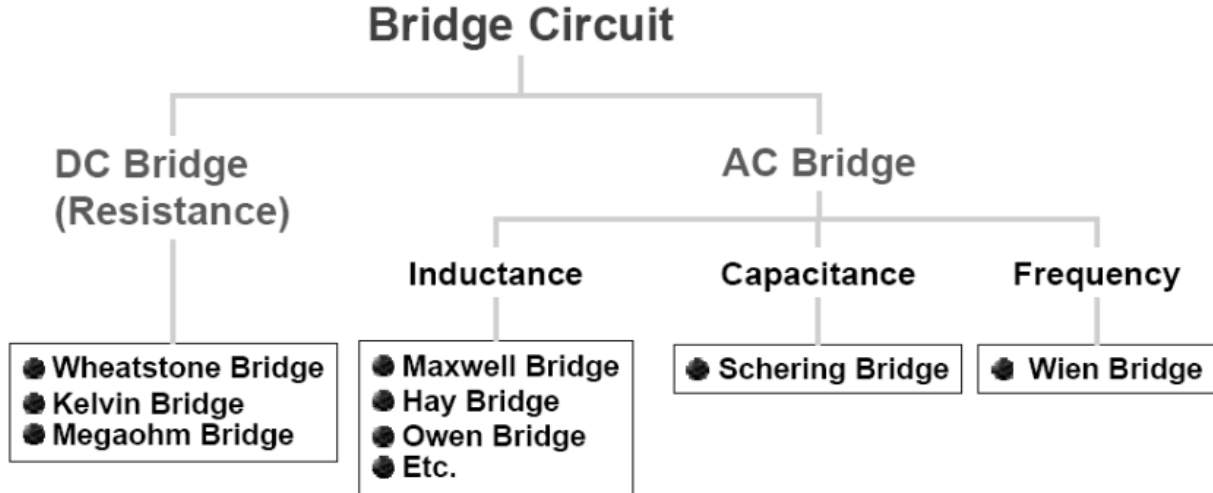


Fig. 3.1 Classification of Bridge Circuits

3.2 Wheatstone Bridge

The Wheatstone bridge is suitable for moderate resistance values: 1 Ω to 10 M Ω .

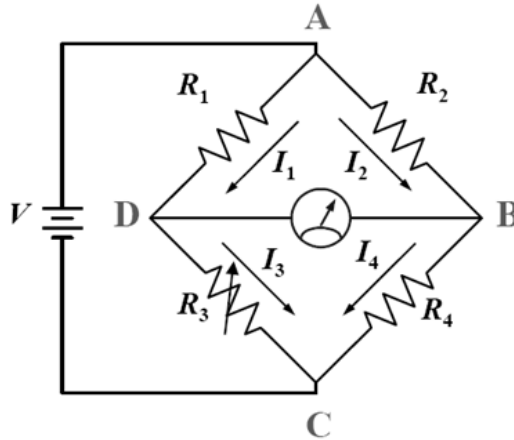


Fig. 3.2 The Wheatstone bridge Circuit

Under *Balance condition* there is **no potential difference** across the galvanometer (there is no current through the galvanometer).

Under this condition:

$$V_{AD} = V_{AB} \quad \rightarrow \quad I_1 R_1 = I_2 R_2$$

And also

$$V_{DC} = V_{BC} \quad \rightarrow \quad I_3 R_3 = I_4 R_4$$

where I_1 , I_2 , I_3 , and I_4 are currents in resistance arms respectively.

Since $I_1 = I_3$ and $I_2 = I_4$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \quad \text{or} \quad R_x = R_4 = R_3 \frac{R_2}{R_1}$$

3.2.1 Measurement Errors

Errors in Wheatstone bridge measurements include:

1. Limiting error of the known resistors.

Using 1st order approximation (Fig. 3.3):

$$R_x = R_4 = (R_3 \pm \Delta R_3) \left(\frac{R_2 \pm \Delta R_2}{R_1 \pm \Delta R_1} \right)$$

$$R_x = R_4 = R_3 \frac{R_2}{R_1} \left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3} \right)$$

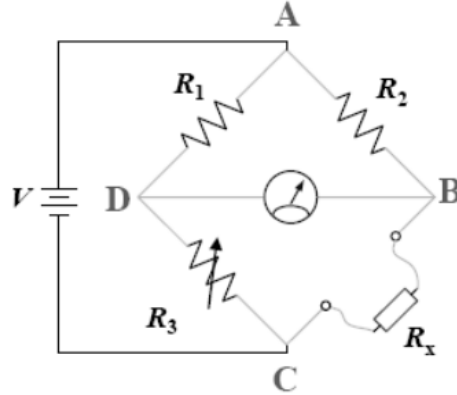


Fig. 3.3 The Wheatstone bridge Circuit using first order approximation

2. Insufficient sensitivity of Detector.
 3. Changes in resistance of the bridge arms due to the heating effect (I^2R) or temperatures.
 4. Thermal emf or contact potential in the bridge circuit.
 5. Error due to the lead connection.
- 3, 4 and 5 play important roles in the measurement of low value resistances.

Example In the Wheatstone bridge circuit, R_3 is a decade resistance with a specified in accuracy of $\pm 0.2\%$ and R_1 and $R_2 = 500 \Omega \pm 0.1\%$. If the value of R_3 at the null position is 520.4Ω , determine the possible minimum and maximum value of R_x .

Solution: Apply the error equation:

$$R_x = R_3 \frac{R_2}{R_1} \left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3} \right)$$

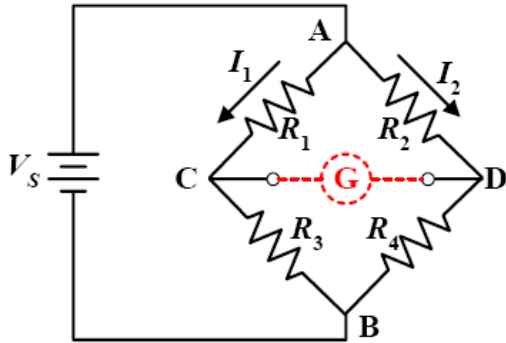
$$R_x = \frac{520.4 \times 500}{500} \left(1 \pm \frac{0.1}{100} \pm \frac{0.1}{100} \pm \frac{0.2}{100} \right) = 520.4(1 \pm 0.004) = 520.4 \pm 0.4 \%$$

Therefore, the possible values of R_x are 518.32 to 522.48Ω .

3.2.2 Sensitivity of Galvanometer

A galvanometer is used to detect an unbalance condition in the Wheatstone bridge circuit. Its sensitivity is governed by: current sensitivity (currents per unit deflection) and internal resistance. Consider a bridge circuit under a small unbalance condition, and apply circuit analysis to solve the current through the galvanometer. NB: Revise applied electricity on the theorem below.

Thévenin Equivalent Circuit



Thévenin Voltage (V_{TH})

$$V_{CD} = V_{AC} - V_{AD} = I_1 R_1 - I_2 R_2$$

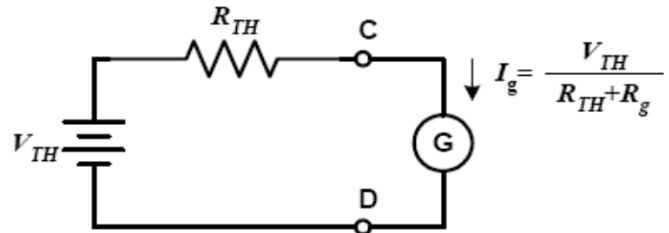
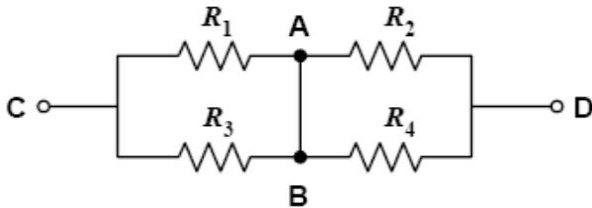
where

$$I_1 = \frac{V}{R_1 + R_3} \quad \text{and} \quad I_2 = \frac{V}{R_2 + R_4}$$

Therefore;

$$V_{TH} = V_{CD} = V \left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4} \right)$$

Thévenin Resistance (R_{TH})



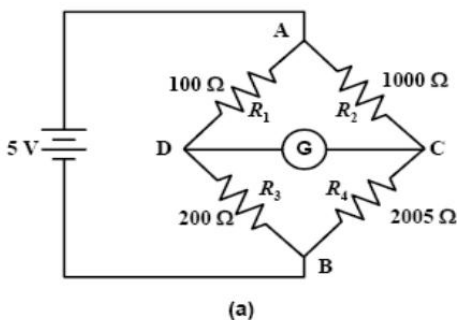
$$R_{TH} = R_1 // R_3 + R_2 // R_4$$

Completed Circuit

where I_g = the galvanometer current and R_g = the galvanometer resistance

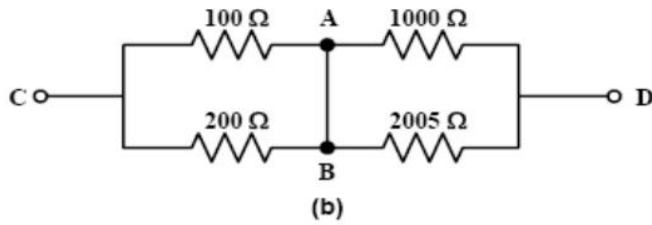
Example: The figure below shows the schematic diagram of a Wheatstone bridge with values of the bridge elements. The battery voltage is 5 V and its internal resistance negligible. The galvanometer has a current sensitivity of 10 mm/ μ A and an internal resistance of 100 Ω . Calculate the deflection of the galvanometer caused by the 5- Ω unbalance in arm BC.

Solution: The bridge circuit is in the small unbalance condition since the value of resistance in arm BC is 2,005 Ω .



