Forward Modelling for

Gravity Anomalies

GOPH 547 L01

Brandon Karchewski

Safian Omar Qureshi

ID 10086638

Contents

Abstract

The purpose of this exercise is to investigate the relationship between mass and distance via gravitational potential (U) and gravitational effect (gz) formulae. Essentially, forward modelling was performed to understand the relationship of U and gz being dependent on the mass of the anomaly within the subsurface and the distance between the observer and anomaly. The gravity effect was also approximated using the first partial derivatives and tested against calculated results. One underlying goal of this lab is to raise awareness of the problem of non-uniqueness within gravity surveys, and ultimately non-uniqueness within the field of geophysics.

Background and Theory

The following equations formed the foundations of this exercise and were employed as functions in the Matlab implementation;

$$U(r) = \frac{Gm}{r} \qquad (1)$$

$$g_z = -\frac{\partial U}{\partial z} = Gm\frac{z - z_m}{r^3} \qquad (2)$$

representing gravity potential and gravity effect respectively. Contributed by a number of physicists, most notably Newton in his *Principa Mathematica* (1687), the first equation, derived from Newton's more general force of attraction between two objects, explains the work done against gravity to bring a mass to a given point in space (a space experiencing a gravitational field). The second equation describes the gravity effect, the component of g that is directed toward the Earth's centre of mass, deriving from taking the negative gradient of the gravitational potential (U).

Some of the assumptions made in the lab that are worth noting are deriving equations in Cartesian coordinates. It is important to note there are other reference frames, such as spherical or cylindrical, but the Cartesian frame was used in the lab. The exercise also dealt with scaler potential, assuming it is constant in the radial direction. Point masses were also used in questions 3 and 4 which do not occur in the natural world. The boundary condition was a 'homogenous' half space with density anomalies embedded, as opposed to multiple different layers within subsurface with density anomalies embedded.

Elaborating more on point masses previously mentioned, the derivation of an arbitrary body was necessary in the later questions of the lab that is worth going over. Further equations such as the following;

