

Regularização

Estrutura

- Problemas Inversos
 - Introdução
- Sistemas lineares
 - Determinante $\neq 0$
 - Determinante $= 0$
 - Determinante ≈ 0
- Problemas lineares
- Problemas não-lineares
- Regularização

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1} \quad \bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
observados

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

Problema não-linear

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

Problema não-linear

Problemas Inversos

(Introdução)

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\begin{bmatrix} \bar{B} & \bar{B} \end{bmatrix}^T \begin{bmatrix} \bar{B} & \bar{B} \end{bmatrix} \right)^{-1} \begin{bmatrix} \bar{B} & \bar{B} \end{bmatrix}^T [\bar{d} - \bar{b}]$$

Problema não-linear

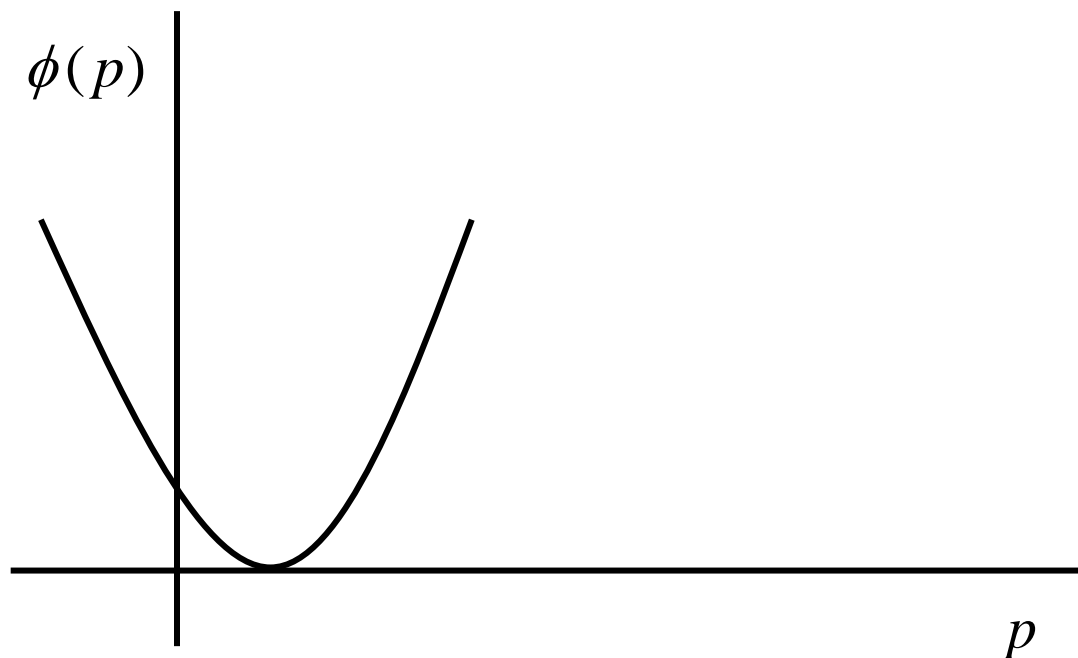
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\Delta \bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas Inversos

(Introdução)

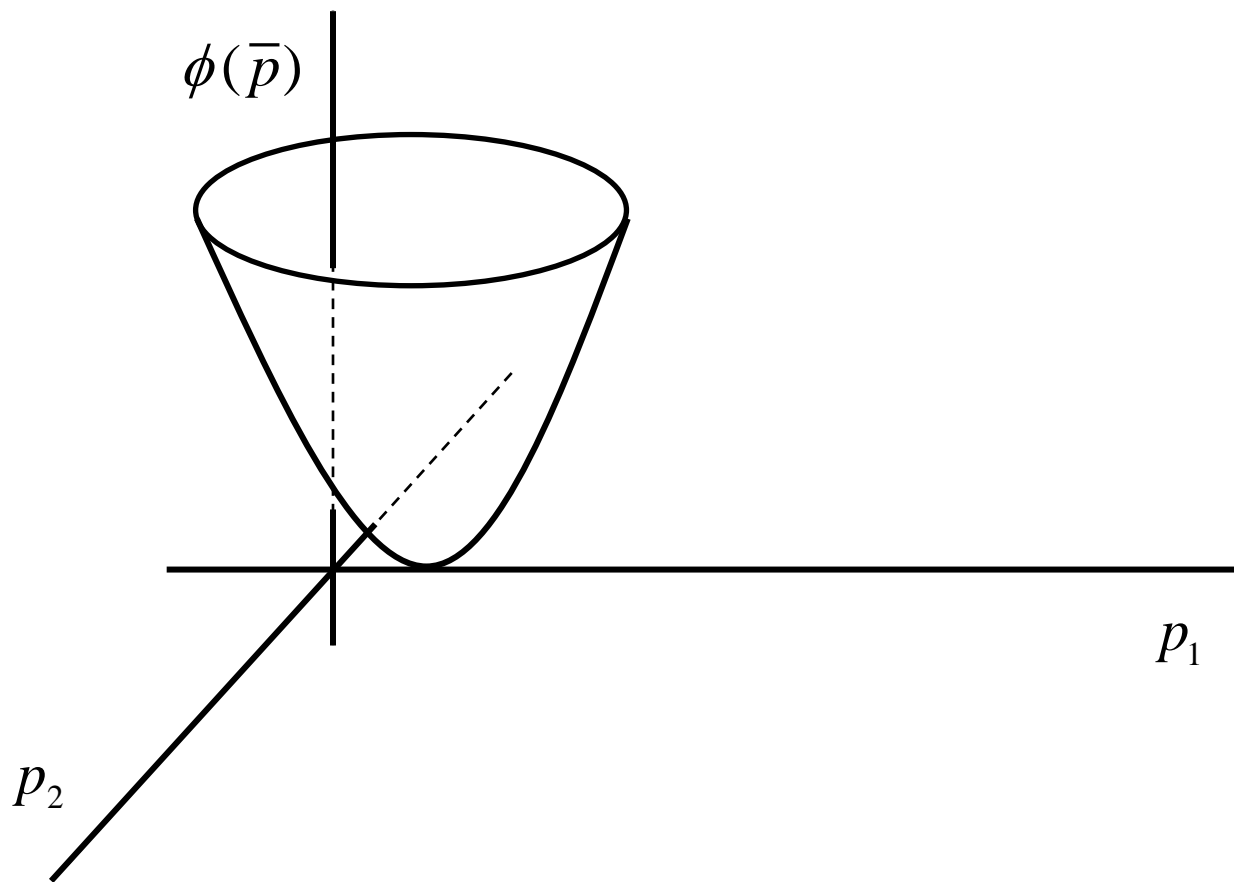
Exemplo
linear 1D



$$\phi(p) = [\bar{d} - \bar{g}(p)]^T [\bar{d} - \bar{g}(p)]$$

Problemas Inversos (Introdução)

Exemplo
linear 2D

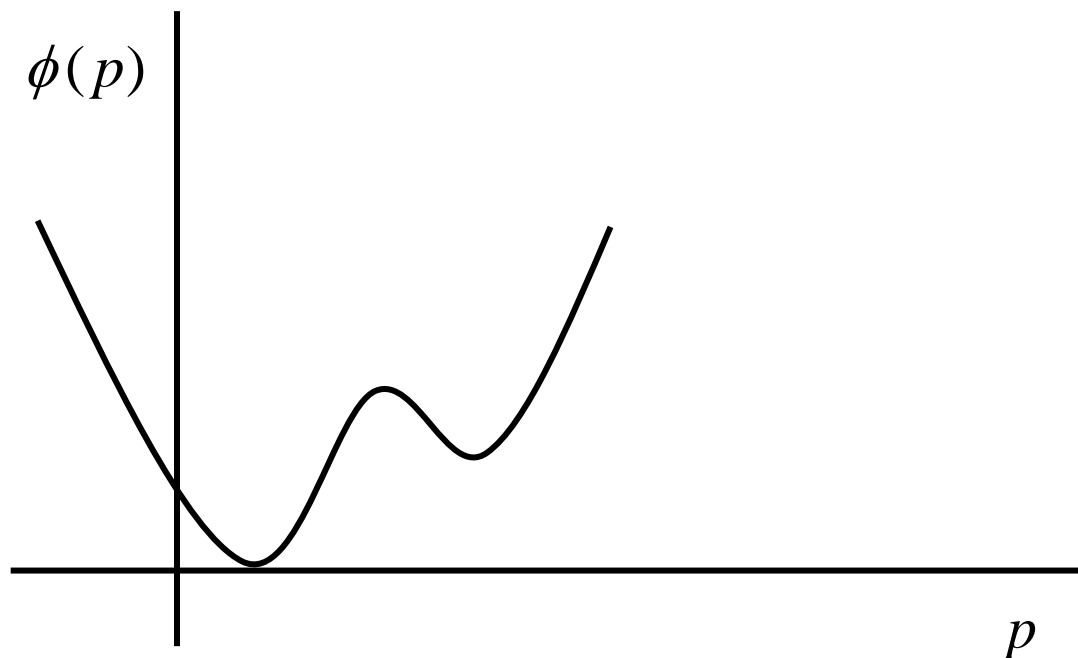


$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Problemas Inversos

(Introdução)

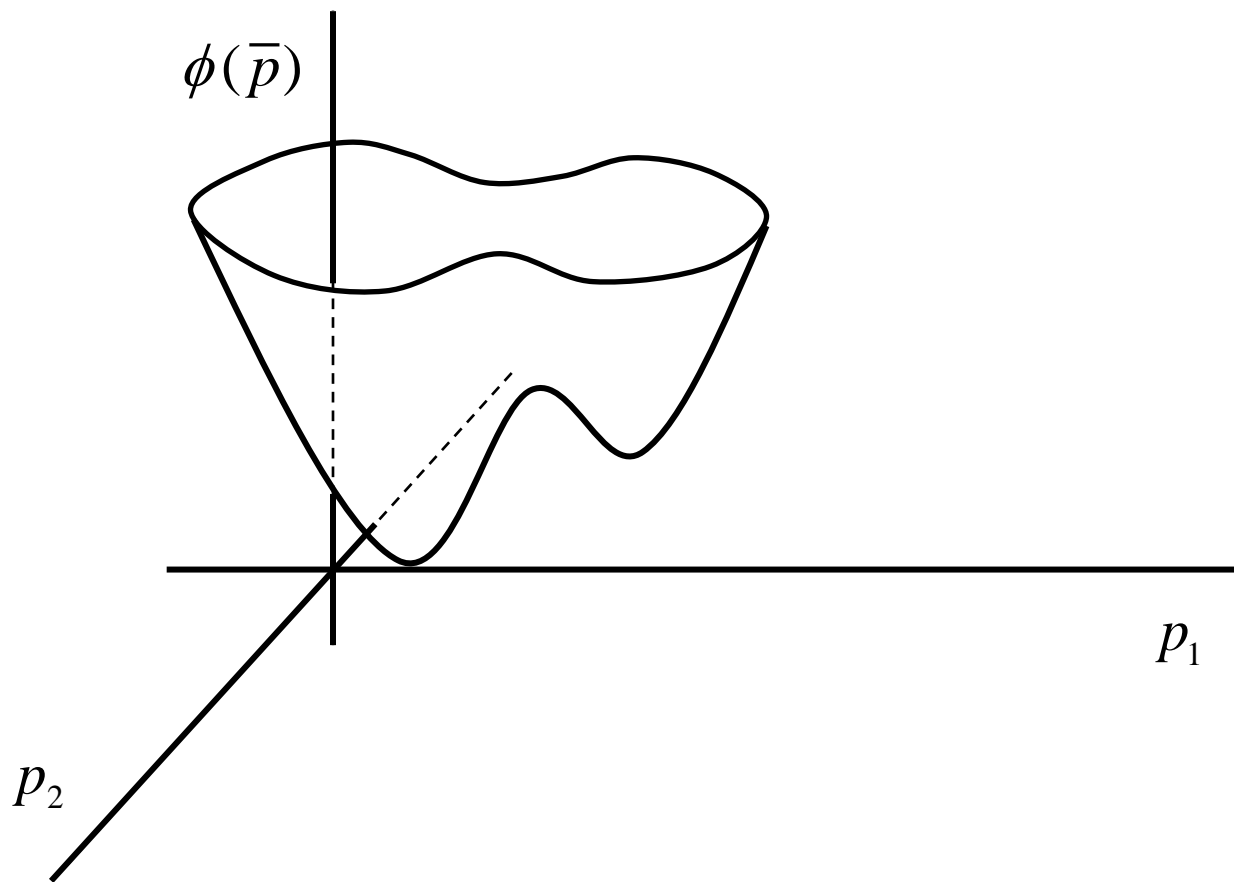
Exemplo
não-linear 1D



$$\phi(p) = [\bar{d} - \bar{g}(p)]^T [\bar{d} - \bar{g}(p)]$$

Problemas Inversos (Introdução)

Exemplo
não-linear 2D

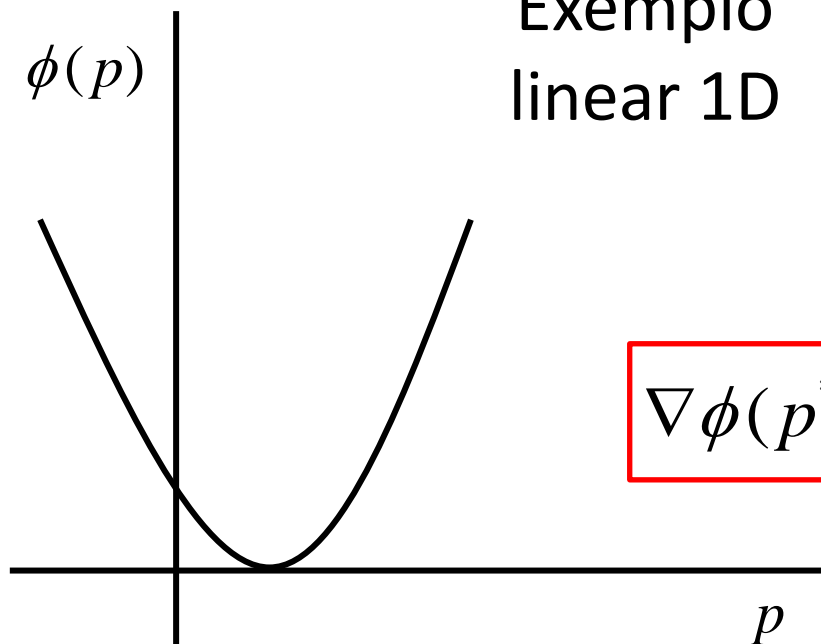


$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

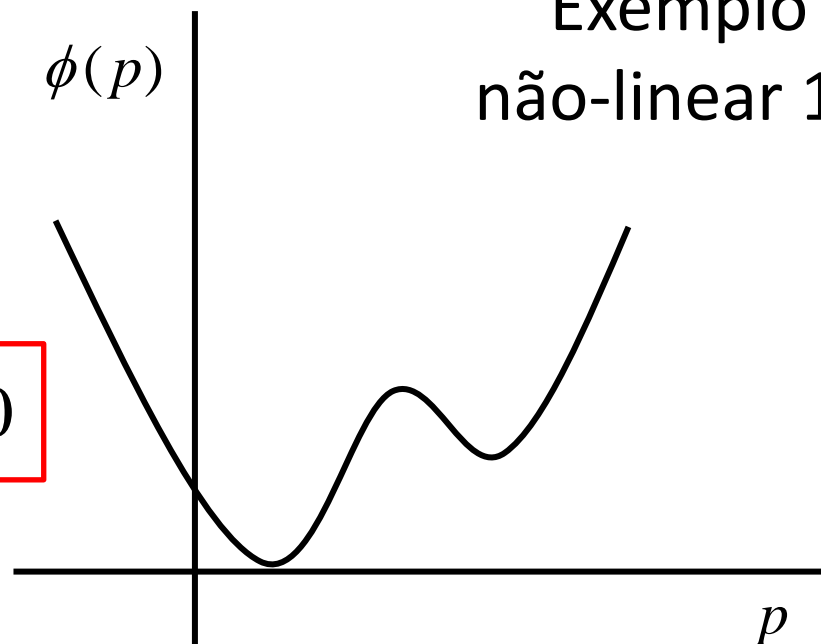
Problemas Inversos

(Introdução)

Exemplo
linear 1D



Exemplo
não-linear 1D

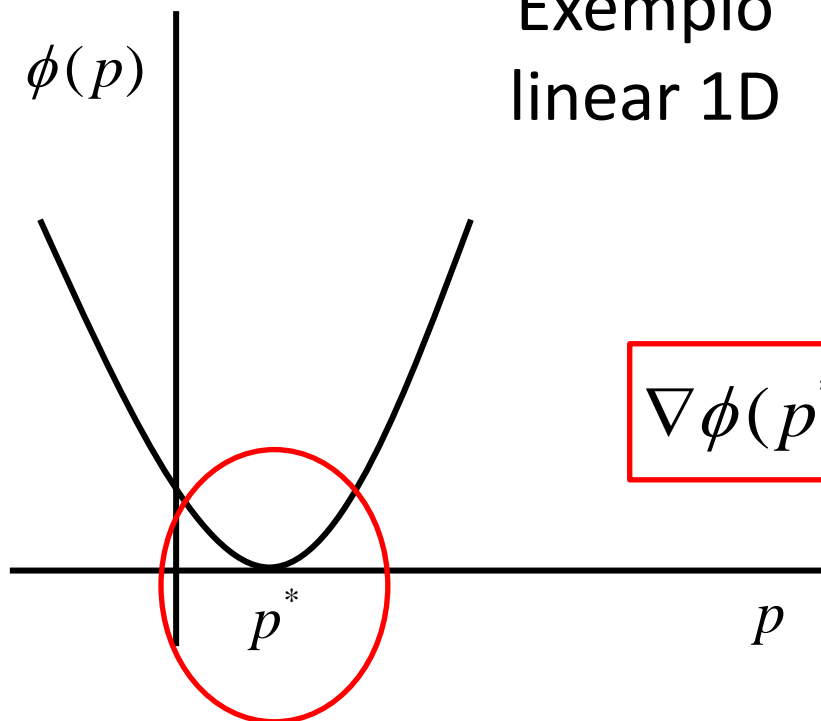


$$\nabla \phi(p^*) = 0$$

Problemas Inversos

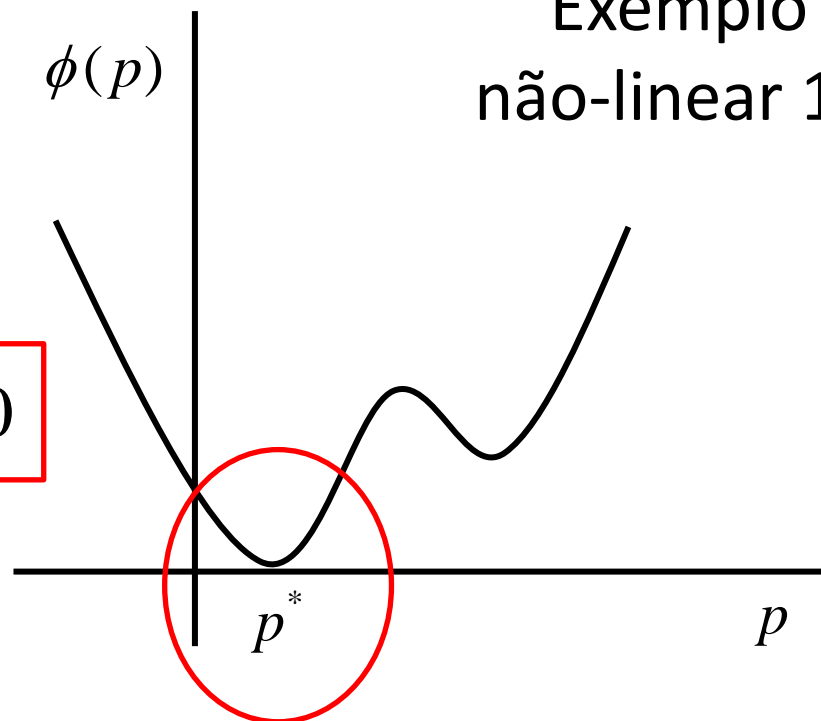
(Introdução)

Exemplo
linear 1D



$$\nabla \phi(p^*) = 0$$

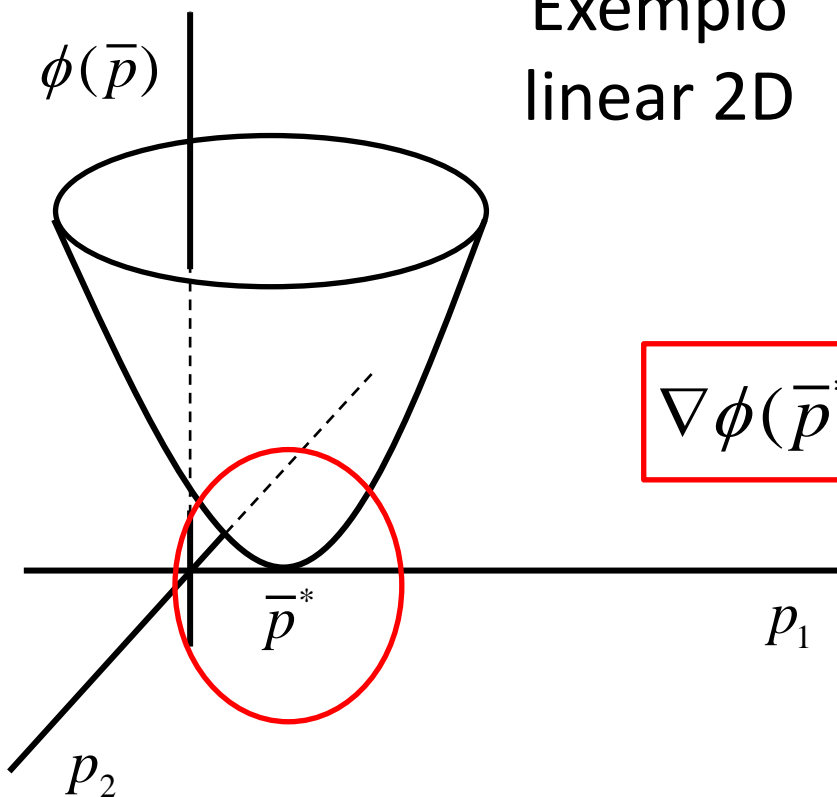
Exemplo
não-linear 1D



Problemas Inversos

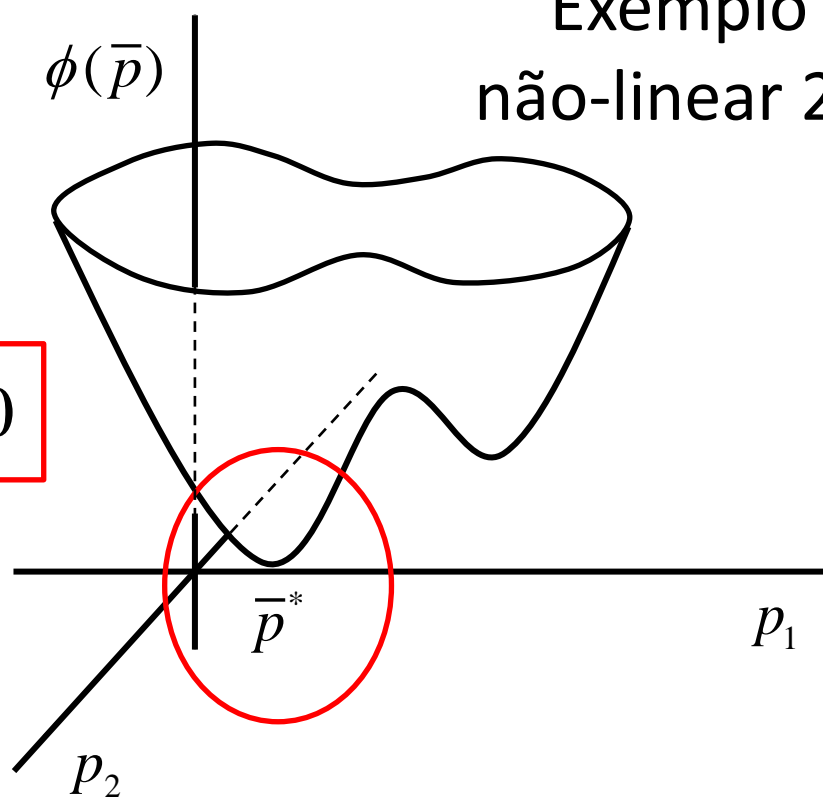
(Introdução)

Exemplo
linear 2D



$$\nabla \phi(\bar{p}^*) = 0$$

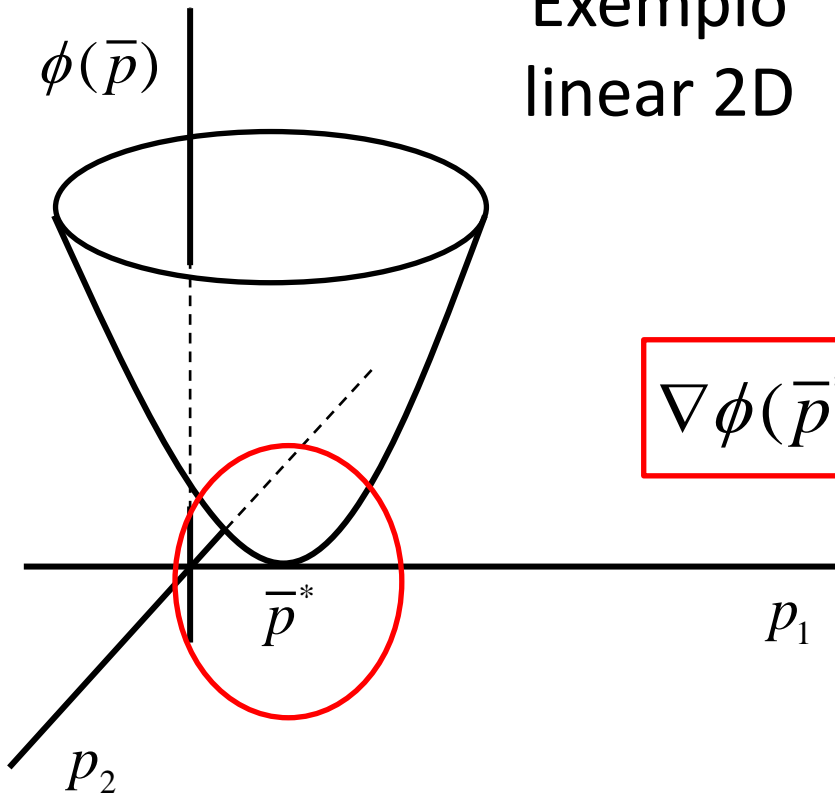
Exemplo
não-linear 2D



Problemas Inversos

(Introdução)

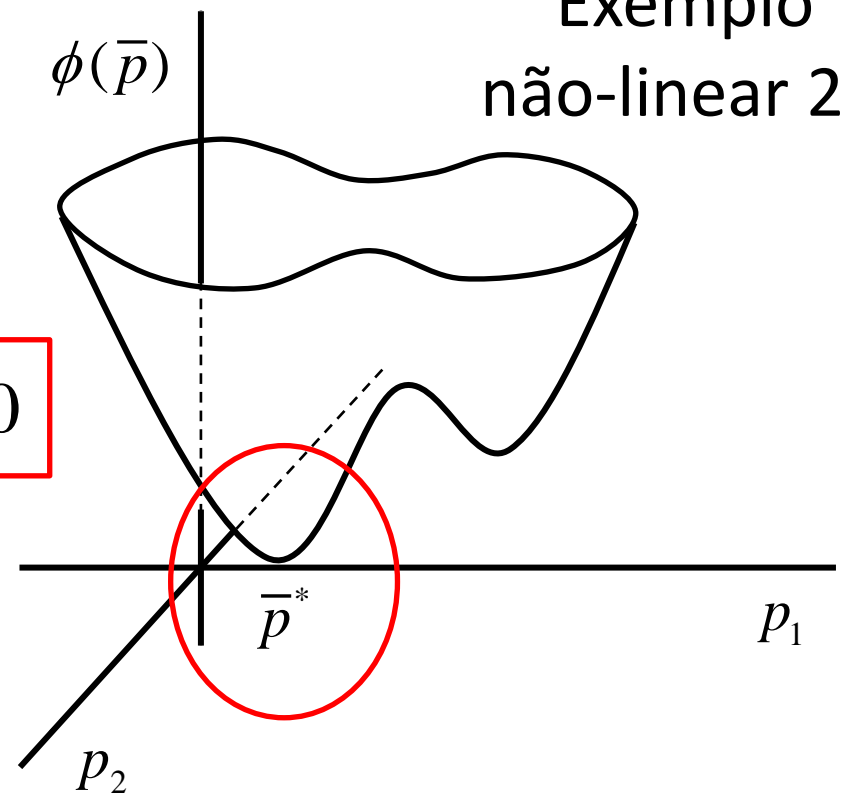
Exemplo
linear 2D



$$\nabla \phi(\bar{p}^*) = 0$$

$$\bar{p}^* = \begin{pmatrix} \overline{\overline{B}}^T & \overline{\overline{B}} \end{pmatrix}^{-1} \overline{\overline{B}}^T [\bar{d} - \bar{b}]$$

Exemplo
não-linear 2D

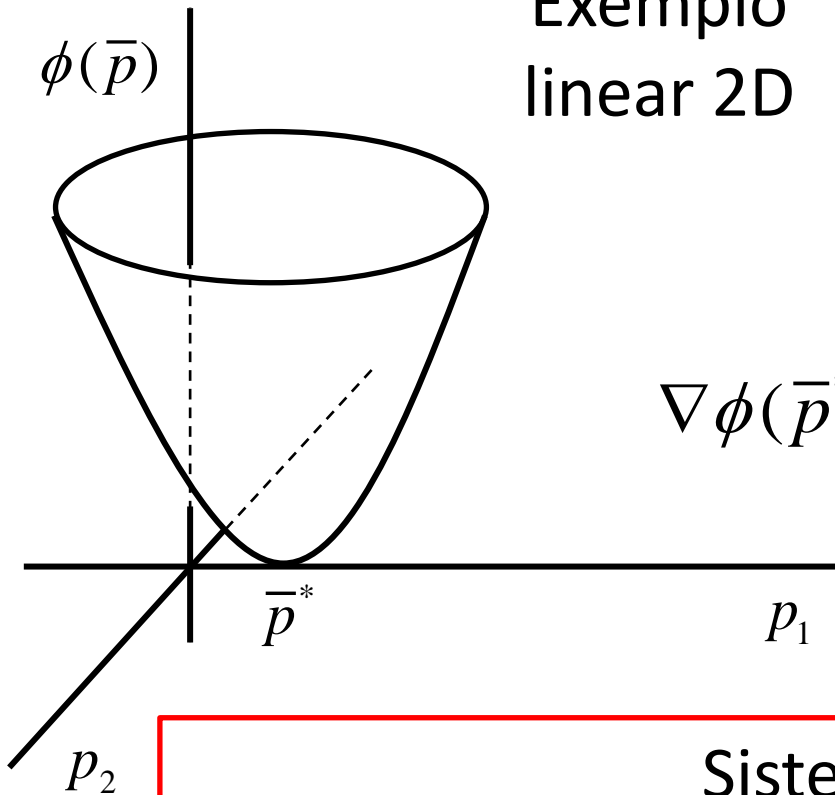


$$\Delta \bar{p} = \left(\overline{\overline{G}}(\bar{p}_0)^T \overline{\overline{G}}(\bar{p}_0) \right)^{-1} \overline{\overline{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas Inversos

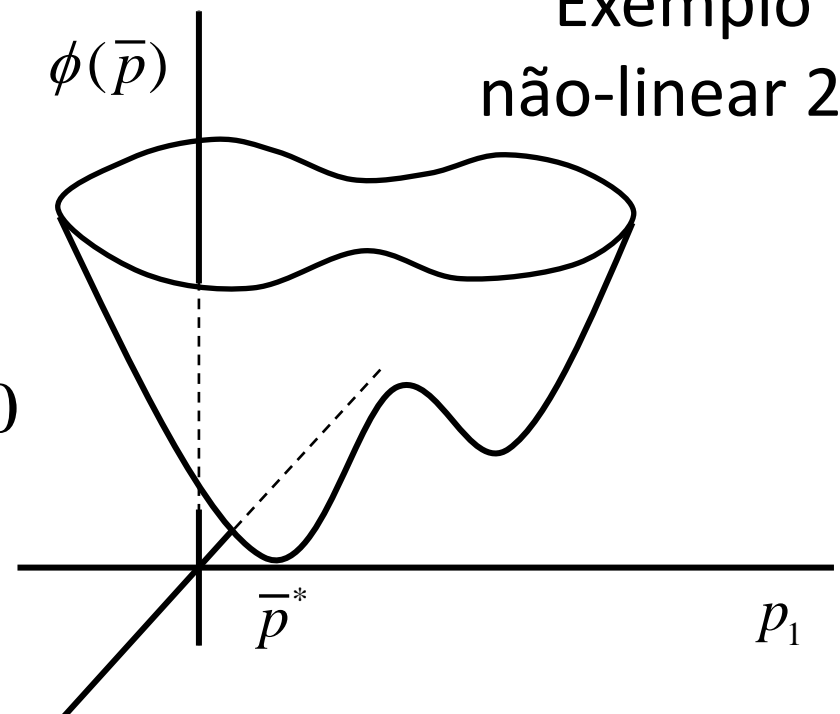
(Introdução)

Exemplo
linear 2D



$$\nabla \phi(\bar{p}^*) = 0$$

Exemplo
não-linear 2D



Sistemas lineares


$$\bar{p}^* = \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix}^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

$$\Delta \bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistemas lineares

$$\bar{b} = \begin{pmatrix} {}^T & {}^T \\ \mathbf{B} & \mathbf{B} \end{pmatrix}^{-1} {}^T \mathbf{B} \bar{r}$$

Sistemas lineares

$$\bar{b} = \begin{pmatrix} {}^T & {}^T \\ \mathbf{B} & \mathbf{B} \end{pmatrix}^{-1} {}^T \mathbf{B} \bar{r}$$


Sistemas lineares

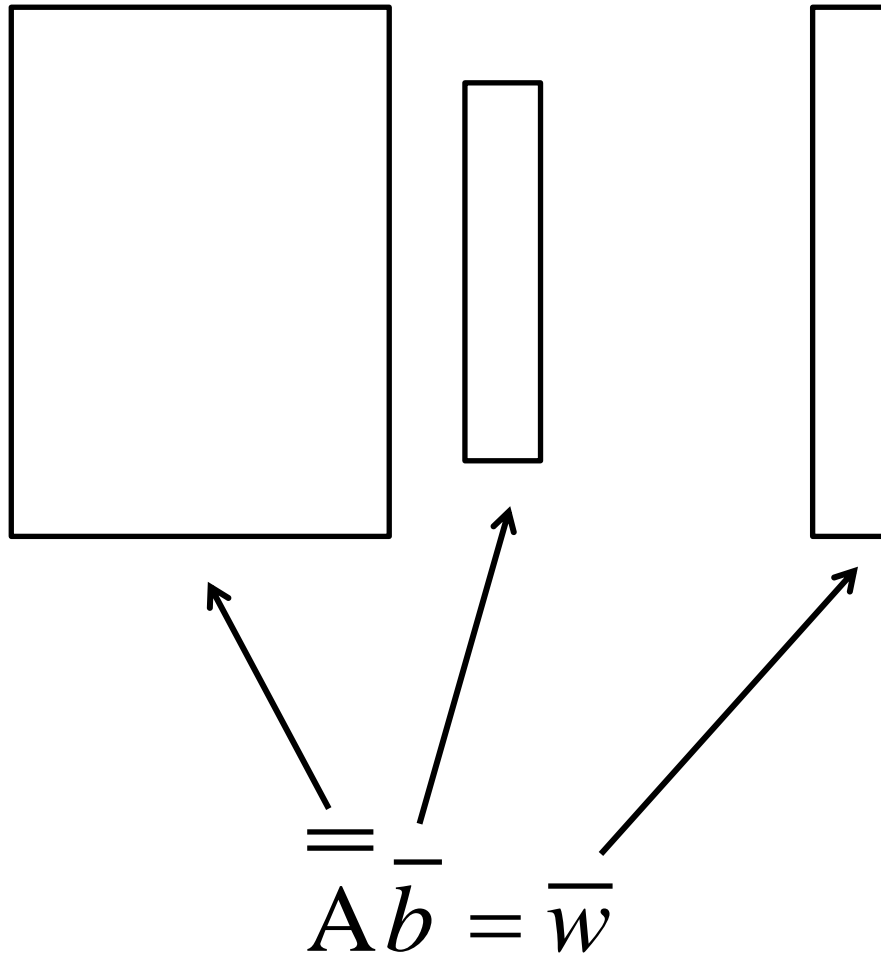
$$\begin{pmatrix} \mathbf{B} & \mathbf{B} \end{pmatrix} \bar{b} = \mathbf{B} \bar{r}$$