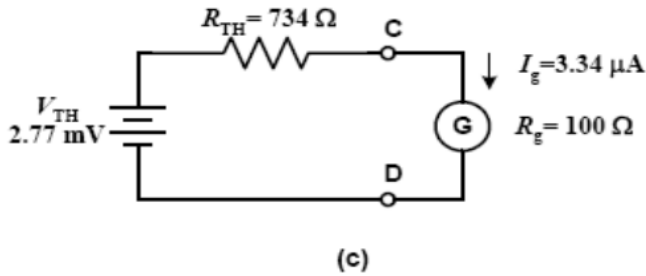
Thévenin Voltage (V_{TH})

$$V_{TH} = V_{AD} - V_{AC} = 5 \text{ V} \times \left(\frac{100}{100 + 200} - \frac{1000}{1000 + 2005} \right) \approx 2.77 \text{ mV}$$



Thévenin Resistance (R_{TH})

$$R_{TH} = 100 // 200 + 1000 // 2005 = 734 \Omega$$



The galvanometer current

$$I_g = \frac{V_{TH}}{R_{TH} + R_g} = \frac{2.77 \text{ mV}}{734 \Omega + 100 \Omega} = 3.32 \mu\text{A}$$

Galvanometer deflection

$$d = 3.32 \mu\text{A} \times \frac{10 \text{ mm}}{\mu\text{A}} = 33.2 \text{ mm}$$

3.3 AC Bridge

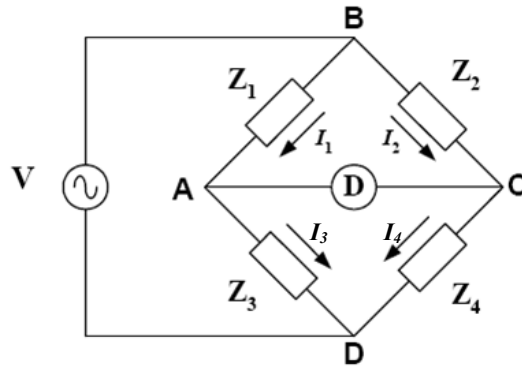


Fig. 3.4 The Wheatstone bridge Circuit using first order approximation

All four arms of the AC Bridge are considered as impedance (frequency dependent components).

The *detector* D is an ac responding device, e.g. headphone and ac meter.

Source: the source voltage V, is an ac voltage at a desired frequency.

Z_1 , Z_2 , Z_3 and Z_4 are the impedances of the bridge arms.

At balance point:

$$E_{BA} = E_{BC} \quad \text{or} \quad I_1 Z_1 = I_2 Z_2$$

and

$$E_{AD} = E_{CD} \quad \text{or} \quad I_3 Z_3 = I_4 Z_4$$

$$I_1 = \frac{V}{Z_1 + Z_3} \quad \text{and} \quad I_2 = \frac{V}{Z_2 + Z_4}$$

General Form of the ac bridge

Complex form: $Z_1 Z_4 = Z_2 Z_3$

Polar form: $Z_1 Z_4 (\angle \theta_1 + \angle \theta_4) = Z_2 Z_3 (\angle \theta_2 + \angle \theta_3).$

Example: The impedances of the basic ac bridge are given as follows:

$$Z_1 = 100 \, \Omega \, \angle 80^\circ \text{ (inductive impedance)}$$

$$Z_3 = 400 \angle 30^\circ \Omega \text{ (inductive impedance)}$$

$$Z_2 = 250 \, \Omega \text{ (pure resistance)}$$

$$Z_4 = \text{unknown.}$$

Determine the constants of the unknown arm.

Solution:

The first condition for bridge balance requires that: $Z_1 Z_4 = Z_2 Z_3$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1,000 \, \Omega$$

The second condition for bridge balance requires that the sum of the phase angles of opposite arms be equal, therefore:

$$\begin{aligned} (\angle \theta_1 + \angle \theta_4) &= (\angle \theta_2 + \angle \theta_3) \\ \angle \theta_4 &= \angle \theta_2 + \angle \theta_3 - \angle \theta_1 = 0 + 30 - 80 = -50^\circ \end{aligned}$$

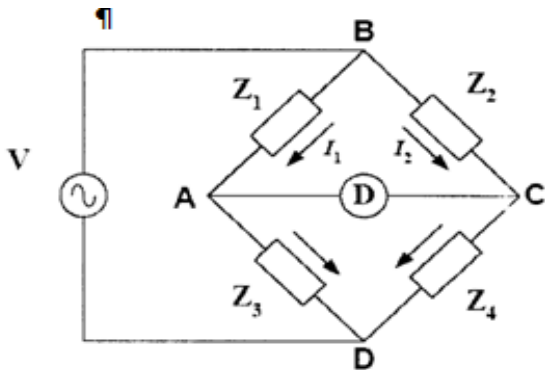
Hence the unknown impedance Z_4 can be written in polar form as:

$$Z_4 = 1,000 \angle -50^\circ \Omega$$

This indicates a capacitive element, possibly consisting of a series combination of a resistor and a capacitor.

Example: An ac bridge is in balance with the following constants: arm AB, $R = 200 \, \Omega$ in series with $L = 15.9 \, \text{mH}$; arm BC, $R = 300 \, \Omega$ in series with $C = 0.265 \, \mu\text{F}$; arm CD, unknown; arm DA = $450 \, \Omega$. The oscillator frequency is $1 \, \text{kHz}$. Find the constants of arm CD.

Solution:



$$Z_1 = R + j\omega L = 200 + j100 \, \Omega$$

$$Z_2 = R + 1/j\omega C = 300 - j600 \, \Omega$$

$$Z_3 = R = 450 \, \Omega$$

$$Z_4 = \text{unknown}$$

The general equation for bridge balance states that: $Z_1 Z_4 = Z_2 Z_3$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{450 \times (200 + j100)}{(300 - j600)} = j150 \, \Omega$$

This result indicates that Z_4 is a pure inductance with an inductive reactance of $150 \, \Omega$ at a frequency of $1 \, \text{kHz}$. Since the inductive reactance is given as $X_L = 2\pi fL$, we can solve for L ; $L = 23.9 \, \text{mH}$.

3.4 Hay Bridge

A Hay Bridge is an AC bridge circuit used for measuring an unknown inductance by balancing the loads of its four arms, one of which contains the unknown inductance. One of the arms of a Hay Bridge has a capacitor of known characteristics, which is the principal component used for determining the unknown inductance value. Fig. 3.5 shows the circuit diagram of the Hay Bridge.

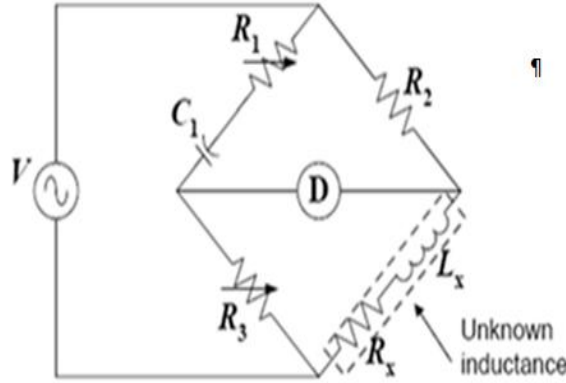


Fig. 3.5 Hay Bridge Circuit

At balance point: $Z_1 Z_x = Z_2 Z_3$

where $Z_1 = R_1 - \frac{1}{\omega C_1}$; $Z_2 = R_2$; and $Z_3 = R_3$

$$Z_1 = \left(R_1 - \frac{1}{\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

which expands to $R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$

Therefore:

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \dots \dots \dots (1)$$

$$\frac{R_x}{\omega C_1} = j\omega L_x R_1 \dots \dots \dots (2)$$

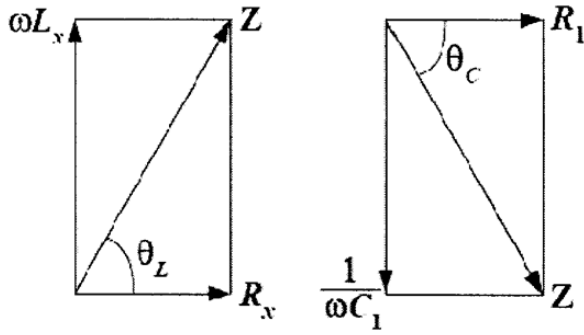
Solving equations 1 and 2 simultaneously gives;

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2} \quad \text{and} \quad L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2}$$

$$\tan \theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q$$

$$\tan \theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1}$$

$$\tan \theta_L = \tan \theta_C \quad \text{or} \quad Q = \frac{1}{\omega C_1 R_1}$$



Phasor diagram of arm 4 and 1

Thus, L_x can be rewritten as:

$$L_x = \frac{R_2 R_3 C_1}{1 + (1/Q^2)}$$

For high Q coil (> 10), the term $1/Q^2$ can be neglected and $L_x \approx R_2 R_3 C_1$

3.5 Wien Bridge

The Wien Bridge is a bridge circuit used for measuring an unknown capacitance by balancing the loads of its four arms, one of which contains the unknown capacitance. It can also be used to measure the unknown frequency of an AC voltage source.

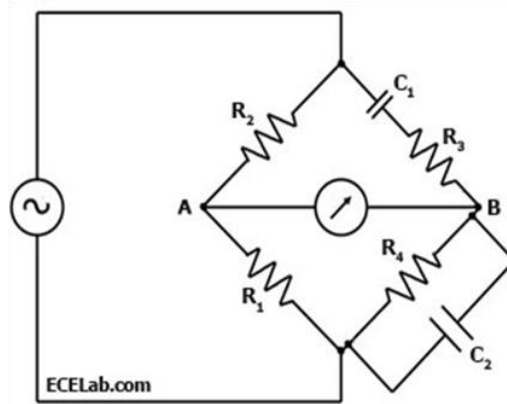


Fig. 3.6 The Wien Bridge

As shown in Fig. 3.6, one arm of a Wien bridge consists of a capacitor in series with a resistor (C_1 and R_3) and another arm consists of a capacitor in parallel to a resistor (C_2 and R_4). The other two arms simply contain a resistor each (R_1 and R_2). The values of R_1 and R_2 are known, and R_4 and C_2 are both adjustable. The unknown values are those of C_1 and R_3 .

Like other bridge circuits, the measuring ability of a Wien Bridge depends on ‘balancing’ the circuit. Balancing the circuit in Fig. 3.6 means adjusting R_4 and C_2 until the current through the ammeter between points A and B becomes zero. This happens when the voltages at points A and B are equal. When the Wien Bridge is balanced, it follows that $\left(\frac{R_2}{R_1} = \frac{Z_1}{Z_2}\right)$ where Z_1 is the impedance of the arm containing C_1 and Z_2 is the impedance of the arm containing C_2 .