$$dU = \frac{Gdm}{r} \tag{3}$$

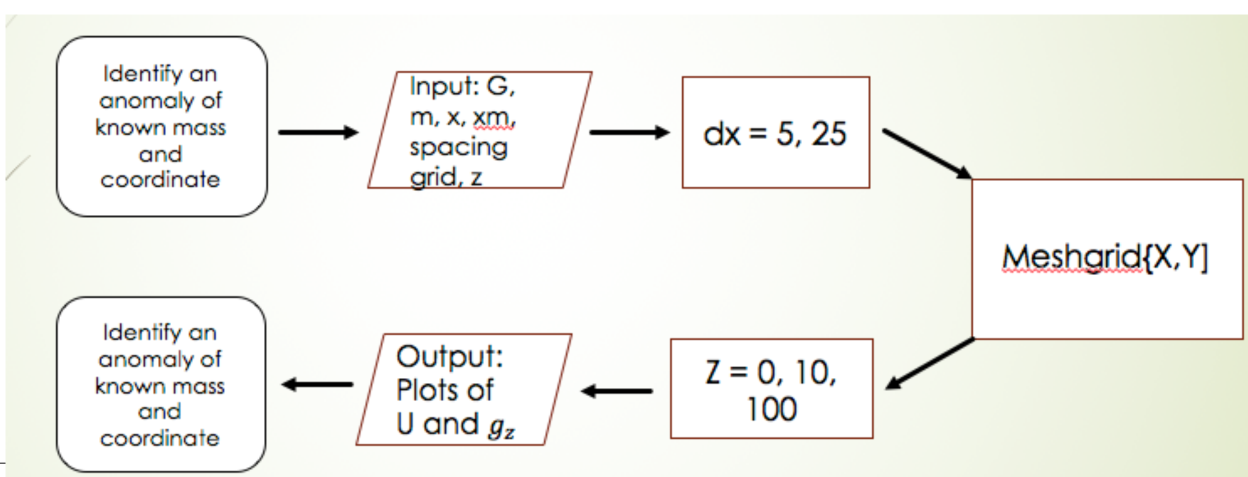
$$dm = \rho dV \tag{4}$$

were needed where equation 3 gives differential potential (dU) resulting from a differential mass (dm). The differential mass is then given by equation 4, where ρ is density and dV is the differential volume. In most cases, density is a function of position but for simplicity, the density of the anomaly is assumed to be a constant average value. The differential potential arising from an irregularly shaped anomaly is thus only dependent on the differential volume, which is dependent on the mass distribution of the anomaly[1]. With the background and theory reviewed, a Matlab code is constructed employing the fundamentals to perform forward modelling.

Method and Algorithm

To begin Matlab exploration, defining the equations as Matlab functions was necessary in order to employ them in our algorithm. This was done quite simply by defining two separate Matlab functions for gravitational potential and gravitational effect by inputting our variables, arranging them into equations and saving the 'package' as a function.

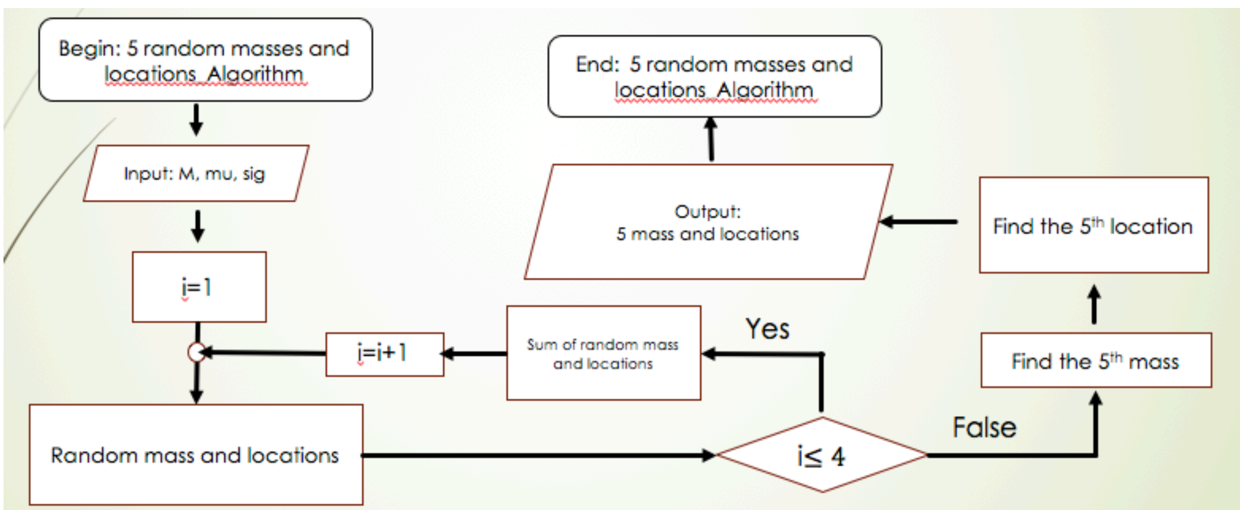
In question 3, certain aspects are given about the anomaly and the spacing interval which are stored as variables. The core of the algorithm, which was heavily used throughout the lab, is loosely defined by the following chart;



[1] Karchewski, B. (2017). GOPH 547 - Gravity and magnetics. Retrieved from https://d2l.ucalgary.ca/

To go over it fairly briefly as the code has comments itself, a grid is created of x and y coordinates representing columns and rows. A for loop is employed that goes overs a depth specified which employs the created functions to return newly created variable of gravity potential and effect. These newly created variables are then used to get min and max which create a color bar that is later used by the plot to help visualize the gravity anomaly within the subsurface. An identical algorithm is used for varying survey spacing.