Sistemas lineares

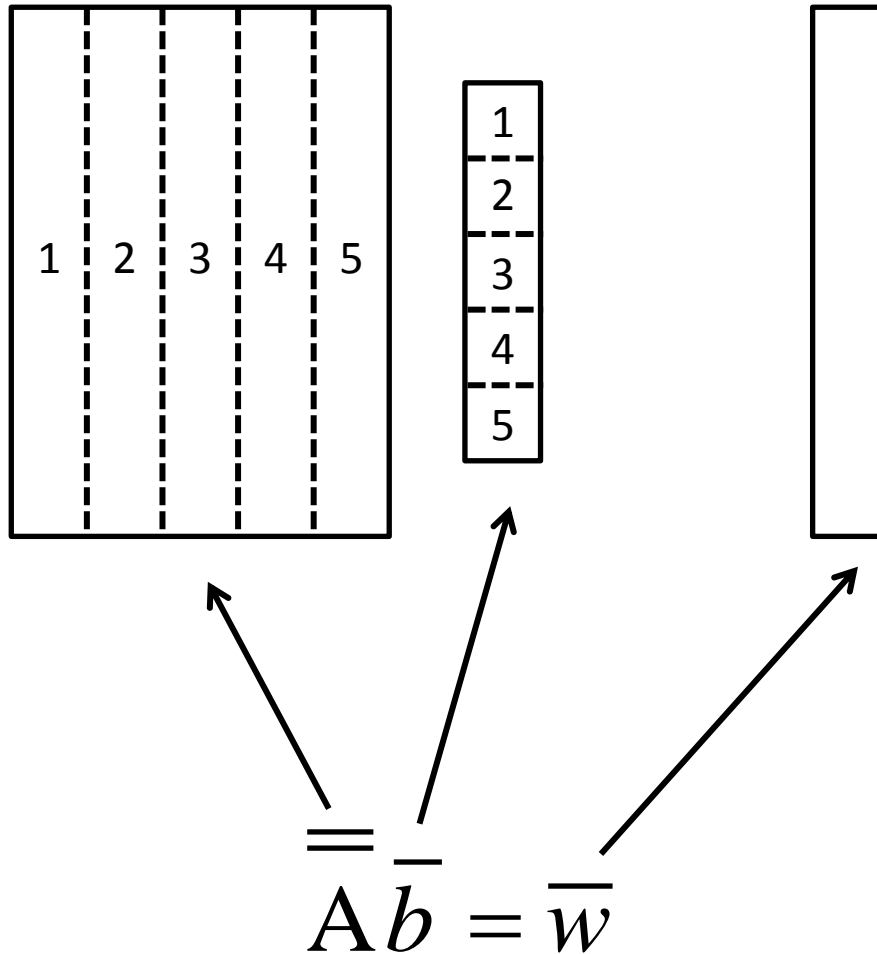
$$\underbrace{\begin{pmatrix} \overline{=T=} \\ \mathbf{B} & \mathbf{B} \end{pmatrix}}_{\downarrow} \overline{b} = \underbrace{\overline{=T=} \mathbf{B}}_{\downarrow} \overline{r}$$

$$\overline{A} \overline{b} = \overline{w}$$

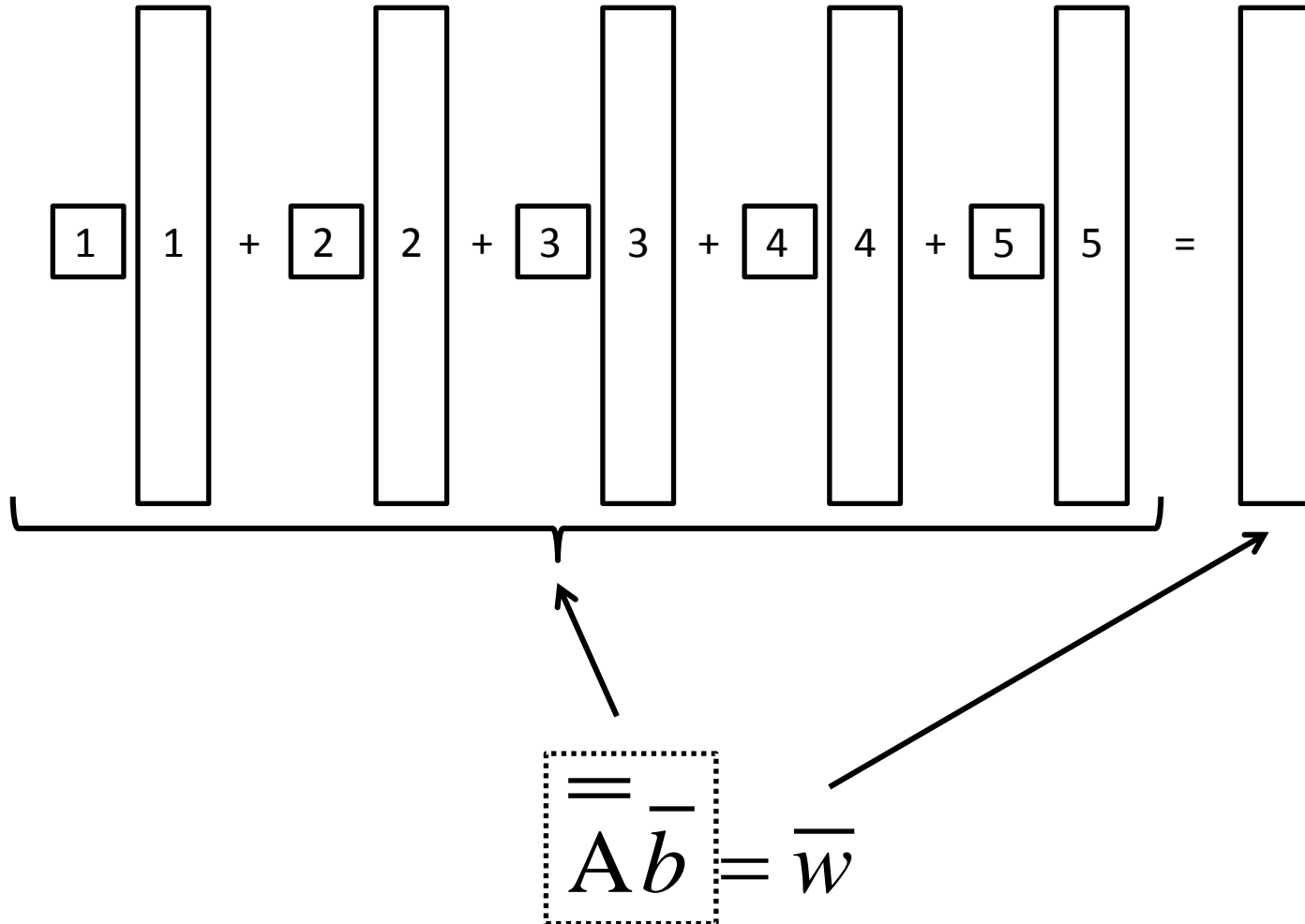
Sistemas lineares



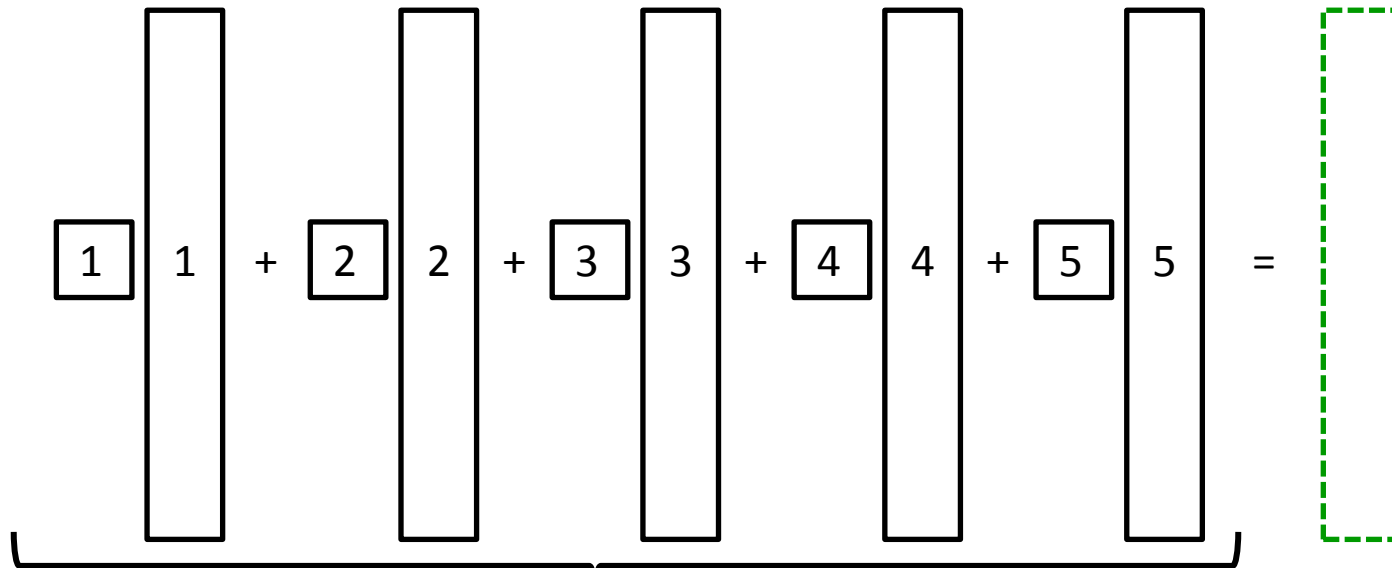
Sistemas lineares



Sistemas lineares



Sistemas lineares

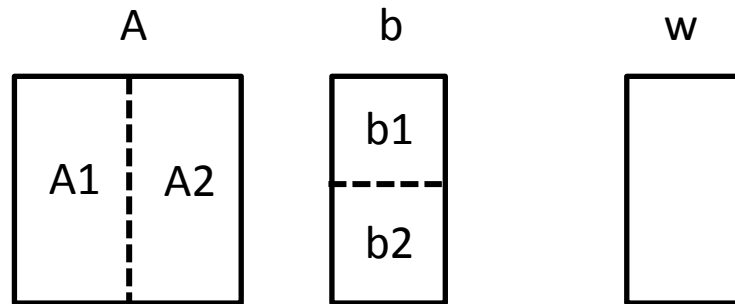


O **vetor do lado direito** é
uma *combinação linear* dos
vetores do lado esquerdo

$$\overline{A} \overline{b} = \overline{w}$$

Sistemas lineares

Exemplo 2D



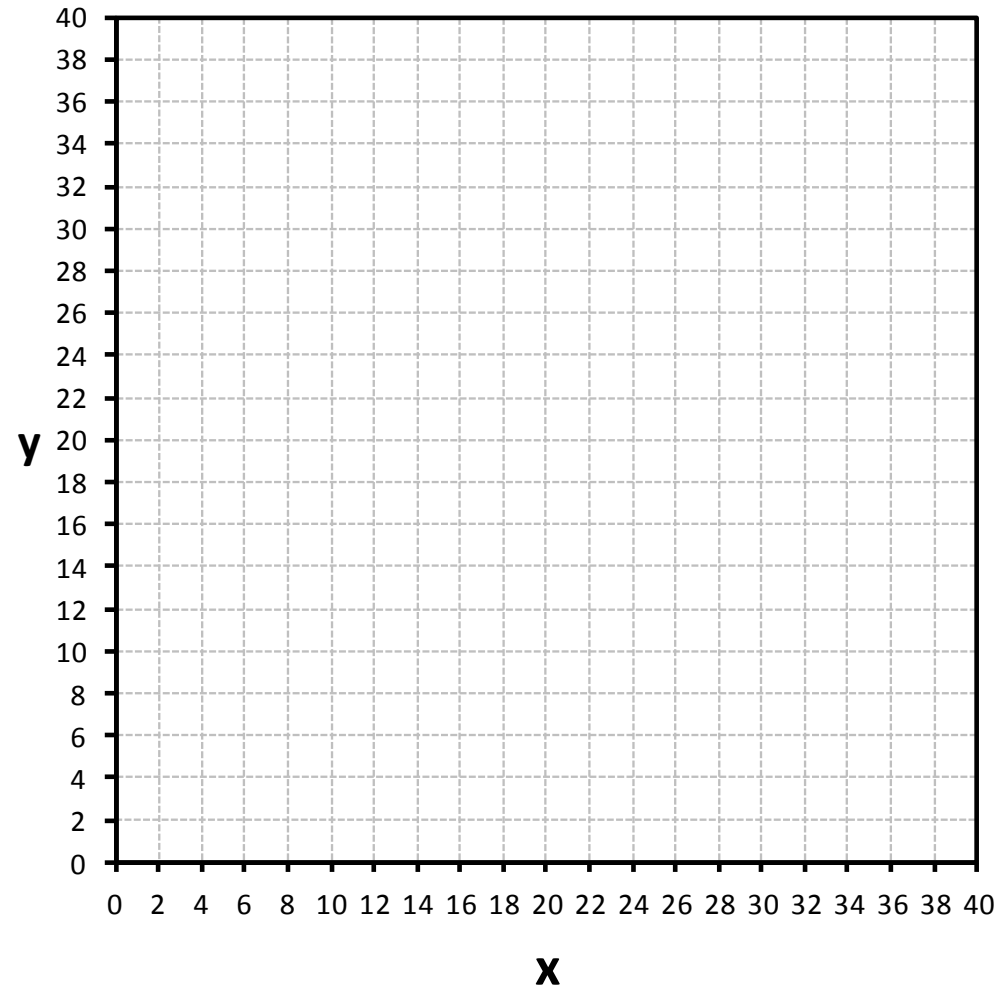
$$\begin{array}{|c|} \hline b1 \\ \hline \end{array} \begin{array}{|c|} \hline A1 \\ \hline \end{array} + \begin{array}{|c|} \hline b2 \\ \hline \end{array} \begin{array}{|c|} \hline A2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

The diagram shows the equation $b_1 A_1 + b_2 A_2 = w$. The vectors b_1 and b_2 are shown as small boxes, and A_1 and A_2 are shown as tall boxes. The result w is shown as a dashed green box.

Sistemas lineares

Exemplo 2D

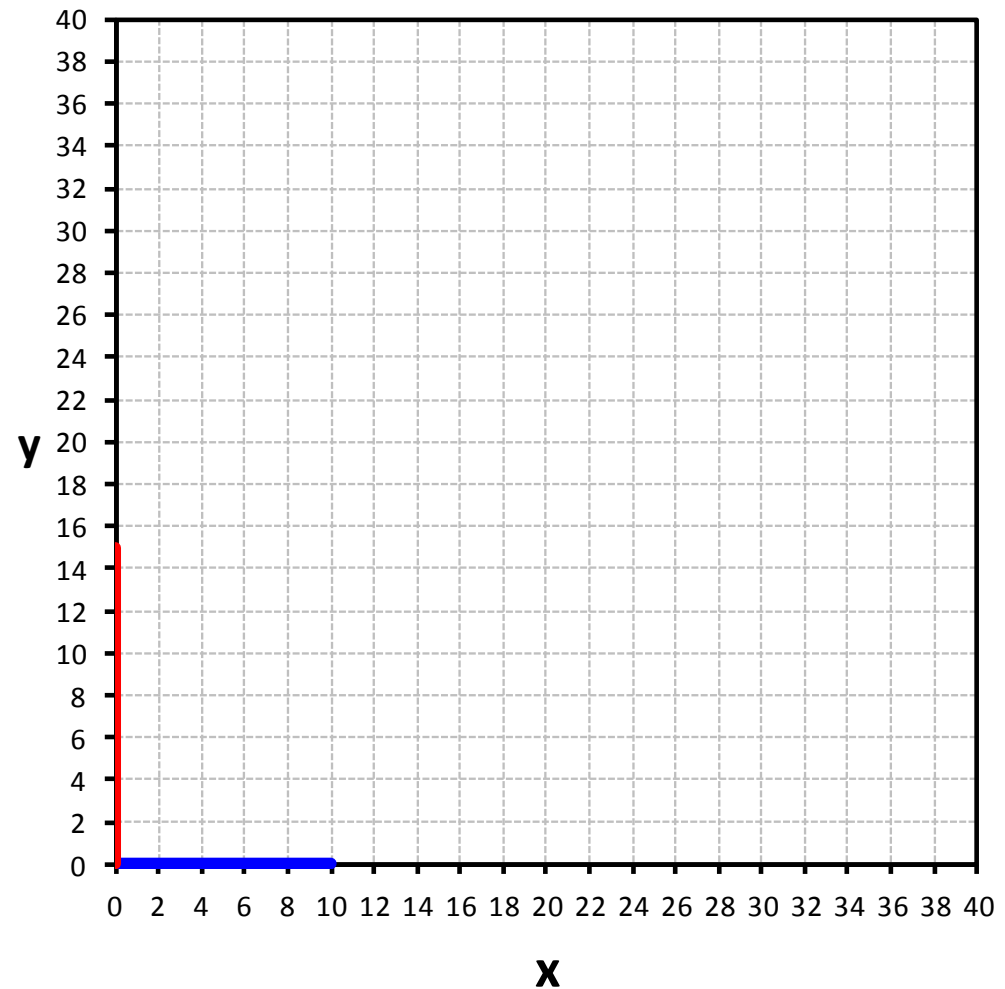
$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$



Sistemas lineares

Exemplo 2D

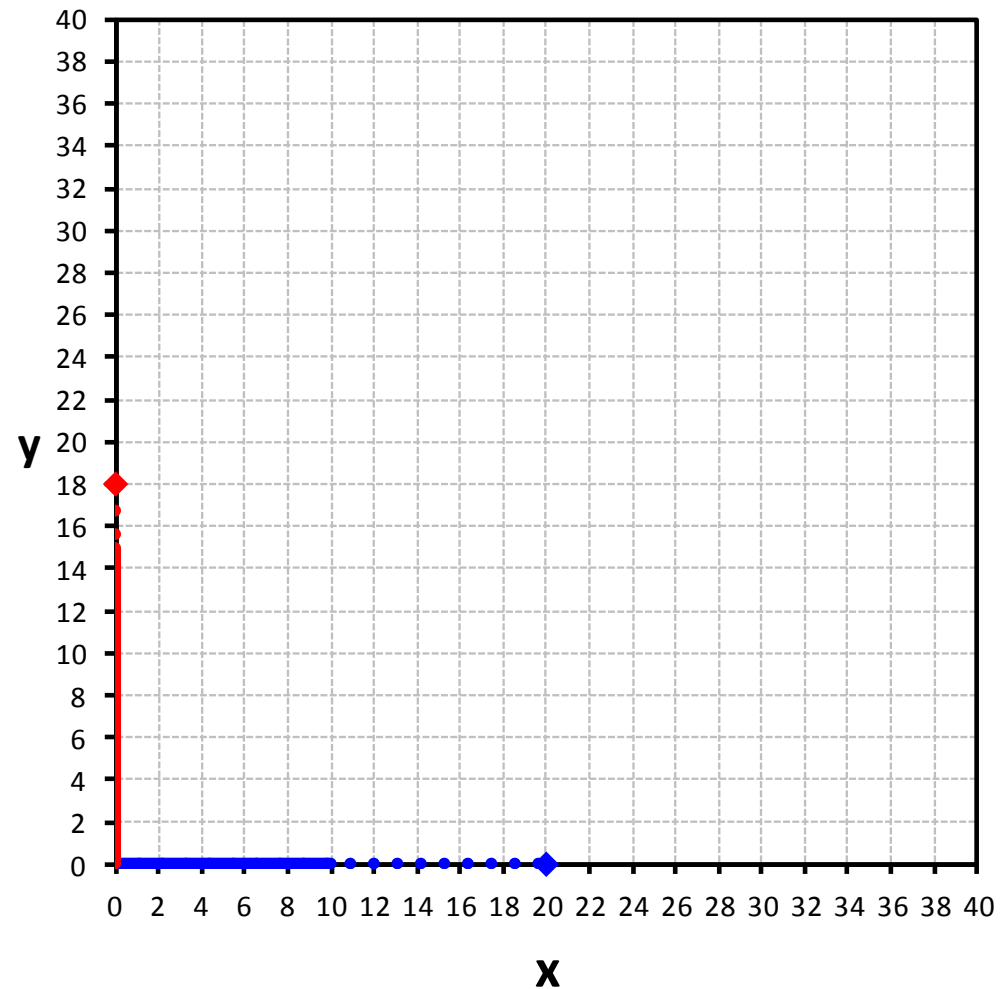
$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$



Sistemas lineares

Exemplo 2D

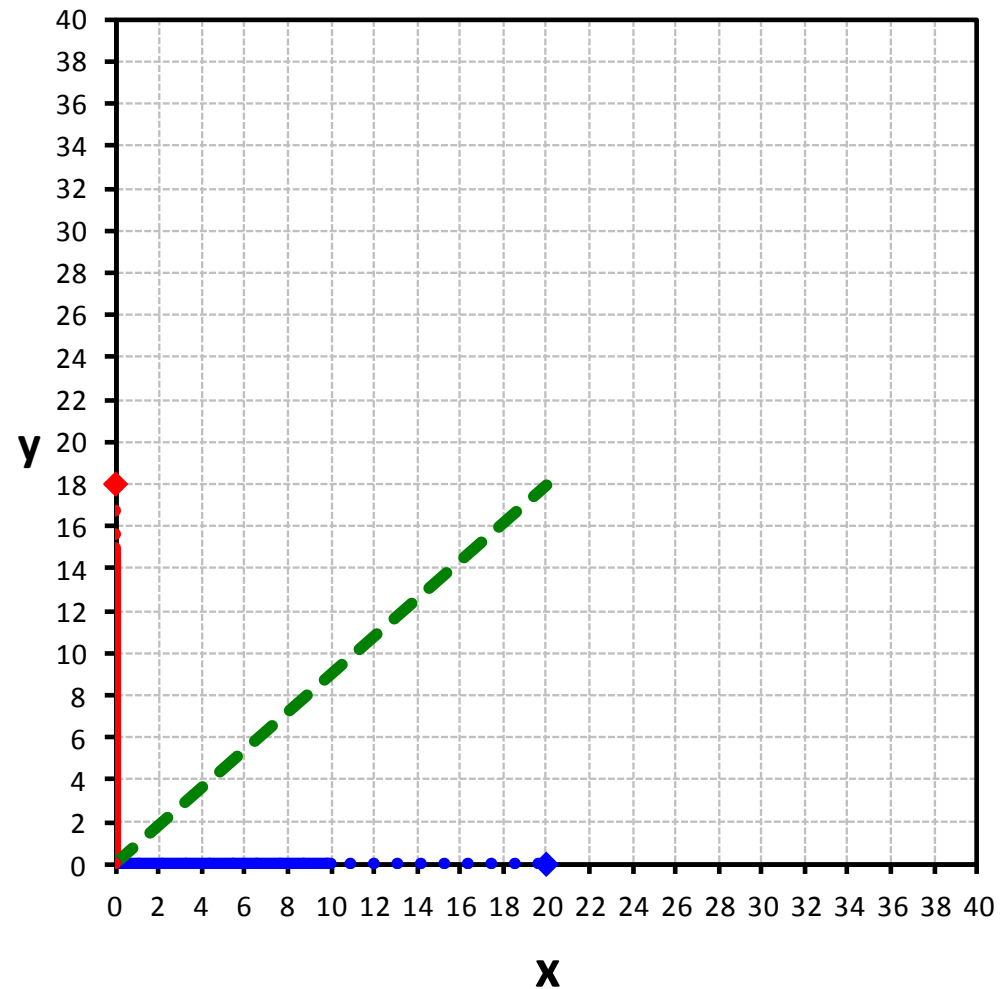
$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$



Sistemas lineares

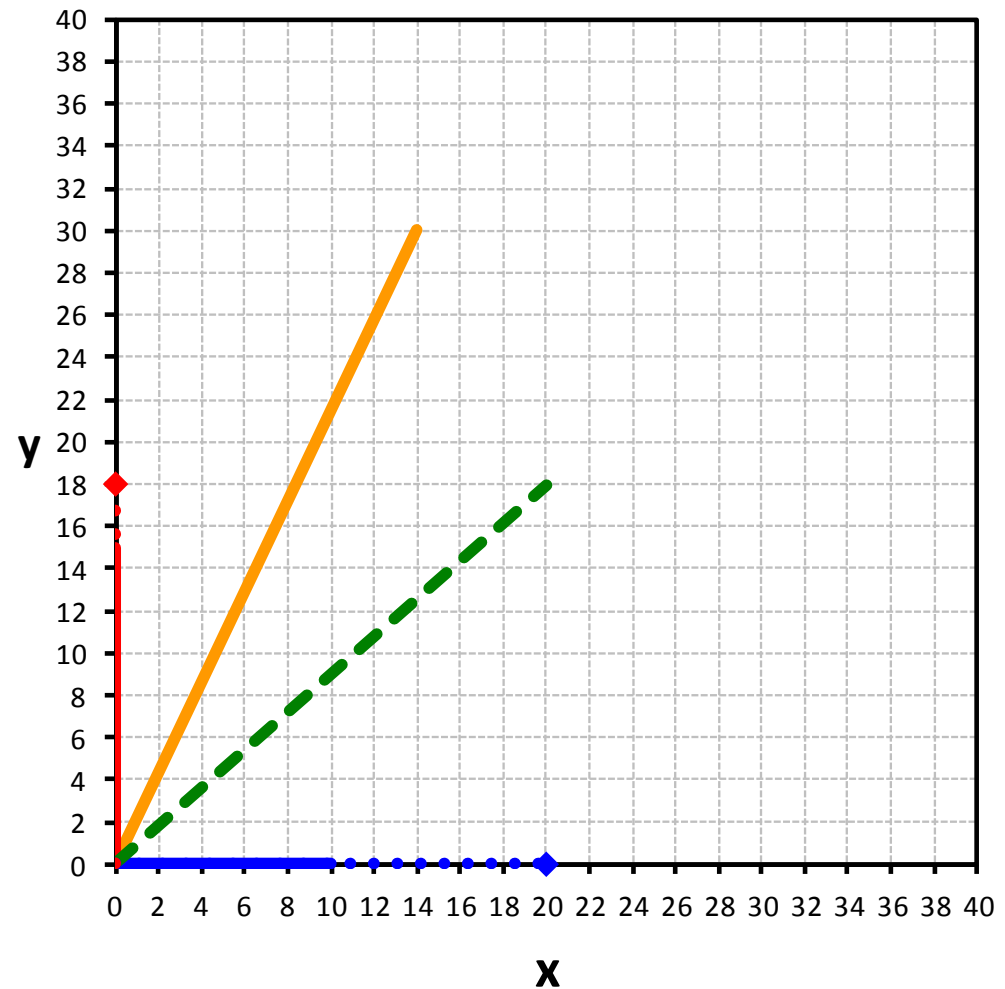
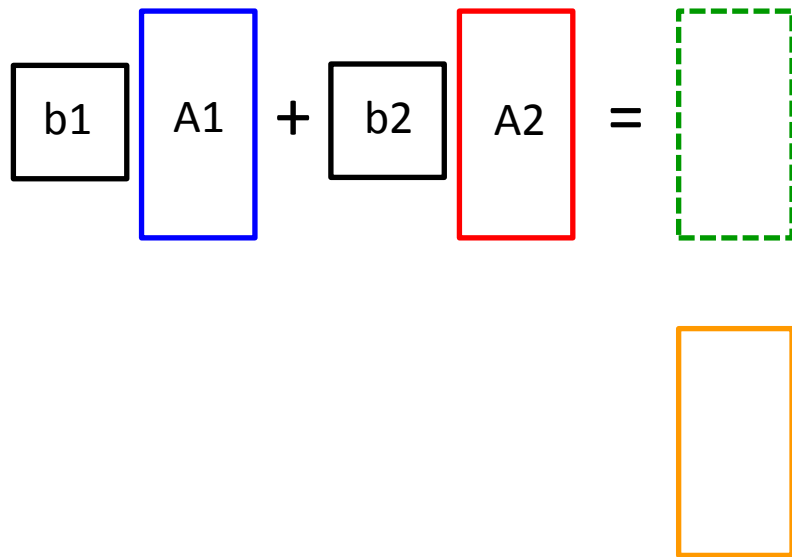
Exemplo 2D

$$\boxed{b_1} \boxed{A_1} + \boxed{b_2} \boxed{A_2} = \boxed{}$$



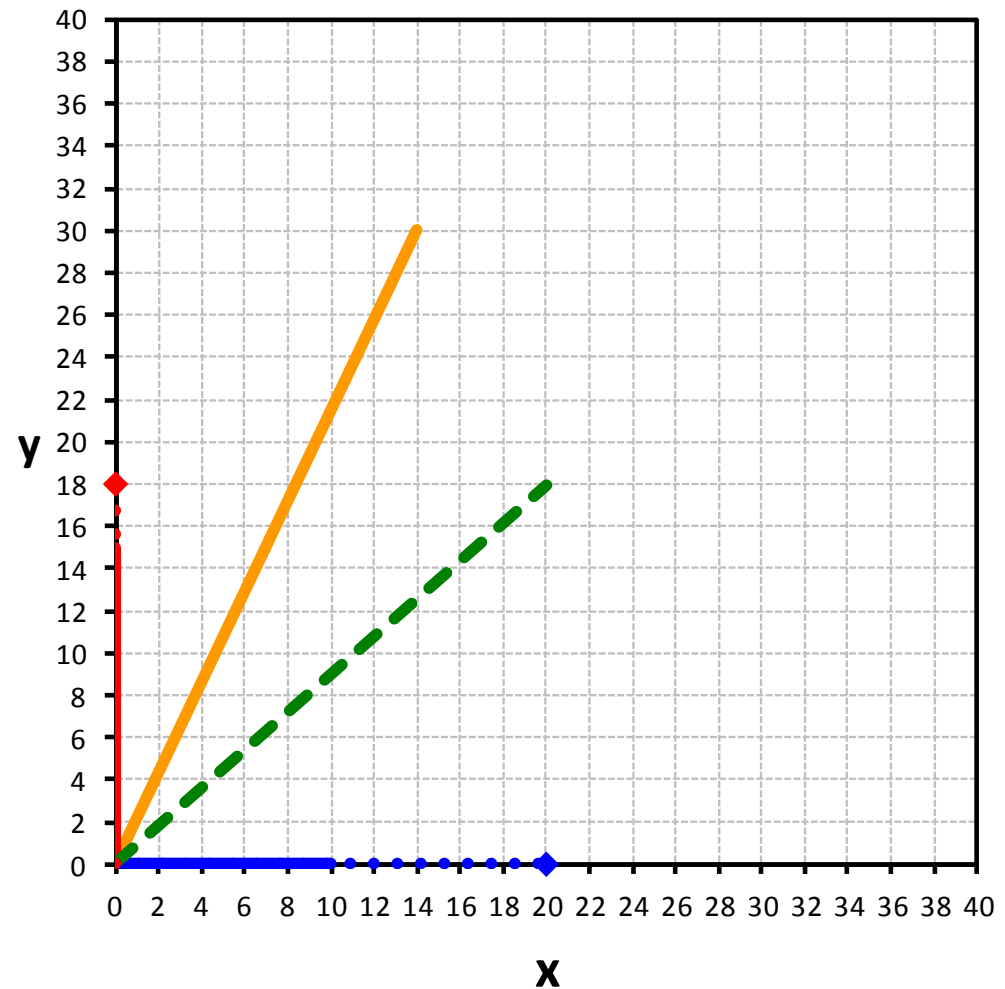
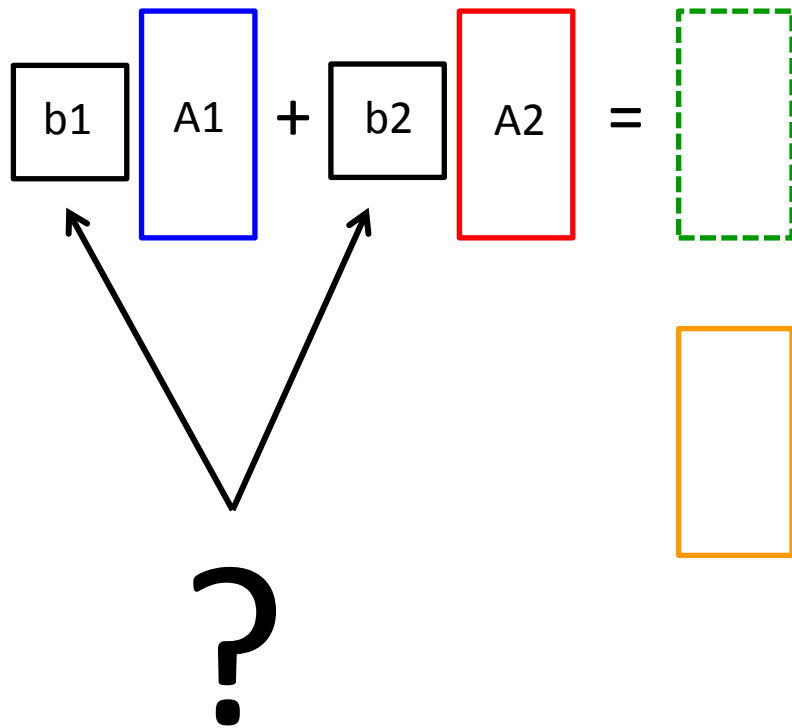
Sistemas lineares

Exemplo 2D



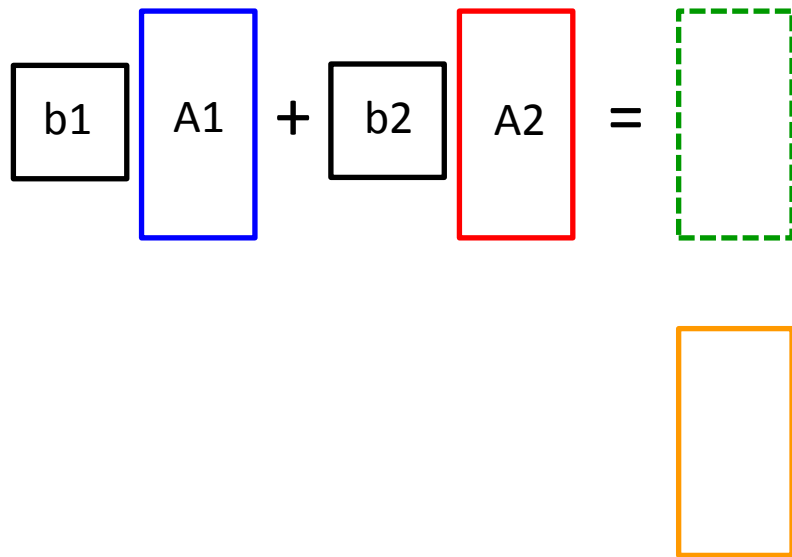
Sistemas lineares

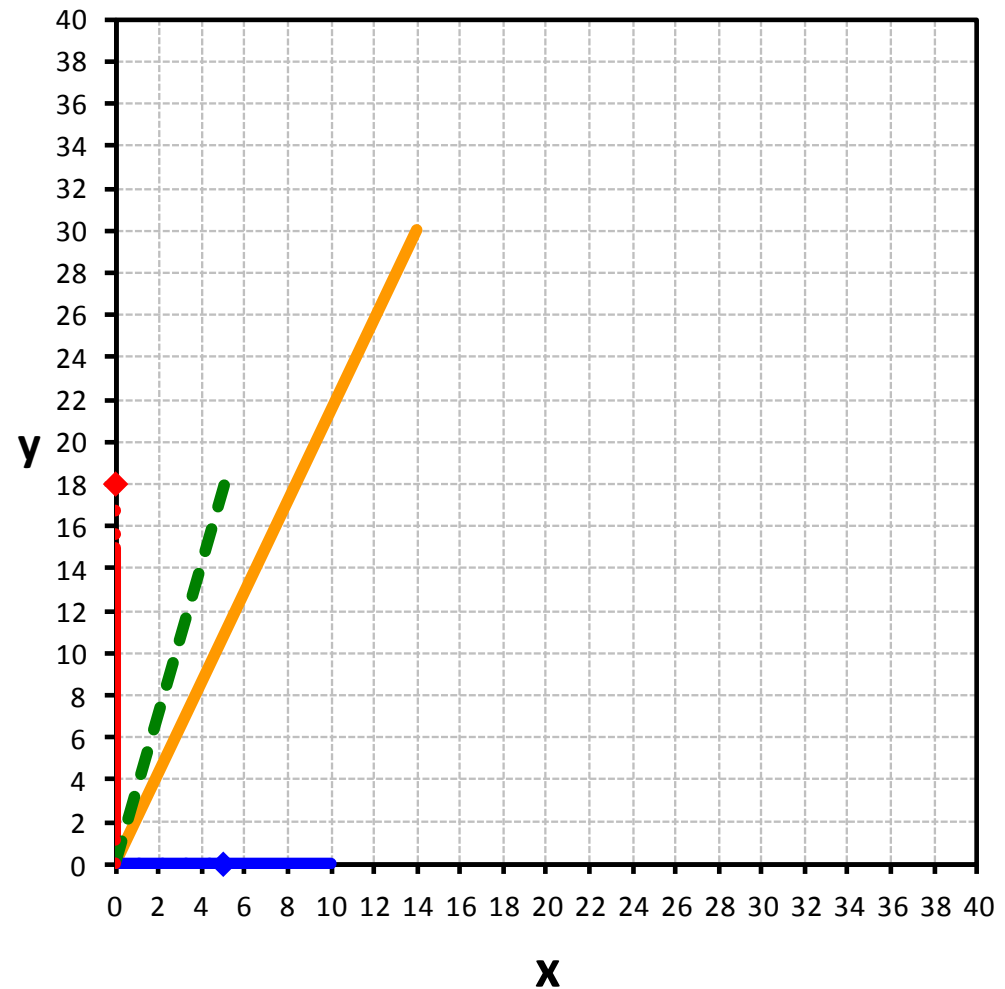
Exemplo 2D



Sistemas lineares

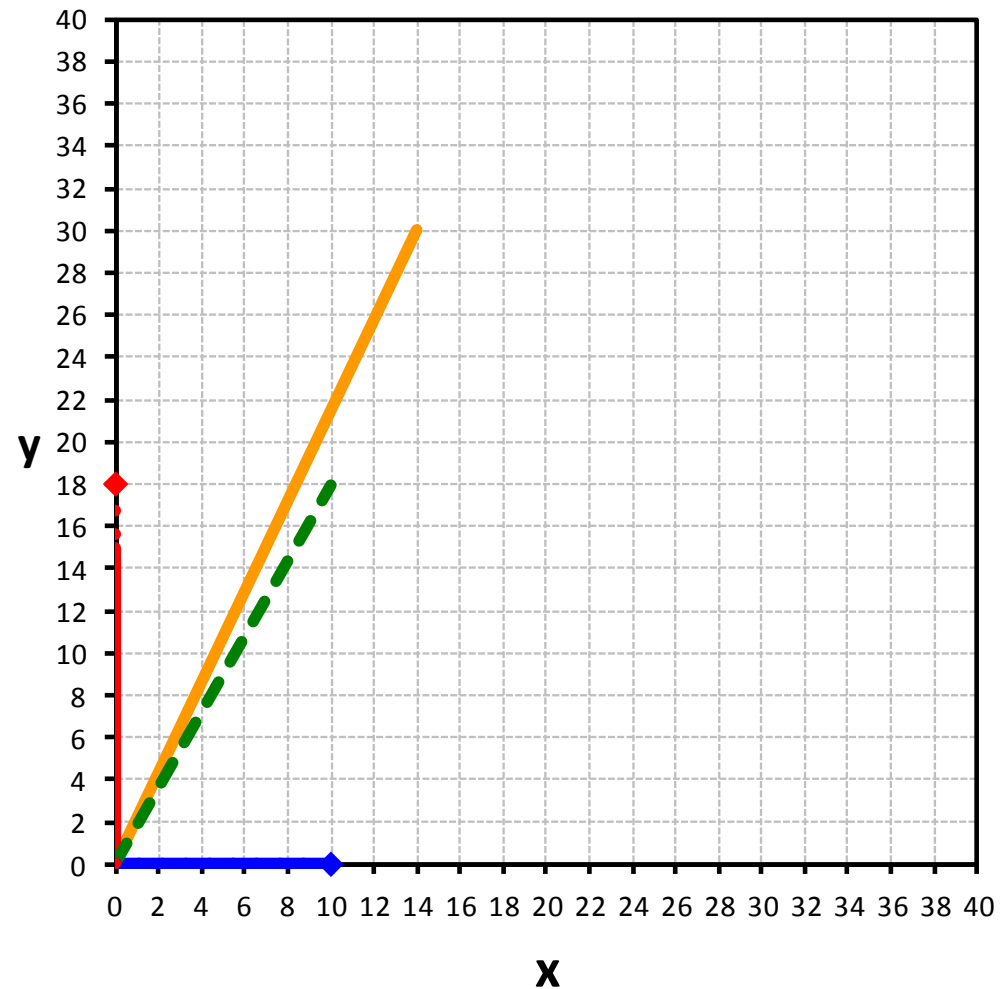
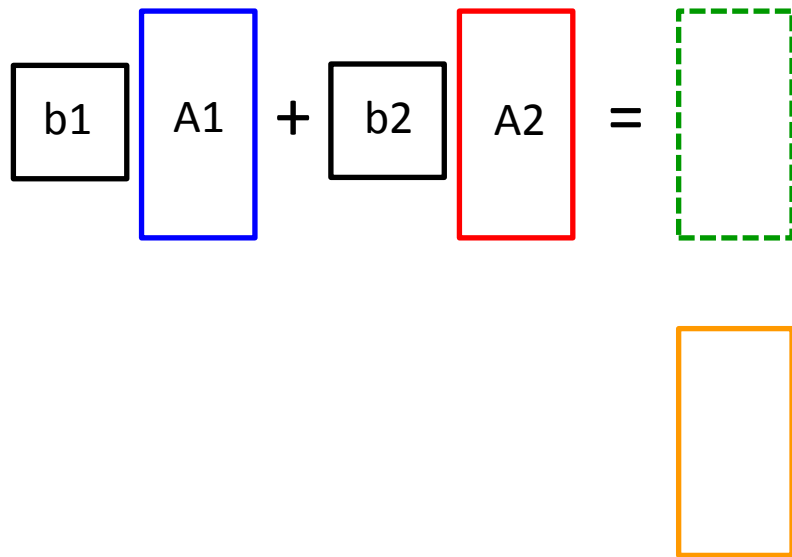
Exemplo 2D