Mathematically, when the bridge is balanced:

$$\frac{R_2}{R_1} = \left(\frac{1}{\omega C_1} + R_3 \right) \div \left(\frac{R_4}{\left[\omega C_2 \left(R_4 + \frac{1}{\omega C_2} \right) \right]} \right)$$

wherein $\omega = 2\pi f$; or

$$\frac{R_2}{R_1} = \left(\frac{1}{\omega C_1} + R_3 \right) \div \left(\frac{R_4}{[\omega C_2 R_4 + 1]} \right)$$

or

$$\frac{R_2}{R_1} = \left(\frac{1}{\omega C_1} + R_3 \right) \left(\omega C_2 + \frac{1}{R_4} \right)$$

or

$$\frac{R_2}{R_1} = \frac{C_2}{C_1} + \omega C_2 R_3 + \frac{1}{\omega C_1 R_4} + \frac{R_3}{R_4}$$

When the bridge is balanced, the capacitive reactance cancels each other out, so

$$\frac{R_2}{R_1} = \frac{C_2}{C_1} + \frac{R_3}{R_4}$$

Thus

$$\frac{C_2}{C_1} = \frac{R_2}{R_1} - \frac{R_3}{R_4}$$

Note that the balancing of a Wien Bridge is frequency-dependent. The frequency f at which the Wien Bridge in Figure 1 becomes balanced is the frequency at which $\omega C_2 R_3 = \frac{1}{\omega C_1 R_4}$, or $2\pi f C_2 R_3 = \frac{1}{2\pi f C_1 R_4}$. Thus, the frequency f is given by the following equation:

$$f = \left(\frac{1}{2\pi} \right) \times \left(\text{sqrt} \left(\frac{1}{[R_3 R_4 C_1 C_2]} \right) \right)$$

CHAPTER 4

DYNAMOMETER AND WATTMETER

4.1 Introduction to Wattmeters

These are instruments for the measurement of electric power, or the rate of supply of electric energy to any circuit. The term is generally applied to describe a particular form of electro-dynamometer, consisting of a fixed coil of wire and an embracing or neighbouring coil of wire suspended so as to be movable. The fixed coil is called the current coil, and the movable coil is called the potential coil, and each of these coils has its ends brought to separate terminals on the base of the instrument. The “wattmeter” is an indicating type of instrument, which is generally used for power measurement of the electrical circuit.

A wattmeter consists of:

- (1) a low resistance current coil which is inserted in series with the line carrying the current and
- (2) a high resistance pressure coil, which is connected across the two points whose potential difference is to be measured. The wattmeter requires polarity markings so that the current in the stationary coils will be in the correct direction relative to the current in the movable coil.

There are two principal types of wattmeter:

- i. Dynamometer Wattmeter – for both dc and ac power; and
- ii. Induction Wattmeter – for ac power only.

4.2 Dynamometer Wattmeter Design

Power in an electric circuit is the product (multiplication) of voltage and current, so any meter designed to measure power must account for both of these variables. A special meter movement designed especially for power measurement is called the dynamometer movement, and is similar to a D’Arsonval in that, a lightweight coil of wire is attached to the pointer mechanism. However, unlike the D’Arsonval movement, another (stationary) coil is used instead of a permanent magnet to provide the magnetic field for the moving coil to react against. The moving coil is generally energized by the voltage in the circuit, while the stationary coil is generally energized by the current in the circuit. A typical dynamometer movement connected in a circuit is shown below (Fig. 4.1).

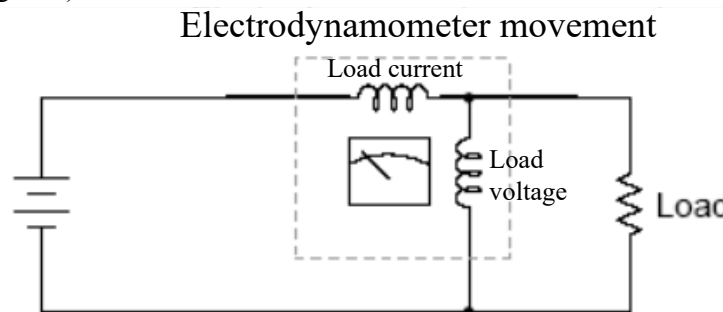


Fig. 4.1 A typical dynamometer movement connected in a circuit

The top (horizontal) coil of the wire measures load current while the bottom (vertical) coil measures the load voltage. Just like the lightweight moving coils of voltmeter movements, the (moving) voltage coil of a dynamometer is typically connected in series with a range resistor so that full load voltage is not applied to it. Likewise, the (stationary) current coil of a dynamometer may have precision shunt resistors to divide the load current around it. Fig. 4.2(a) shows the schematic diagram of the arrangement and Fig. 4.2(b) show the circuit diagram.

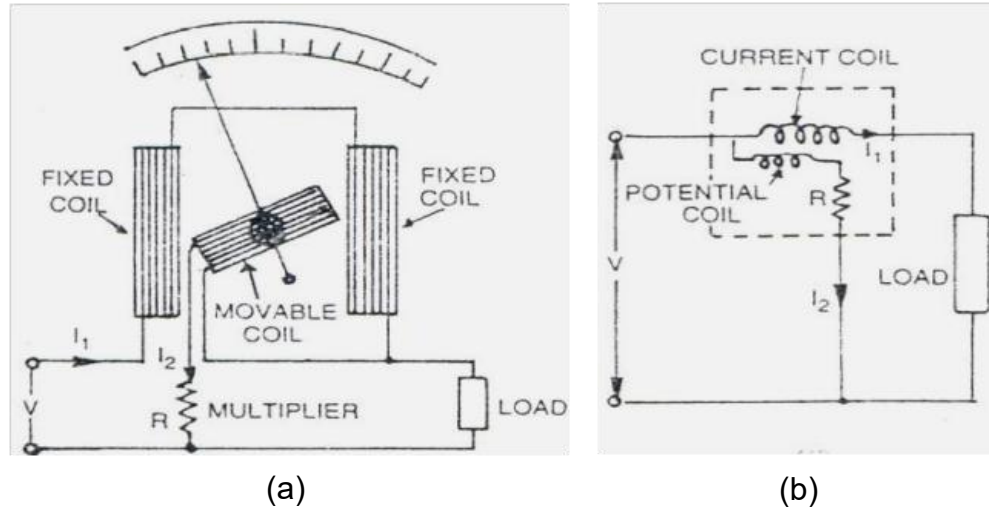


Fig. 4.2(a) shows the schematic diagram of the arrangement and (b) show the circuit diagram

With custom-built dynamometer movements, shunt resistors are less likely to be needed because the stationary coil can be constructed with heavy wire as needed without impacting meter response, unlike the moving coil which must be constructed of lightweight wire for minimum inertia.

4.2.1 Operation

When the wattmeter is connected in the circuit to measure power the current (stationary coil) which is wound with a larger-diameter wire carries the load current and potential (moving coil) coil carries current proportional to the load voltage. Due to currents in the coils, mechanical force exists between them. The result is that movable coil moves the pointer over the scale. The pointer comes to rest at a position when deflecting torque is equal to the controlling torque. The moving coil is used to detect the magnitude of the circuit voltage. The stationary coils are referred to as the current coils. The circuit current is detected by the current coils, which are connected in series with the load. The stationary current is wound with larger diameter. This keeps the resistance that is in series with the load as low as possible. The moving coil is wound with thin wire to keep it as high as possible. Since the movable coil responds to voltage, it has a multiplier (a high non-inductive resistance) connected in series with the moving coil to limit the current flowing through the moving coil to a small value, usually up to 100 mA. Such instruments can be used for the measurement of dc as well as ac power. The circuits below (Fig. 4.3) show two ways of connecting the dynamometer wattmeter into circuit.

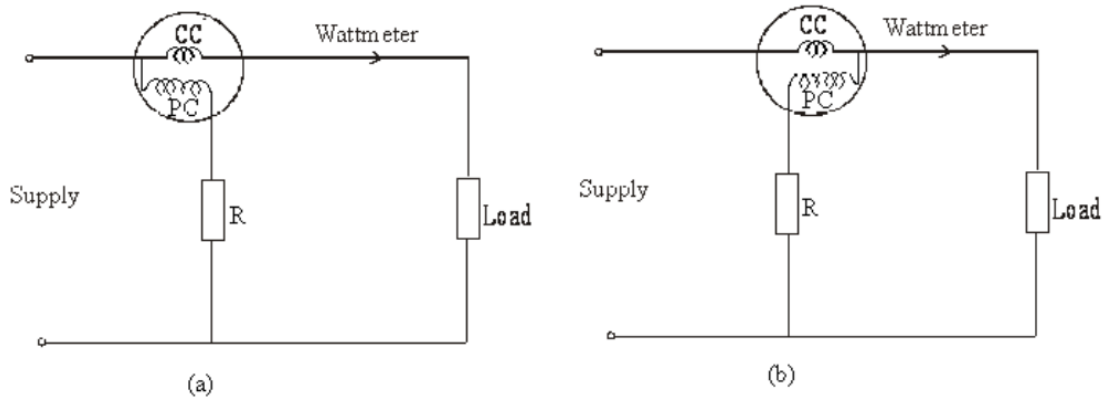


Fig. 4.3 (a and b) Shows two ways of connecting the dynamometer wattmeter into circuit

In connection (a), the pressure coil is connected on the supply side ((i.e. CC (current coil) on the load side)) and therefore the voltage applied to the pressure coil is the voltage across the load plus the voltage drop across the current coil. Thus the wattmeter measures the power loss in its current coil in addition to the power consumed by load.

Power indicated by wattmeter = power consumed by load + power loss in current coil ($I^2 R_c$) = $P_L + P_C$.

In connection (b) the current coil is on supply side and, therefore it carries the pressure coil current plus the load current. Hence the wattmeter reads the power consumed by the load plus the power loss in pressure coil.

Power indicated by wattmeter = power consumed by load + power loss in pressure coil (V^2/R_p). If the load current is small, the voltage drop in the current coil is small, so that connection (a) introduces a very small as compared with the load current and hence power loss in pressure coil will be very small as compared with the load power and, therefore, connection of fig (b) is preferable.

Note that the connection in (a) is used for small current high voltage load and (b) high current low voltage loads.

4.2.2 Advantages and Disadvantages of Dynamometer Wattmeter

Advantages

- Such instruments can be made to give a very high degree of accuracy. Hence, they are used as a standard for calibration purposes.
- They are equally accurate on dc as well as ac measurements.
- It can be used on both ac and dc supply, for any waveform of voltage and current, and is not restricted to sinusoidal waveforms.

Disadvantages

At low power factor, the inductance of the voltage coil causes serious error unless special precautions are taken to reduce this effect.

4.3 Induction Wattmeter

The **induction type wattmeter** can be used to measure ac power only in contrast to dynamometer wattmeter which can be used to measure dc as well as ac power. However, it also differs because two separate coils are used to produce the rotating magnetic field in place of one coil with a phase split arrangement. The figure below (Fig. 4.4) shows the arrangement of the various parts of an induction wattmeter.