In question 4, sets of masses are to be generated using an algorithm of itself, satisfying certain constraints. The following flowchart loosely represents the algorithm employed, with further comments in the code itself;



After defining the constraints given, the Matlab function 'rand' generates four random values that the constraints are applied to, stored as variables. The fifth value is calculated which when combined, generates the first mass set. Additional 2 mass sets are also generated using the same algorithm. Later on, the exact positions of these 5 mass points are determined by knowing the original centre of mass position. a for loop is employed in a manner very similar to the previous equation but we instead sum up the total potential and effect for the location. Eventually, further plots are created to visualize the gravity potential and effect of the 5 mass point anomalies.

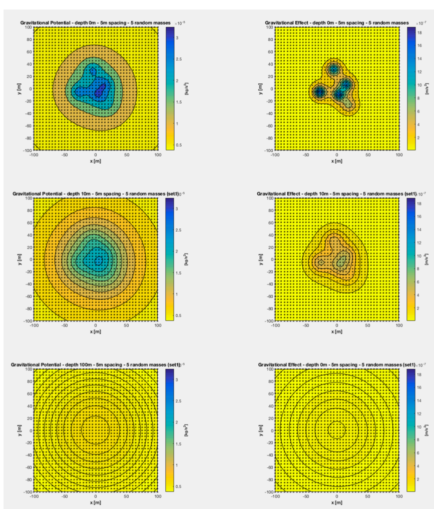
For the last question we used Matlab to perform forward modelling to simulate survey results for an anomaly with randomly distributed density. This would simulate a case which would be more applicable to the natural world as there are no individual point source masses within the subsurface in reality. An anomaly loaded given file was used to described a 'cavity' with randomly distributed density. To visualize the distribution of the density within the region, multiple cross-sections were produced. The given mass was then integrated by looping over i, j and k indices to determine the total mass of the anomaly by the density-volume-mass relation, using the 2x2x2 cube given fact in the exercise. Later on in question 5, similar procedures as in 4 were performed with this anomaly to simulate gravitation surveys with varying depth and grid spacing.

A section in this question that is worth going over in a little more detail was when we approximated the first and second partial derivatives by the Finite Difference Method. Survey results at z=1m and z=100m were used in conjunction to estimate the first partial derivative. Similarly, the second derivative was to be generated using the Laplacian, using z=0 and z=100 (although I, in specific, was unable to evaluate the second partial derivative with respect to z).

Once the method and algorithm was employed in Matlab, it produced interesting resulting plots and images that help visualize the subsurface anomalies and how they varied with depth and grid spacing which is worth discussing.

Results and Discussion

It is fairly intuitive to expect that the survey results with minimal distance would produce the strongest potentials and effects observed, given our background equations and theory state that they are inversely proportional to distance . This indeed turned out to be the case where the potential and effects were strongest at the shallower depths, as seen in the following figures;



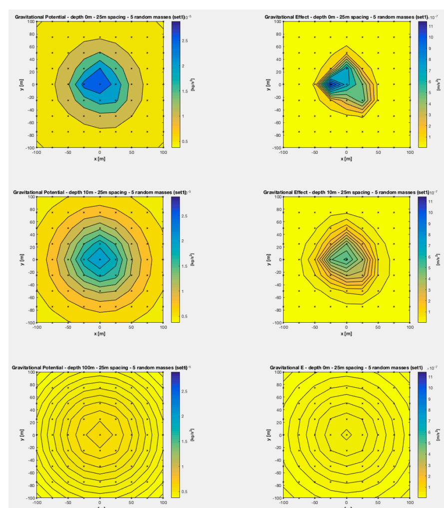


Figure set 1: grid spacing 5m **|varying elevations|** **Figure set 2: grid spacing 25m**

A clear demonstration of shallower depth detects the potential and effect as much stronger, while the closer grid spacing helps resolve the anomalies better. On the right hand side, an encounter with non-uniqueness in geophysical surveys is observed were difficulty in resolving masses is experienced.