$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array}$$




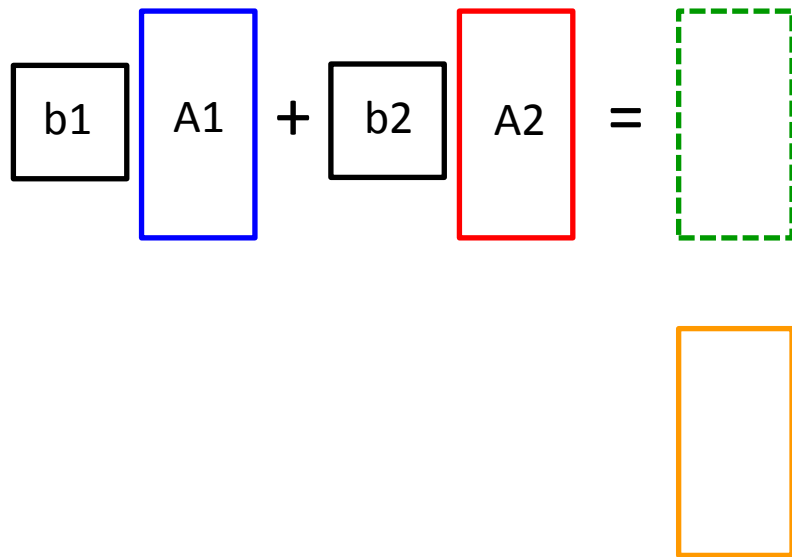
Sistemas lineares

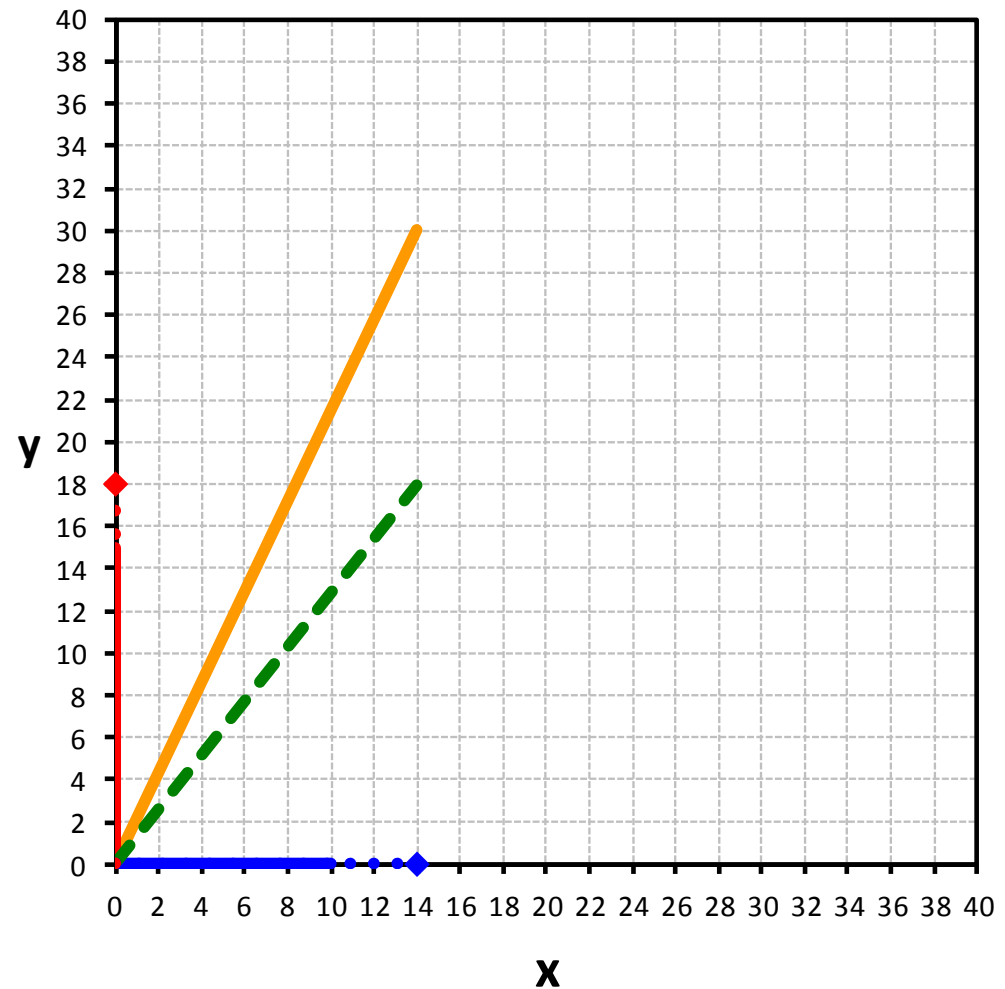
Exemplo 2D



Sistemas lineares

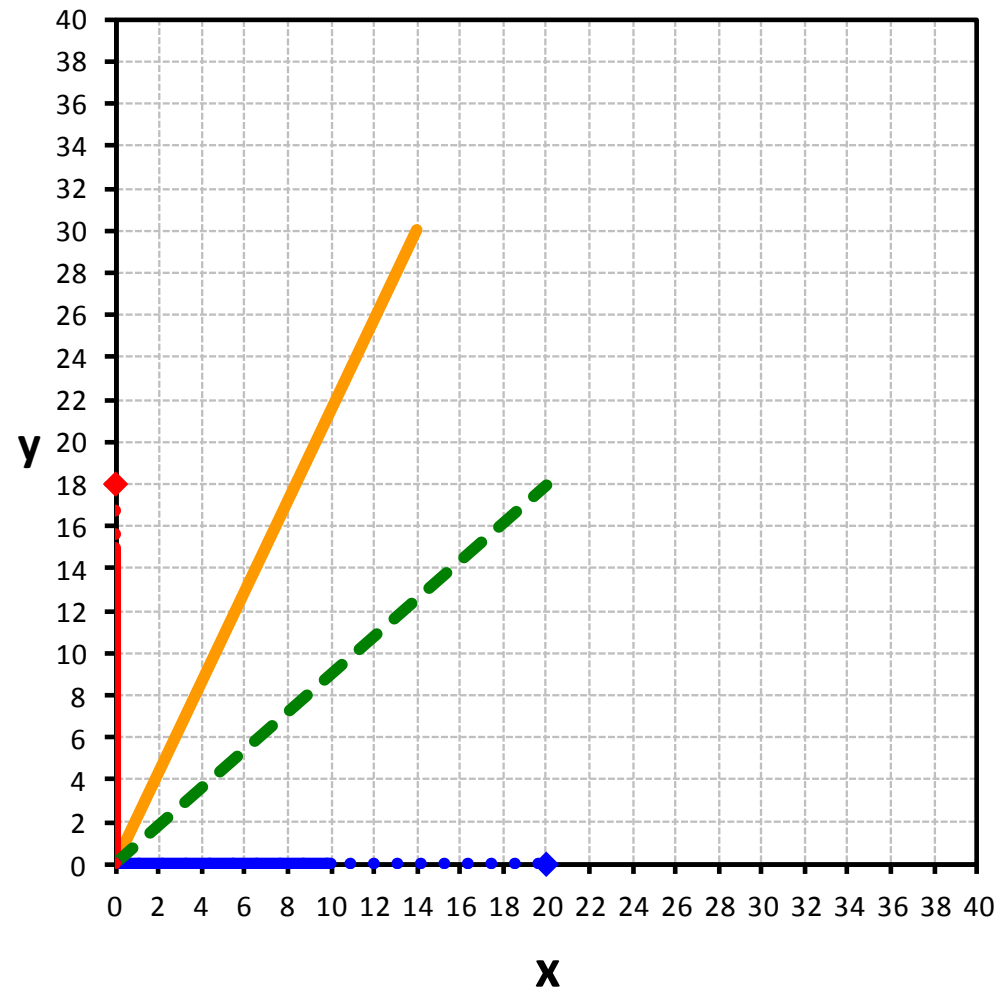
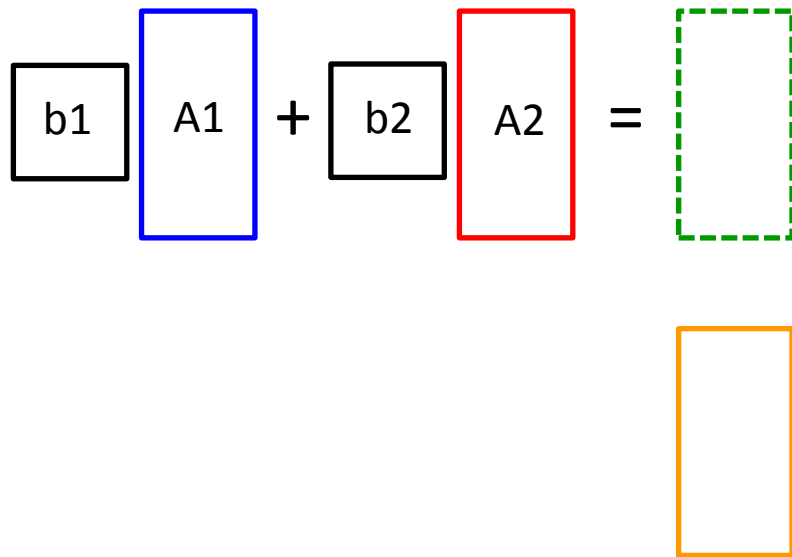
Exemplo 2D

$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array}$$




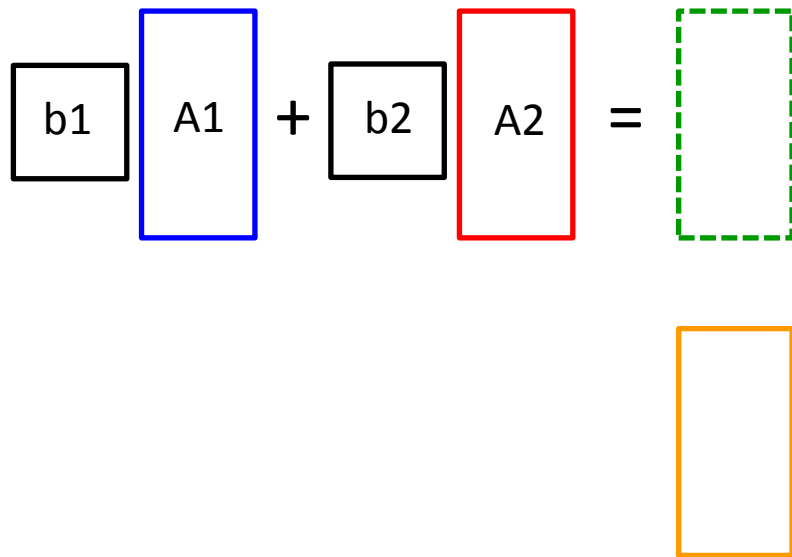
Sistemas lineares

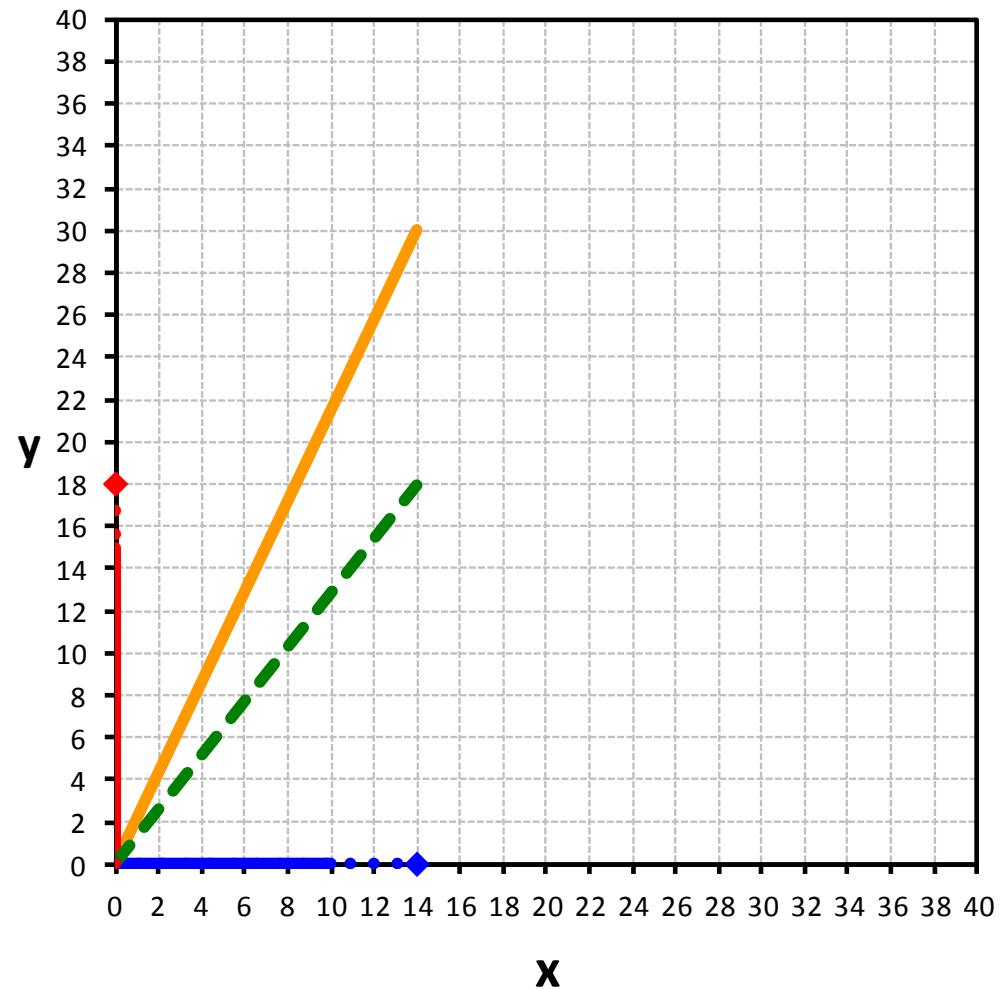
Exemplo 2D



Sistemas lineares

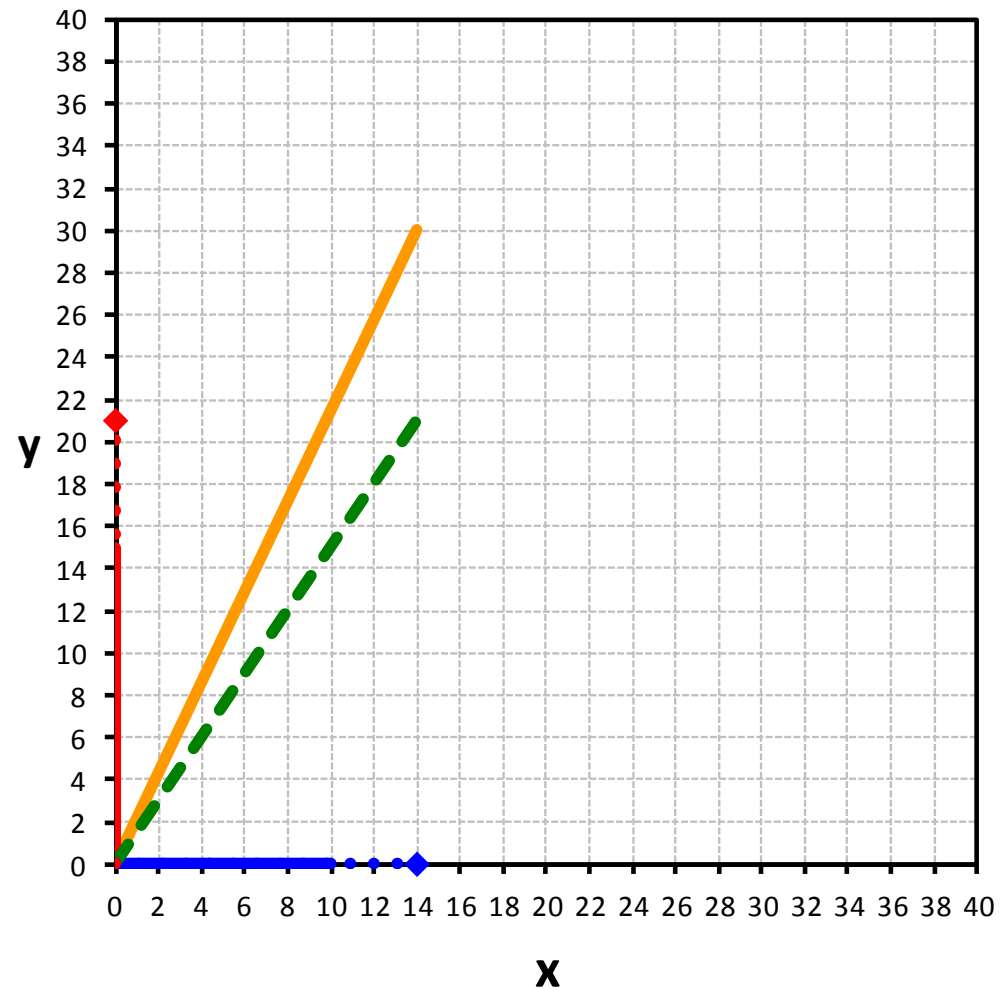
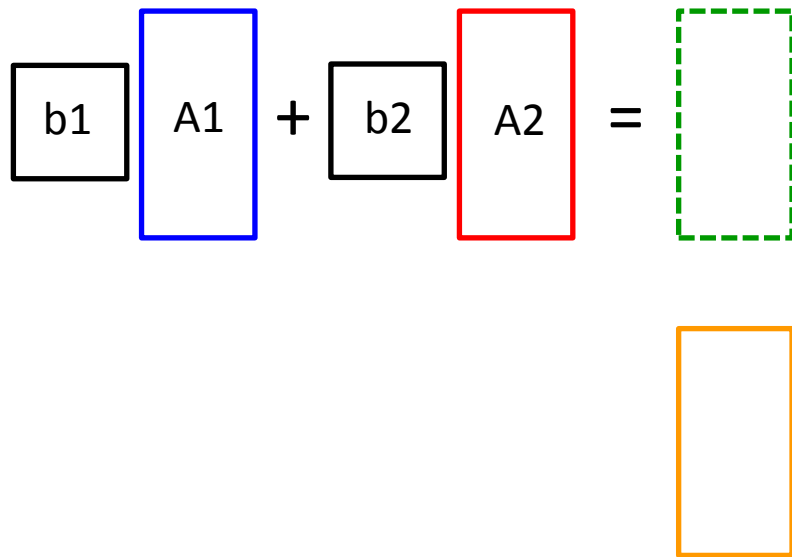
Exemplo 2D

$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array}$$




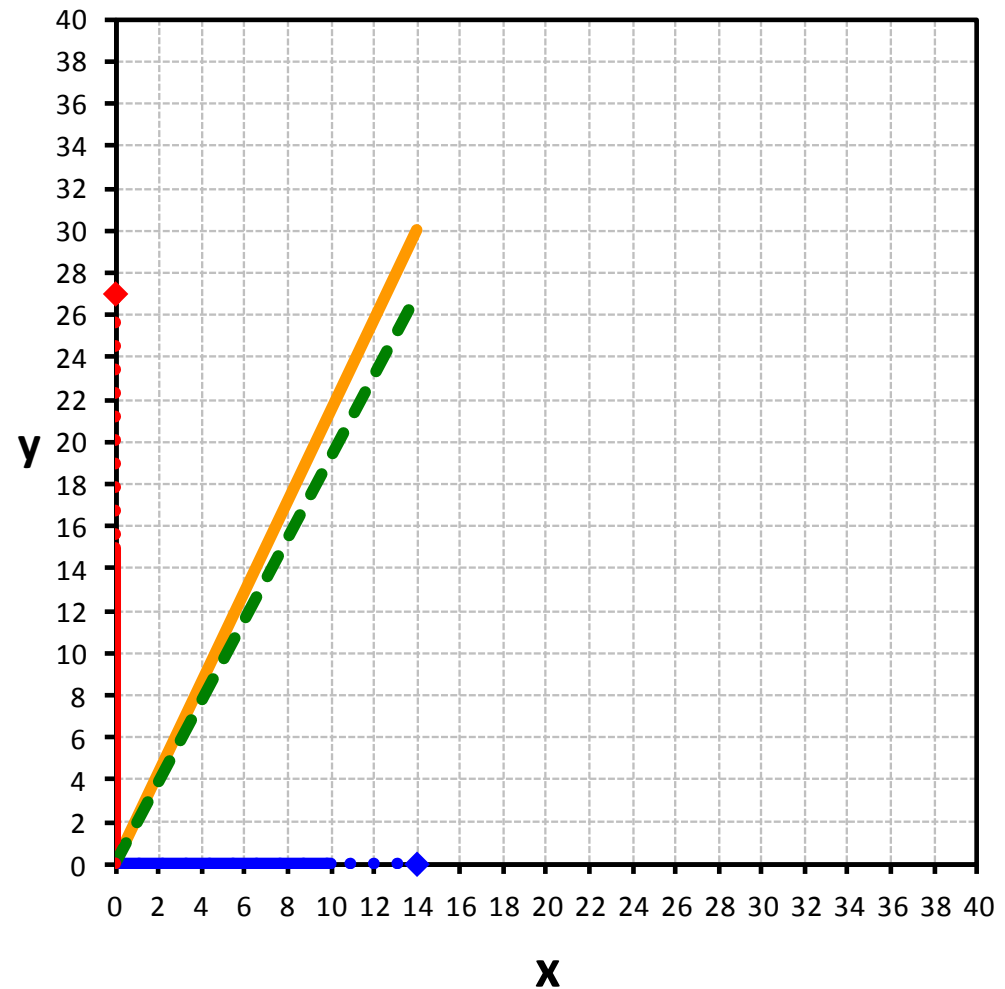
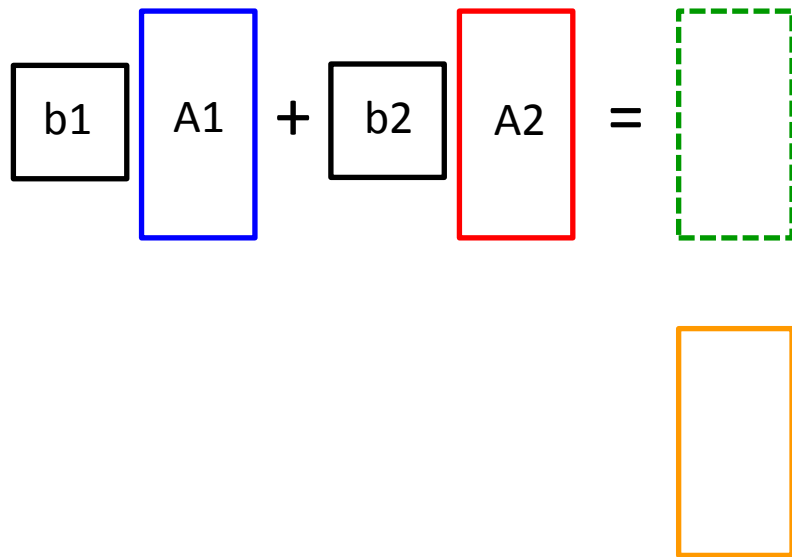
Sistemas lineares

Exemplo 2D



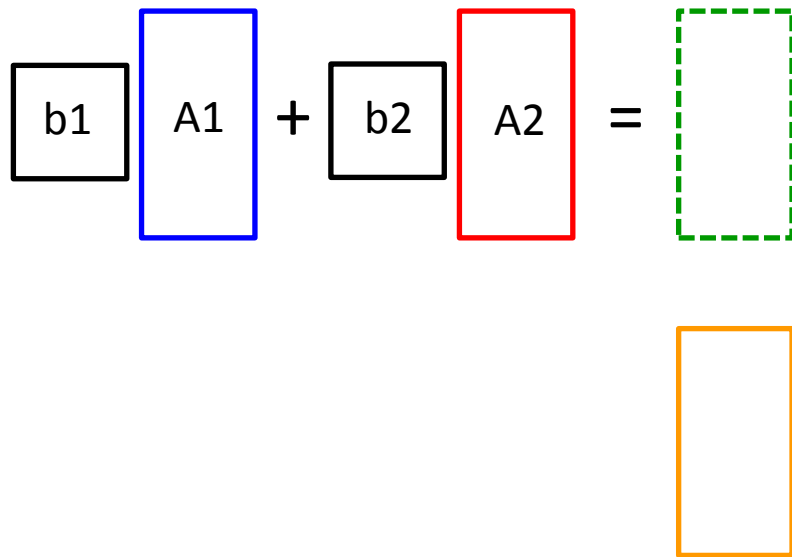
Sistemas lineares

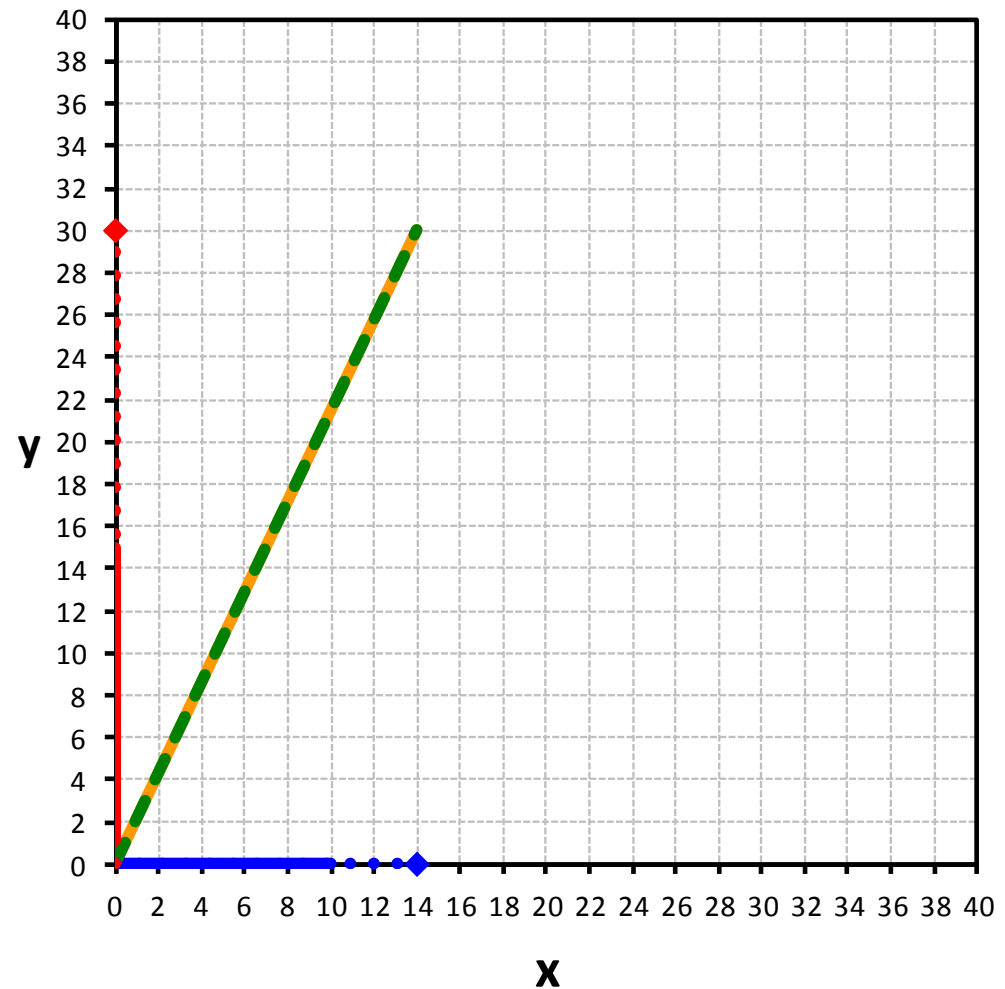
Exemplo 2D



Sistemas lineares

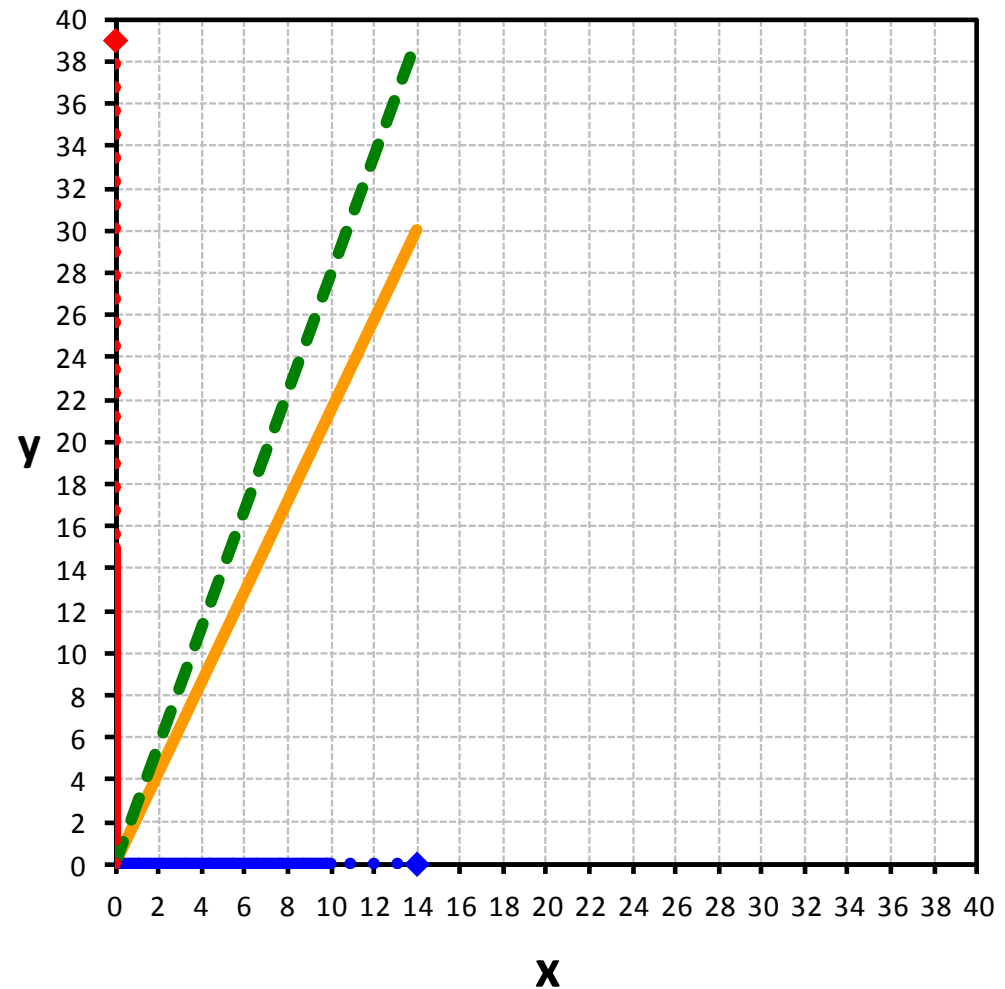
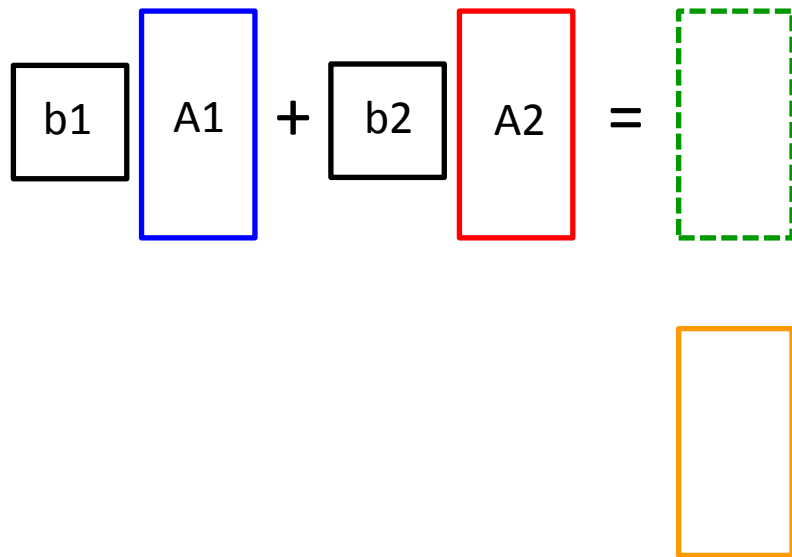
Exemplo 2D

$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array}$$




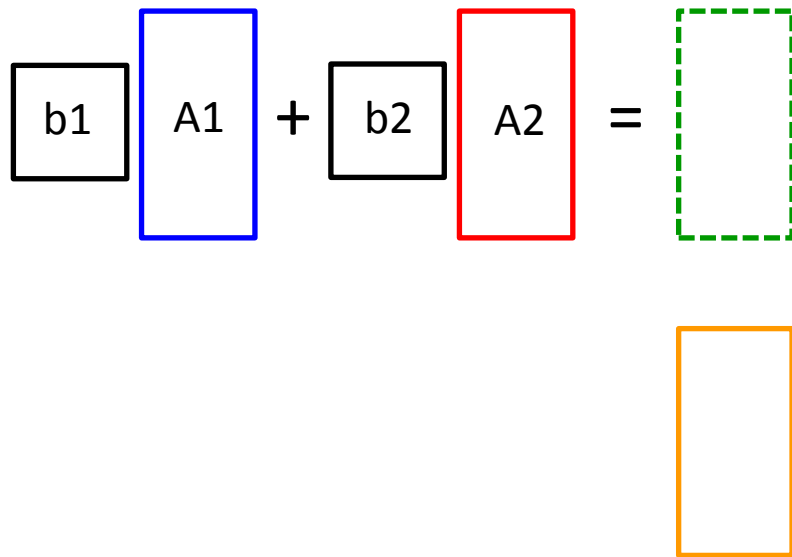
Sistemas lineares

Exemplo 2D

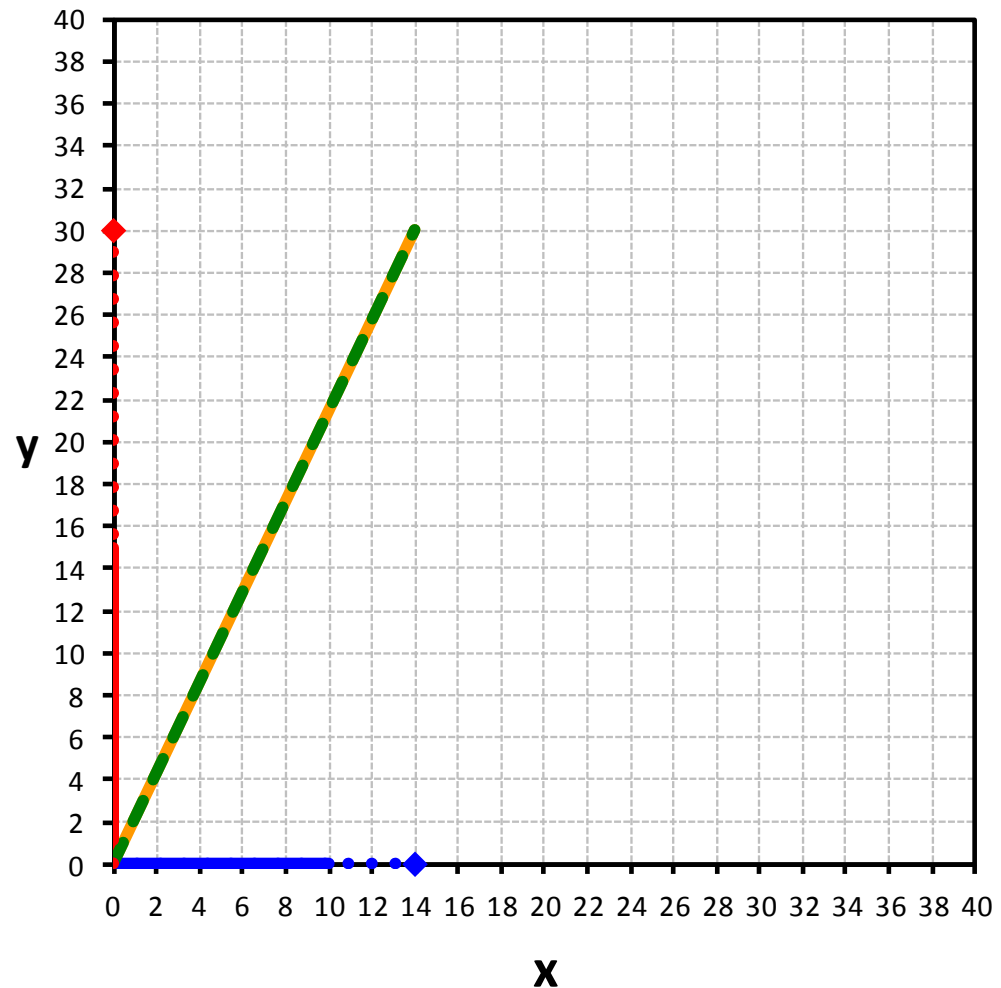


Sistemas lineares

Exemplo 2D

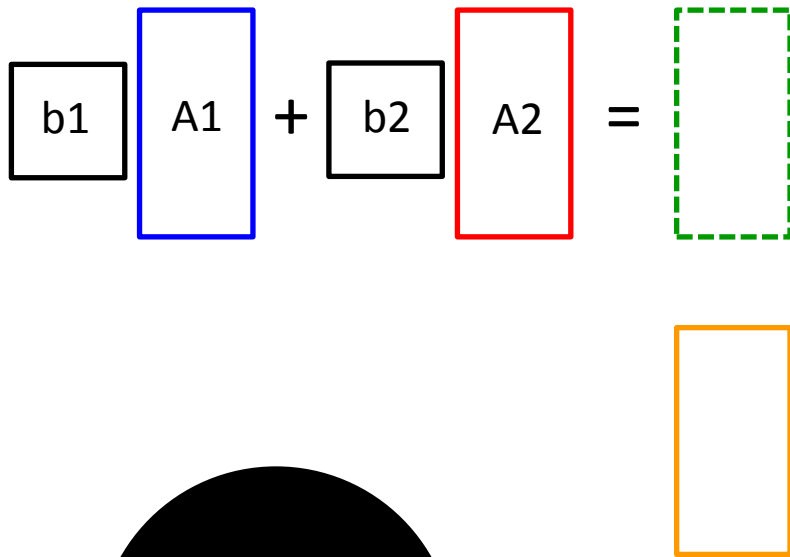
$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$