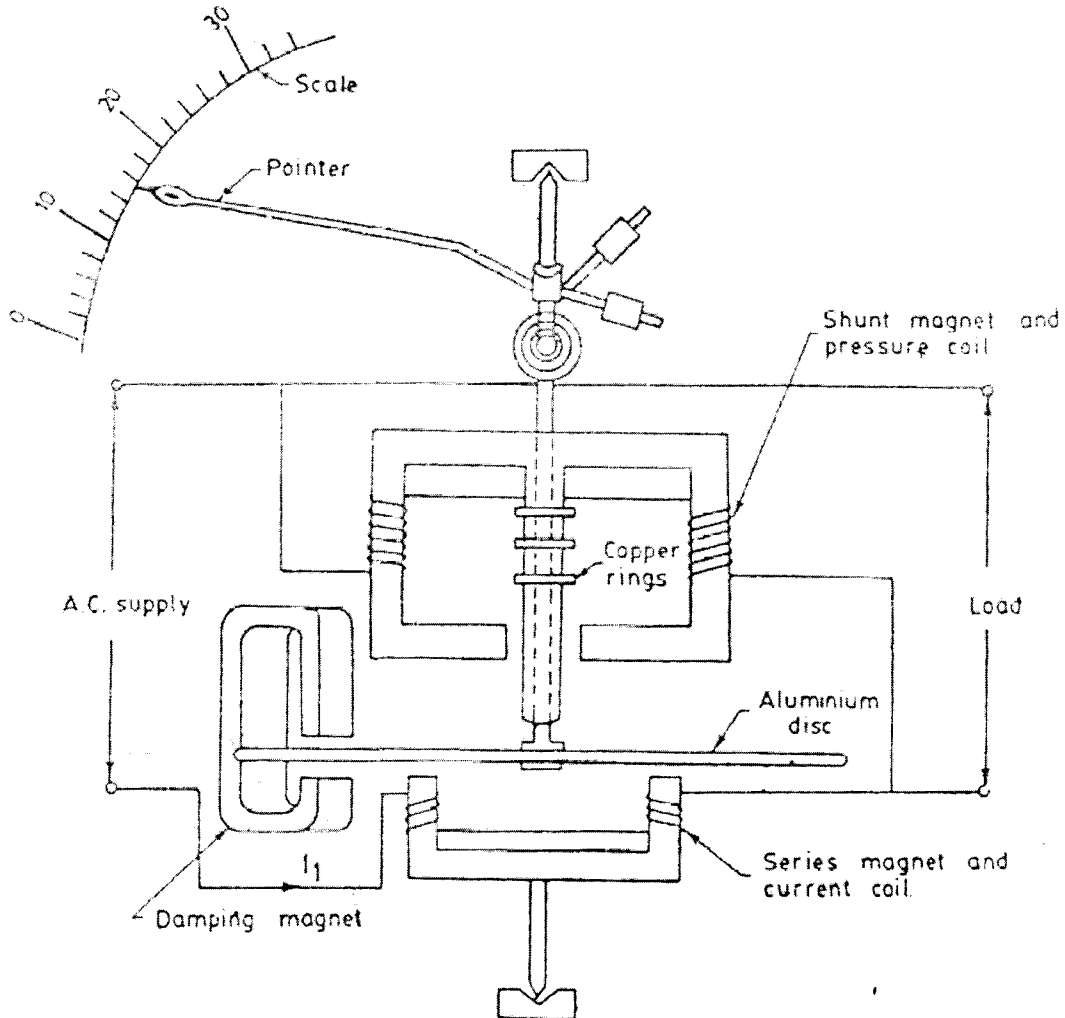


Fig. 4.4 Arrangement of the various parts of an induction wattmeter

4.3.1 Operations

When the wattmeter is connected in the circuit to measure ac power, the shunt magnet carries current proportional to the supply voltage and the series magnet carries the load current. The two fluxes produced by the magnets induce eddy currents in the aluminum disc. The interaction between the fluxes and eddy currents produces the deflecting torques on the disc, causing the pointer connected to the moving system to move over the scale. The pointer comes to rest at a position where deflecting torque is equal to the controlling torque. The current I_V in the shunt magnet lags the supply voltage by 90° and so does the flux Φ_V produced by it. This current I_C in the series magnet is the load current and hence lags behind the supply voltage V by Φ . The flux, Φ_C produced by this current (i.e. I_C) is in phase with it.

4.3.2 Advantages and Disadvantages of Induction Wattmeter

Advantages

- i. They have a uniform scale
- ii. They are free from the effects of stray fields
- iii. They provide very good damping

Disadvantages

- i. They can be used to measure ac power only
- ii. They cause series error due to temperature variation
- iii. They have high power consumption

Induction wattmeters have their chief application as panel instruments where the variations in frequency are not too much.

4.4 Power Measurement in a Single Phase Circuit

In a dc circuit, power is the product of the ammeter and voltmeter readings, and a wattmeter is not absolutely necessary. In an ac circuit, the product of ammeter and voltmeter readings gives the apparent power or volt-amperes but does not take power factor into account. The wattmeter is necessary to indicate actual power in watts. Fig. 4.5 show how a wattmeter is inserted in a single phase circuit.

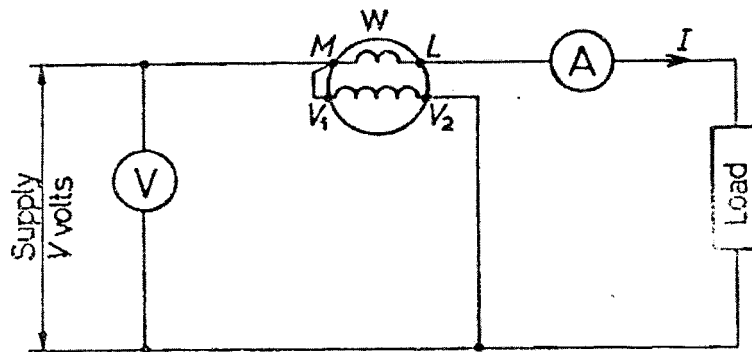


Fig. 4.5 Wattmeter is inserted in a single phase circuit

Wattmeter reading = actual power $P = VI \cos \phi$ [watts].

The product of V and A = apparent power, $S = VI$ volt-amperes.

From these instrument readings, the power factor of the load can be calculated.

$$\text{Power factor} = \frac{VI \cos \phi}{VI} = (\cos \phi) = \frac{P}{S}$$

From this, $\sin \phi$ can be found and the reactive power, in reactive volt amperes (var), can also be calculated.

$$\text{Reactive power} = Q = VI \sin \phi \text{ var}$$

If the reactance is X ohms

Then

$$Q = \frac{V^2}{X} \quad \text{or} \quad X = \frac{V^2}{Q}$$

4.5 Power Measurement in a Three Phase Circuit

Power in a three phase circuit could be measured by any of the following methods depending on the type of circuit.

4.5.1 One Wattmeter Method

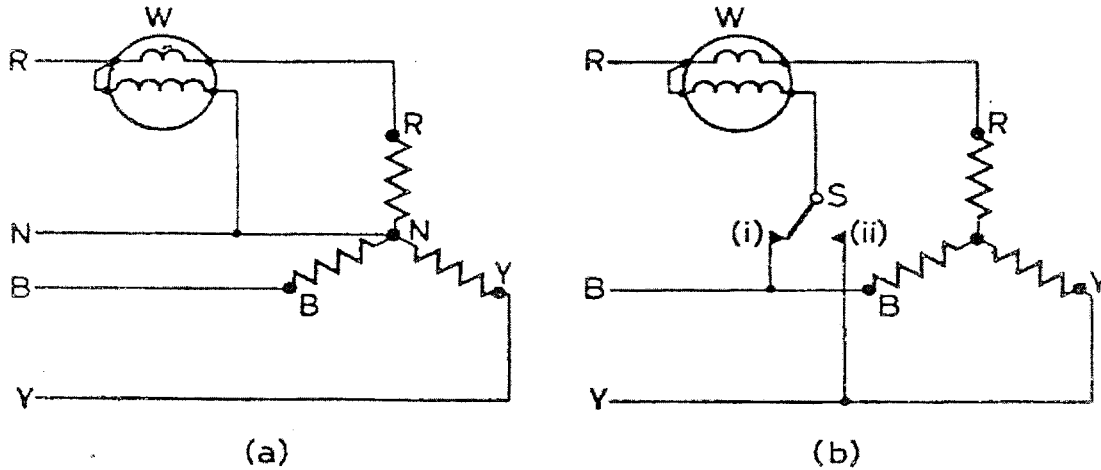


Fig. 4.6 Measurement of Power in a Three Phase Circuit

This method of measuring power in a three - phase circuit can only be used if the load is balanced. Total Power = $3 \times$ wattmeter reading (Fig.4.6a). In Fig. 4.6b, there is no neutral so the potential coil of the wattmeter is connected between lines with a switch S in circuit so that connection can be made to both lines in turn. The potential across the wattmeter coil will be 1.73 times the phase voltage and will have a 30° phase shift. Two readings are taken, first with S on position (i) then with S on position (ii).

The total three - phase power is the sum of these two readings.

Thus,

$$\text{Total power} = P(i) + P(ii)$$

4.5.2 Two Wattmeter Method

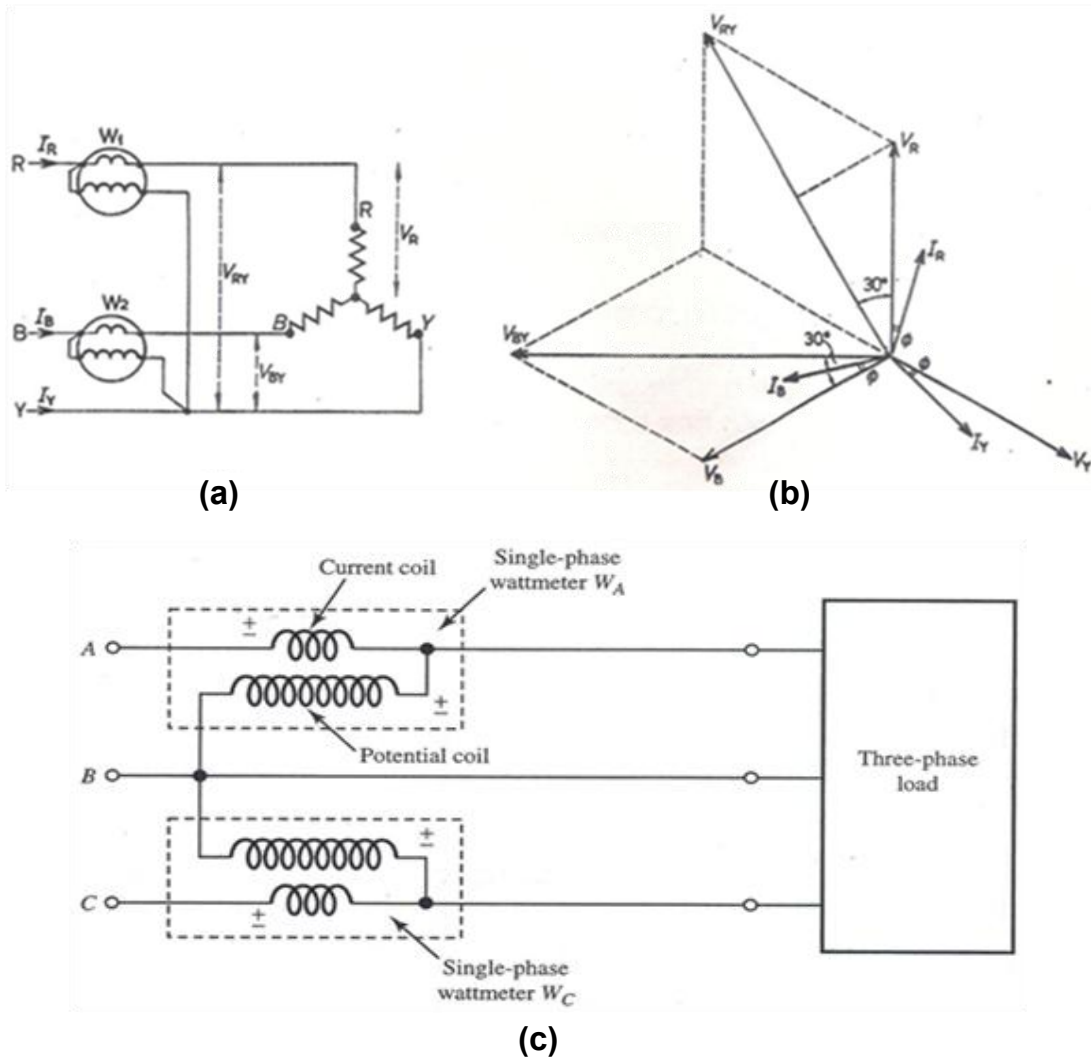


Fig. 4.7 Three phase wattmeter (a) Schematic diagram, (b) Phasor diagram and (c) connection diagram for two-wattmeter method of measuring three-phase power