A major assumption that should be again restated is that the anomalies in the figures generated in question 4 were treated as point sources. This is not the case in the physical subsurface of the Earth. Rather, anomalies will tend to have varying body dimensions with non uniform densities.

This is where question 5 required us to generate survey results for a given anomaly with randomly distributed density. The following figure set shows multiple density cross-sections cutting through the anomaly;

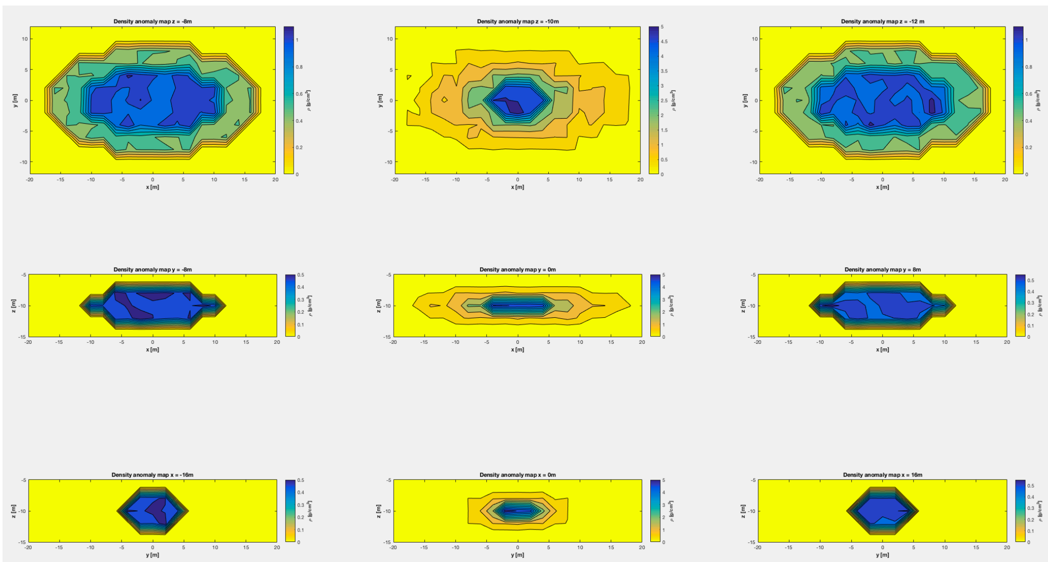
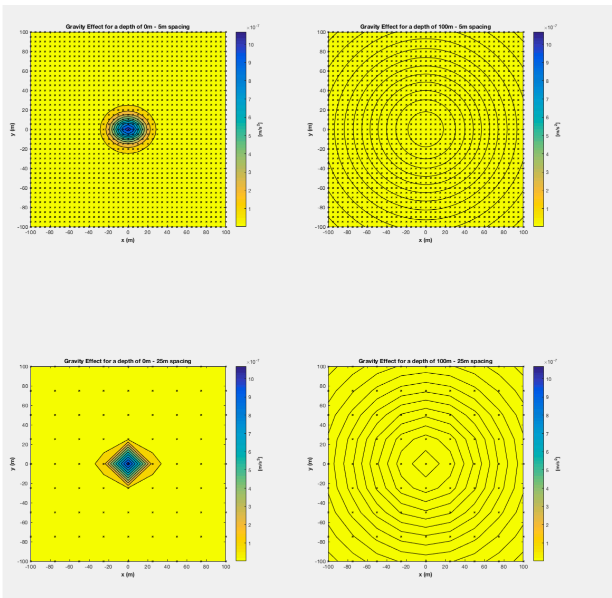


Figure set 3: random anomaly of varying density 'cut' at varying x, y and z planes

which correlates more to real world examples. Integration of the anomaly data was made by using nested for loops that gave the total mass of the anomaly (in specific, result being **$2.84*10^6$ g** after unit conversions).