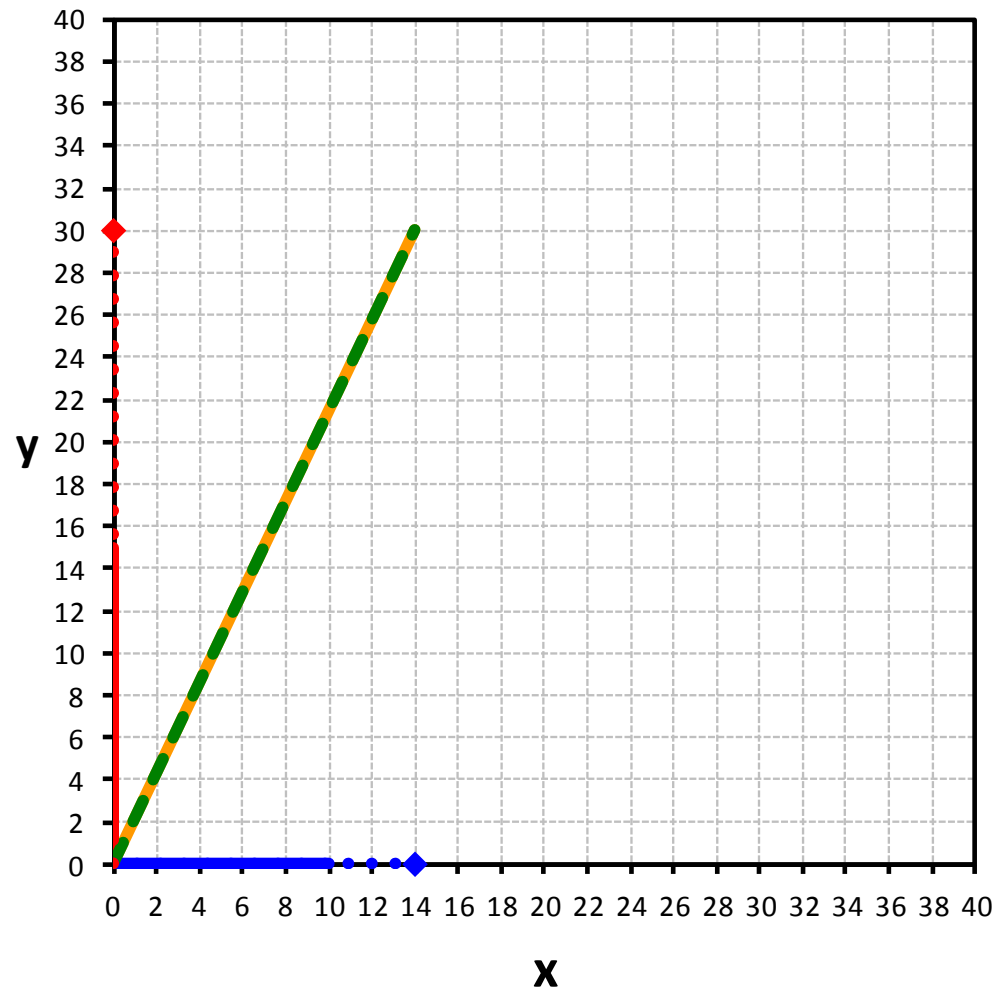
Neste caso, os vetores **A1** e **A2** são *linearmente independentes* e os coeficientes $b1$ e $b2$ são únicos



Sistemas lineares

Exemplo 2D

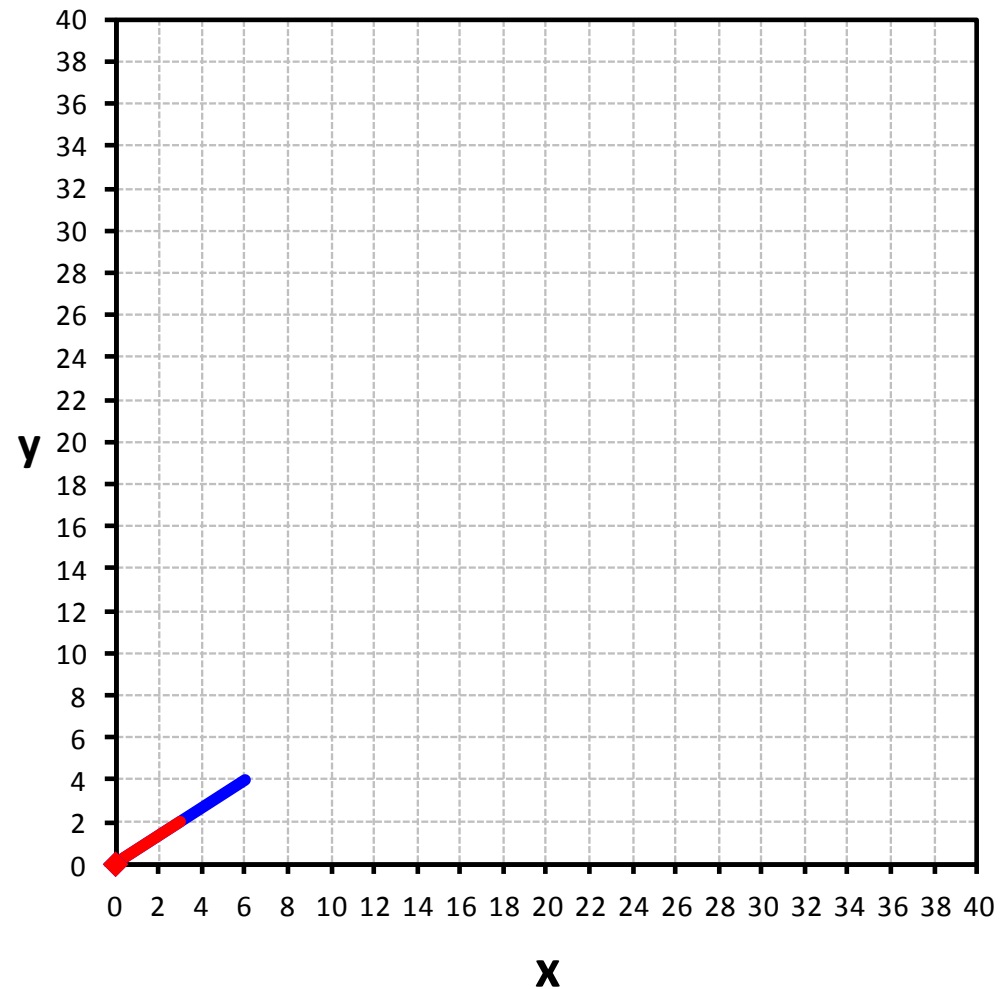
$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$




Sistemas lineares

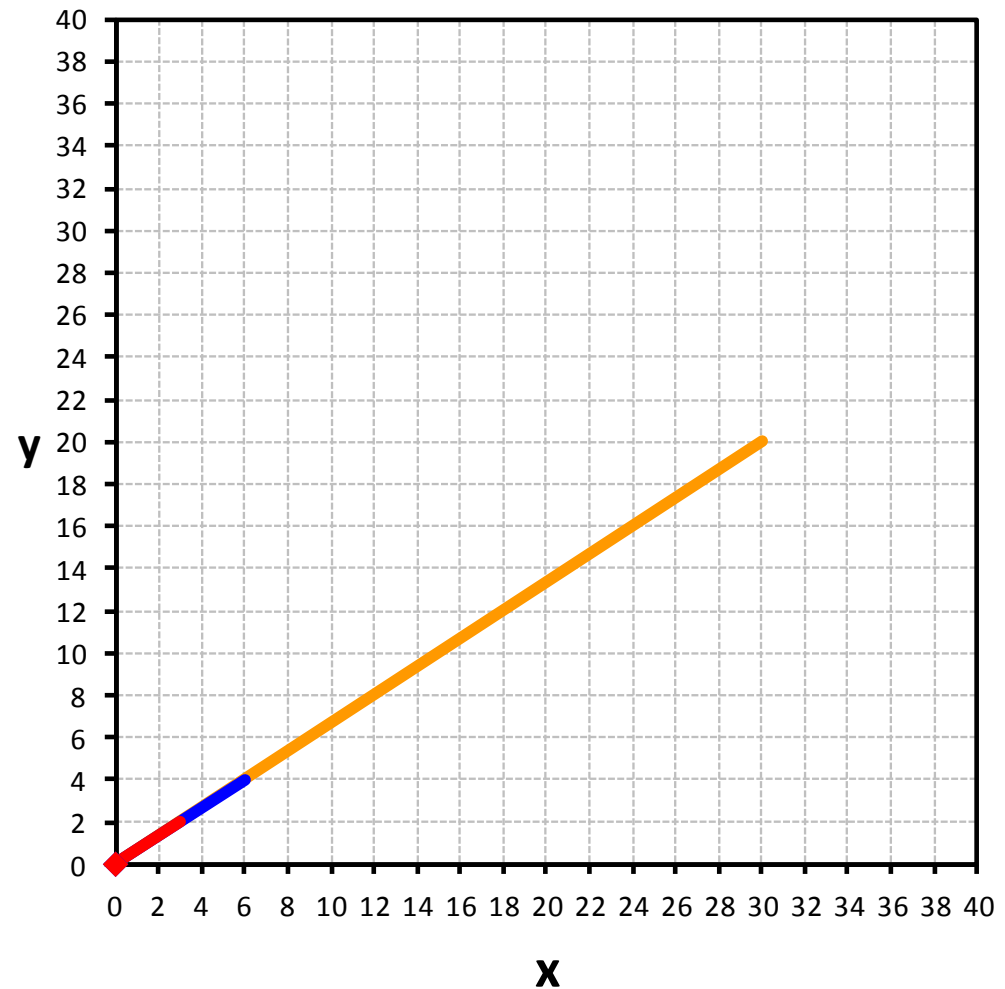
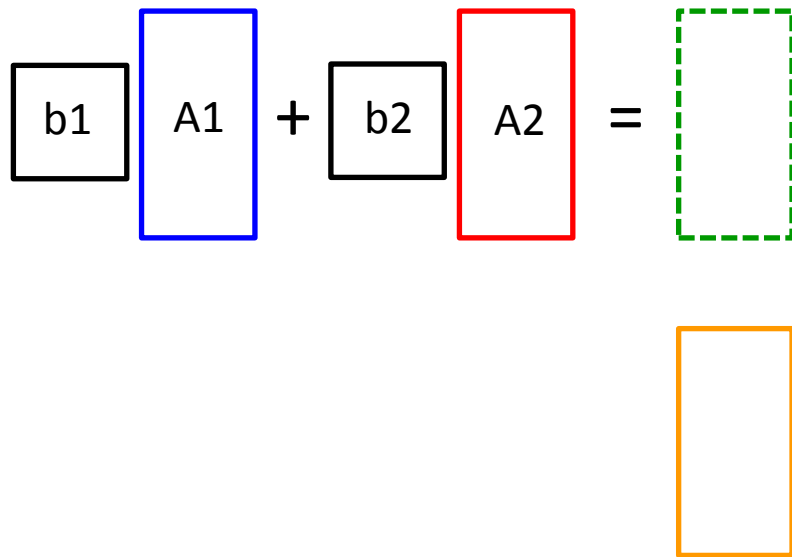
Exemplo 2D

$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$



Sistemas lineares

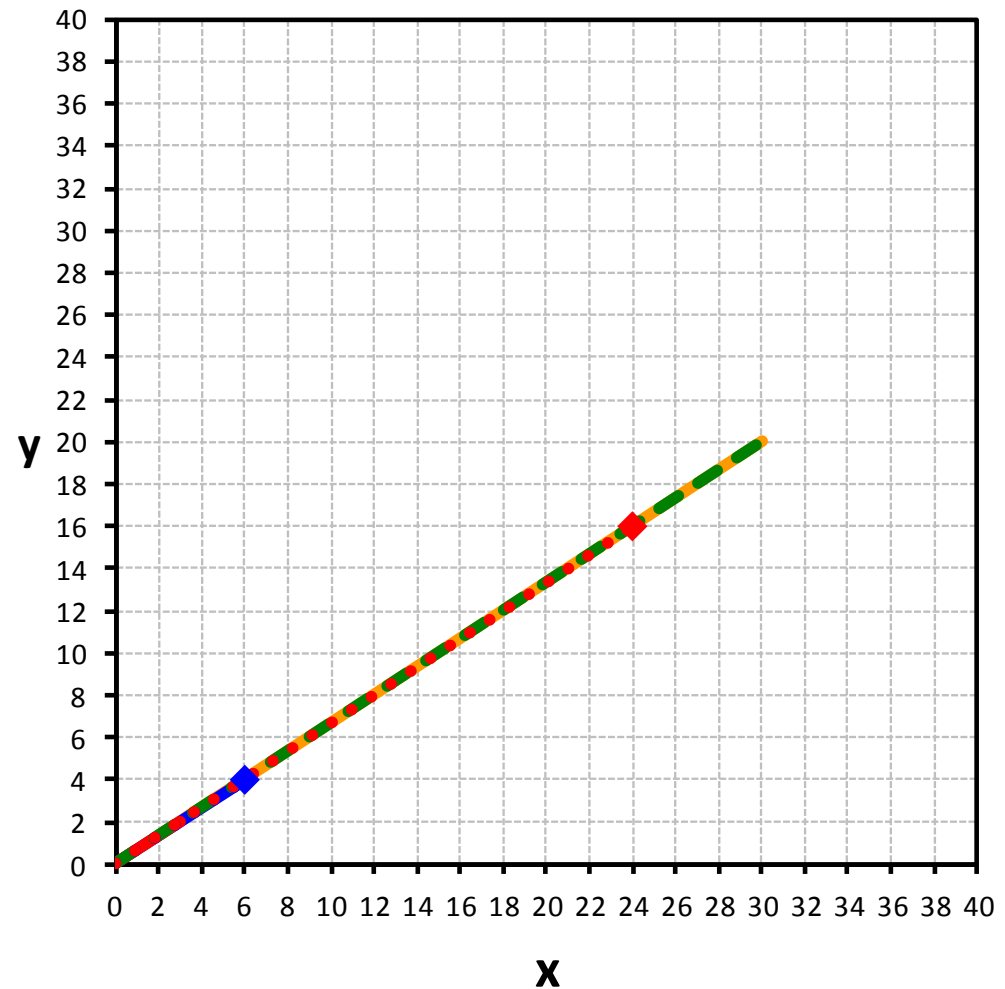
Exemplo 2D



Sistemas lineares

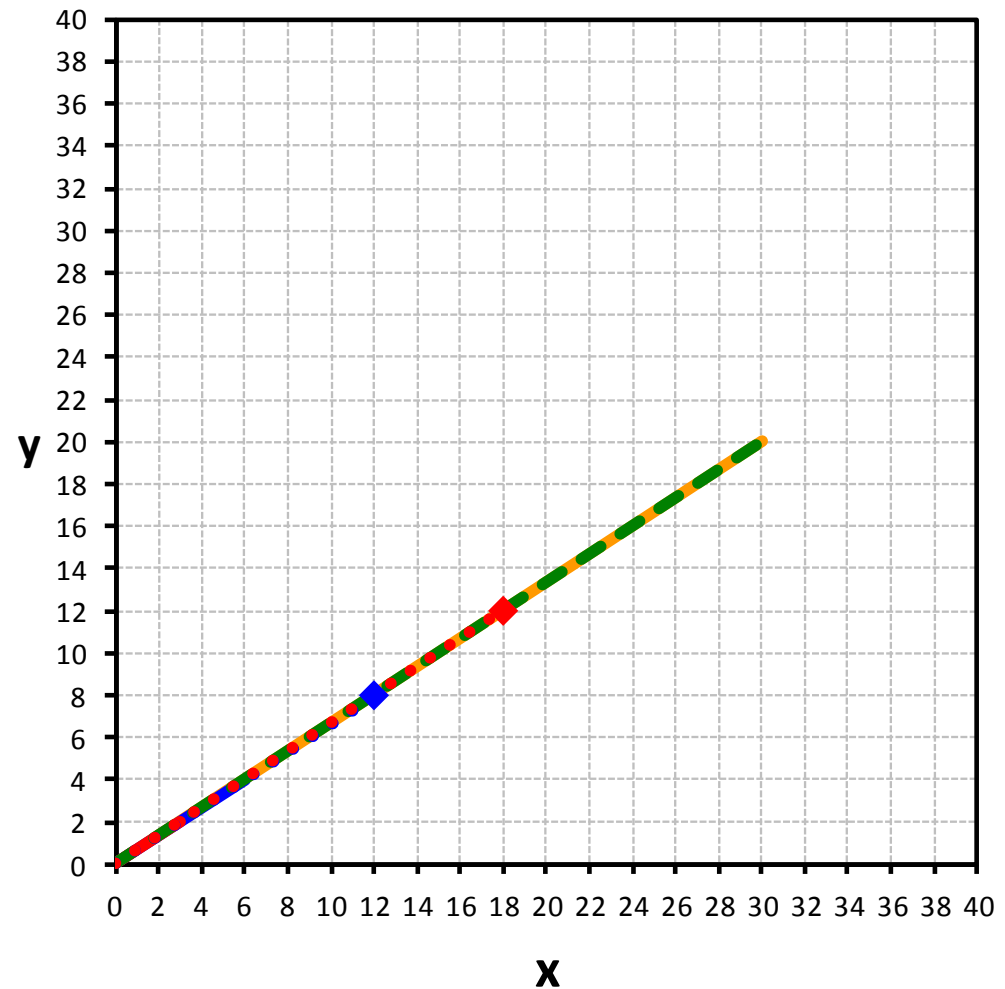
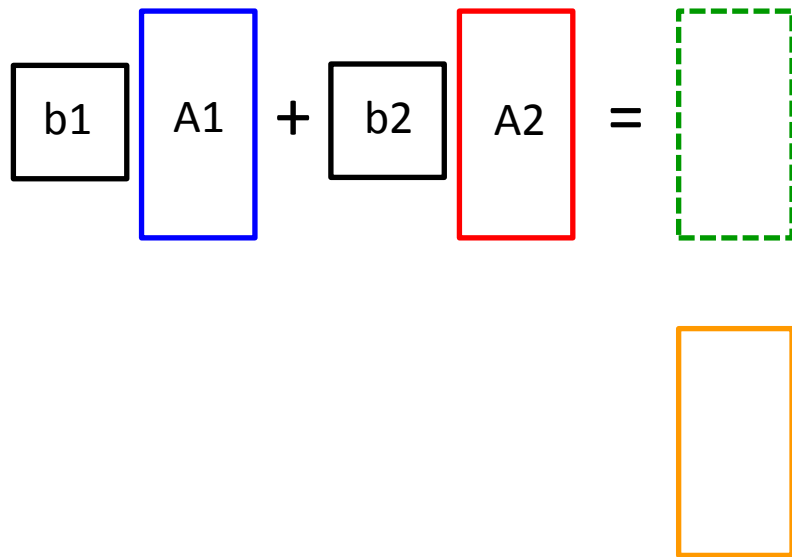
Exemplo 2D

Diagram illustrating the addition of two vectors A_1 and A_2 . The vectors are represented by rectangles. The first vector A_1 is blue-outlined, and the second vector A_2 is red-outlined. The sum is represented by a green dashed-outlined rectangle. The diagram shows the vectors b_1 and b_2 , and the resulting vector $b_1 + b_2$.



Sistemas lineares


Exemplo 2D

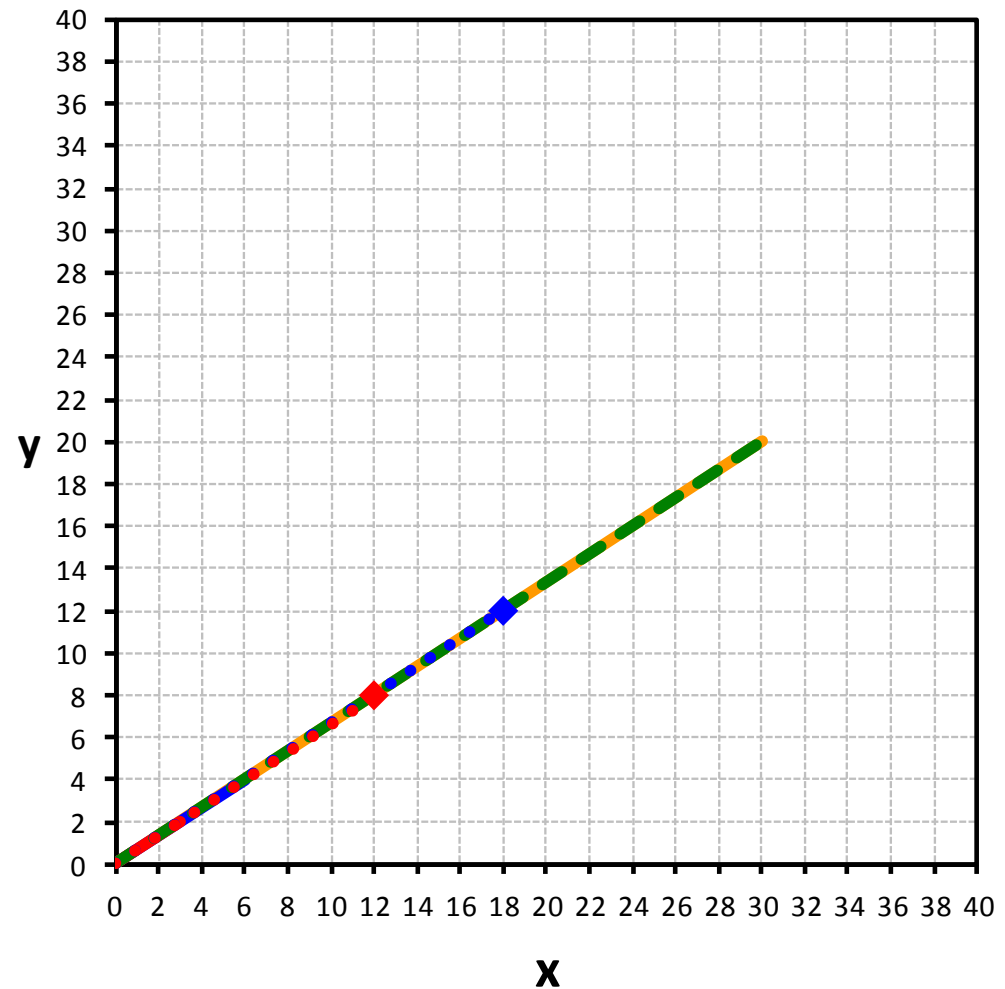


Sistemas lineares

Exemplo 2D

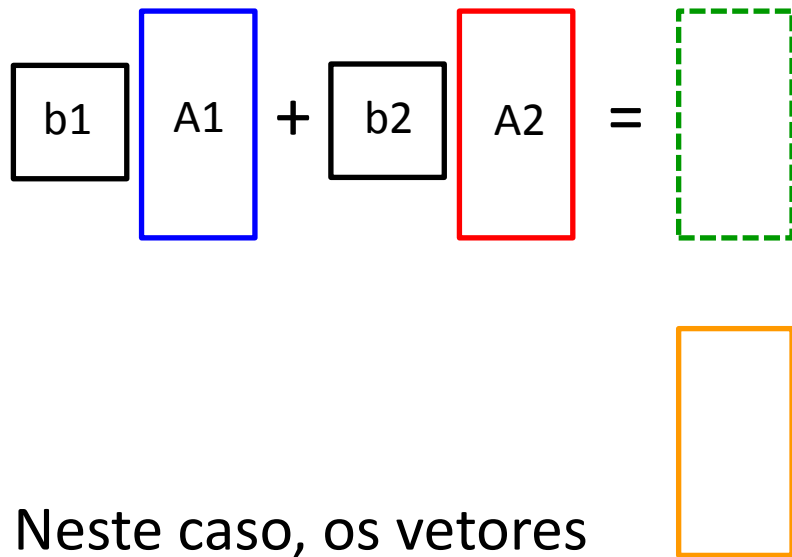
$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array}$$





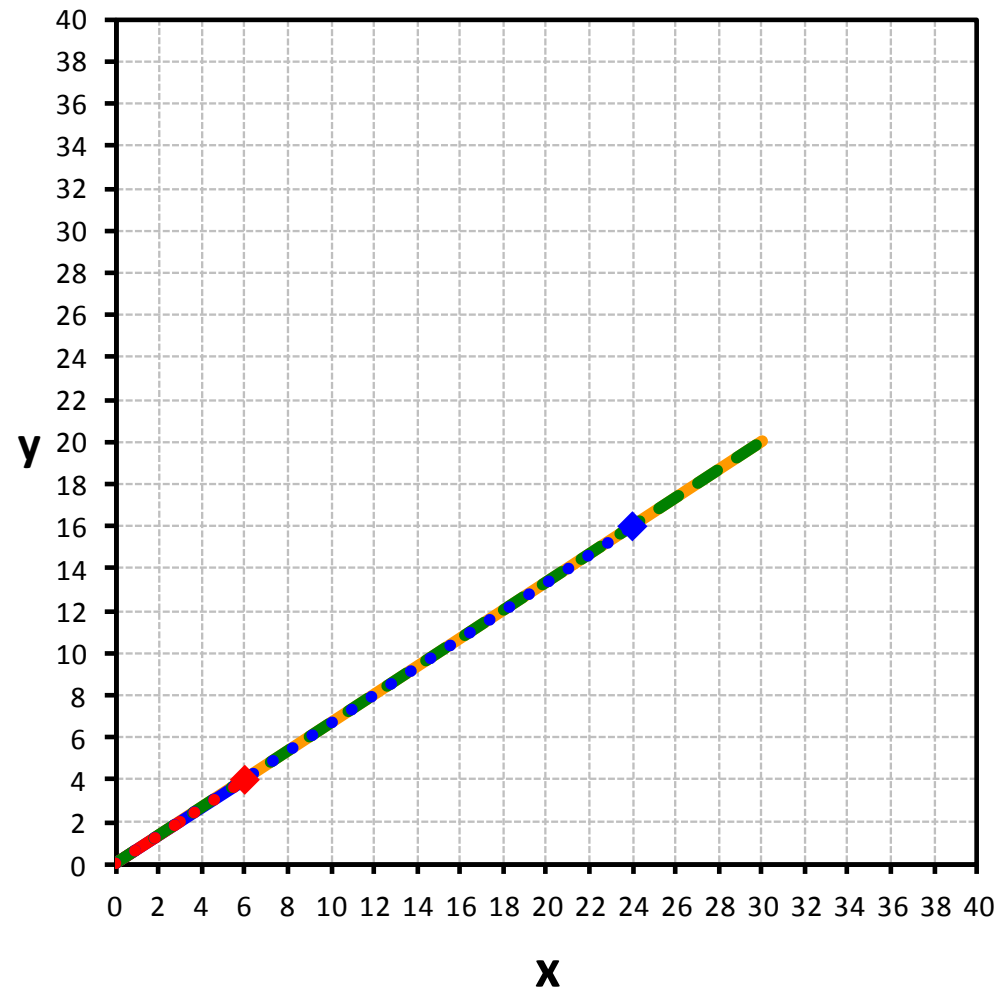
Sistemas lineares

Exemplo 2D

$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$


The diagram illustrates the linear combination of two vectors, A1 and A2, to produce a result vector. Vector A1 is represented by a blue-outlined rectangle, and vector A2 is represented by a red-outlined rectangle. They are added together, as indicated by the plus sign and the equals sign. The result is shown in a green dashed-outlined rectangle. Below this, an orange-outlined rectangle represents the same result vector, demonstrating that different combinations of b1 and b2 can produce the same result when A1 and A2 are linearly dependent.

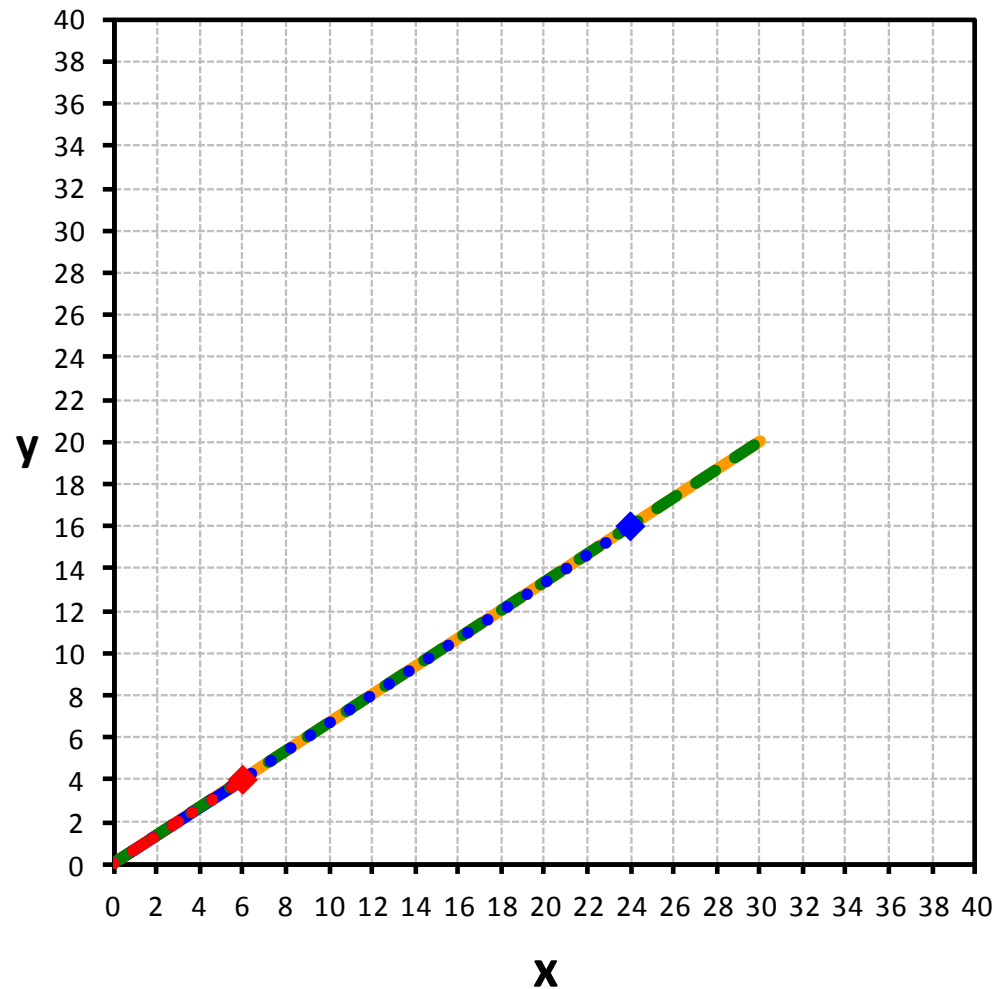
Neste caso, os vetores **A1** e **A2** são *linearmente dependentes* e existem infinitos pares $b1$ e $b2$ que produzem o mesmo resultado



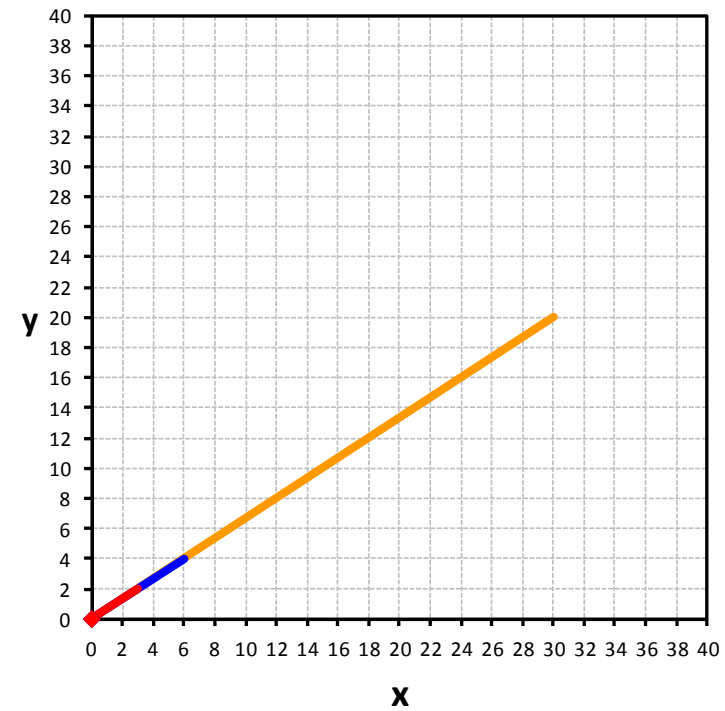
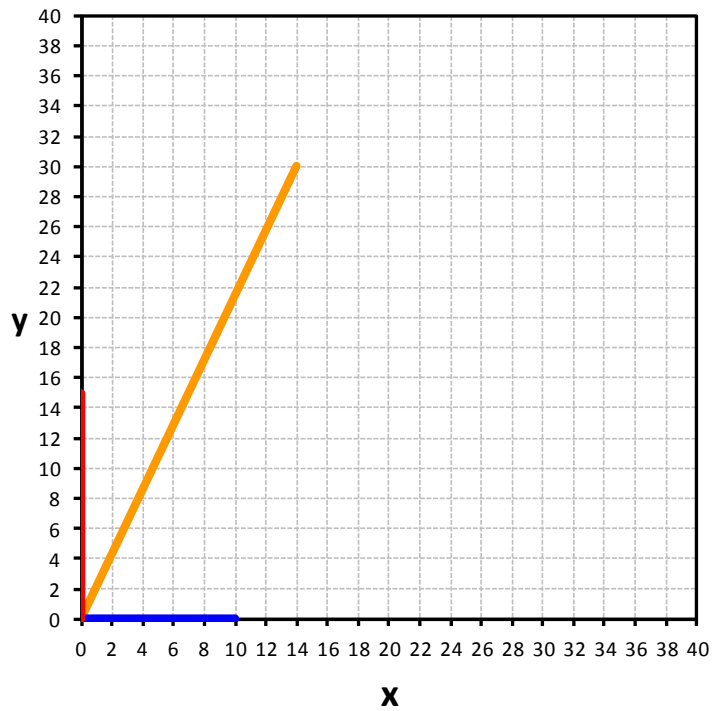
Sistemas lineares