This is the most commonly used method for measuring power in a three - wire system, since it can be used for both balanced and unbalanced, star or delta loads. The current coils of the wattmeters carry the current in two lines and the potential coils are connected across to the third line, as shown in the above diagram. The phasor diagram for a balance load is also as shown.

The current in wattmeter W_1 is I_R and the potential is V_{RY} . In W_2 the current is I_B and the potential is V_{BY} . The phasor diagram shows that the phasor V_{RY} leads V_R by 30° and V_{BY} lags 30° behind V_B . If the load power factor is $\cos\phi$ lagging, then phase angle between the current and voltage in W_1 is $30^\circ + \phi$, and in W_2 is $30^\circ - \phi$.

$$\therefore W_1 \text{ measures } V_{RY} I_R \cos(30 + \phi) = P_1$$

$$W_2 \text{ measures } V_{BY} I_B \cos(30 - \phi) = P_2$$

For balanced conditions

$$V_{RY} = V_{BY} = V$$

$$I_R = I_B = I$$

$$W_1 + W_2 = VI \cos(30 + \phi) + VI \cos(30 - \phi)$$

$$= VI (\cos 30 \cos \phi - \sin 30 \sin \phi + \cos 30 \cos \phi + \sin 30 \sin \phi)$$

$$= VI \times 2\cos 30 \cos \phi$$

But $2\cos 30 = 1.73$

Therefore

$$W_1 + W_2 = 1.73VI \cos \phi = \text{Total power in Watts.}$$

This is the equation for power in a three-phase circuit.

Therefore, total power is given by the sum of the wattmeter readings. This is also true if the load is unbalanced and there is no fourth wire in the system.

If the phase angle ϕ is 60° then the angle between V_{RY} and I_R will be 90° . This is similar to a pure inductive circuit and wattmeter W_1 , will read zero. In this case, W_2 will indicate all the power in the circuit. If ϕ is greater than 60° , wattmeter W_1 will give negative readings which must be subtracted from the reading W_2 . When W_1 gives this negative reading, it is necessary to reverse the potential coil connections to correct the indication on the scale. In some instruments a changeover switch is included for this purpose.

For an unbalanced load, the phase angle may be different for each phase, but when the load is balanced the power factor can be found.

From the above equation:

$$\begin{aligned} W_1 + W_2 &= 1.73VI \cos \phi \\ W_2 - W_1 &= VI(2\sin 30 \sin \phi) = VI \sin \phi \end{aligned}$$

Therefore

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{VI \sin \phi}{1.73VI \cos \phi} = \frac{1}{1.73} \tan \phi$$

and

$$\tan \phi = \frac{1.73(W_2 - W_1)}{W_2 + W_1}$$

From the above equation the power factor $\cos \phi$ can be found.

The following in a balanced system may be noted:

i)	if $W_2 = W_1$	$\phi = 0$	p. f. = 1
ii)	if $W_2 = 2W_1$	$\phi = 30$	p. f. = 0.866
iii)	if $W_2 = \text{zero}$	$\phi = 60$	p. f. = 0.5
iv)	if W_1 is negative	$\phi > 60$	p. f. < 0.5

4.5.3 Three Wattmeter Method

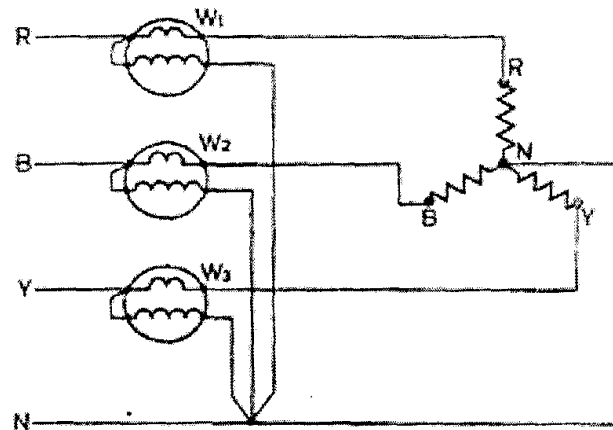


Fig. 4.8 Three phase wattmeter connection with four-wire and unbalanced load

If the system is four-wire and the load is unbalanced, three wattmeters are necessary. These are connected as shown above. Each wattmeter measures the power in one phase. The total power will be the sum of the three wattmeter readings.

4.6 Three-Phase Wattmeters

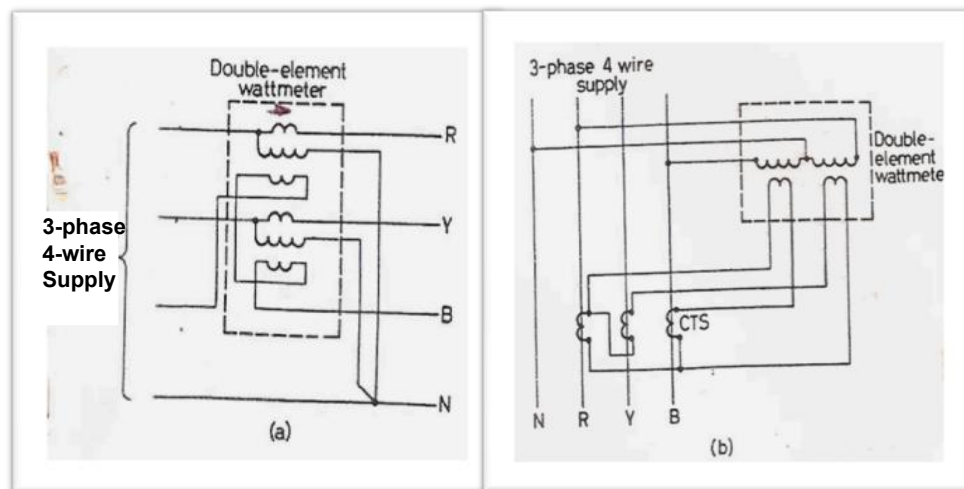


Fig. 4.9 Three phase wattmeters

Power in a three-phase circuit is often measured by a single instrument which contains two wattmeter elements. The two elements are connected in a similar way to the two-wattmeter method. The two moving coils are mounted on the same spindle and the resultant torque is the algebraic sum of the torques of the two elements, and therefore gives a direct measure of the total power. Power in a four - wire system can be measured by a similar instrument if the current coil of each element is halved and the current fed from each phase, or alternatively CTs can be used. The circuits are as shown above.

The circuits for three-phase energy meters are similar.

CHAPTER 5

OSCILLOSCOPE

5.1 Introduction

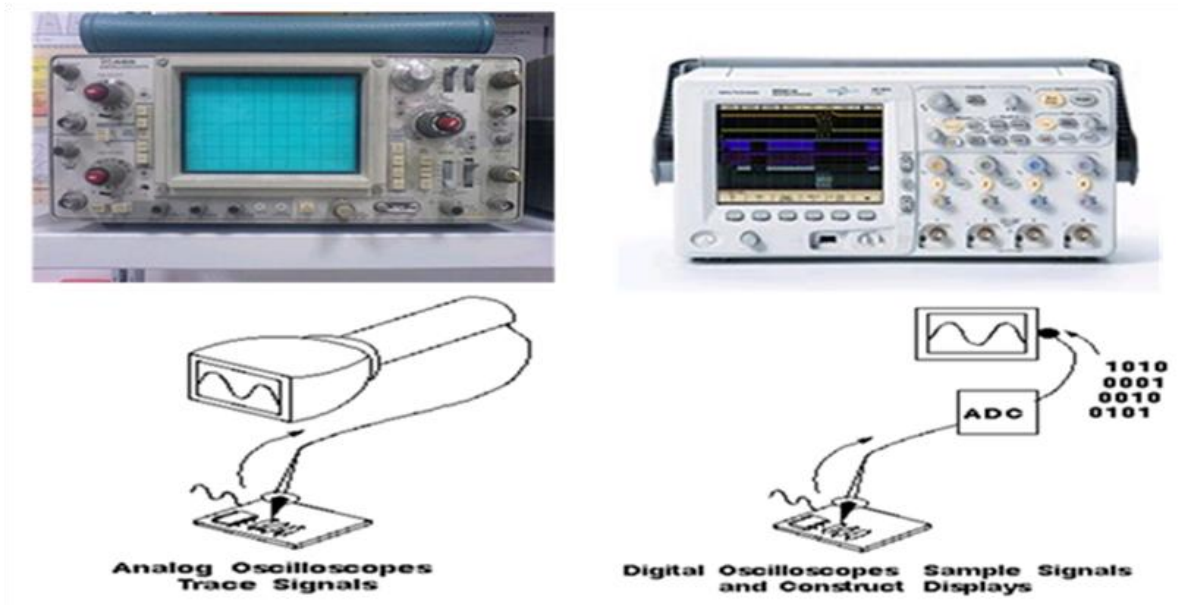
The CRO (Cathode Ray Oscilloscope), generally referred to as the oscilloscope or simply “scope” is probably the most versatile electrical measuring instrument available. The CRO can measure the following electrical parameters:

- i) AC or DC voltage;
- ii) AC or DC current;
- iii) Time;
- iv) Phase relationship; and
- v) Frequency.

The scope consists of the following parts:

- i) CRT – Cathode Ray Tube (Heart of Instrument);
- ii) Vertical amplifier;
- iii) Horizontal amplifier;
- iv) Sweep generator;
- v) Trigger circuit; and
- vi) Associated power supply.

