2019品管第一次期中考答案

1.

(a) $\bar{\bar{X}} = \frac{\sum \bar{X}}{m} = \frac{157.85175}{35} = 4.51005$

$\bar{R} = \frac{\sum R}{m} = \frac{2.18750}{35} = 0.0625$

$\hat{\sigma}_x = \frac{\bar{R}}{d_2} = \frac{0.0625}{2.326} = 0.02687016$

$$\bar{X}\ Chart: \begin{cases} UCL = \bar{\bar{X}} + \frac{3\left(\frac{\bar{R}}{d_2}\right)}{\sqrt{5}} = 4.51005 + \frac{3(0.02687016)}{\sqrt{5}} = 4.5461 \\ CL = \bar{\bar{X}} = 4.51005 \\ LCL = \bar{\bar{X}} - \frac{3\left(\frac{\bar{R}}{d_2}\right)}{\sqrt{5}} = 4.51005 - \frac{3(0.02687016)}{\sqrt{5}} = 4.474 \end{cases}$$

$$R\ Chart: \begin{cases} UCL = \bar{R} + 3d_3\left(\frac{\bar{R}}{d_2}\right) = 0.0625 + 3(0.864)(0.02687016) = 0.132147 \\ CL = \bar{R} = 0.0625 \\ LCL = \bar{R} - 3d_3\left(\frac{\bar{R}}{d_2}\right) = 0.0625 - 3(0.864)(0.02687016) = -0.00714 \\ \text{(負不合)} \therefore LCL = 0 \end{cases}$$

(b) $P\left(LSL < X < USL \mid \hat{\mu} = \bar{\bar{X}} = 4.51005,\ \hat{\sigma}_x = 0.02687016\right)$

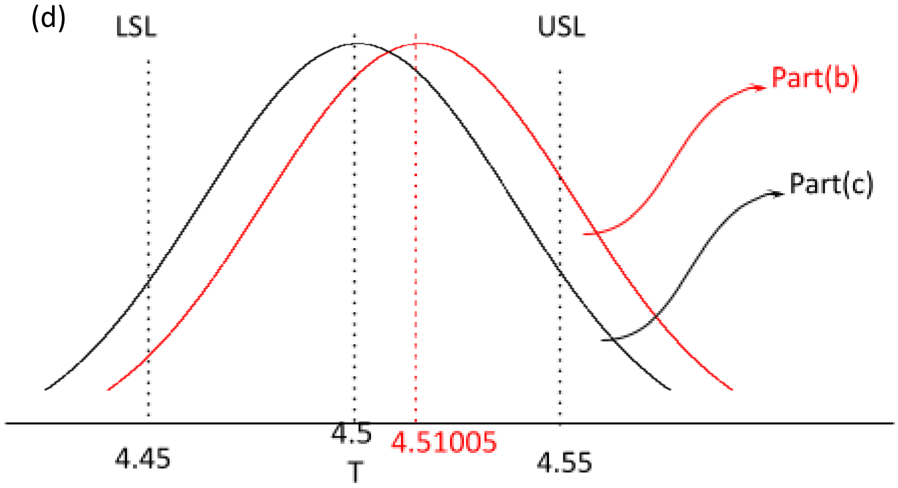
$= P\left(\frac{4.45-4.51005}{0.02687016} < Z < \frac{4.55-4.51005}{0.02687016}\right)$

$= P(-2.234821 < Z < 1.486779) = 0.9187 \approx 0.919 = 91.9\%$

(c) $P\left(LSL < X < USL \mid u = T = 4.5,\ \hat{\sigma}_x = 0.02687016\right)$

$= P\left(\frac{4.45-4.5}{0.02687016} < Z < \frac{4.55-4.5}{0.02687016}\right) = 0.93722 \approx 0.9372 = 93.72\%$

(d)



2. n=4, N=25

(a) $A_3 = 1.628,\ B_4 = 2.266,\ \bar{\bar{X}} = \frac{13050}{25} = 522,\ \bar{S} = \frac{660}{25} = 26.4$

$$\bar{X}\ Chart: \begin{cases} UCL = CL + A_3\bar{S} = 522 + 1.628 \times 26.4 = 564.9792 \\ CL = 522 \\ LCL = CL - A_3\bar{S} = 522 - 42.9792 = 479.0208 \end{cases}$$

$$S\ Chart: \begin{cases} UCL = B_4\bar{S} = 2.266 \times 26.4 = 59.8224 \\ CL = 26.4 \end{cases}$$

$LCL = 0$

$\hat{\sigma} = \frac{26.4}{C_4} = \frac{26.4}{0.921} = 28.66$

(b) $P(x < 450) = P\left(Z < \frac{450-522}{28.66}\right) = P(Z <- 2.51221) = 0.5 - 0.494 = 0.006$