Exemplo 2D

$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$



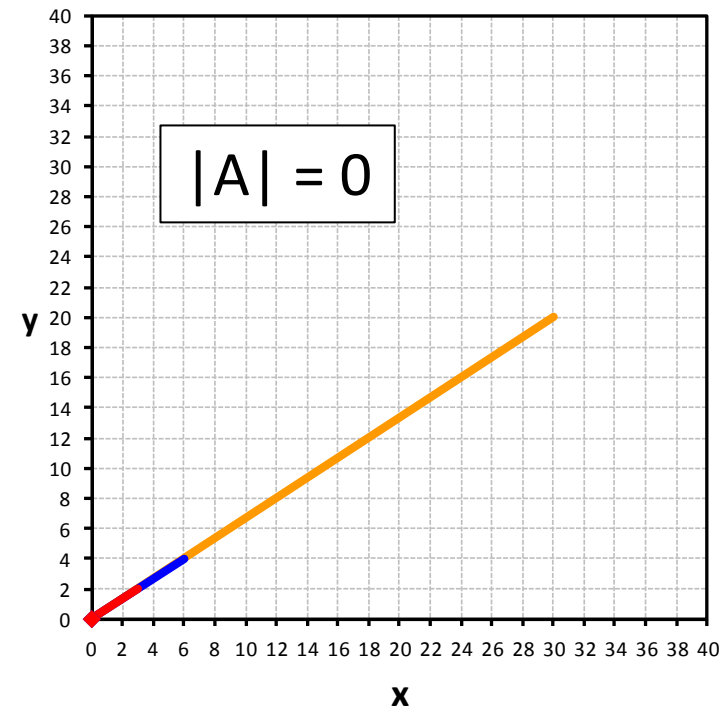
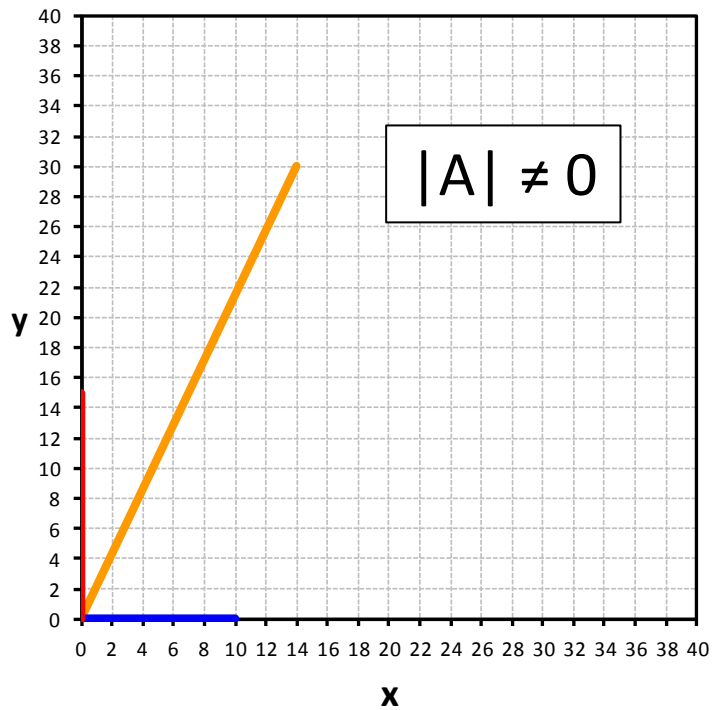
Sistemas lineares



$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{} \boxed{}$$



Sistemas lineares



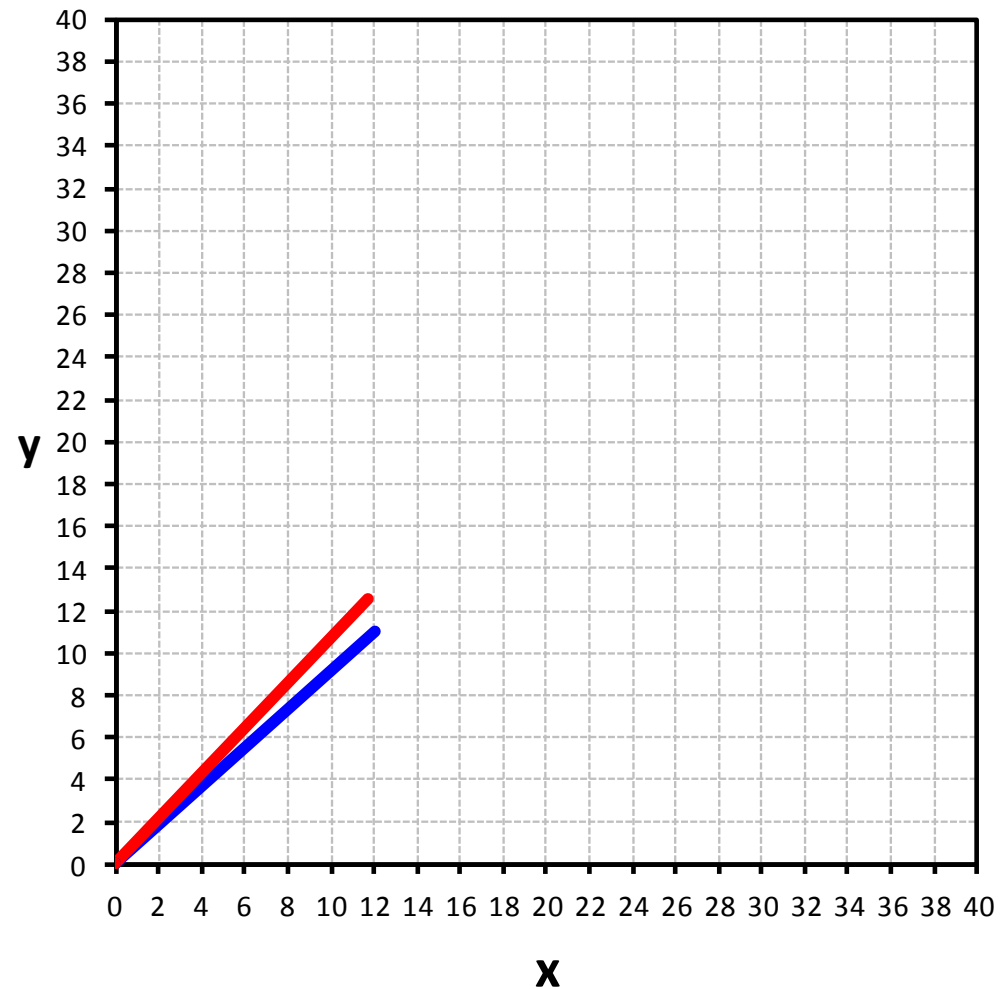
$$\begin{array}{|c|} \hline b1 \\ \hline \end{array} \begin{array}{|c|} \hline A1 \\ \hline \end{array} + \begin{array}{|c|} \hline b2 \\ \hline \end{array} \begin{array}{|c|} \hline A2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array} \begin{array}{|c|} \hline \\ \hline \end{array}$$



Sistemas lineares

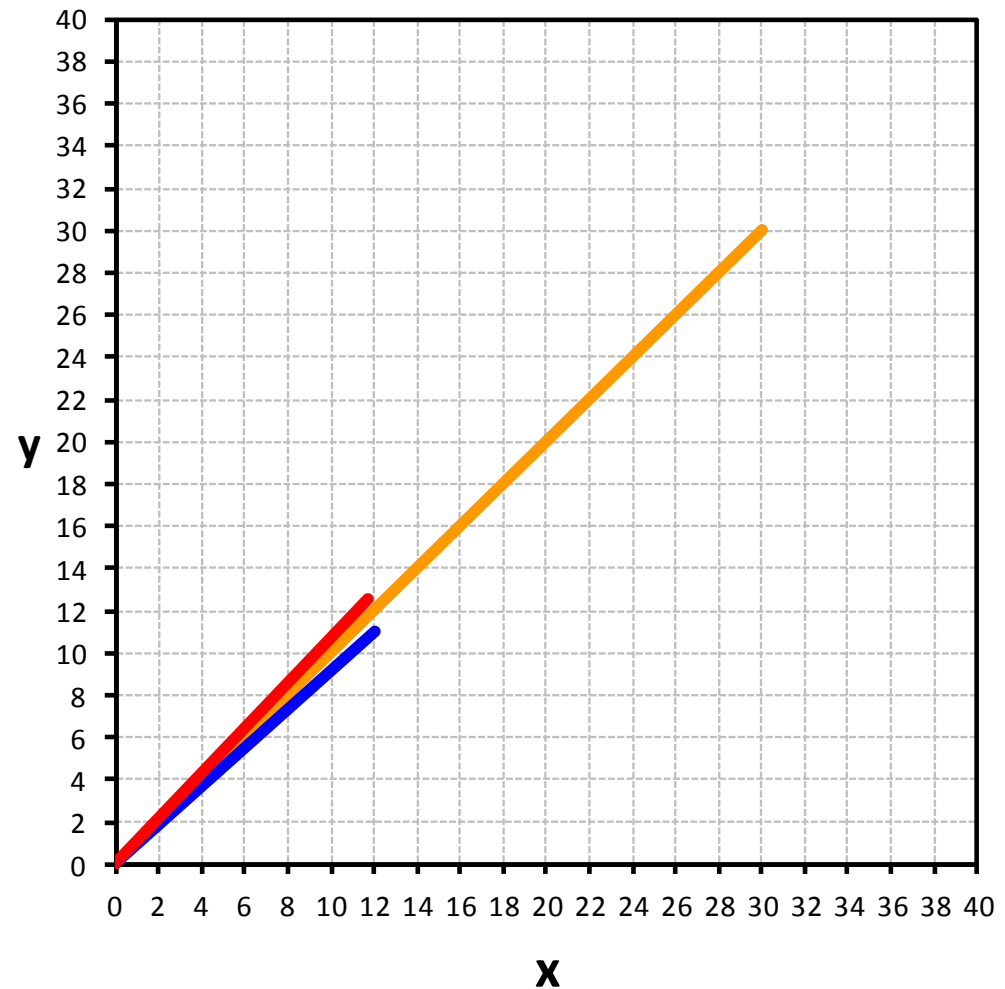
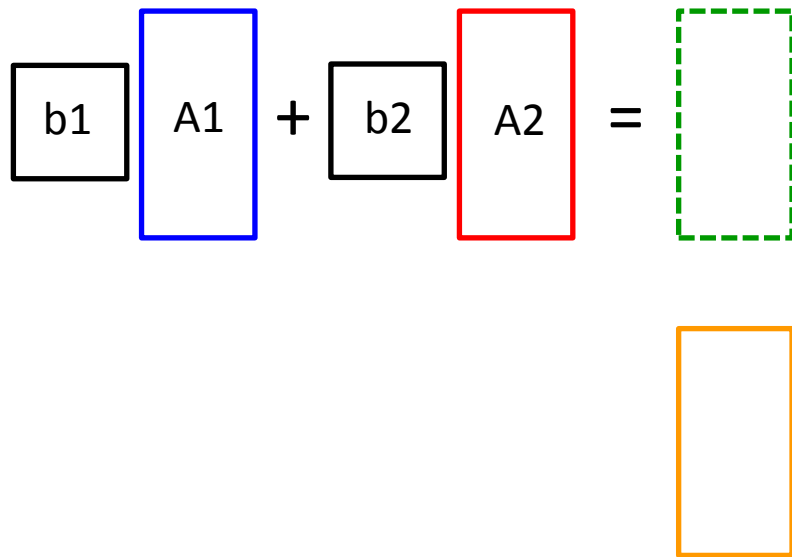
Exemplo 2D

$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{}$$



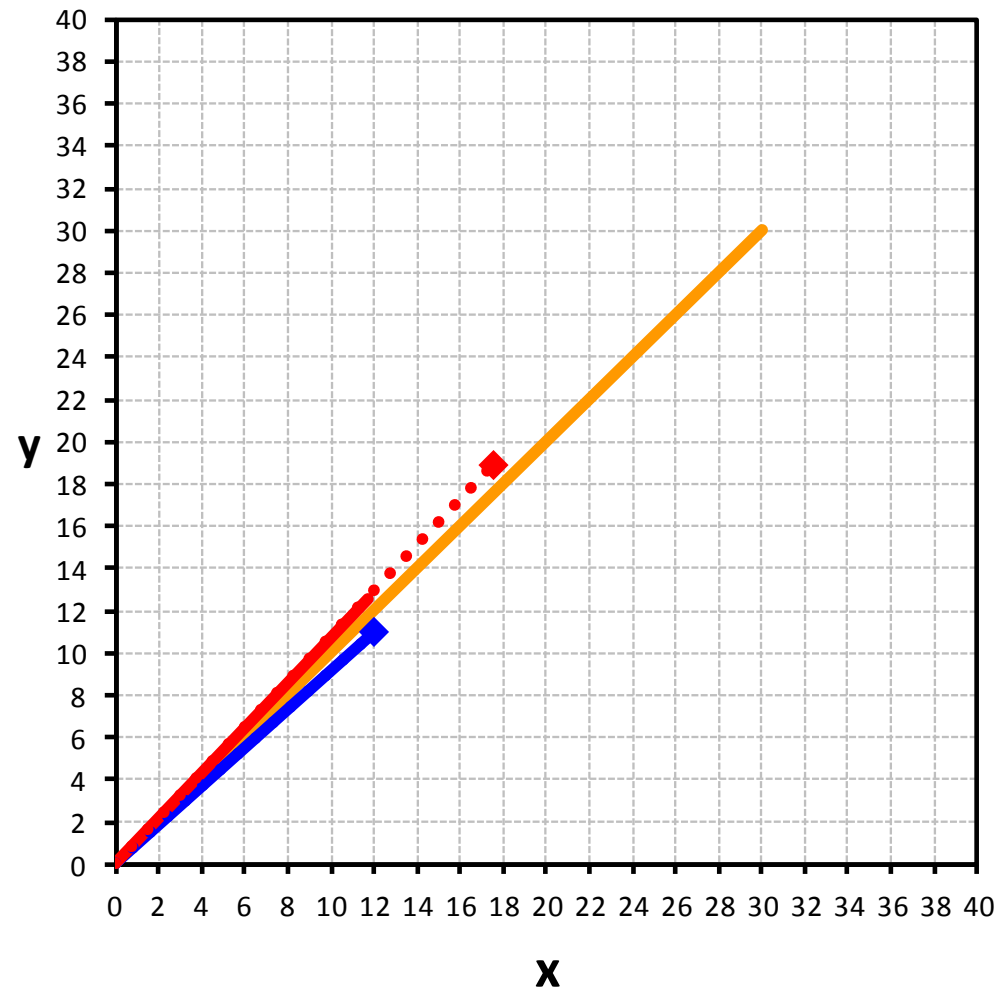
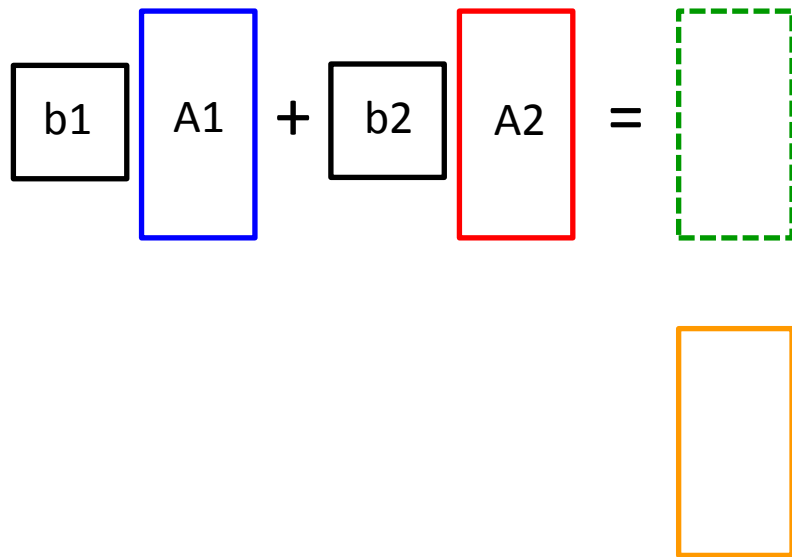
Sistemas lineares

Exemplo 2D



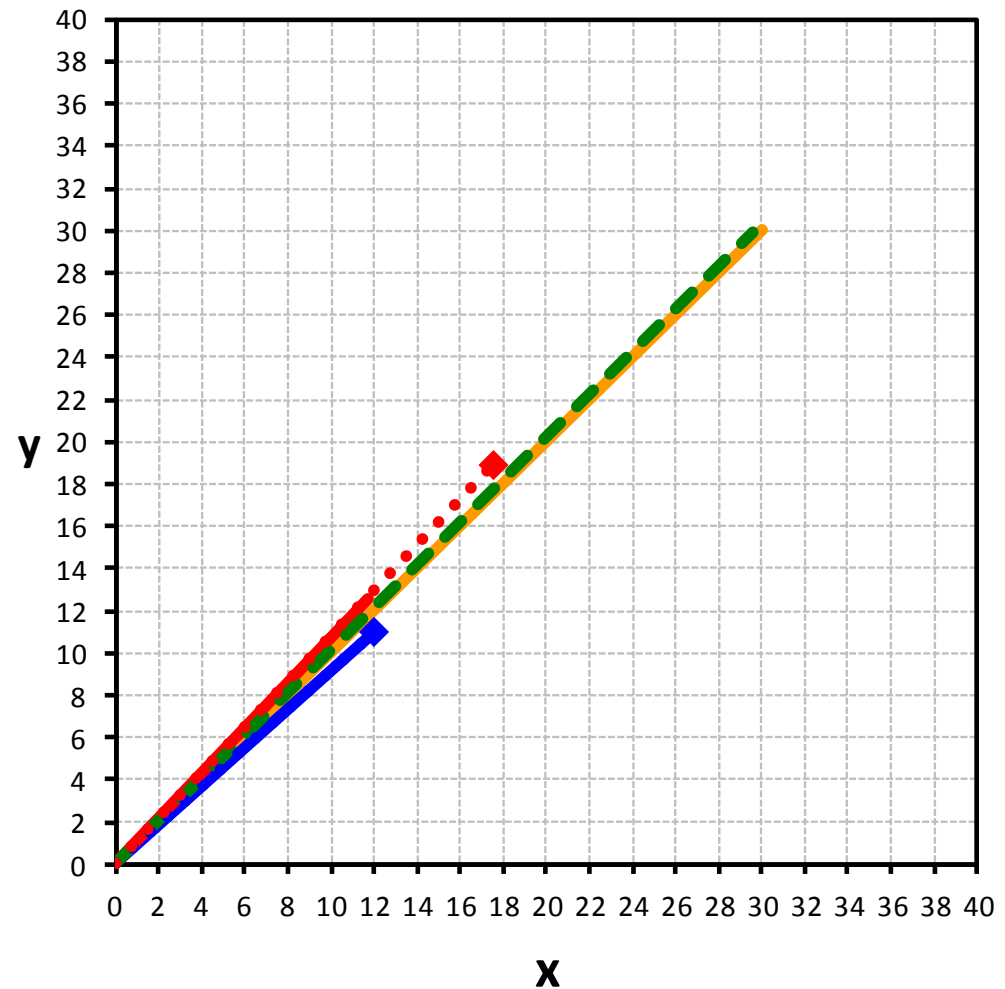
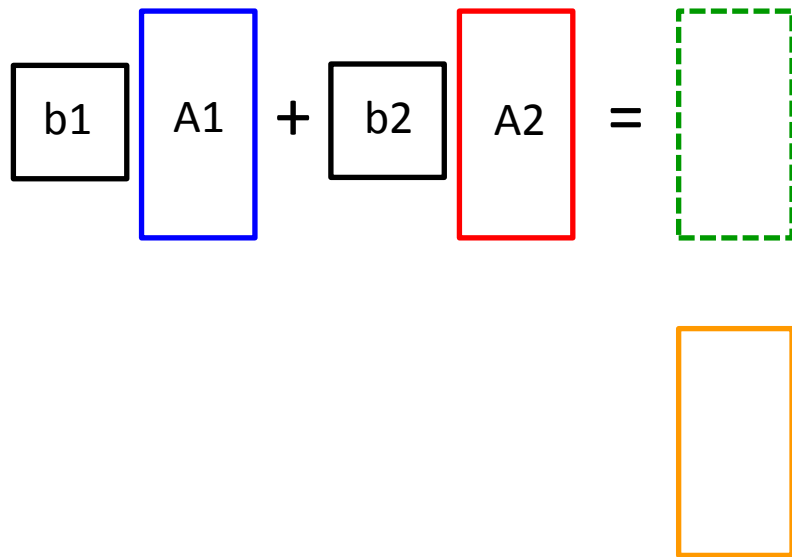
Sistemas lineares

Exemplo 2D



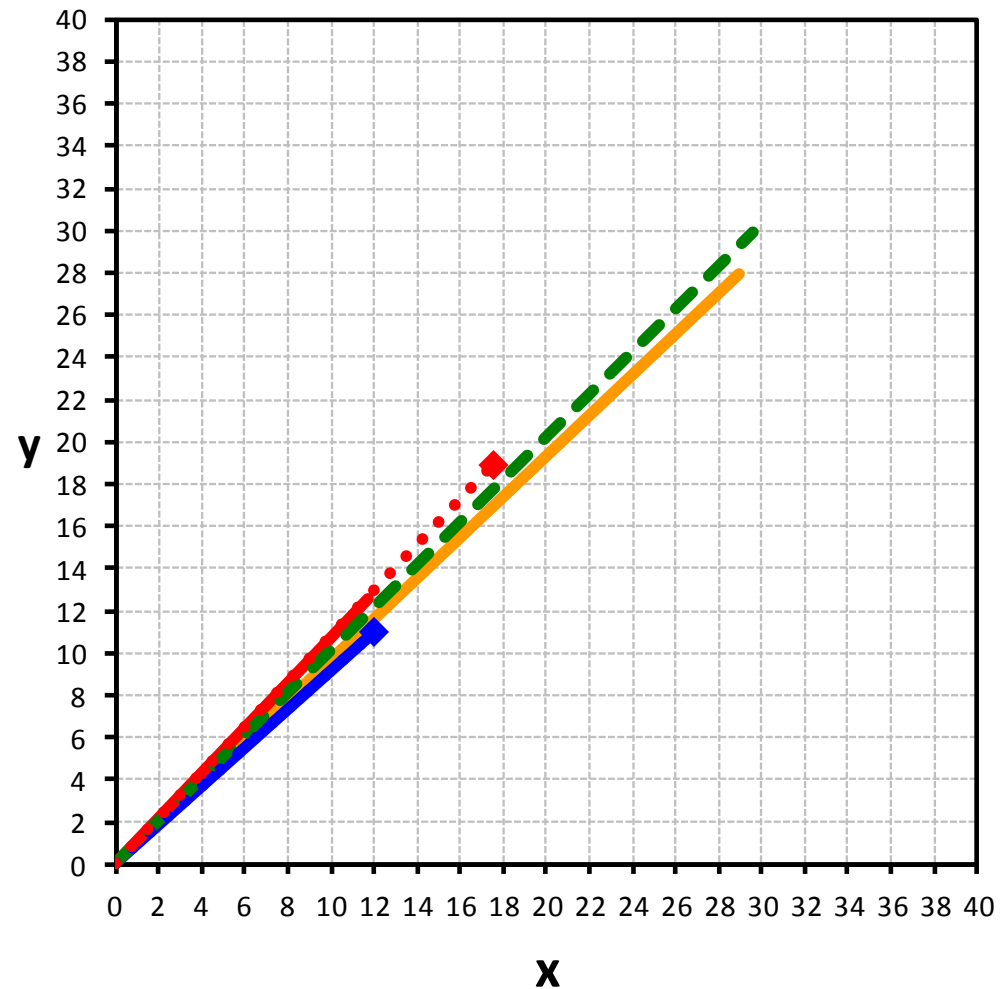
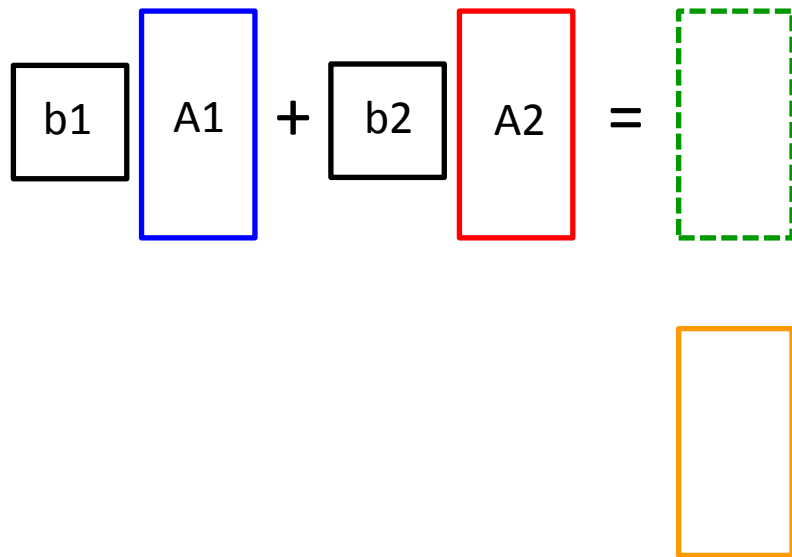
Sistemas lineares

Exemplo 2D



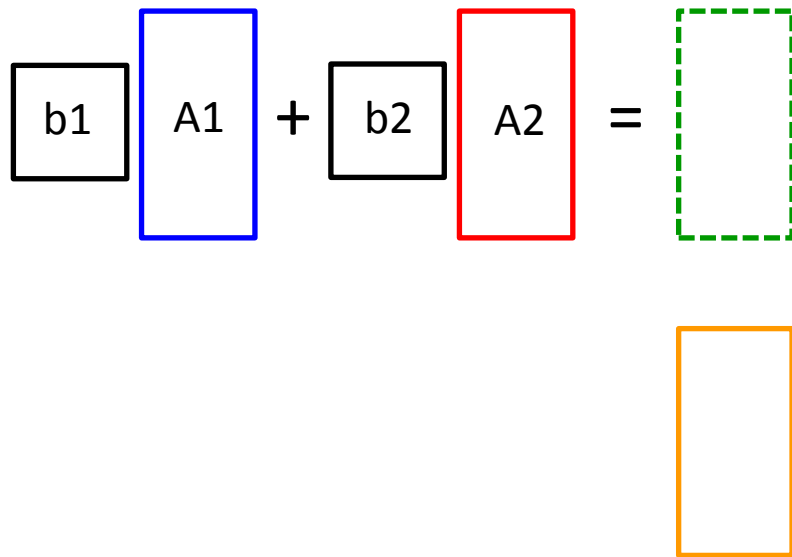
Sistemas lineares

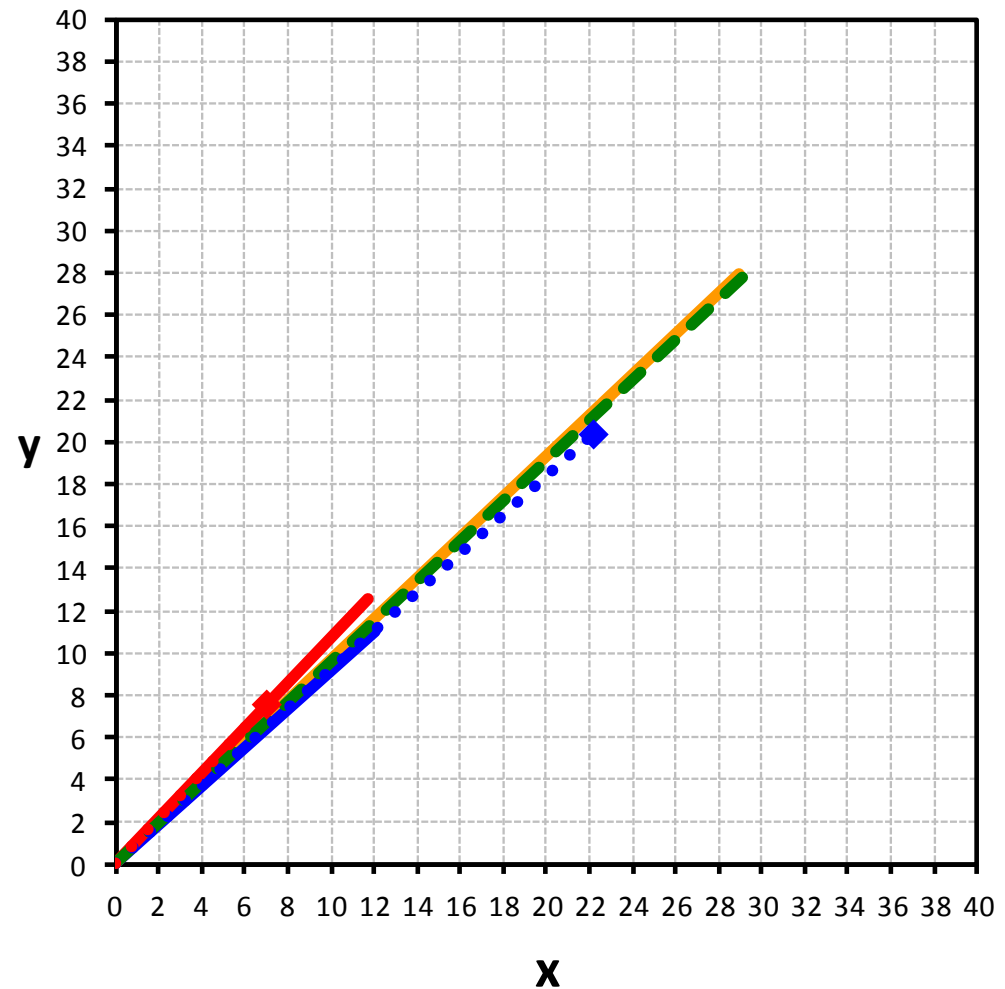
Exemplo 2D



Sistemas lineares

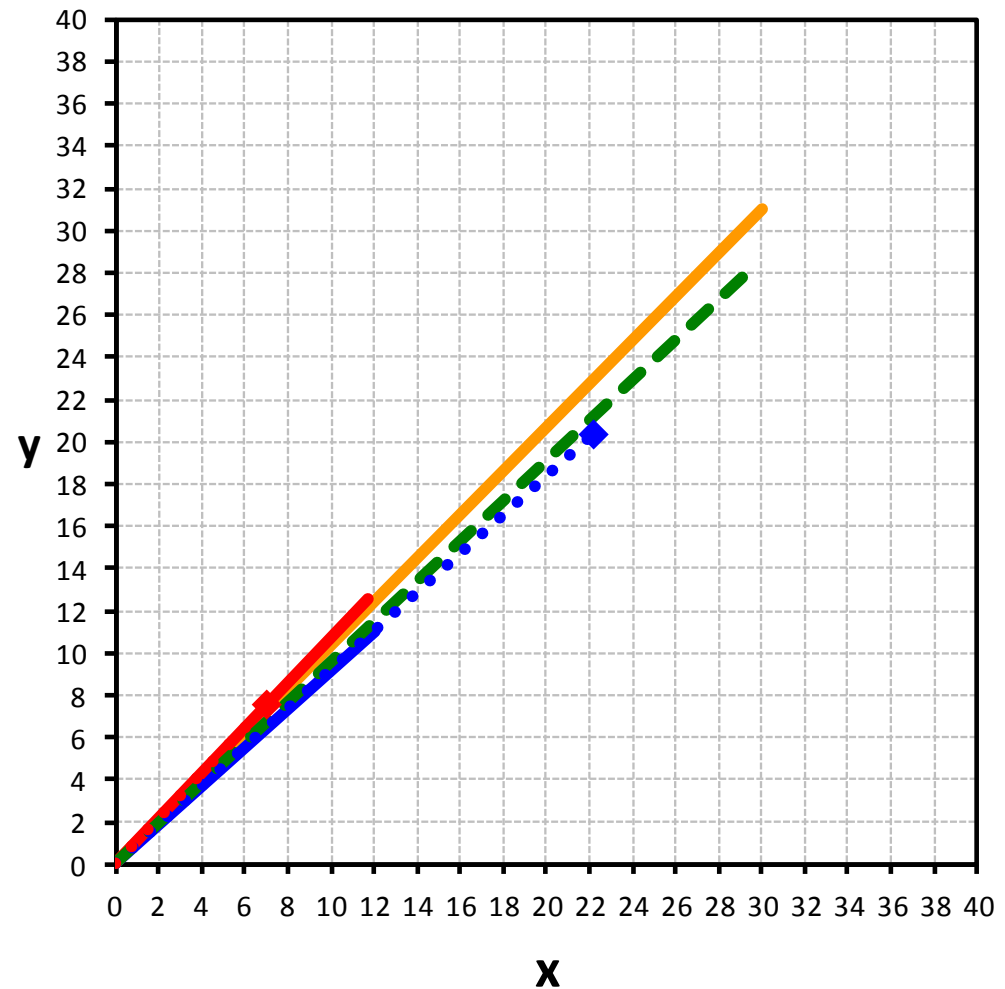
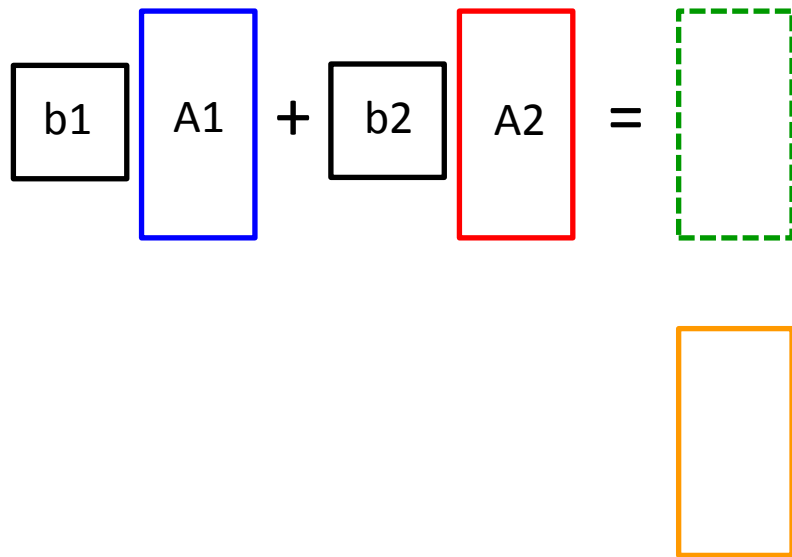
Exemplo 2D

$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array}$$




Sistemas lineares


Exemplo 2D

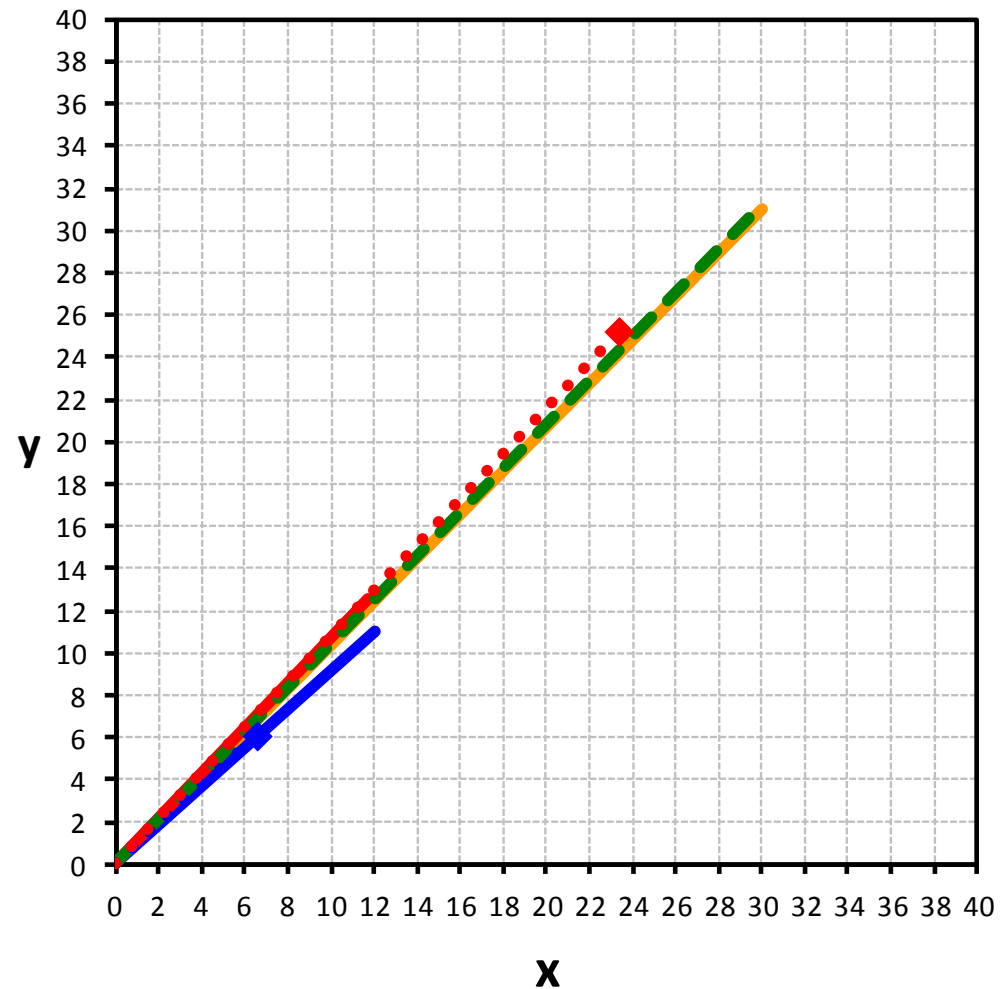


Sistemas lineares

Exemplo 2D

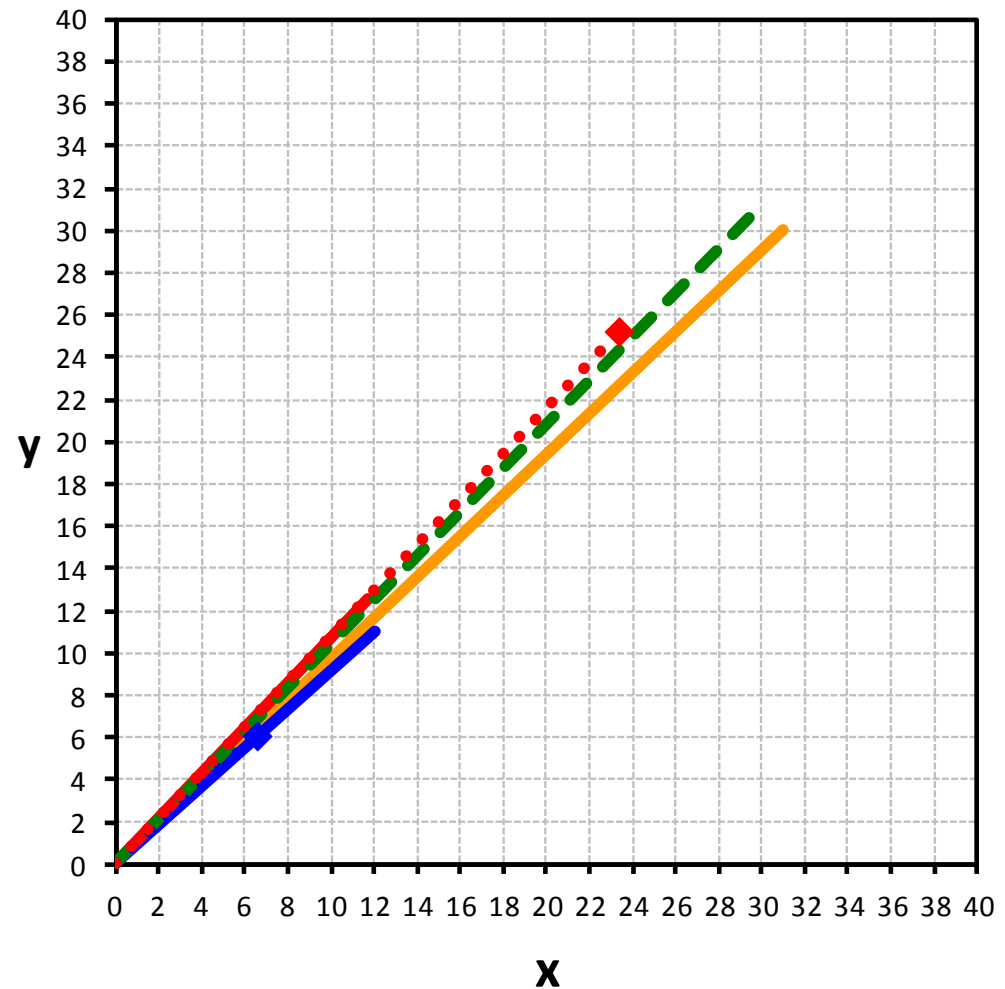
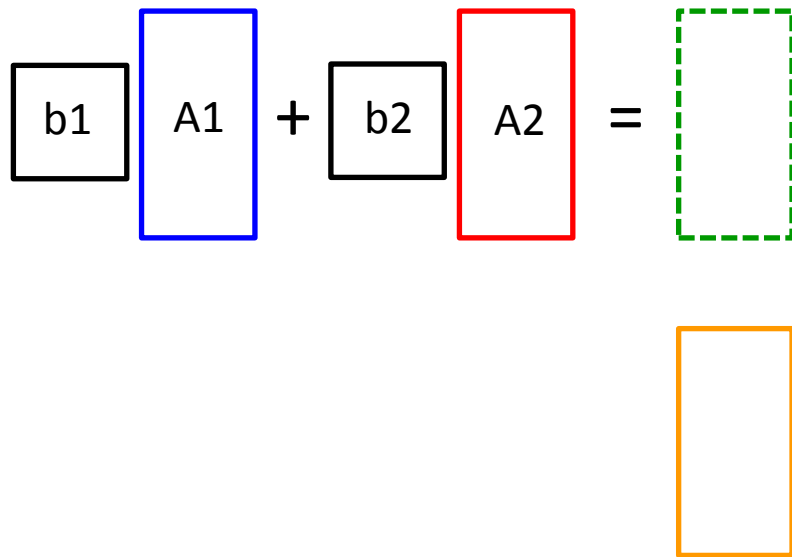
$$\begin{array}{|c|} \hline b_1 \\ \hline \end{array} \begin{array}{|c|} \hline A_1 \\ \hline \end{array} + \begin{array}{|c|} \hline b_2 \\ \hline \end{array} \begin{array}{|c|} \hline A_2 \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array}$$