There are 2 types of oscilloscope: analog and digital. Their principle and basic characteristics are the same.



5.2 Basic Construction of CRO

The various parts of the CRO are summarized in the block diagram shown below.

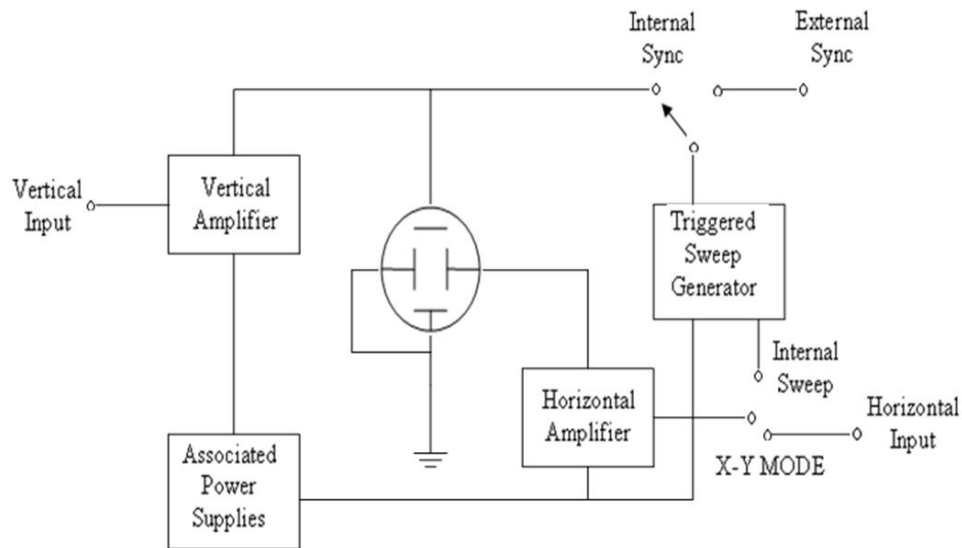


Fig. 5.1 Basic components of a CRO

5.3 Cathode Ray Tube (CRT)

CRT is the heart of the CRO providing visual display of an input signal waveform.

A CRT contains four basic parts:

- i) An electron gun to provide stream of electrons;
- ii) Focusing and accelerating elements to produce a well-defined beam of electrons;
- iii) Horizontal and vertical deflecting plates to control the path of the electron beam; and
- iv) An evacuated glass envelope with a phosphorescent screen which glows visibly when struck by electron beam.

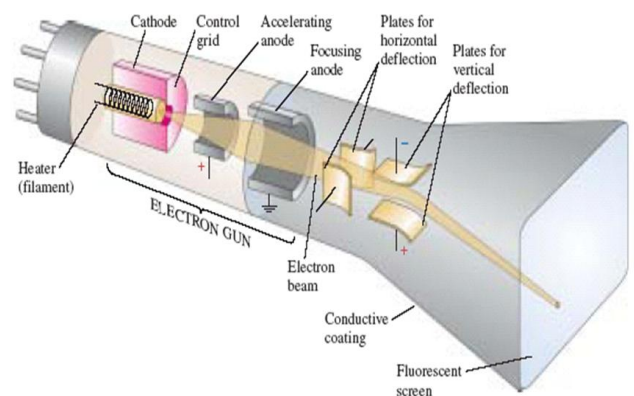
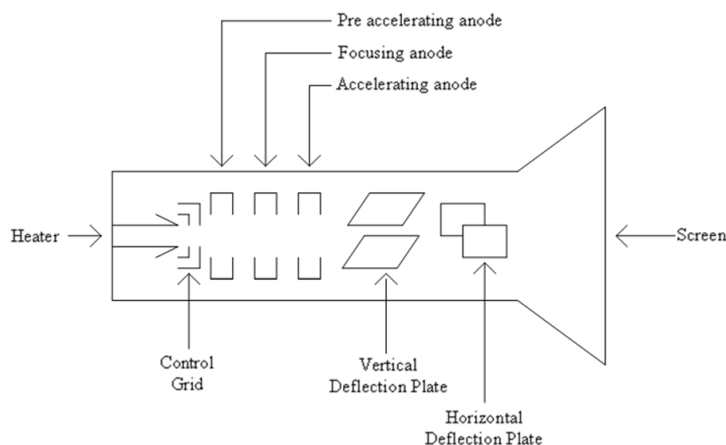


Fig. 5.1 Basic components of a CRT

Control Grid

It regulates the number of electrons that reach the anode and hence control the brightness of the spot on the screen.

Focusing Anode

Ensures that the electrons leaving the cathode in slightly different directions are focused down to a narrow beam and all arrive at the same spot on the screen.

Electron Gun

The electron gun consists of cathode, control grid, focusing anode and accelerating anode.

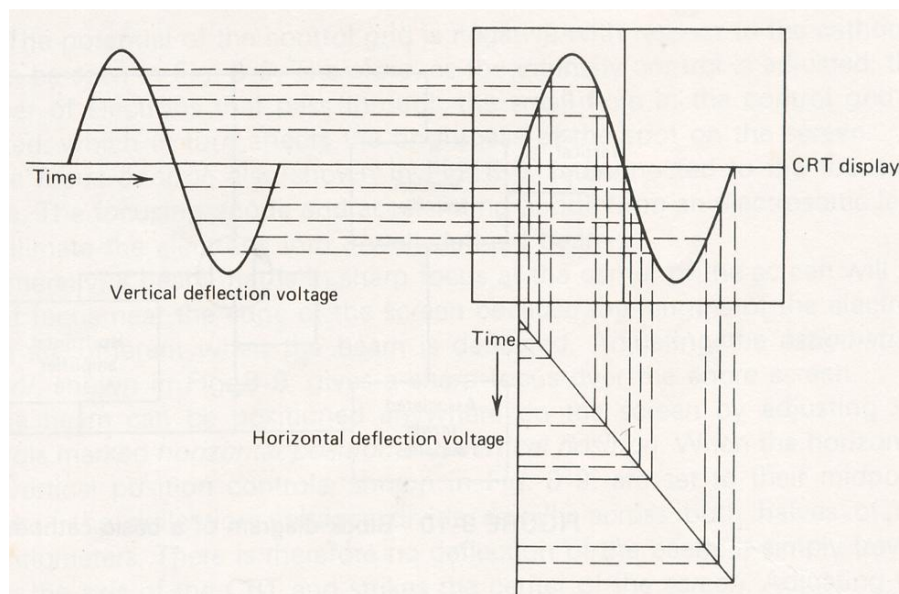
Deflecting Plates

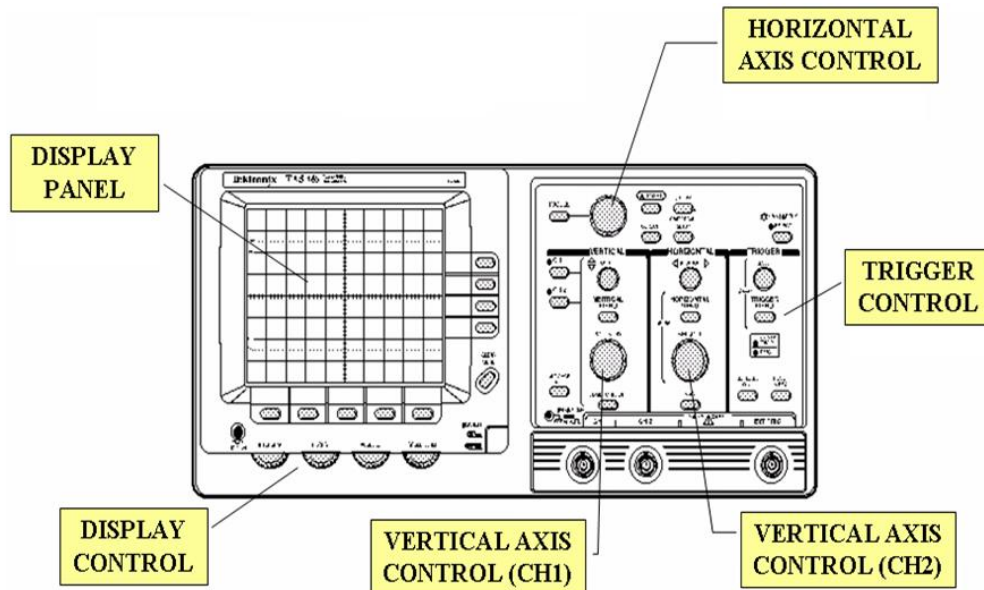
Electric fields between the first pair of plates deflect the electrons horizontally and electric field between the second pair deflects them vertically. If no deflecting fields are present, the electrons travel in a straight line from the hole in the accelerating anode to the center of the screen, where they produce a bright spot.

5.4 Principal Elements of the CRT

- i) The interior of the tube is a vacuum, with a pressure of around 0.01 Pa (10^{-7} atm) or less.
- ii) The *cathode* is raised to a high temperature by the *heater*, and electrons evaporate from the surface of the cathode.
- iii) The *accelerating anode*, with a small hole at its center, is maintained at a high positive potential V_1 , of the order of 1 to 20 kV, relative to the cathode.
- iv) This potential difference gives rise to an electric field directed from right to left in the region between the accelerating anode and the cathode.
- v) Electrons passing through the hole in the anode form a narrow beam and travel with constant horizontal velocity from the anode to the *fluorescent screen*.

Signal on the CRT





5.5 Measurements of the Oscilloscope

The CRO can carry out the following measurements:

- i) Voltage Measurements;
- ii) Period and Frequency Measurements; and
- iii) Phase Measurements or Time Delay.

5.5.1 Voltage Measurements

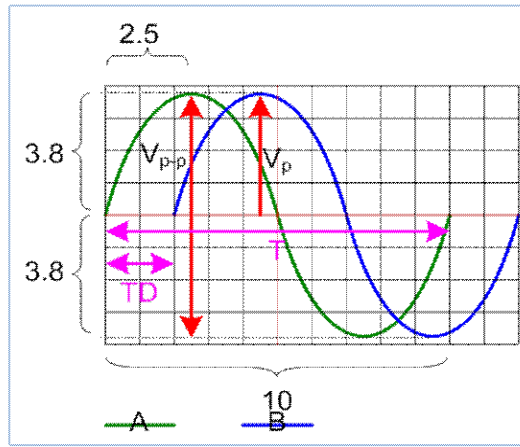
The vertical scale is calibrated in either volts per division or millivolts per division.