The results for a gravity effect survey for this specific random density anomaly are shown in figure set 4. Figure set 5 then shows the gravity effect approximations using the first partial derivative between z=0 and z=1m and between 100m and 110m. The general shape and magnitude of the anomaly is very similar between these figure sets, implying that the first partial derivative approximation may be a good estimate of the gravity effect for the anomaly



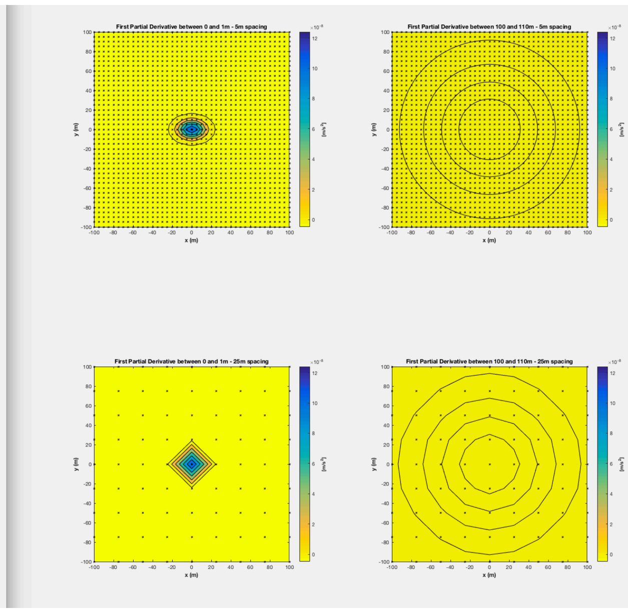


Figure set 4: gravity effect at varying depth, grid 5 **Figure set 5: first derivative approximation, grid 5**

Conclusion

To wrap up, the exercise required to perform forward modelling for gravity anomalies that varied in depth and innate property. Differing mass distribution parameters were given to observe how these variables can affect the gravity potential and gravity effect a gravimeter would experience. Varying survey grids were employed to help visualize how changing grid spacing can affect the perceived shape a geophysicist may encounter.

The magnitude of both U and gz was strongest for the shallowest depth (or lowest elevation) and barely detectable when the depth was 100m, implying any higher elevation of airborne survey would be quite meaningless. The anomalies are detectable for depths of 10m or less for either small or largely spaced grids though it is evident the smaller grid produces increasingly accurate results with respect to the exact geometry of the anomalies. The meshgrid spacing in Matlab was an extremely crucial parameter as it is the defining aspect of trying to resolve multiple closely spaced anomalies. The 5 separate mass sets which were generated in question 4 appeared as one ambiguous anomaly for all the plots produced with the larger spacing, which gives rise to the problem of non-uniqueness.

We then dealt with a more physical world example where an anomaly with an unevenly distributed density was given via the anomaly data set file. This contrasted with our previous assumption that treated them as point mass sources. Various density cross-sections through the anomaly were produced to observe the general geometry of the anomaly. Further, the gravity effect was approximated using first partial derivatives using Finite Difference Method (FDM). The approximation that was produced very workable results but with slightly lower resolution then the calculated results.

Ultimately, an underlying goal of this lab was to help raise awareness on the problem of non-uniqueness within gravity surveys and to spark further thought into the concept of non-uniqueness within the field of geophysics as a whole.

References

[1,2]Karchewski, B. (2017). GOPH 547 - Gravity and magnetics. Retrieved from https://d2l.ucalgary.ca/d2l/le/content/169330/viewContent/2346958/View

W.M. Telford, L.P. Geldart and R.E. Sheriff. (1990). *Applied Geophysics, Second Edition*. Cambridge University Press.

"Newton's law of universal gravitation" from *Wikipedia: The Free Encyclopedia*. Link: https://en.wikipedia.org/wiki/Newton%27s_law_of_universal_gravitation.

"Gravitational potential" from *Wikipedia: The Free Encyclopedia*. Link: https://en.wikipedia.org/wiki/Gravitational_potential.