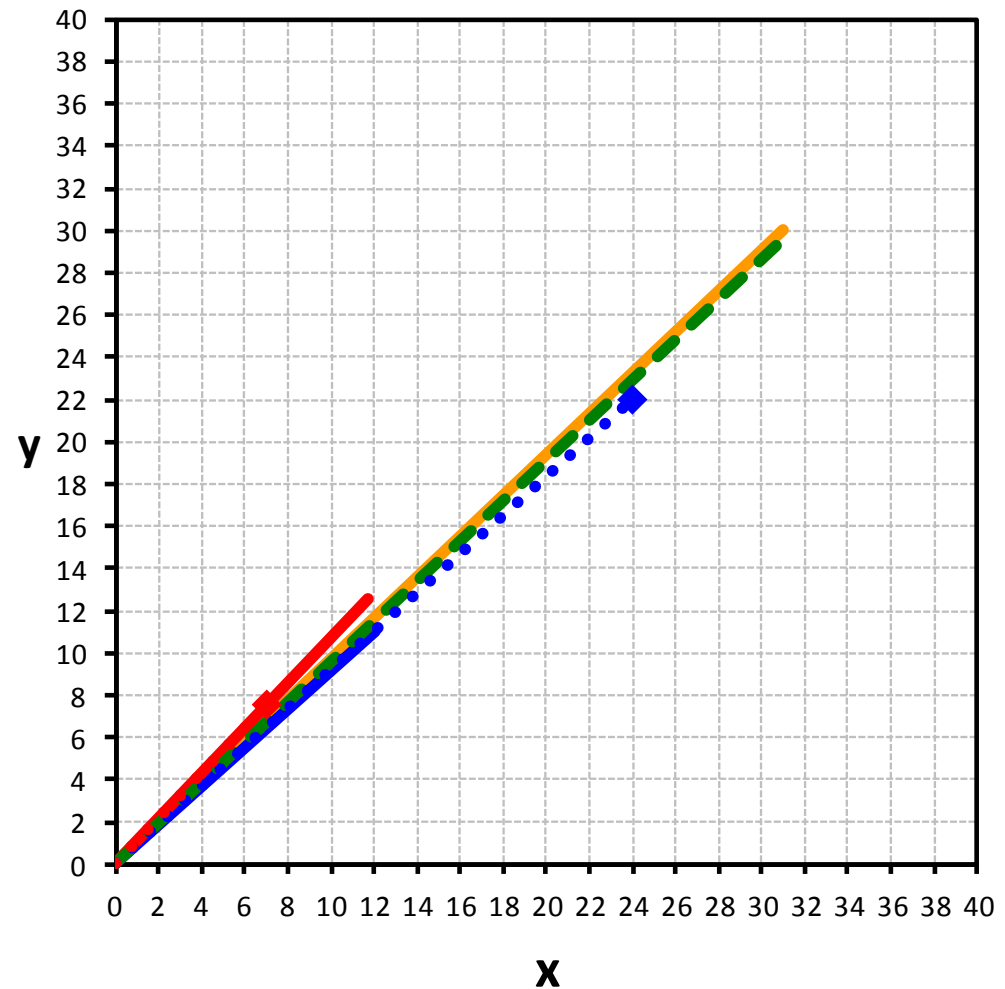
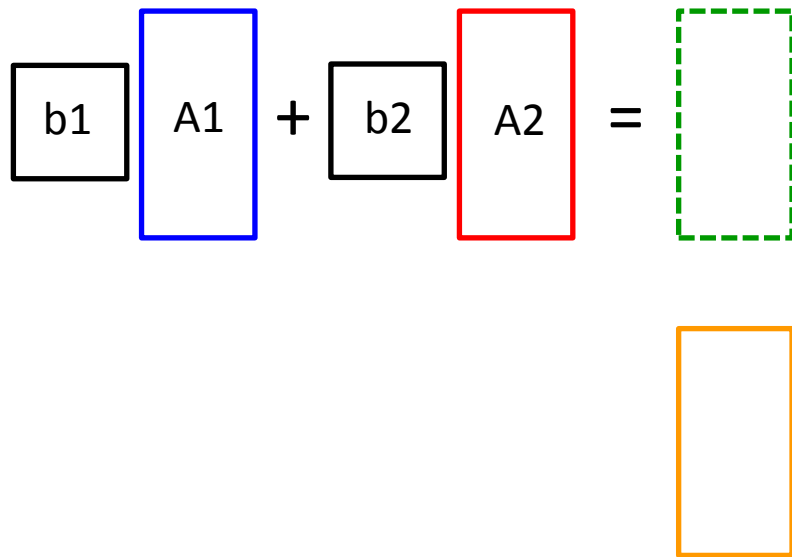
Sistemas lineares

Exemplo 2D



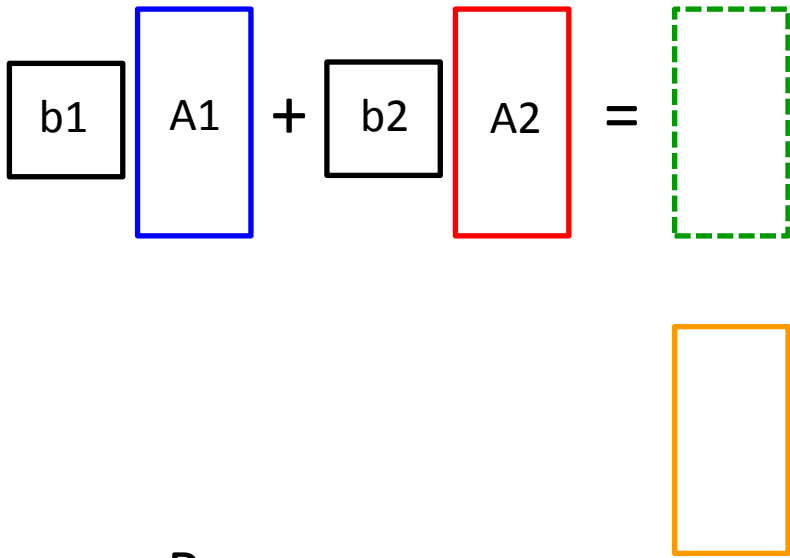
Sistemas lineares

Exemplo 2D

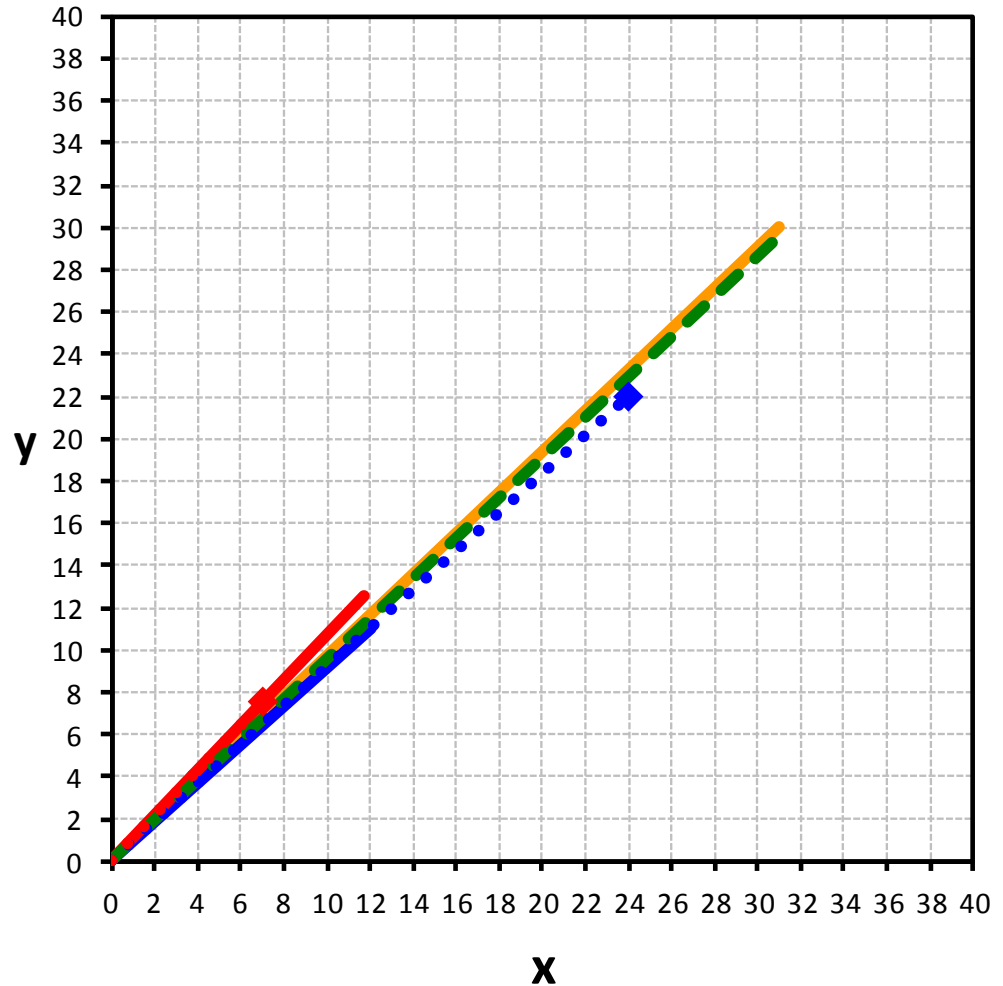


Sistemas lineares

Exemplo 2D

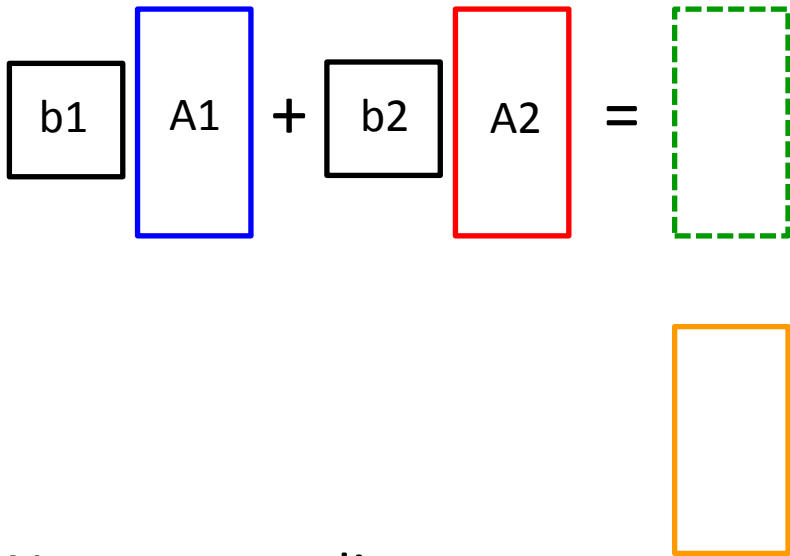


Pequenas
perturbações do lado
direito causam grandes
perturbações nos
coeficientes b_1 e b_2

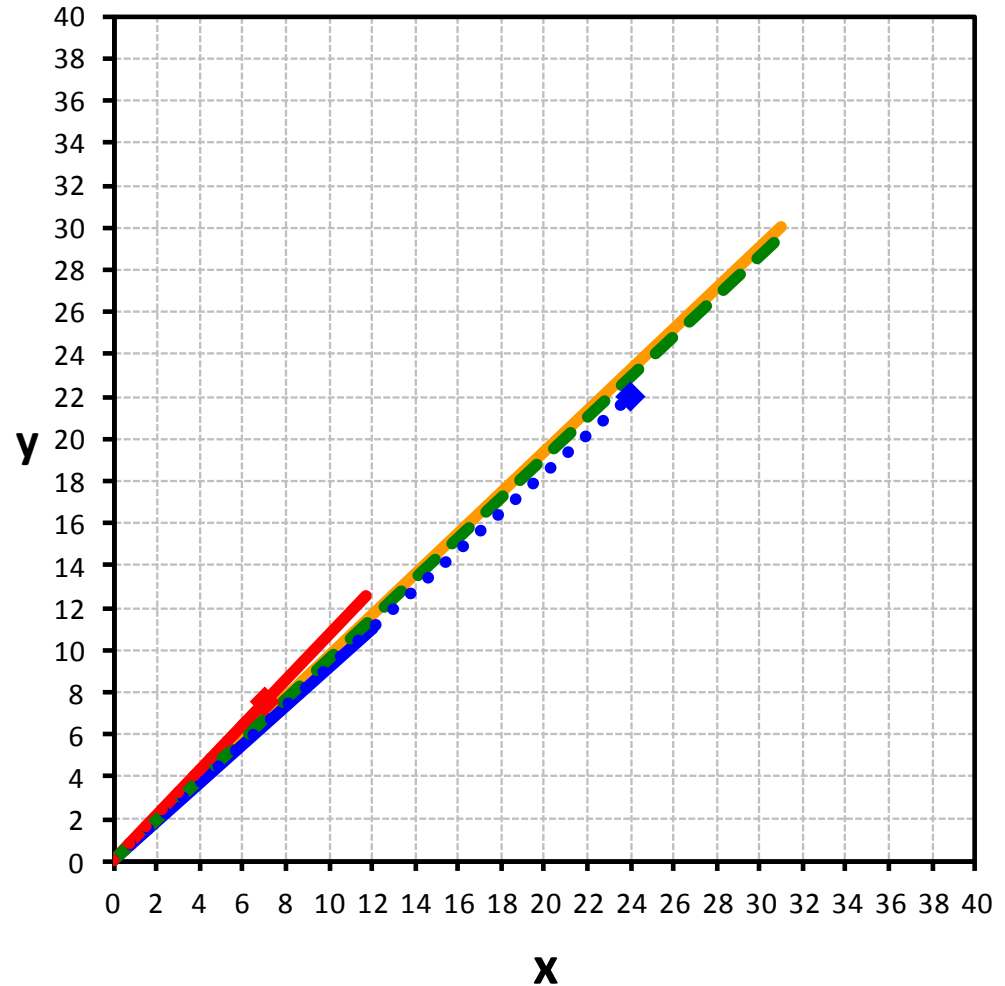


Sistemas lineares

Exemplo 2D

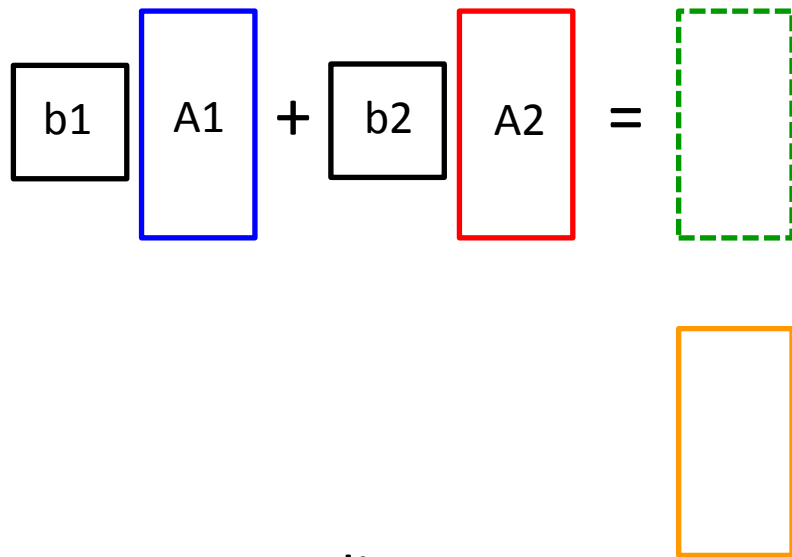


Nesse caso, diz-se que o sistema linear é *instável* e que a matriz é *mal-condicionada*

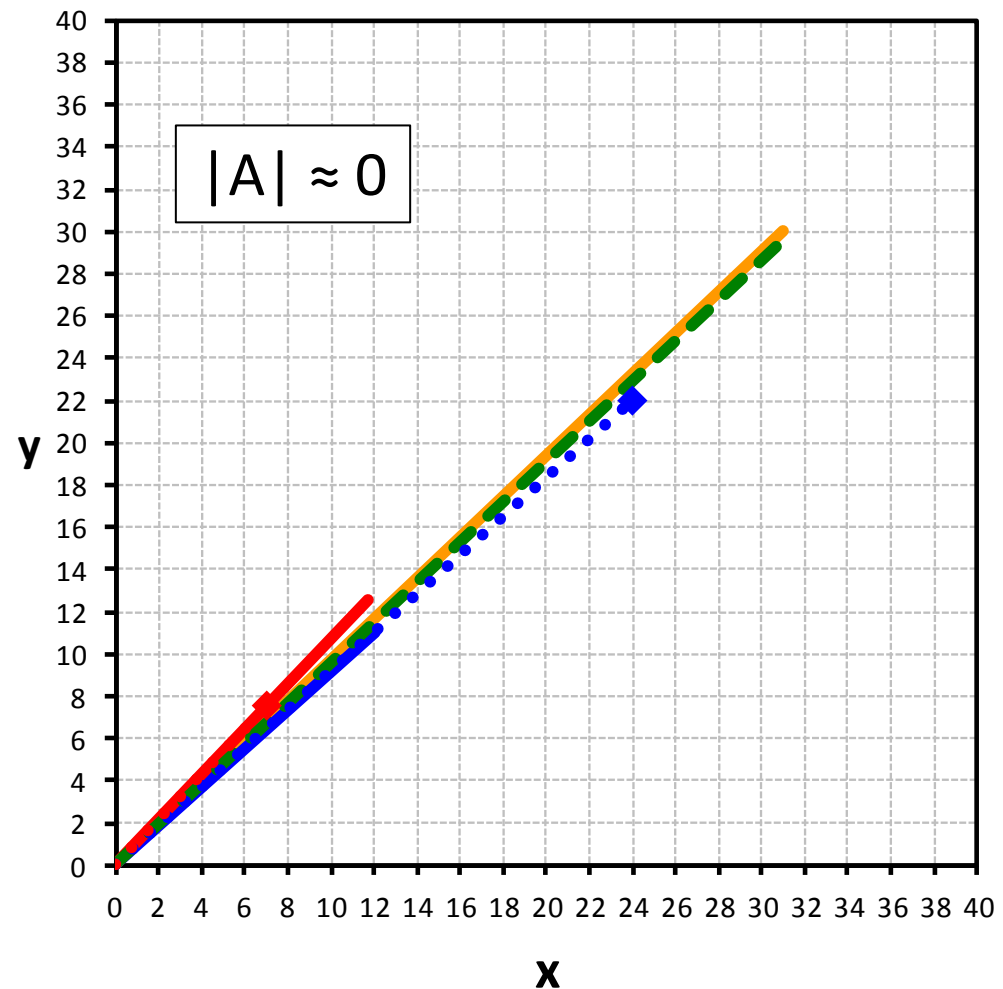


Sistemas lineares

Exemplo 2D

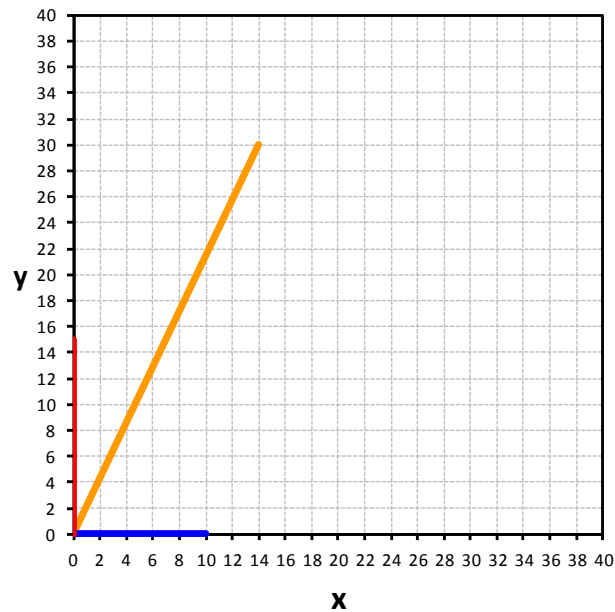
$$\begin{bmatrix} b1 \end{bmatrix} \begin{bmatrix} A1 \end{bmatrix} + \begin{bmatrix} b2 \end{bmatrix} \begin{bmatrix} A2 \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$


Nesse caso, diz-se que o sistema linear é *instável* e que a matriz é *mal-condicionada*

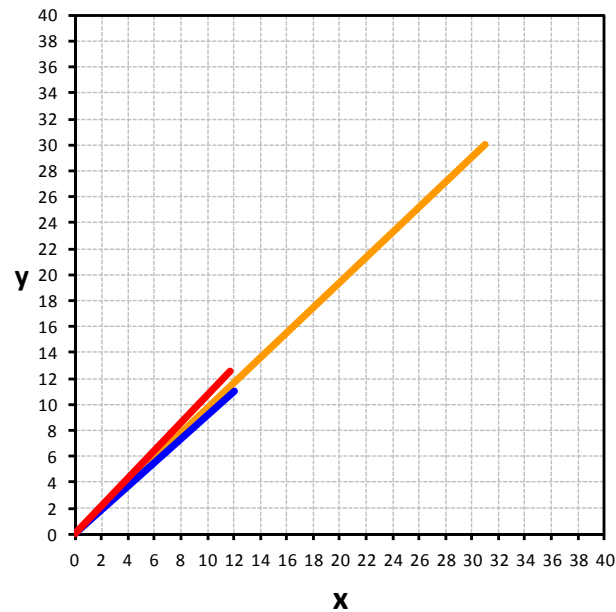


Sistemas lineares

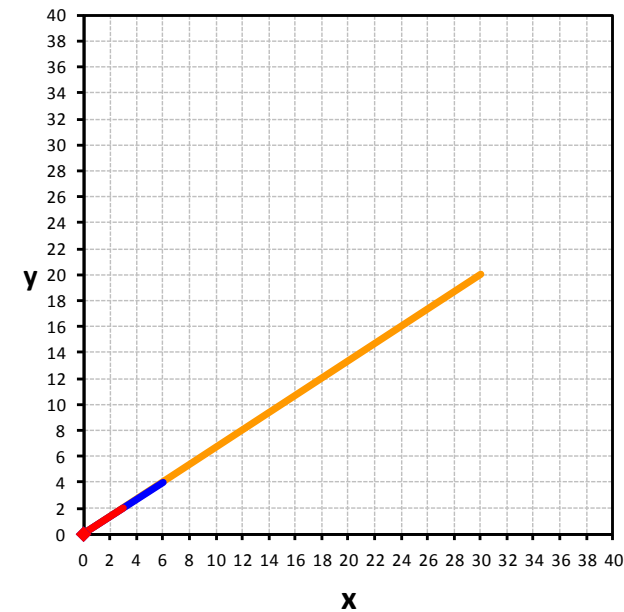
$$|A| \neq 0$$



$$|A| \approx 0$$



$$|A| = 0$$



$$\boxed{b1} \boxed{A1} + \boxed{b2} \boxed{A2} = \boxed{} \boxed{}$$



Problemas lineares

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\begin{bmatrix} \bar{B} & \bar{B} \end{bmatrix}^T \right)^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problema não-linear

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\Delta \bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$


$$\bar{p}^* = \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix}^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p}^* = \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix}^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$


Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

Problemas lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

Caso 1)

$$\det \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \neq 0$$

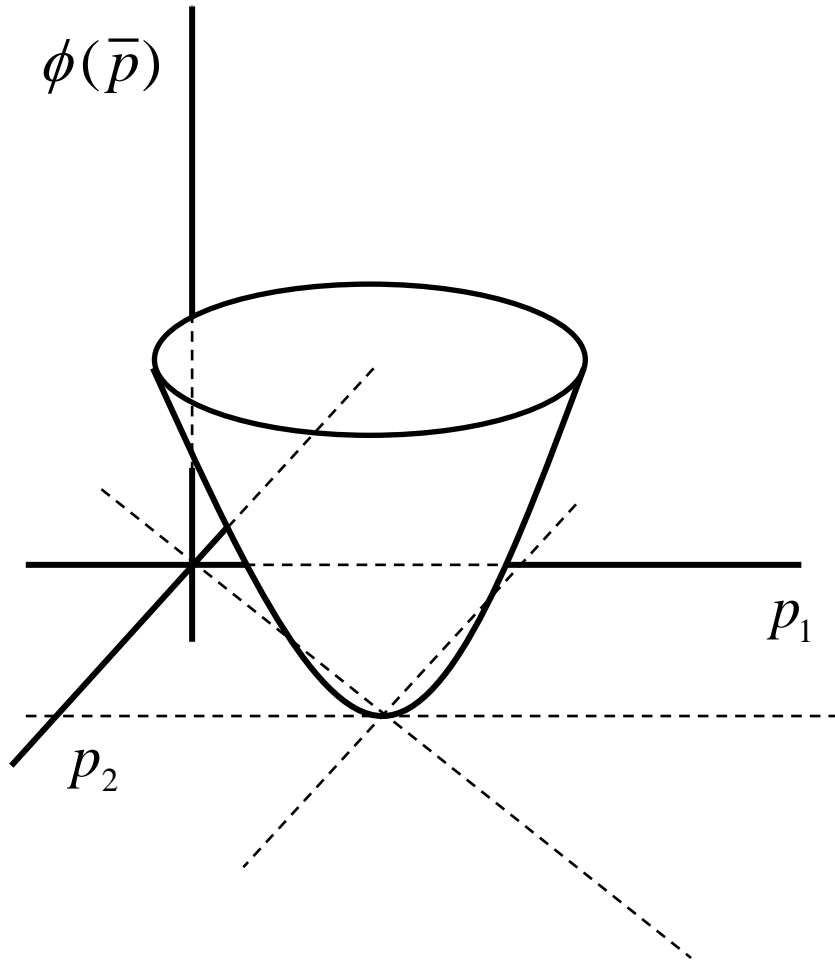
Caso 2)

$$\det \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} \text{---}^T \text{---} \\ B & B \end{pmatrix} \neq 0$$

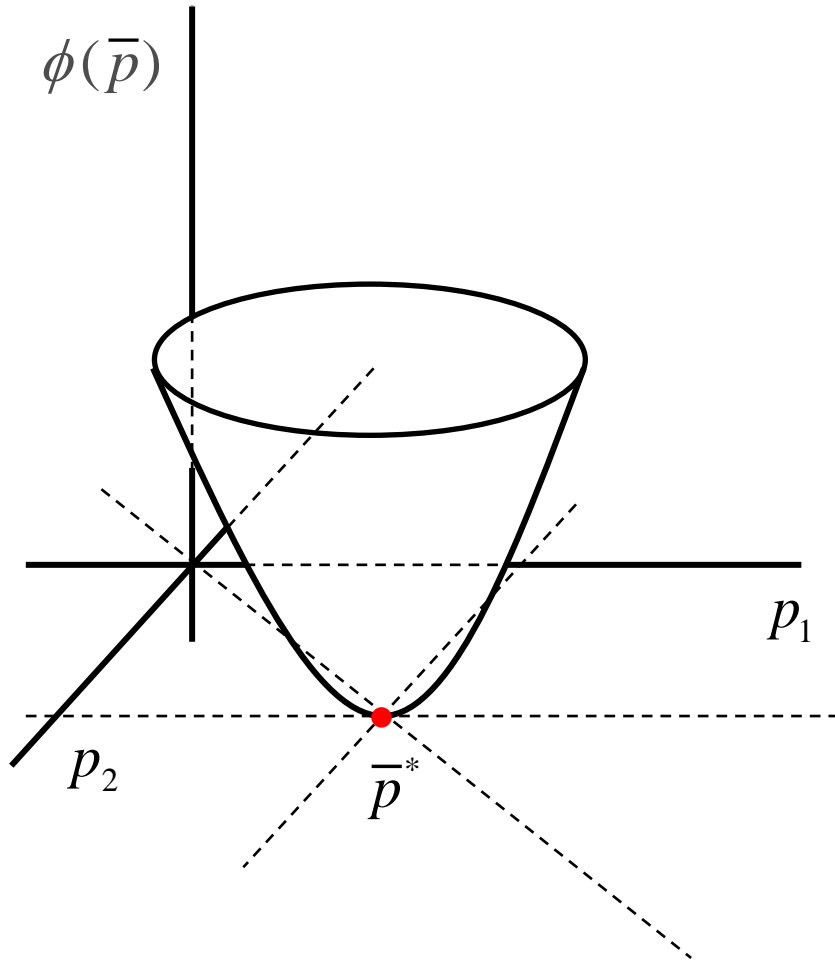
Caso 2)

$$\det \begin{pmatrix} \text{---}^T \text{---} \\ B & B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \text{---}^T \text{---} \\ B & B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \neq 0$$

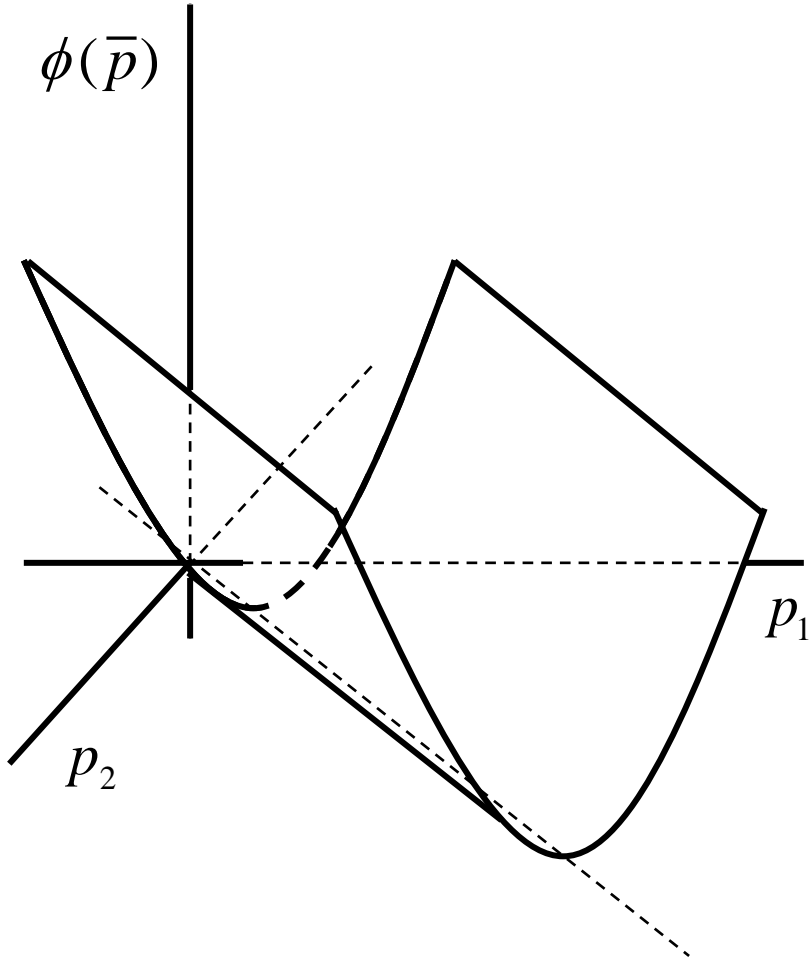
Caso 2)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \neq 0$$

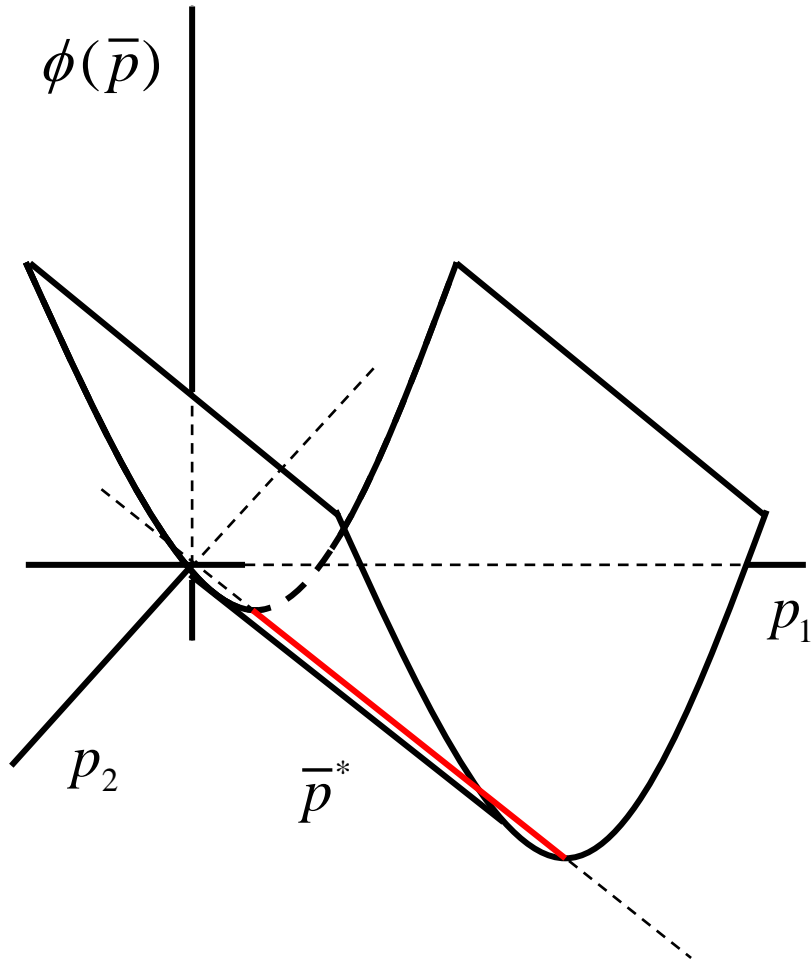
Caso 2)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \neq 0$$

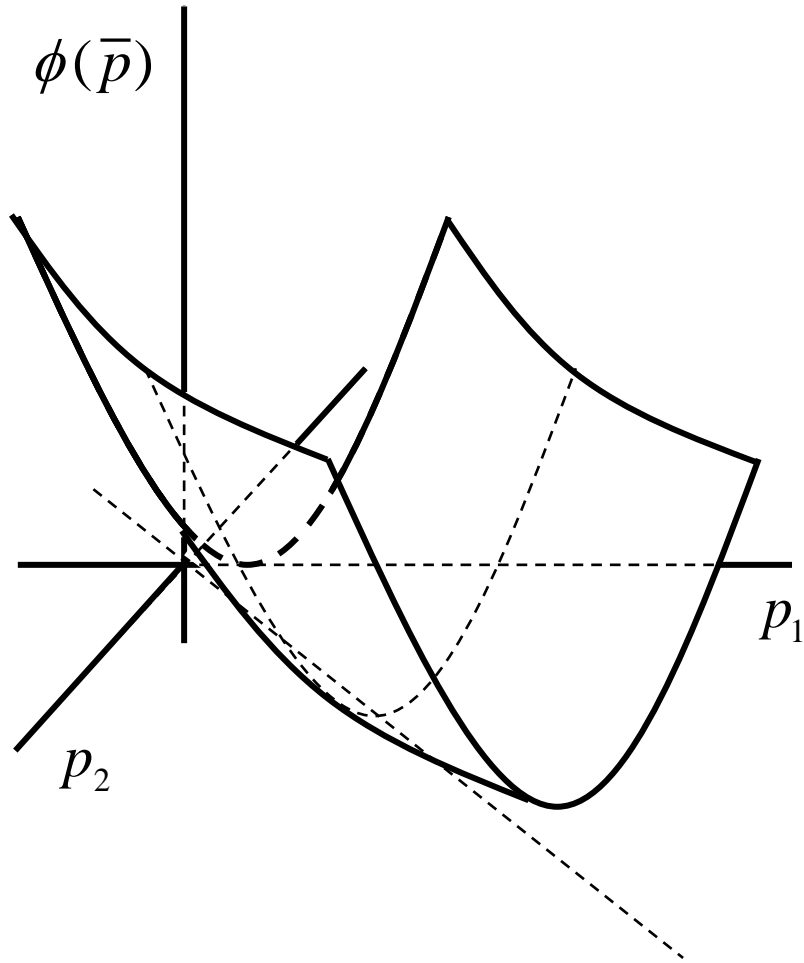
Caso 2)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \neq 0$$