Using the scale setting of the scope and the signal measured off the face of the scope, then it can measure peak-to-peak voltage for an ac signal.

$$V_{p-p} = (\text{vertical } p-p \text{ division}) \times (\text{volts/div})$$

OR

$$V_{p-p} = (\text{no. of vertical division}) \times (\text{volts/div})$$



a) Voltage Peak-to-Peak

$$\begin{aligned} V_{p-p} &= (V/Div) \times \text{No. of vert. div.} \\ &= 100 \text{ mV/div} \times (3.8 \times 2) \\ &= \underline{0.76 \text{ V}} \end{aligned}$$

b) Voltage Peak

$$\begin{aligned} V_p &= (V/Div) \times \text{No. of vert. div.} \\ &= 100 \text{ mV/div} \times (3.8) \\ &= \underline{0.38 \text{ V}} \end{aligned}$$

(Volt/Div : 100mV/Div, Time/Div : 0.5ms/Div)

5.5.2 Period and Frequency Measurements

Period

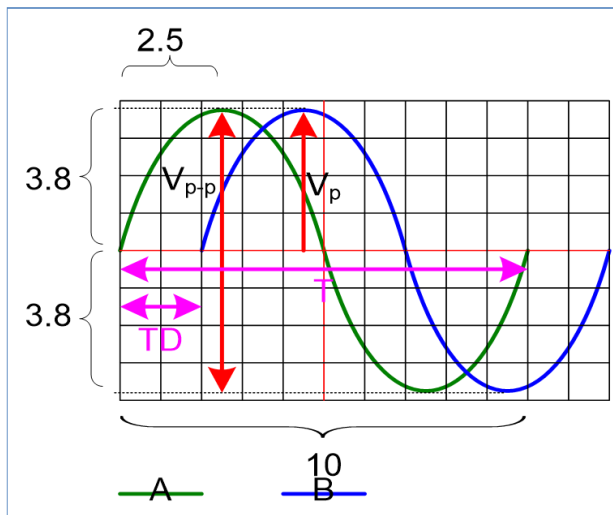
Horizontal scale of the scope can be used to measure time in second, millisecond or nanosecond. The interval of a pulse from start to end is the period of the pulse.

$$\text{Period} = (\text{horizontal } p - p \text{ division}) \times (\text{time/div})$$

Frequency

The measurement of a repetitive waveform period can be used to calculate the signal frequency.

$$F = 1/T$$



a) Period, T

$$\begin{aligned} T &= (\text{Time/Div}) \times (\text{no. div/cycle}) \\ &= 0.5\text{ms/div} \times 10 \\ &= \underline{5\text{ms}} \end{aligned}$$

b) Frequency, f

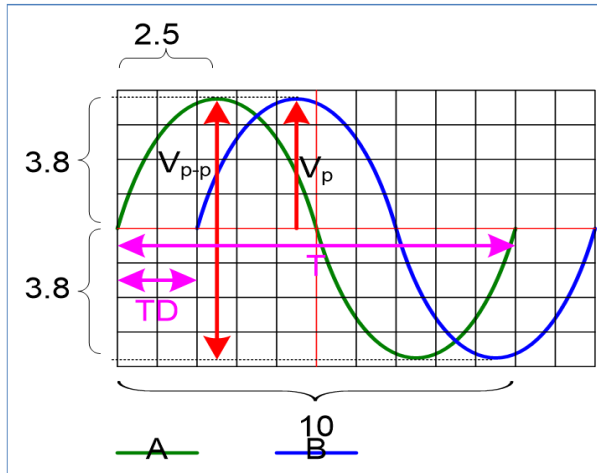
$$\begin{aligned} f &= 1/T \\ &= 1/5\text{ms} \\ &= \underline{200 \text{ Hz}} \end{aligned}$$

(Time/Div : 0.5ms/Div)

5.5.3 Phase Shift (Phase Difference) Measurements

The time interval between pulses is called pulse delay. The pulse delay is measured between the midpoint at the start of each pulse.

$$\text{Phase difference, } \theta = (\text{phase difference in division}) \times (\text{degrees/div})$$



(Time/Div : 0.5ms/Div)

$$1 \text{ cycle} = 10 \text{ div}$$

$$\text{TD} = 2 \text{ div}$$

Therefore,

$$1 \text{ cycle} : 10 \text{ div} = 360^\circ$$

$$1 \text{ div} = 360^\circ / 10 = 36^\circ$$

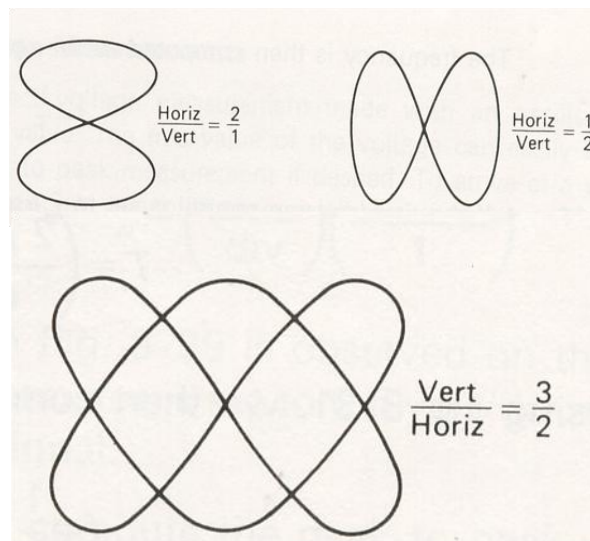
$$2 \text{ div} = 2 \times 36^\circ = 72^\circ$$

5.5 Lissajous Patterns

5.6.1 Frequency Measurement

The alternative way of using oscilloscope to measure frequency is to use Lissajous patterns. In order to generate a Lissajous pattern a known reference frequency sine wave is applied to one of the deflection plates of the oscilloscope and the unknown sinusoidal signal to the other deflection plates. A Lissajous pattern is produced on the screen according to the frequency ration between the two signals.

$$\frac{F_y}{F_x} = \frac{\text{Number of positive peaks}}{\text{Number of right hand side peaks}}$$



5.6.2 Phase Angle Measurement

Oscilloscope can be used in the X-Y mode to determine the phase angle between two signals. This useful technique is limited to small frequency. The formula for phase angle measurement is given by:

$$\sin \theta = \frac{Y_1}{Y_2} = \frac{X_1}{X_2}$$

where

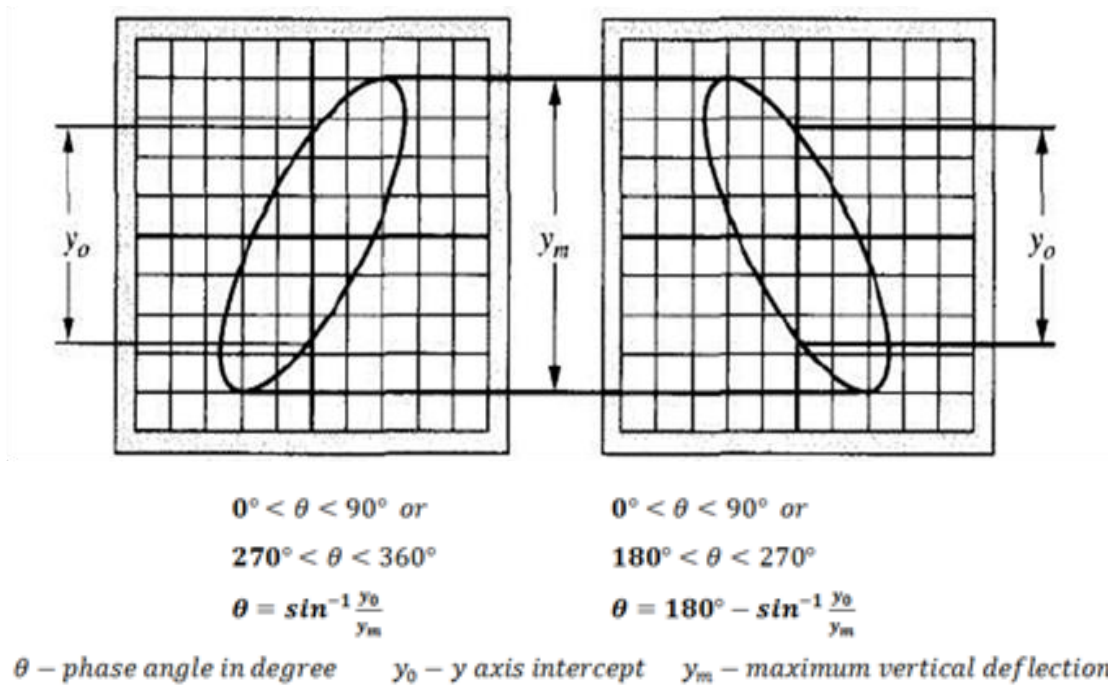
θ = phase angle in degree;

Y_1 = the distance from X-axis to the point where the Lissajous pattern crosses Y-axis;

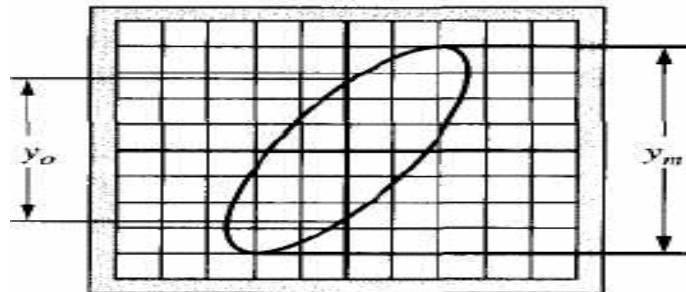
Y_2 = the maximum vertical distance on the Lissajous;

X_1 = the distance from Y-axis to the point where the Lissajous pattern crosses X-axis; and

Y_2 = the maximum vertical distance on the Lissajous.



EXAMPLE



If, in the figure above, the distance y_0 is 1.8 cm and $y_m = 2.3$ cm, what is the phase angle?

CHAPTER SIX

MISCELLANEOUS MEASURING INSTRUMENTS

6.1 Thermocouples

When two wires with dissimilar electrical properties are joined at both ends and one junction is made hot and the other cold, a small electric current is produced proportional to the difference in the temperature. Seebeck discovered this effect. It is true no matter how the ends are joined, so the cold end may be joined at a sensitive millivolt meter. The hot junction forms the sensor end (Fig. 6.1).

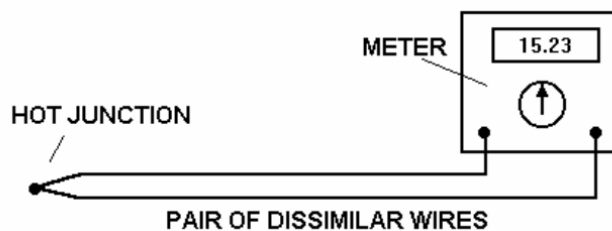


Fig. 6.1 A thermocouple



Fig. 6.2 A typical industrial probe

Fig. 6.2 shows a typical industrial probe with a flexible extension and standard plug.