3. μ $and\ \sigma\ are\ known,\ n = 2$

$$\bar{X}\ Chart: \begin{cases} UCL = \mu_0 + \frac{3\sigma_0}{\sqrt{n}} \\ CL = \mu_0 \\ LCL = \mu_0 - \frac{3\sigma_0}{\sqrt{n}} \end{cases}$$

(a)$Power = 1 - \beta = 1 - P(\mu_0 - \frac{3\sigma_0}{\sqrt{n}} \le \bar{X} \le \mu_0 + \frac{3\sigma_0}{\sqrt{n}} \mid \mu_1 = \mu_0 - \sigma_0)$

$= 1 - P\left(\frac{\left(\mu_0 - 3\frac{\sigma_0}{\sqrt{n}}\right) - \left(\mu_0 - \sigma_0\right)}{\frac{\sigma_0}{\sqrt{n}}} \le Z \le \frac{\left(\mu_0 + 3\frac{\sigma_0}{\sqrt{n}}\right) - \left(\mu_0 - \sigma_0\right)}{\frac{\sigma_0}{\sqrt{n}}}\right)$

$= 1 - P\left(- 3 + \sqrt{n} \le Z \le 3 + \sqrt{n}\right)$

$= 1 - P\left(- 3 + \sqrt{2} \le Z \le 3 + \sqrt{2}\right)$

$= 1 - P(- 1.585 \le Z \le 4.41) = 1 - 0.943601 = 0.05639 = 5.639\%$

(b)n=4

$Power = 1 - \beta = 1 - P\left(- 3 + \sqrt{4} \le Z \le 3 + \sqrt{4}\right)$

$= 1 - P(- 1 \le Z \le 5) = 1 - 0.8413445 = 0.1586555 \approx 15.865\%$

n=8

$Power = 1 - \beta = 1 - P\left(- 3 + \sqrt{8} \le Z \le 3 + \sqrt{8}\right) = 0.4318867 \approx 43.188\%$

(c)**當n愈大，偵測到製程平均偏移的機率愈大**

$\Rightarrow as\ \beta = type\ II\ error\ decrease,\ power = 1 - \beta\ increase$

4.

(a) $\bar{X} = 64.75,\ \sigma = 0.15$

$P(64.775 \le \bar{X} \le 65.225) = P\left(\frac{64.775-64.75}{\frac{0.15}{\sqrt{4}}} \le Z \le \frac{65.225-64.75}{\frac{0.15}{\sqrt{4}}}\right)$

$= P(0.33 \le Z \le 6.33) = 0.5 - 0.1293 = 0.3707$

(b) $P(x > 65.5) + P(x < 64.5) = P\left(Z > \frac{65.5-64.75}{0.15}\right) + P\left(Z < \frac{64.5-64.75}{0.15}\right)$

$= P(Z > 5) + P(Z <- 1.67) = 0.5 - 0.4525 = 0.0475$

(c) $65.5 - 64.5 = 1$

$C_p = \frac{1}{6\times0.15} = 1.11$

$C_{pk} = \left\{\frac{64.75-64.5}{3\times0.15},\ \frac{65.5-64.75}{3\times0.15}\right\} = \{0.56,\ 1.67\} = 0.56$

$or\ \left|C_a\right| = \left|\frac{64.75-65}{0.5}\right| = 0.5\ ,\ C_{pk} = (1 - 0.5)\times1.11 = 0.55$

5.

(a)$C_{pk} = \left[\frac{USL-\bar{X}}{3\sigma},\ \frac{\bar{X}-LSL}{3\sigma}\right] \ge 1.25$

$\frac{0.0015}{3\sigma} \ge 1.25,\ \sigma \le \frac{0.0005}{1.25} = 0.0004 = \frac{1}{2500}$

(b)$\frac{0.001}{3\sigma} \ge 1.25,\ \sigma \le \frac{0.001}{3\times1.25} = 0.000267 = \frac{1}{3750}$

(c) $C_p = \frac{USL-LSL}{6\sigma} \ge 1.25,\ \sigma \le \frac{0.003}{6\times1.25} = 0.0004 = \frac{1}{2500}$

$\bar{\bar{X}} = 0.301\ ,\ and\ \bar{\bar{X}} = 0.298$ 沒影響

(d)C_p指標不會被平均偏移影響

6.

(a) $P(X = 0) = \frac{\binom{100}{0}\binom{900}{25}}{\binom{1000}{25}} = 0.069396$

(b) $p = 0.1$

$P(X = 0) = \binom{25}{0}(0.1)^{0}(0.9)^{25} = 0.0717898$

(c) $\lambda = np = 25\times 0.1 = 2.5$

$P(X = 0) = \frac{2.5^{0}e^{-2.5}}{0!} = e^{-2.5} = 0.082085$