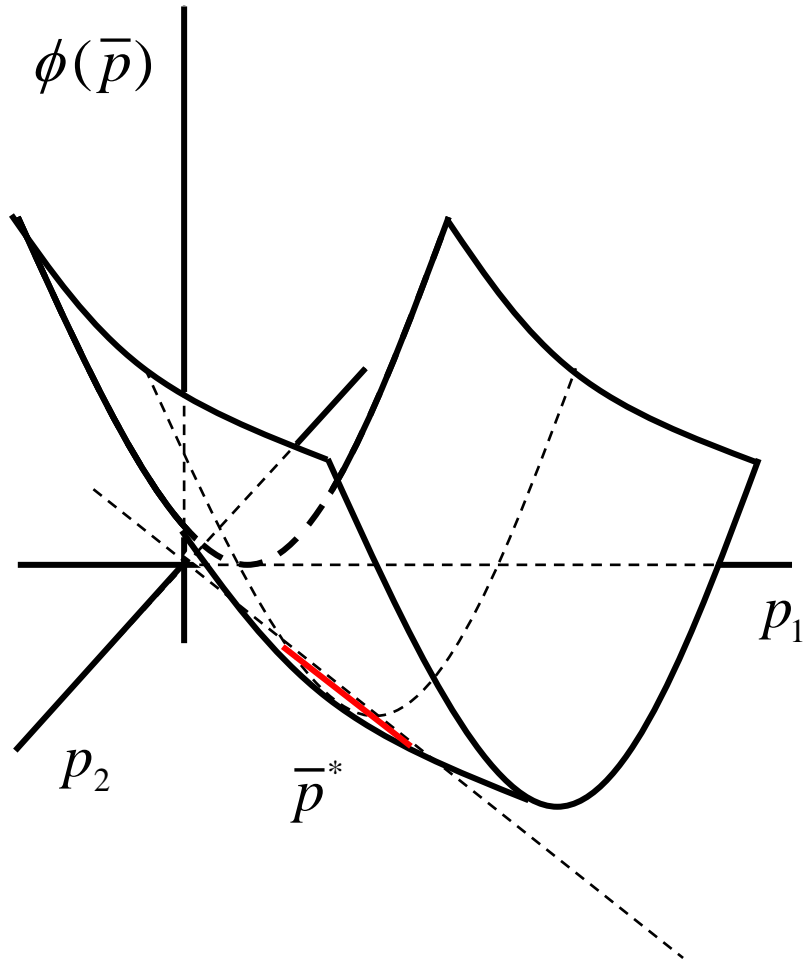
Caso 2)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \begin{matrix} = & T & = \end{matrix} \\ B & B \end{pmatrix} \approx 0$$

Problemas lineares



Caso 1)

$$\det \begin{pmatrix} \begin{smallmatrix} = & T & = \end{smallmatrix} \\ B & B \end{pmatrix} \neq 0$$

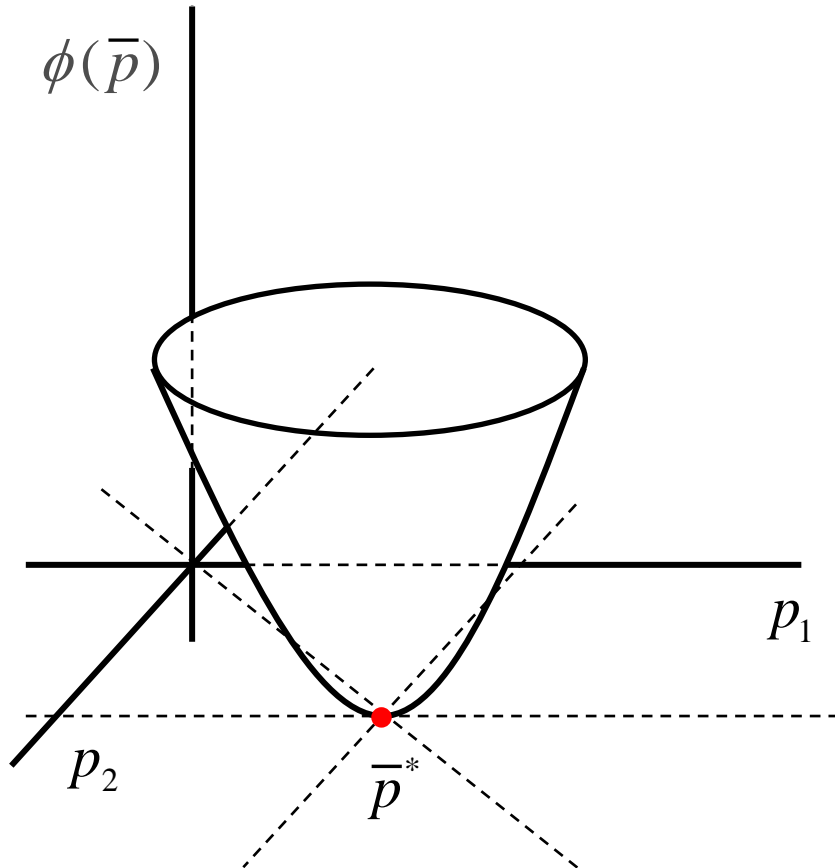
Caso 2)

$$\det \begin{pmatrix} \begin{smallmatrix} = & T & = \end{smallmatrix} \\ B & B \end{pmatrix} = 0$$

Caso 3)

$$\det \begin{pmatrix} \begin{smallmatrix} = & T & = \end{smallmatrix} \\ B & B \end{pmatrix} \approx 0$$

Problemas lineares



$$\begin{pmatrix} \overline{\overline{B}}^T & \overline{\overline{B}} \end{pmatrix} \overline{p}^* = \overline{\overline{B}}^T [\overline{d} - \overline{b}]$$

Solução única

Caso 1)

$$\det \begin{pmatrix} \overline{\overline{B}}^T & \overline{\overline{B}} \end{pmatrix} \neq 0$$

Caso 2)

$$\det \begin{pmatrix} \overline{\overline{B}}^T & \overline{\overline{B}} \end{pmatrix} = 0$$

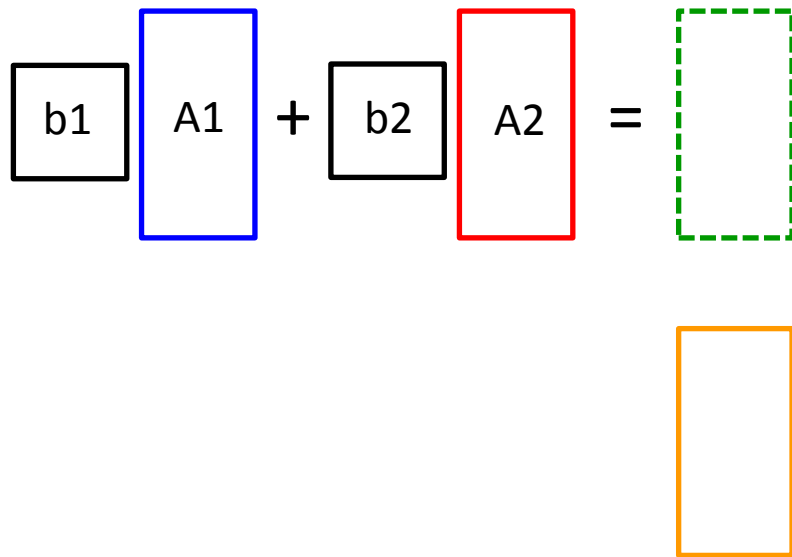
Caso 3)

$$\det \begin{pmatrix} \overline{\overline{B}}^T & \overline{\overline{B}} \end{pmatrix} \approx 0$$

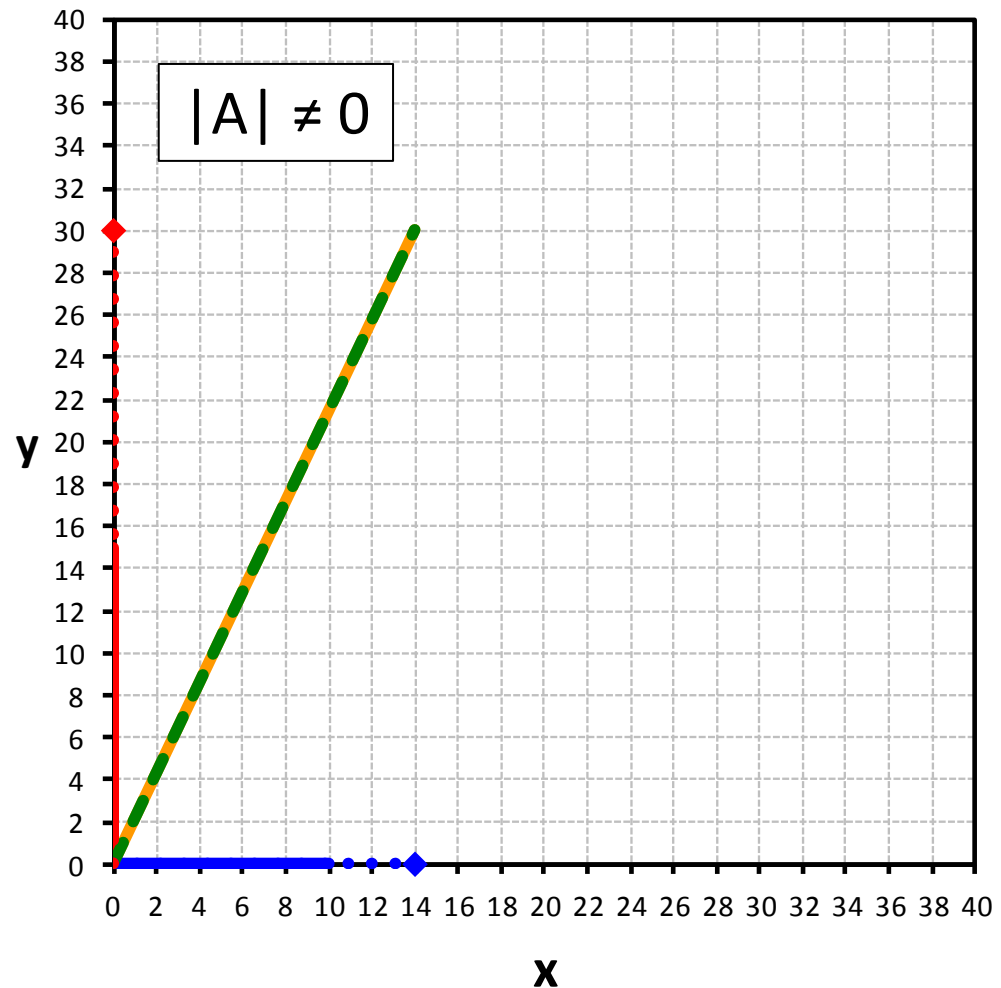
Problemas lineares

Exemplo 2D

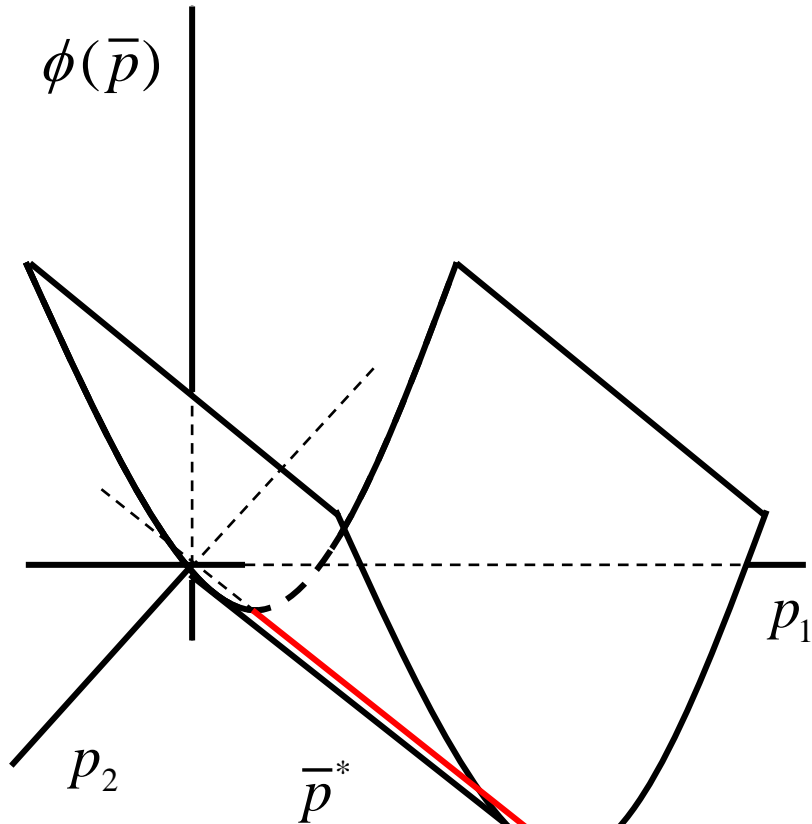
$$\boxed{b_1} \boxed{A_1} + \boxed{b_2} \boxed{A_2} = \boxed{}$$



Neste caso, os vetores A_1 e A_2 são *linearmente independentes* e os coeficientes b_1 e b_2 são únicos



Problemas lineares



$$\begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Infinitas soluções

Caso 1)

$$\det \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} \neq 0$$

Caso 2)

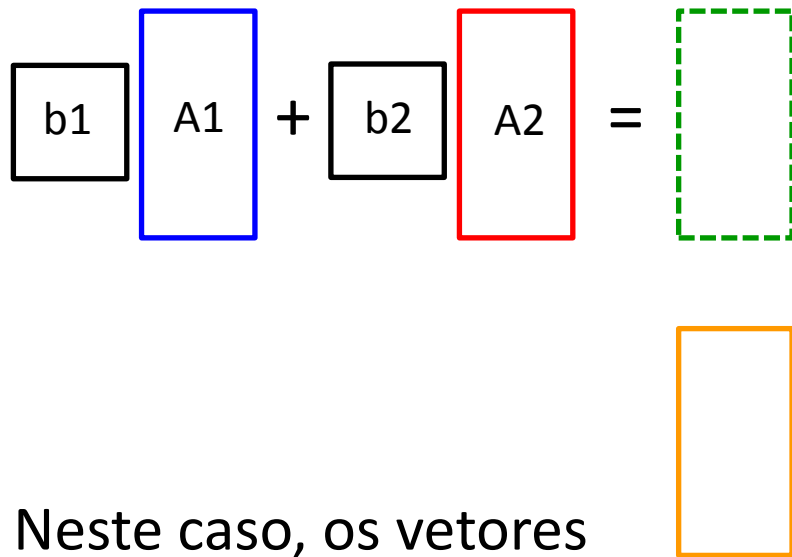
$$\det \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} = 0$$

Caso 3)

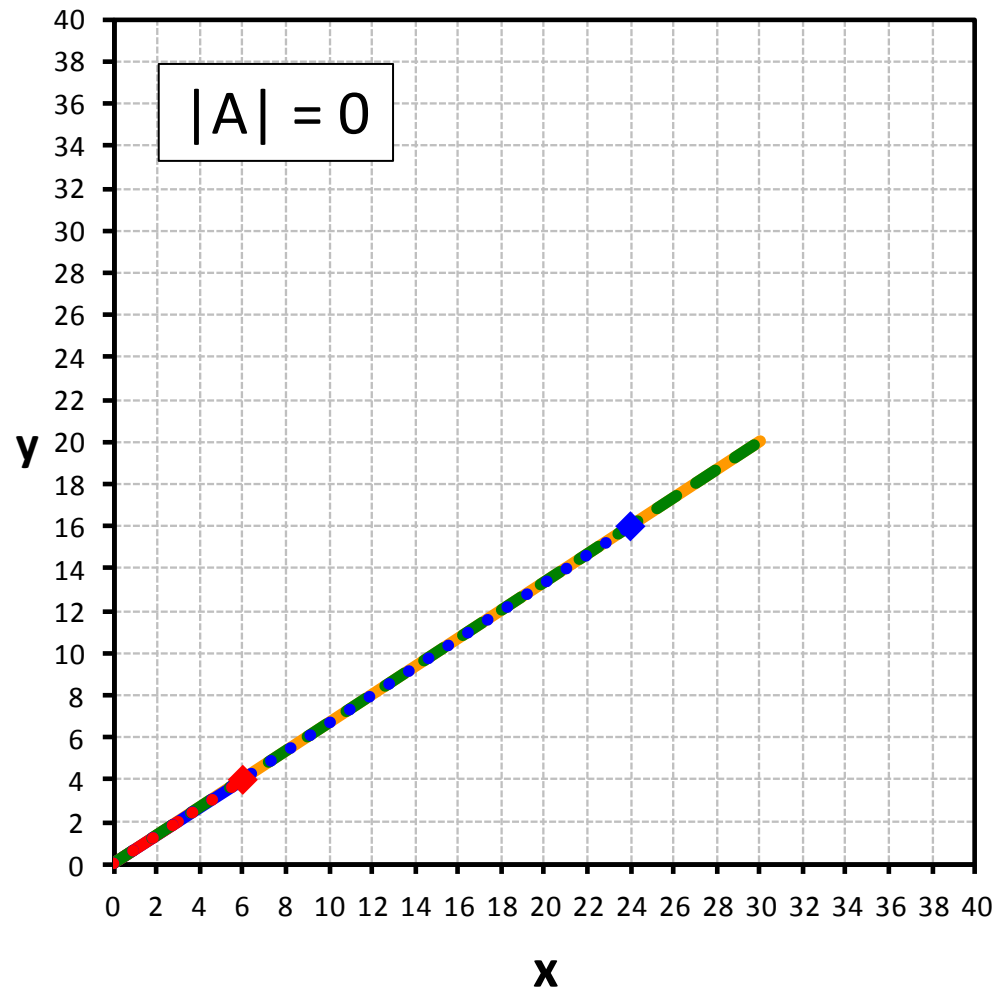
$$\det \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} \approx 0$$

Problemas lineares

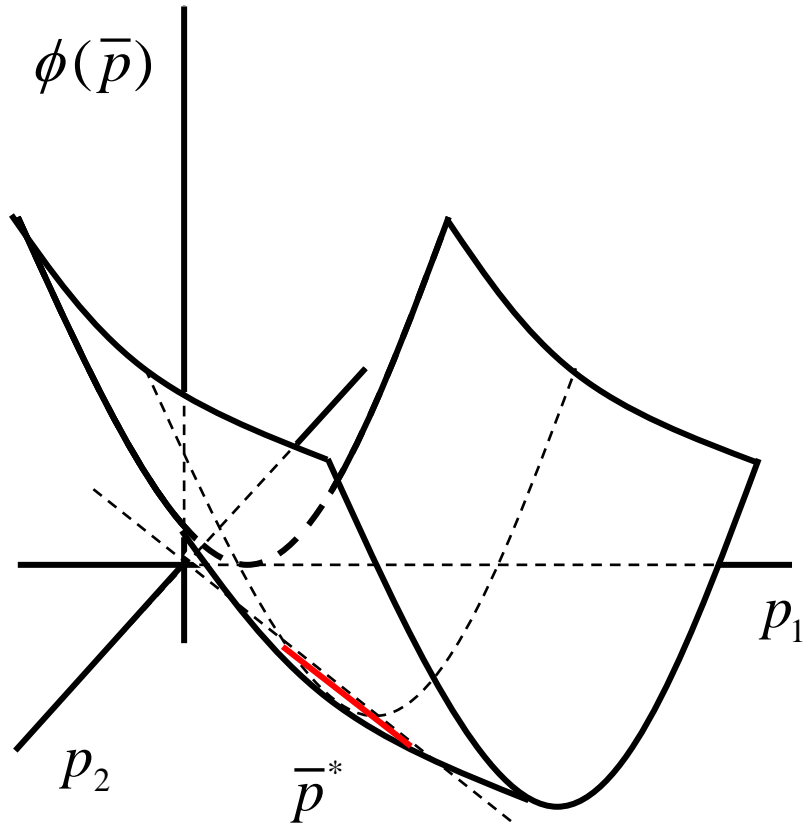
Exemplo 2D



Neste caso, os vetores $A1$ e $A2$ são *linearmente dependentes* e existem infinitos pares $b1$ e $b2$ que produzem o mesmo resultado



Problemas lineares



$$\begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema instável

Caso 1)

$$\det \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} \neq 0$$

Caso 2)

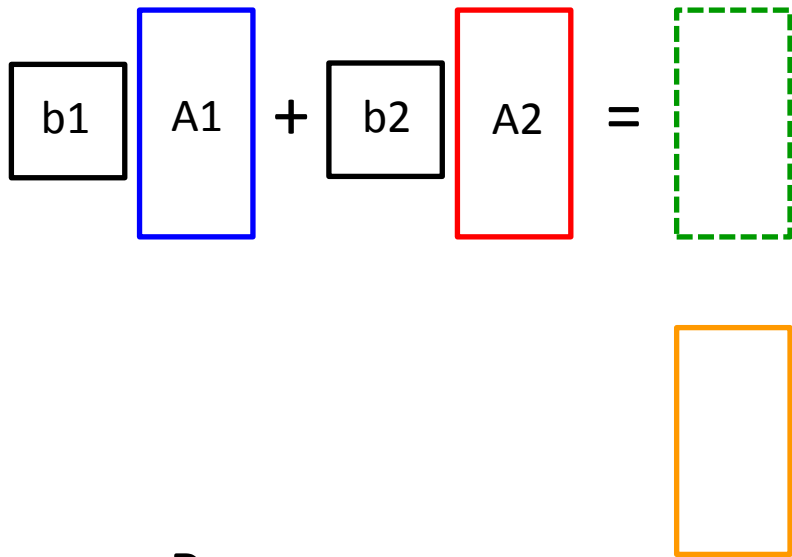
$$\det \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} = 0$$

Caso 3)

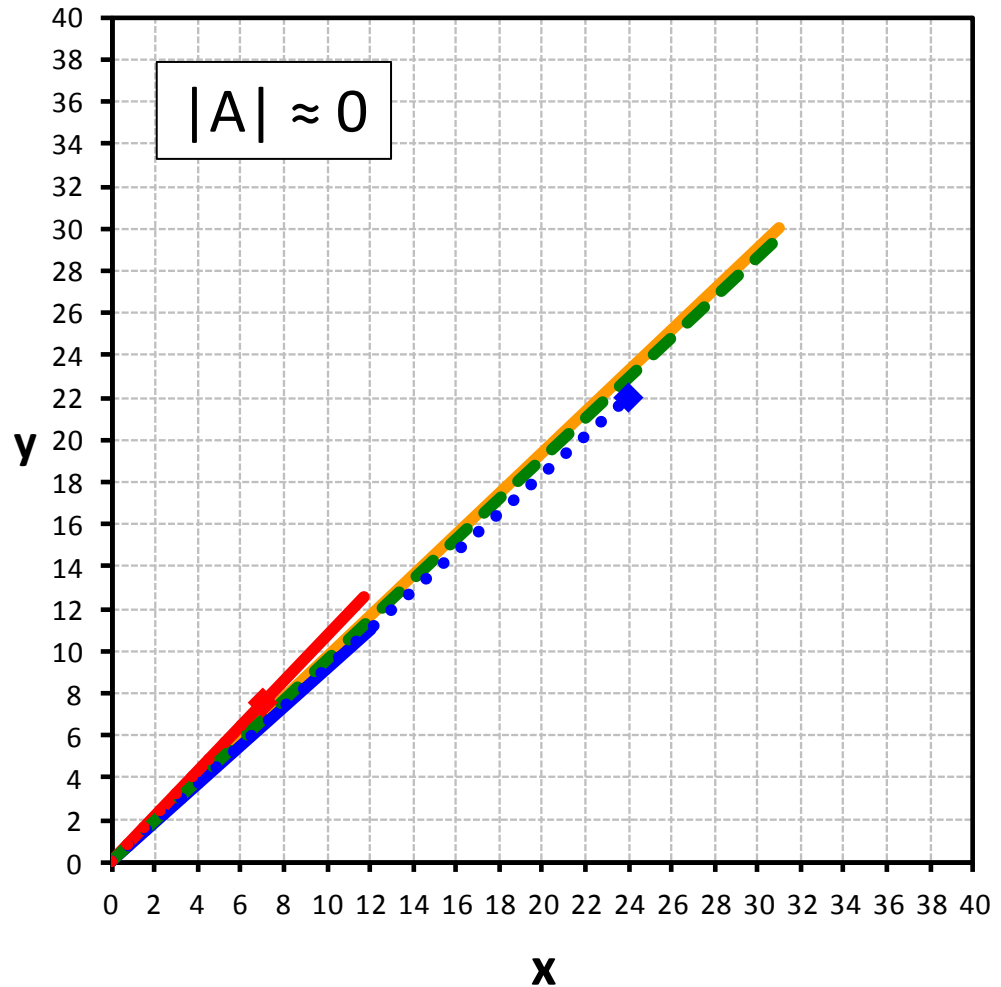
$$\det \begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} \approx 0$$

Problemas lineares

Exemplo 2D



Pequenas
perturbações do lado
direito causam grandes
perturbações nos
coeficientes b_1 e b_2



Problemas não-lineares

$$\bar{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_M \end{bmatrix}_{M \times 1}$$

parâmetros

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N \times 1}$$

dados
observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N \times 1}$$

dados
preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

norma L2
(função escalar)

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

Problema linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{p}^* = \left(\begin{bmatrix} \bar{B} & \bar{B} \end{bmatrix}^T \right)^{-1} \bar{B}^T [\bar{d} - \bar{b}]$$

Problema não-linear

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\Delta \bar{p} = \left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right)^{-1} \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) \neq \overline{\overline{B}}\bar{p} + \overline{\overline{b}}$$

$$\overline{\nabla} \phi(\bar{p}^*) = \overline{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\Delta \bar{p} = \left(\overline{\overline{G}}(\bar{p}_0)^T \overline{\overline{G}}(\bar{p}_0) \right)^{-1} \overline{\overline{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$


Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) \neq \overline{B}\bar{p} + \bar{b}$$

$$\overline{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta\bar{p}$$

$$\Delta\bar{p} = \left(\overline{G}(\bar{p}_0)^T \overline{G}(\bar{p}_0) \right)^{-1} \overline{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$


Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear

Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

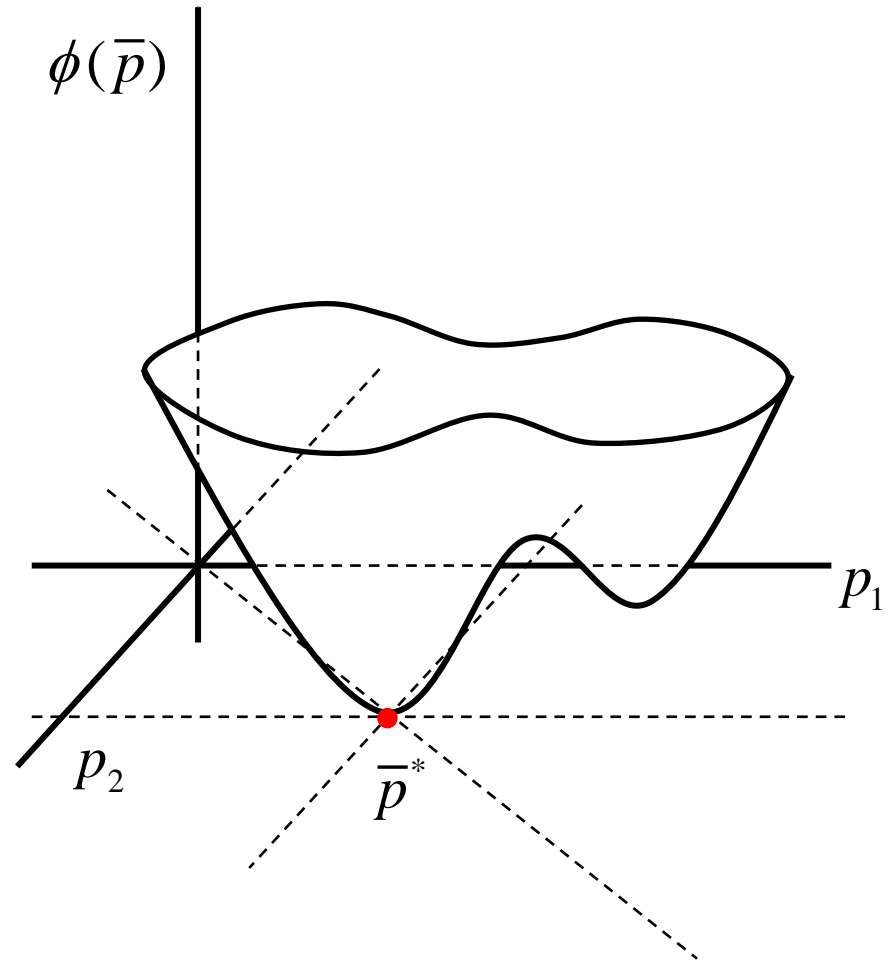
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

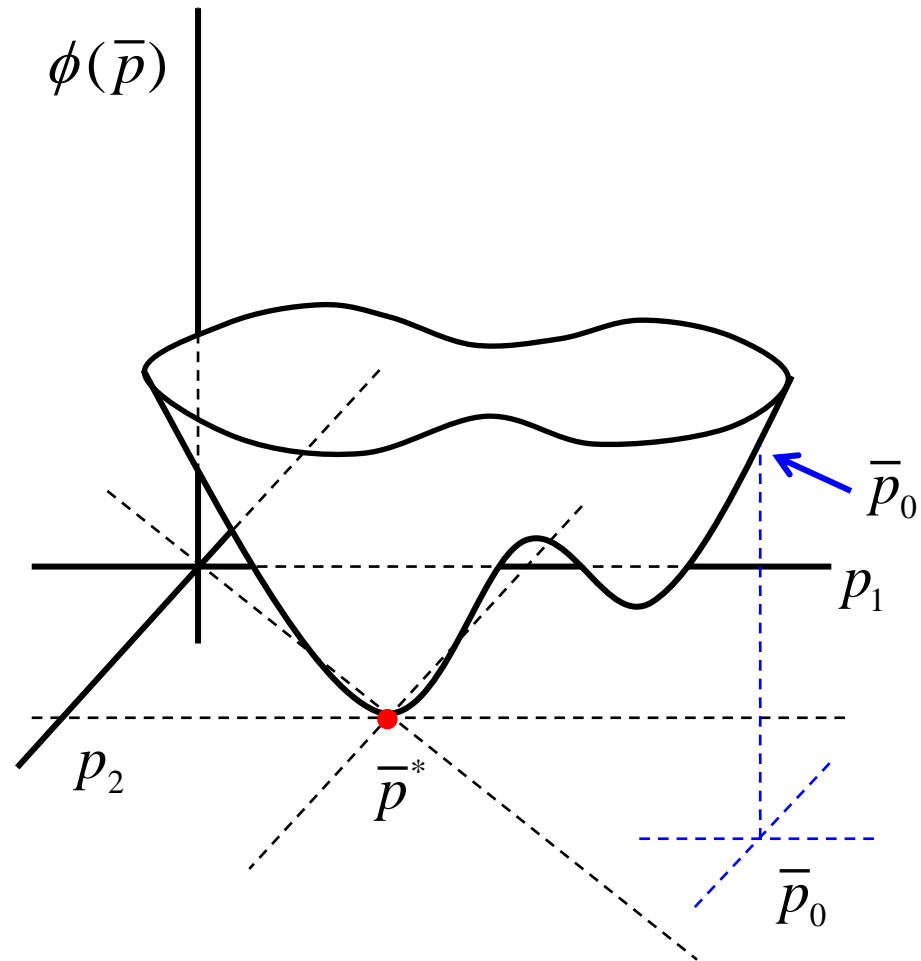
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

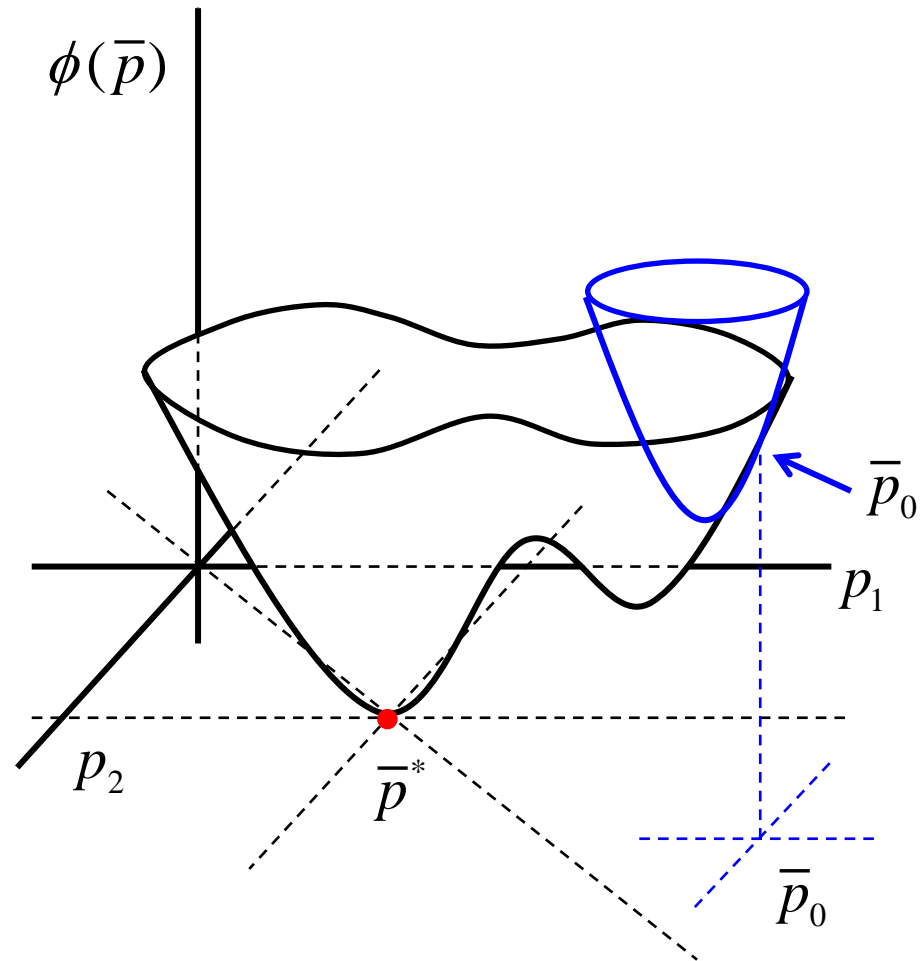
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

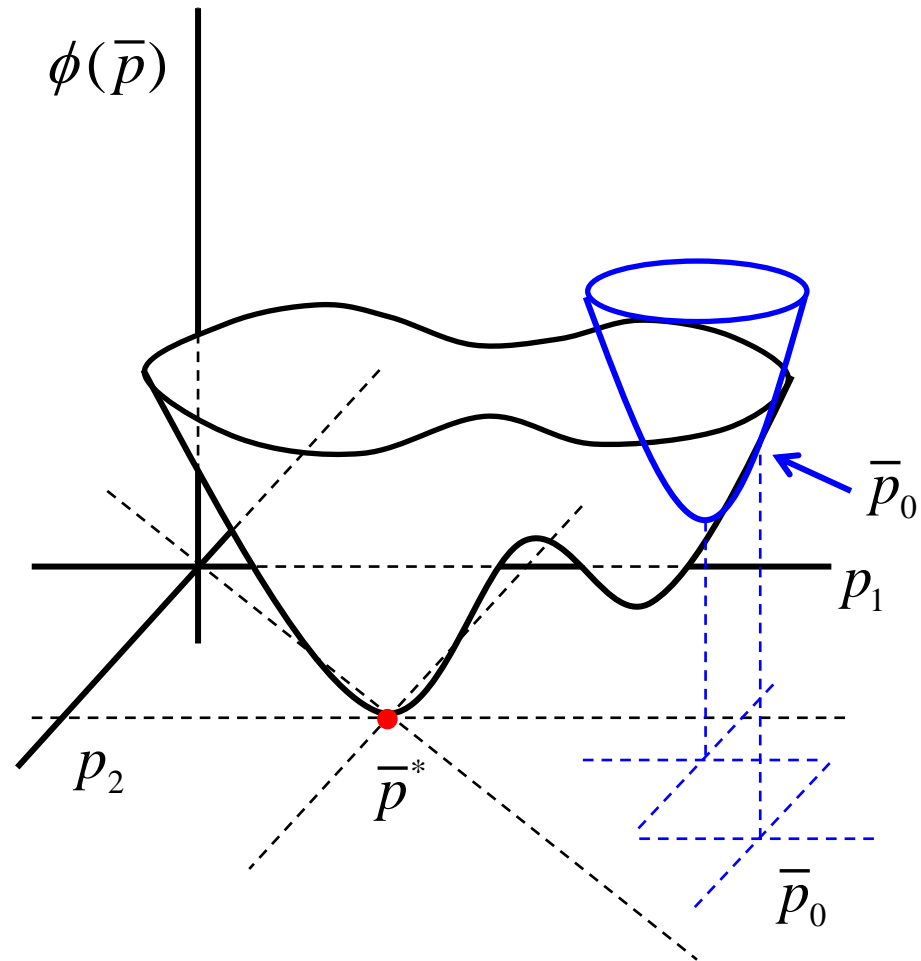
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