Peltier showed that heat is absorbed at the hot end and rejected at the cold end. Thompson showed that part of the emf is due to the temperature gradient in the wire as well as the temperature difference between the junctions. Most thermocouple metals produce a relationship between the two temperatures and the emf as follows:

$$e = \alpha(\theta_1 - \theta_2) + \beta(\theta_1^2 - \theta_2^2)$$

α and β are constants for the type of thermocouple. The relationship is nearly linear over the operating range. The actual characteristic and suitable operating temperatures depends upon the metals used in the wires. The various types are designated in international and national standards. Typical linear operating ranges are shown for standard types. It is important that thermocouples are standard so that the same emf will always represent the same temperature.

Type J	0 to 800 °C
Type K	0 to 1200 °C
Type T	-199 to 250 °C
Type E	0 to 600 °C
Type R/S	0 to 1600 °C
Type B	500 to 1800 °C
Type N	0 to 1200 °C
Type L	0 to 800 °C

Thermocouples come in several forms. They may be wires insulated from each other with plastic or glass fibre materials. For high temperature work, the wire pairs are put inside a tube with mineral insulation. For industrial uses, the sensor comes in a metal enclosure such as stainless steel.

Types of Thermocouples

Direct type: This has the advantage of good sensitivity and quick activity (Fig. 6.3).

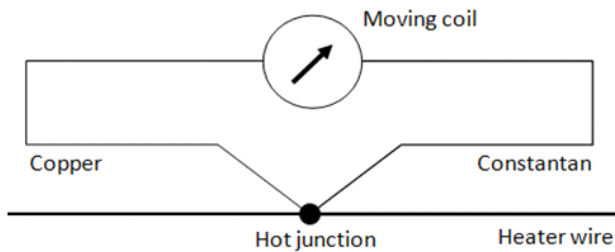


Fig. 6.3 A type of a *Thermocouple, direct type*

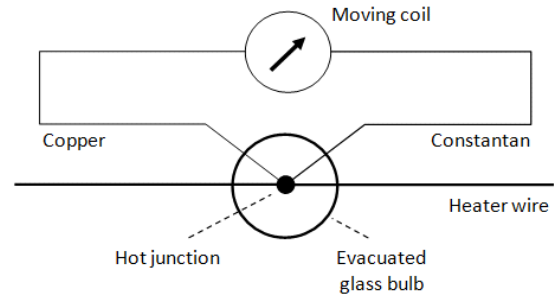


Fig. 6.4 A type of a *Thermocouple, indirect type*

Indirect type: Is less sensitive and more sluggish in operation, but has applications for high currents (Fig 6.4). The meter is isolated from the heater circuit. The heater wire and the thermocouple are usually enclosed in an evacuated glass bulb to prevent leakage of heat at the junction by convection. The deflecting torque is proportional to the heat produced which is also proportional to the square of the current (I^2) making the scale non-linear. The instrument reads rms value and is particularly used for high frequencies such as radio frequency up to say 100 MHz.

6.2 Clamp Ammeter

Clamp meters are a very convenient testing instrument that permits current measurements on a live conductor without circuit interruption. “A clamp meter” (clamp –on meter) is a type of ammeter that measures electrical current without the need to disconnect the wiring through which the current is flowing. A clamp-on ammeter can have either a digital or an analog readout. Many clamp meters also measure other quantities (voltage, resistance, and so on) by using test leads rather than the clamp-on mechanism. Using the clamp meter, however, we can measure current by simply clamping on a conductor as illustrated in the figure below. One of the advantages of this method is that we can even measure a large current without shutting off the circuit being tested. Clamp meters are a very convenient testing instrument that permits current measurements on a live conductor without circuit interruption. When making current measurements with the ordinary multimeter, we need to cut the wiring and connect the instrument to the circuit under test (Fig. 6.5).

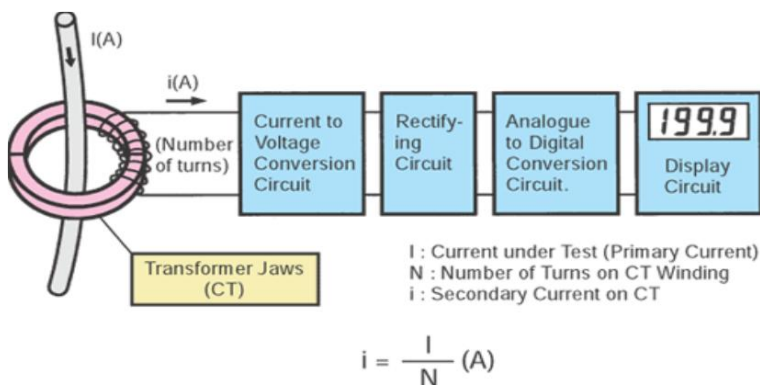


Fig. 6.5 Operation of Clamp a Ammeter

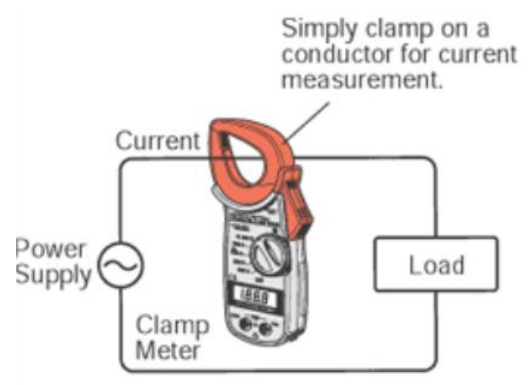


Fig. 6.6 A Typical Clamp Ammeter in a Circuit

Fig. 6.6 shows how measurement could be done using the clamp meter. Open the jaws of the clamp meter and clamp it on to a conductor. Release the jaw opening lever to allow the secondary windings of the clamp meter close up.

6.3 The Field Mill (Electric Field Measurement)

The electric field mill is a device whose principle of operation is based on electrostatic induction (Fig. 6.7). It consists of one or two electrodes which either rotate in an electrostatic field or become periodically exposed to a field by rotating vanes. The figure below illustrates a cylindrical field mill which consists of two cylinder halves that are electrically insulated from each other. An electric motor rotates the two halves in the electric field to be measured so that they become alternately exposed to both the positive and negative direction of the field. The result is that an alternating (ac) signal is generated across the two halves which can be easily amplified. The rotating shutter field mill, on the other hand, comprises a stationary electrode, which becomes periodically exposed to the external electric field through a rotating grounded disc. A variation on this type of field mill is a stationary grounded cover plate with a rotating disc electrode.

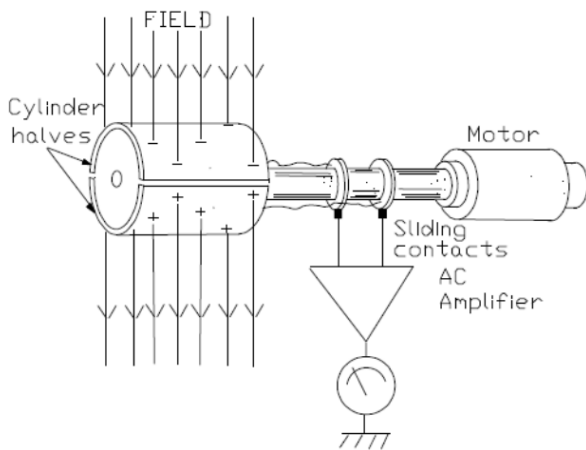


Fig. 6.7 Cylindrical Electric Field Mill

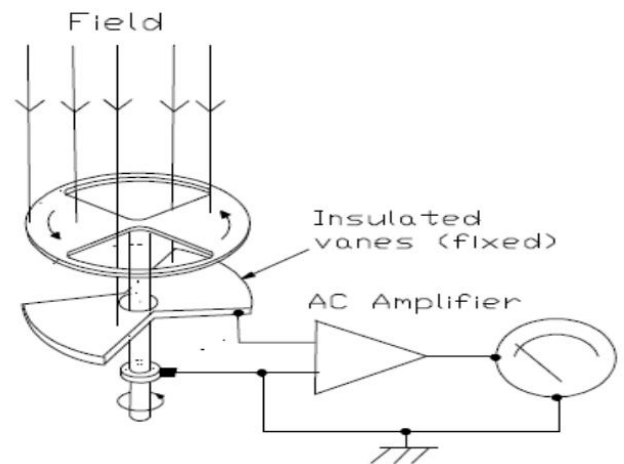


Fig. 6.8 Rotating Shutter Electric Field Mill

Although not commonly used, the cylindrical field mill has the advantage that when mounted in a fixed position it can also indicate the direction of the field. This is accomplished by measuring the phase shift of the ac signal relative to the orientation of electric fields needed to be measured. The rotating shutter type field mill has to be pointed towards the source of the field in order to obtain a maximum reading (Fig. 6.8). This feature is useful when searching for static charges that might cause problems in the laboratory or industrial plant.

6.4 Insulation Resistance Tester (Meggar)

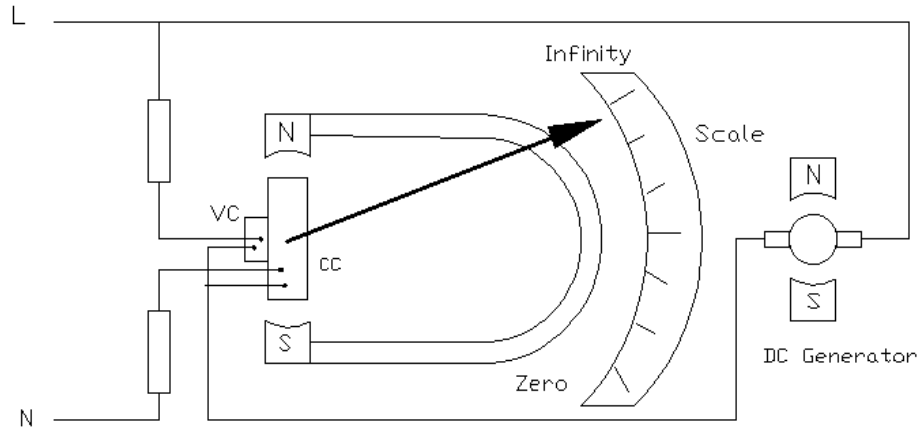


Fig. 6.9 The Insulation Resistance Tester (Meggar)

The insulation resistance tester usually known as meggar consist of a hand driven dc generator, which circulates current through a moving coil instrument when the handle is turned. The voltage coil is connected across the dc generator and the current coil is connected in series with the external resistance which will be under test.

Both coils are fitted on a common steel spindle to which the pointer is attached.

- i) When the external circuit is opened during the test i.e. (when no resistance is connected) only the voltage coil is energized. The field of this coil lines up with that of the permanent magnet. Thus giving infinity reading when the external circuit is not closed.
- ii) If a resistor is connected in the external circuit, current flows through both coils. The interaction between the fields of both coils causes the pointer to move towards zero position.

Assignment:

1. Students should present short notes on tachometers and tachogenerators and clearly indicate the difference between them.
2. Stroboscopes and explain briefly their areas of application.

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