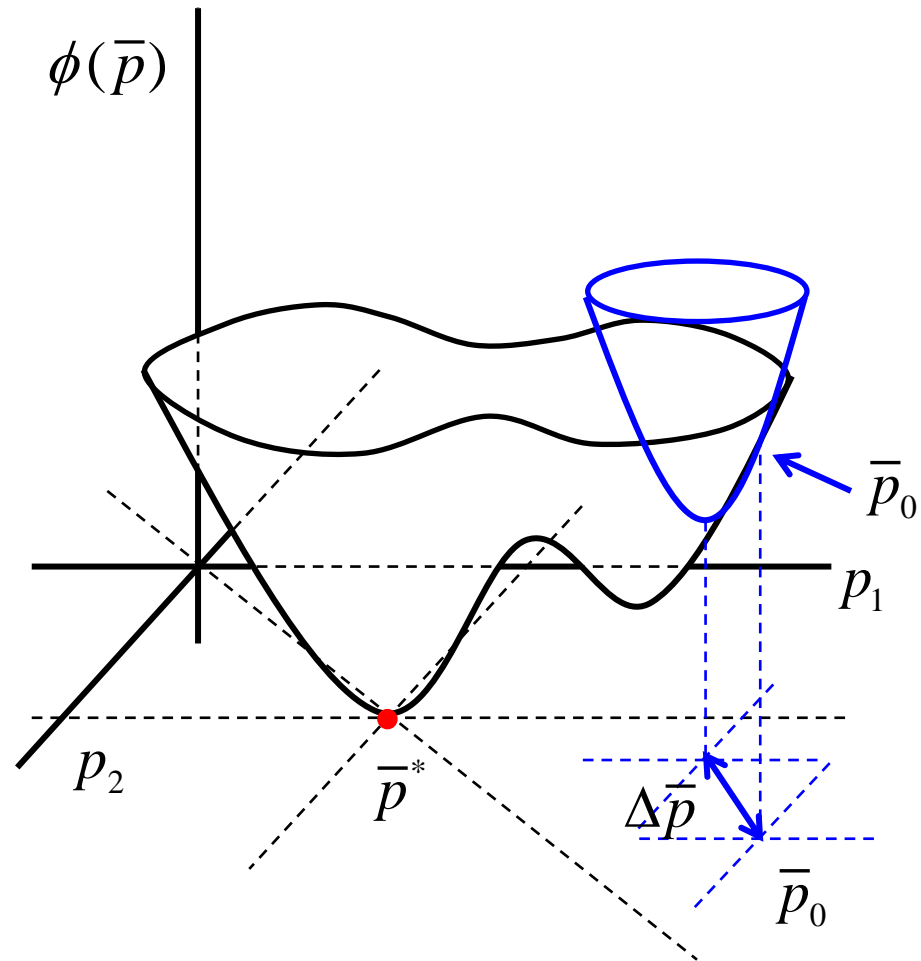
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

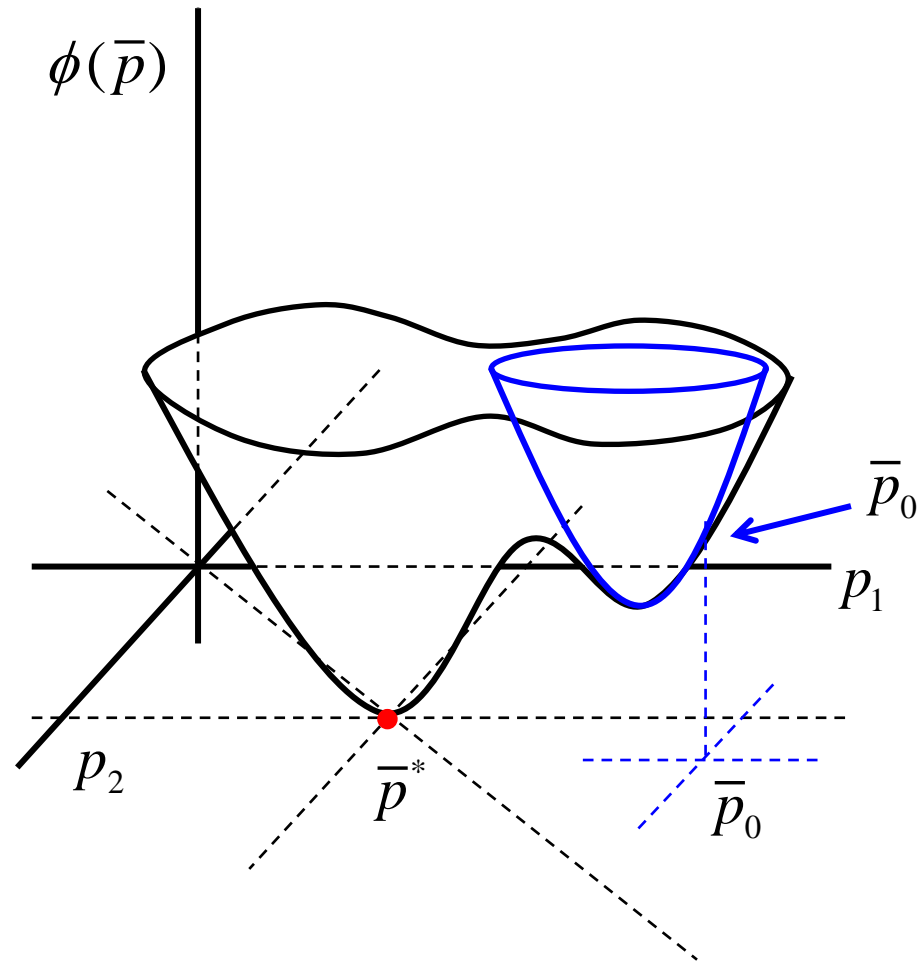
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

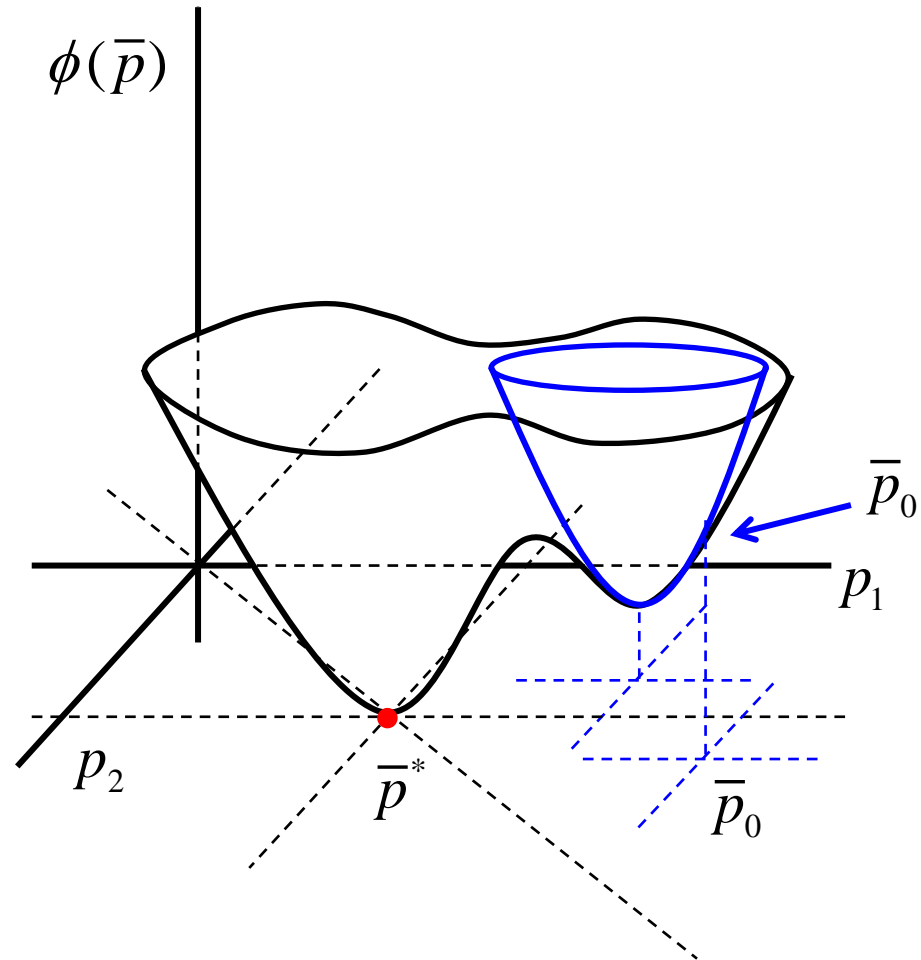
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

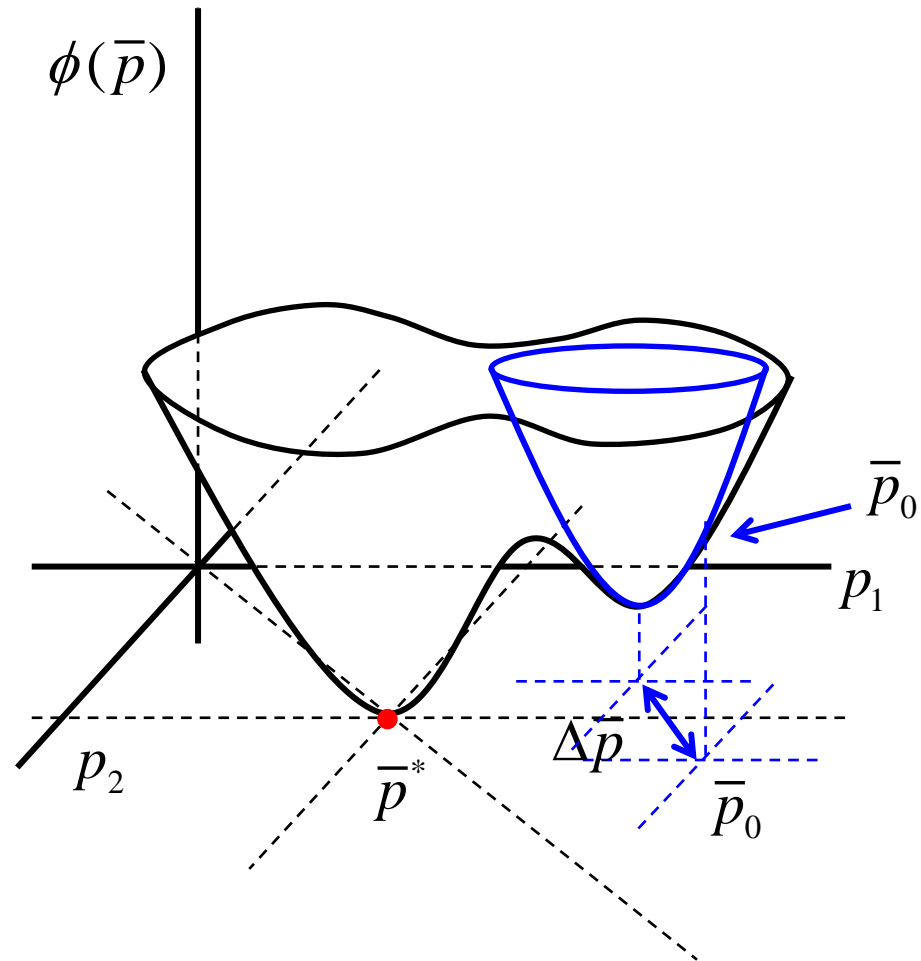
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

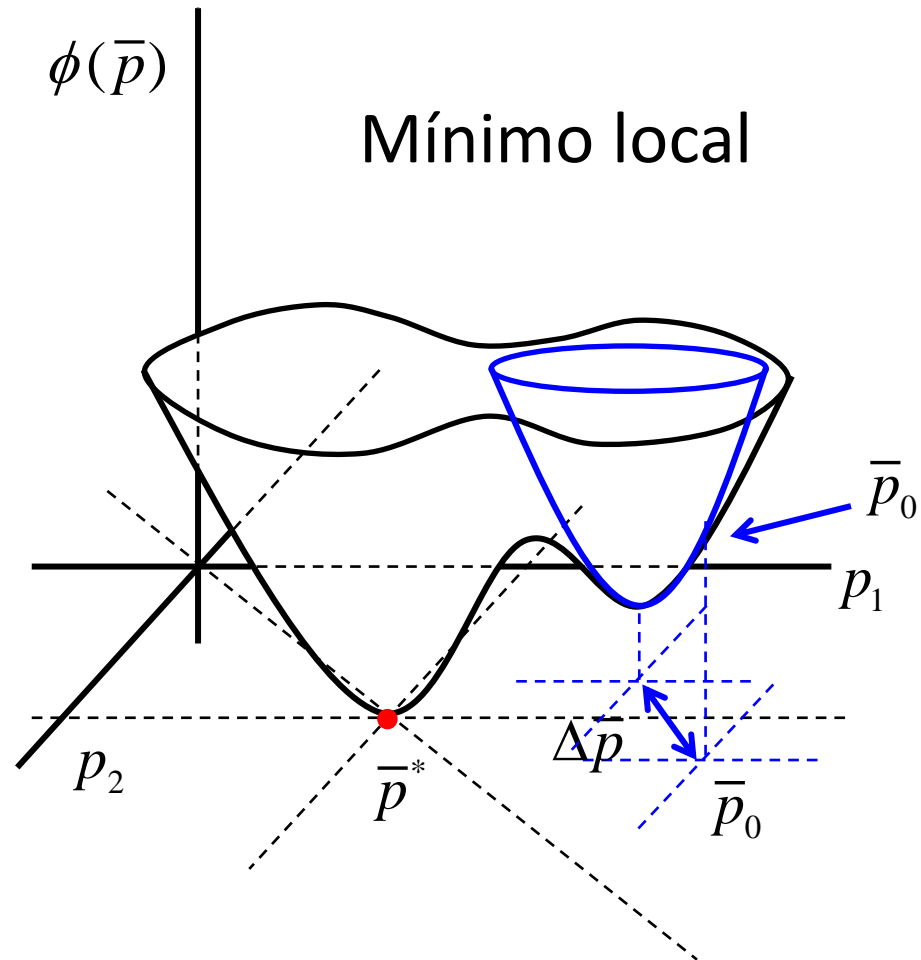
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

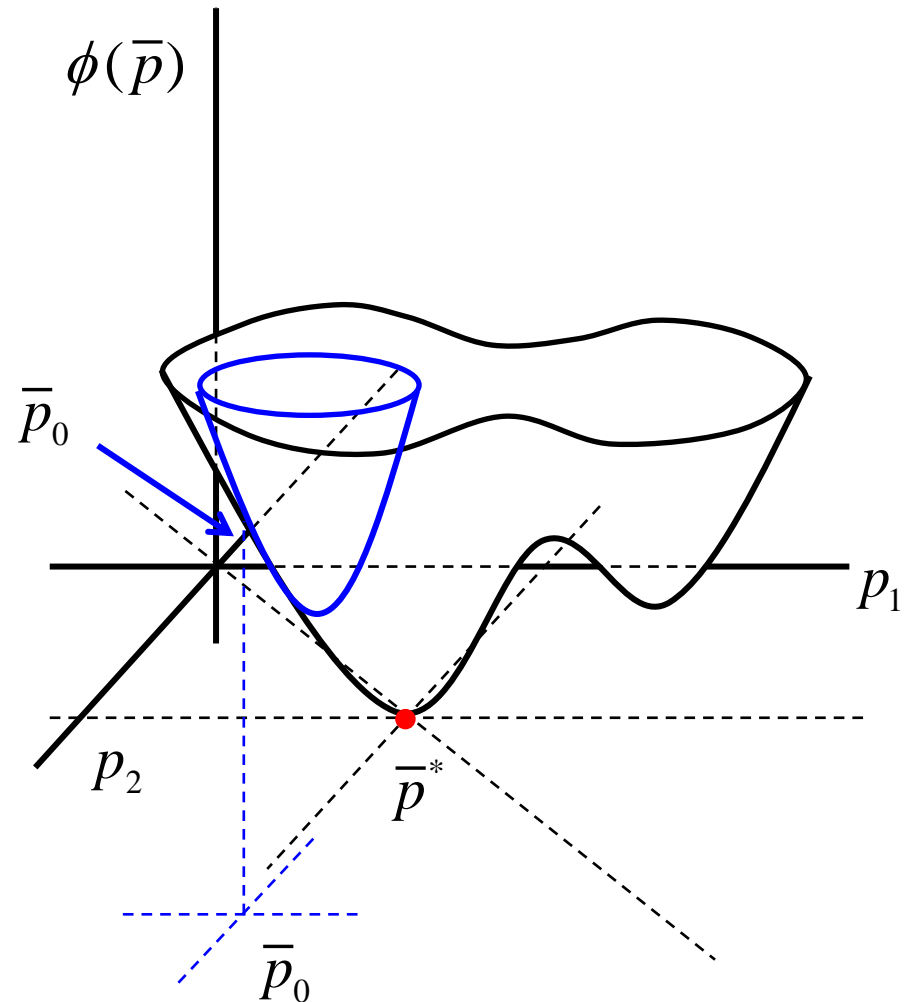
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

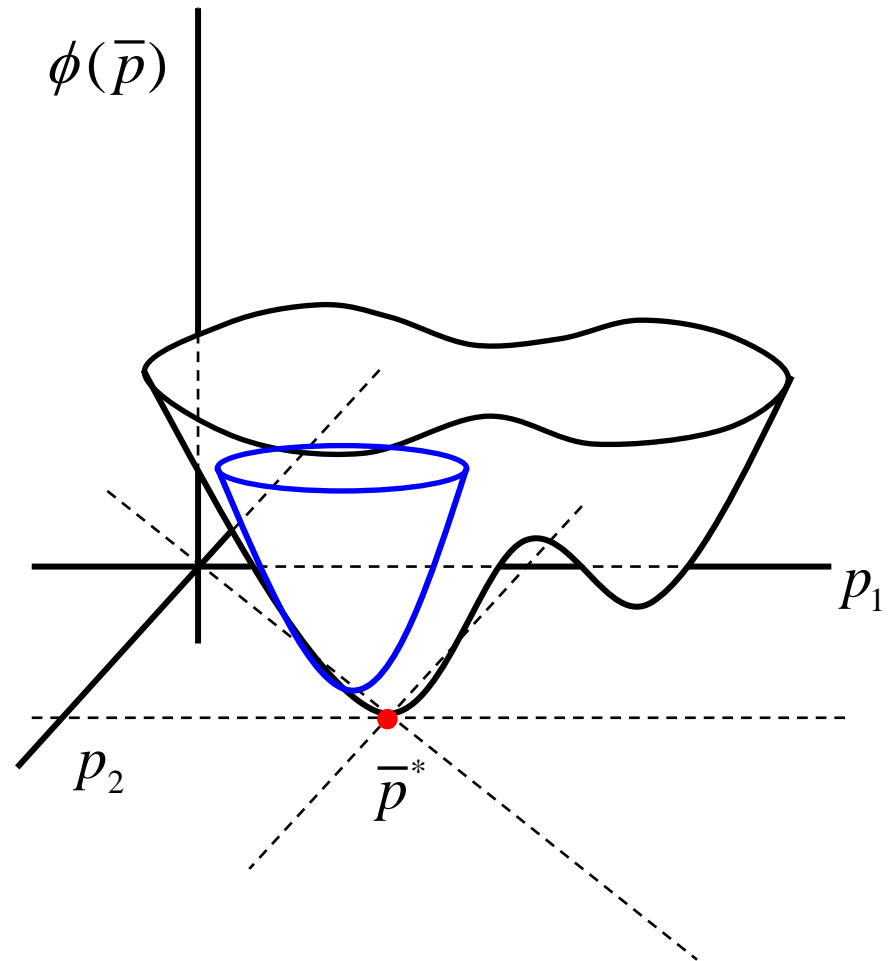
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

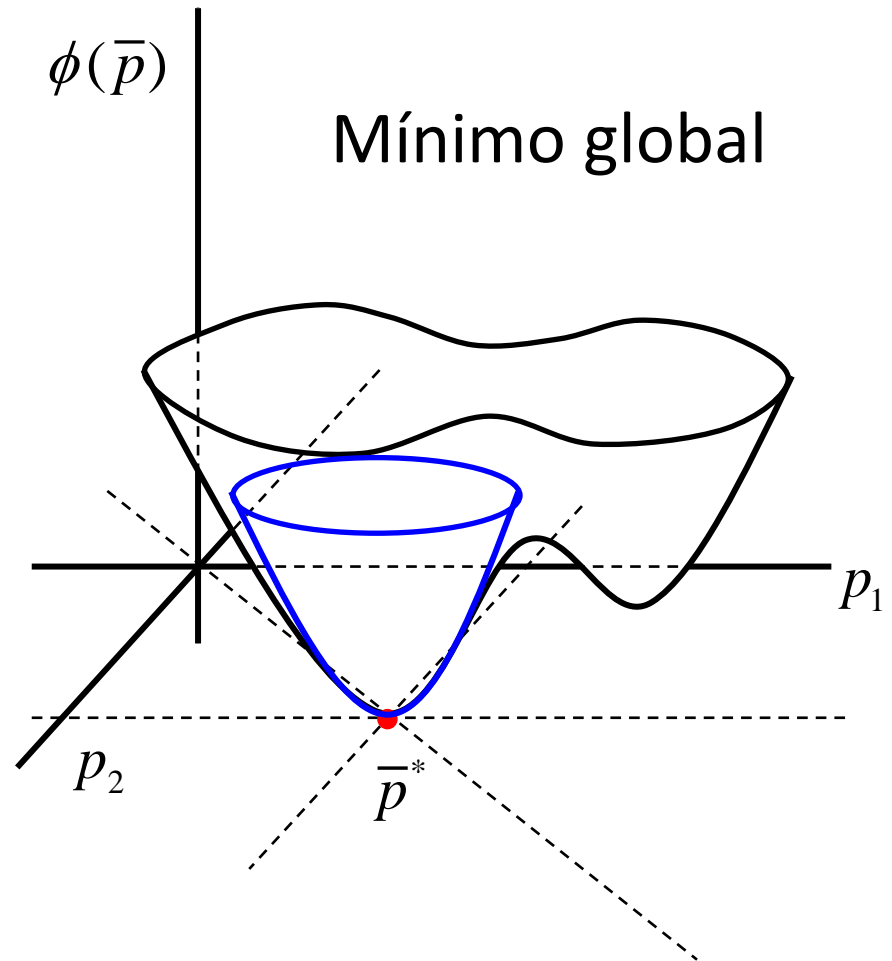
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

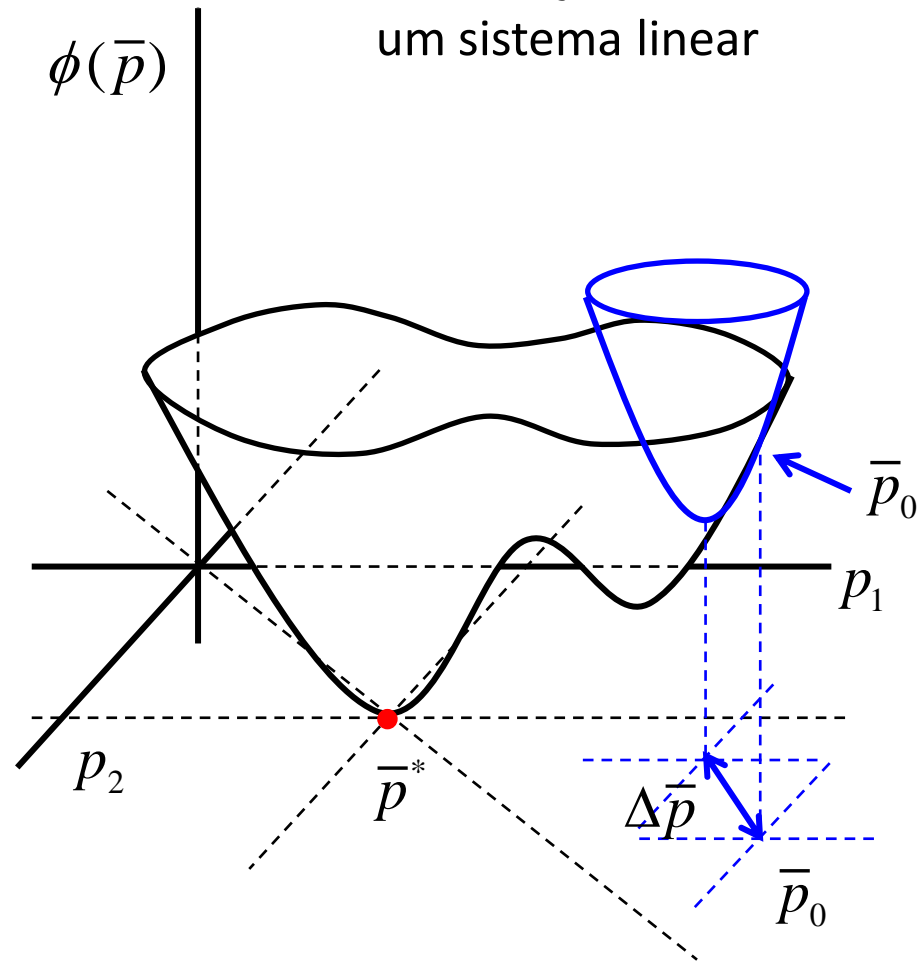
$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear

Em cada iteração é resolvido
um sistema linear



Problemas não-lineares

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

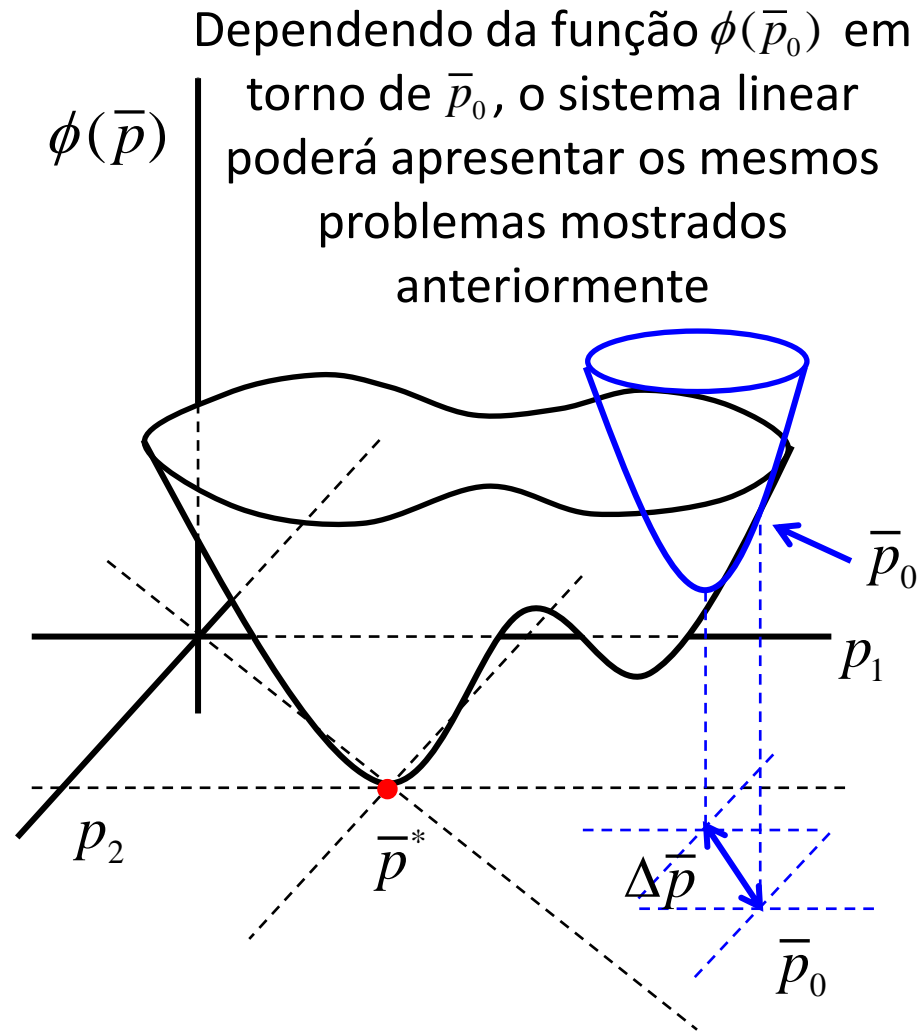
$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear



Regularização

Problema
linear

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Problema
não-linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\begin{pmatrix} \bar{B}^T & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear

Regularização

Problema
linear

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

Problema
não-linear

$$\bar{g}(\bar{p}) = \bar{B}\bar{p} + \bar{b}$$

$$\det \approx 0$$

$$\bar{g}(\bar{p}) \neq \bar{B}\bar{p} + \bar{b}$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\det = 0$$

$$\bar{\nabla} \phi(\bar{p}^*) = \bar{0}_{M \times 1}$$

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\begin{pmatrix} \bar{B} & \bar{B} \end{pmatrix} \bar{p}^* = \bar{B}^T [\bar{d} - \bar{b}]$$

Sistema linear

$$\left(\bar{G}(\bar{p}_0)^T \bar{G}(\bar{p}_0) \right) \Delta \bar{p} = \bar{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Sistema linear

Regularização

A regularização é um procedimento que objetiva tornar bem condicionadas as matrizes envolvidas na solução dos problemas inversos lineares e não-lineares

Regularização

Problema linear

Problema não-linear

$$\begin{pmatrix} \overline{B}^T & \overline{B} \end{pmatrix} \bar{p}^* = \overline{B}^T [\bar{d} - \bar{b}] \qquad \left(\overline{G}(\bar{p}_0)^T \overline{G}(\bar{p}_0) \right) \Delta \bar{p} = \overline{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Estas equações levam ao vetor de parâmetros que ajustam os dados, ou seja, estimam um vetor de parâmetros que produz os dados preditos mais próximos possíveis aos dados observados

Regularização

Problema linear

Problema não-linear

$$\begin{pmatrix} \overline{B}^T & \overline{B} \end{pmatrix} \overline{p}^* = \overline{B}^T [\overline{d} - \overline{b}]$$

$$\left(\overline{G}(\overline{p}_0)^T \overline{G}(\overline{p}_0) \right) \Delta \overline{p} = \overline{G}(\overline{p}_0)^T [\overline{d} - \overline{g}(\overline{p}_0)]$$

E se quiséssemos que o vetor de parâmetros ajustasse um vetor \overline{h} (diferente dos dados observados) dado pela relação linear abaixo?

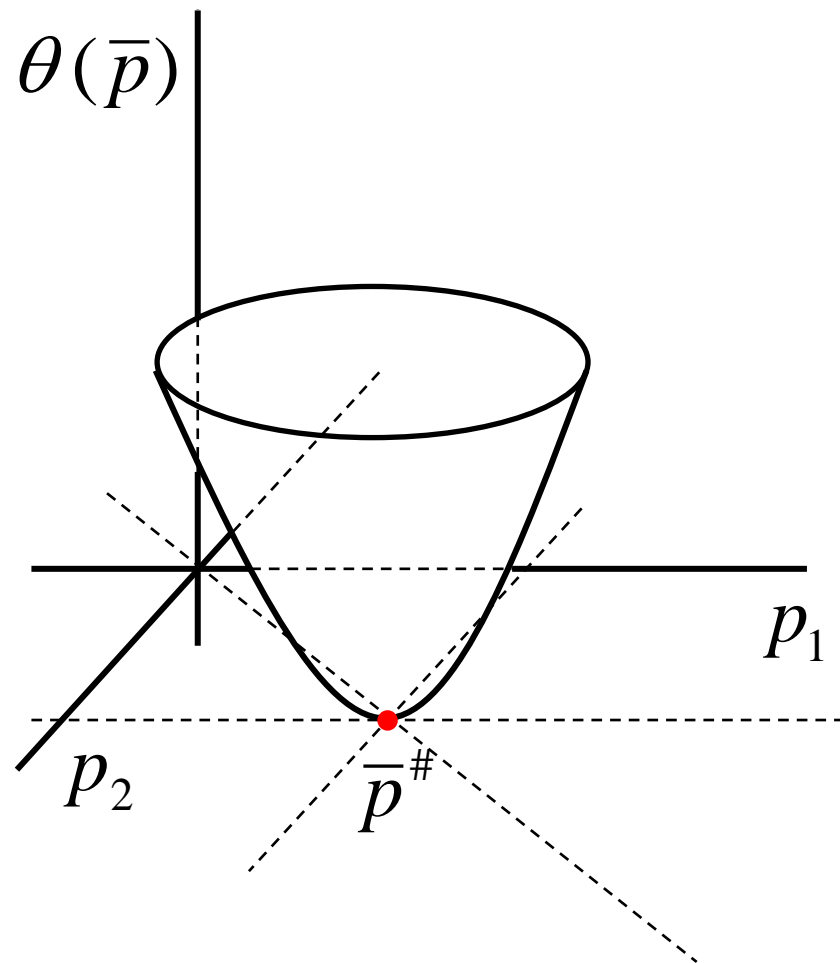
$$\overline{H} \overline{p} = \overline{h}$$

Regularização

$$\theta(\bar{p}) = [\bar{h} - \bar{\bar{H}}\bar{p}]^T [\bar{h} - \bar{\bar{H}}\bar{p}]$$

$$\bar{\nabla} \theta(\bar{p}^\#) = \bar{0}_{M \times 1}$$

$$\begin{pmatrix} \bar{\bar{H}}^T & \bar{\bar{H}} \end{pmatrix} \bar{p}^\# = \bar{\bar{H}}^T \bar{h}$$



Regularização

Problema linear

Problema não-linear

$$\begin{pmatrix} \overline{B}^T & \overline{B} \end{pmatrix} \bar{p}^* = \overline{B}^T [\bar{d} - \bar{b}] \qquad \left(\overline{G}(\bar{p}_0)^T \overline{G}(\bar{p}_0) \right) \Delta \bar{p} = \overline{G}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

E se agora quiséssemos que o vetor de parâmetros ajustasse o vetor \bar{h} e também o vetor de dados observados ao mesmo tempo?

$$\overline{H} \bar{p} = \bar{h}$$

Regularização

$$\overline{\overline{B}} \overline{p} + \overline{b} \approx \overline{d}$$

$$\overline{\overline{H}} \overline{p} \approx \overline{h}$$

Regularização

$$\overline{\overline{B}} \overline{p} + \overline{b} \approx \overline{d}$$

$$\overline{\overline{H}} \overline{p} \approx \overline{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \overline{\overline{H}} \end{bmatrix}}_{\overline{\overline{A}}} \overline{p} \approx \underbrace{\begin{bmatrix} \overline{d} - \overline{b} \\ \overline{h} \end{bmatrix}}_{\overline{\overline{w}}}$$

Regularização

$$\overline{\overline{B}} \bar{p} + \bar{b} \approx \bar{d}$$

$$\overline{\overline{H}} \bar{p} \approx \bar{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \overline{\overline{H}} \end{bmatrix}}_{\overline{\overline{A}}} \bar{p} \approx \underbrace{\begin{bmatrix} \bar{d} - \bar{b} \\ \bar{h} \end{bmatrix}}_{\bar{w}}$$

$$\Omega(\bar{p}) = \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]^T \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]$$

Regularização

$$\overline{\overline{B}} \bar{p} + \bar{b} \approx \bar{d}$$

$$\overline{\overline{H}} \bar{p} \approx \bar{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \overline{\overline{H}} \end{bmatrix}}_{\overline{\overline{A}}} \bar{p} \approx \underbrace{\begin{bmatrix} \bar{d} - \bar{b} \\ \bar{h} \end{bmatrix}}_{\bar{w}}$$

$$\Omega(\bar{p}) = \phi(\bar{p}) + \theta(\bar{p})$$

$$\Omega(\bar{p}) = \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]^T \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]$$

$$\phi(\bar{p}) = \left[\bar{d} - \bar{g}(\bar{p}) \right]^T \left[\bar{d} - \bar{g}(\bar{p}) \right]$$

$$\theta(\bar{p}) = \left[\bar{h} - \overline{\overline{H}} \bar{p} \right]^T \left[\bar{h} - \overline{\overline{H}} \bar{p} \right]$$

Regularização

$$\overline{\overline{B}} \bar{p} + \bar{b} \approx \bar{d}$$

$$\overline{\overline{H}} \bar{p} \approx \bar{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \overline{\overline{H}} \end{bmatrix}}_{\overline{\overline{A}}} \bar{p} \approx \underbrace{\begin{bmatrix} \bar{d} - \bar{b} \\ \bar{h} \end{bmatrix}}_{\bar{w}}$$

$$\Omega(\bar{p}) = \phi(\bar{p}) + \theta(\bar{p})$$

$$\Omega(\bar{p}) = \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]^T \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]$$

$$\phi(\bar{p}) = \left[\bar{d} - \bar{g}(\bar{p}) \right]^T \left[\bar{d} - \bar{g}(\bar{p}) \right]$$

$$\theta(\bar{p}) = \left[\bar{h} - \overline{\overline{H}} \bar{p} \right]^T \left[\bar{h} - \overline{\overline{H}} \bar{p} \right]$$

Regularização

$$\overline{\overline{B}} \bar{p} + \bar{b} \approx \bar{d}$$

$$\alpha \overline{\overline{H}} \bar{p} \approx \alpha \bar{h}$$

$$\underbrace{\begin{bmatrix} \overline{\overline{B}} \\ \alpha \overline{\overline{H}} \end{bmatrix}}_{\overline{\overline{A}}} \bar{p} \approx \underbrace{\begin{bmatrix} \bar{d} - \bar{b} \\ \alpha \bar{h} \end{bmatrix}}_{\bar{w}}$$

$$\mu = \alpha^2 \rightarrow$$

$$\Omega(\bar{p}) = \phi(\bar{p}) + \mu \theta(\bar{p})$$

$$\Omega(\bar{p}) = \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]^T \left[\bar{w} - \overline{\overline{A}} \bar{p} \right]$$

$$\phi(\bar{p}) = \left[\bar{d} - \bar{g}(\bar{p}) \right]^T \left[\bar{d} - \bar{g}(\bar{p}) \right]$$

$$\theta(\bar{p}) = \left[\bar{h} - \overline{\overline{H}} \bar{p} \right]^T \left[\bar{h} - \overline{\overline{H}} \bar{p} \right]$$

Regularização

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\left[\bar{\bar{\nabla}} \phi(\bar{p}_0) + \mu \bar{\bar{\nabla}} \theta(\bar{p}_0) \right] \Delta \bar{p} = - \left[\bar{\nabla} \phi(\bar{p}_0) + \mu \bar{\nabla} \theta(\bar{p}_0) \right]$$

Regularização

$$\bar{p} = \bar{p}_0 + \Delta \bar{p}$$

$$\underbrace{\left[\bar{\nabla} \phi(\bar{p}_0) + \mu \bar{\nabla} \theta(\bar{p}_0) \right]}_{\text{Espera-se que essa matriz resultante seja bem-condicionada}} \Delta \bar{p} = - \left[\bar{\nabla} \phi(\bar{p}_0) + \mu \bar{\nabla} \theta(\bar{p}_0) \right]$$

Espera-se que essa
matriz resultante seja
bem-condicionada