Formulação matemática do Problema Inverso

Estrutura

- Problema Inverso
  - Otimização
- Problema Inverso linear
  - Mínimos Quadrados
- Problema Inverso não-linear
  - Método de Gauss-Newton
- Aspectos geométricos
  - Problema linear 1D
  - Problema não-linear 1D
- Exercícios

Problema Inverso
(Otimização)

$$\bar{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}_{N\times 1}$$

dados observados

$$\bar{g}(\bar{p}) = \begin{bmatrix} g_1(\bar{p}) \\ \vdots \\ g_N(\bar{p}) \end{bmatrix}_{N\times 1}$$

dados preditos

Problema Inverso
(Otimização)

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dados preditos

$$\phi(\bar{p}) = [\bar{d} - \bar{g}(\bar{p})]^T [\bar{d} - \bar{g}(\bar{p})]$$

$$\phi(\bar{p}) = \sum_{i=1}^{N} [d_i - g_i(\bar{p})]^2$$

norma L2
(função escalar)

Problema Inverso
(Otimização)

O Problema Inverso consiste em determinar um vetor de parâmetros $\bar{p}^*$, M-dimensional, que minimiza a função $\phi(\bar{p})$

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$$\bar{\nabla}\phi(\bar{p}^*) = \bar{0}_{M\times 1}$$

norma L2
(função escalar)

Problema Inverso
(Otimização)

$$\overline{\nabla}\phi(\overline{p}) = -2\overline{\overline{G}}(\overline{p})^T[\overline{d} - \overline{g}(\overline{p})]$$

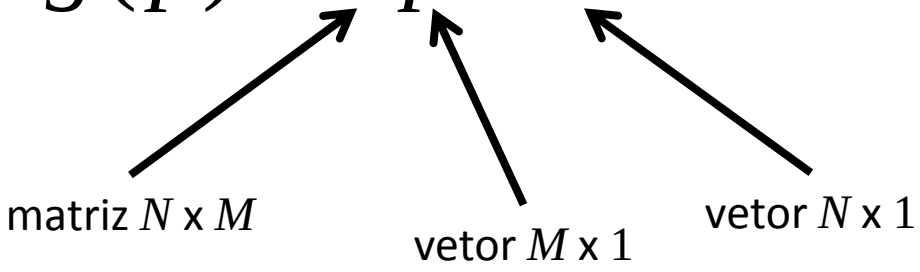
matriz N x M
transposta

vetor N x 1

Problema Inverso linear

(Mínimos Quadrados)

$$\bar{g}(\bar{p}) = \bar{\bar{B}}\bar{p} + \bar{b}$$



Problema Inverso linear

(Mínimos Quadrados)

$$\bar{g}(\bar{p}) = \bar{\bar{B}}\bar{p} + \bar{b}$$

matriz N x M — vetor M x 1 — vetor N x 1

$$\bar{p}^{*} = \left(\bar{\bar{B}}^{T} \bar{\bar{B}} \right)^{-1} \bar{\bar{B}}^{T} [\bar{d} - \bar{b}]$$

Estimador de Mínimos Quadrados

Problema Inverso não-linear

(Método de Gauss-Newton)

$$\bar{g}(\bar{p}) \neq \bar{\bar{B}}\bar{p} + \bar{b}$$

diferente

Problema Inverso não-linear

(Método de Gauss-Newton)

$$\bar{g}(\bar{p}) \neq \bar{\bar{B}}\bar{p} + \bar{b}$$

$$\bar{p} = \bar{p}_0 + \Delta\bar{p}$$

$$\Delta\bar{p} = \left(\bar{\bar{G}}(\bar{p}_0)^T \bar{\bar{G}}(\bar{p}_0) \right)^{-1} \bar{\bar{G}}(\bar{p}_0)^T [\bar{d} - \bar{g}(\bar{p}_0)]$$

Método de Gauss-Newton

Aspectos geométricos
(Problema linear 1D)

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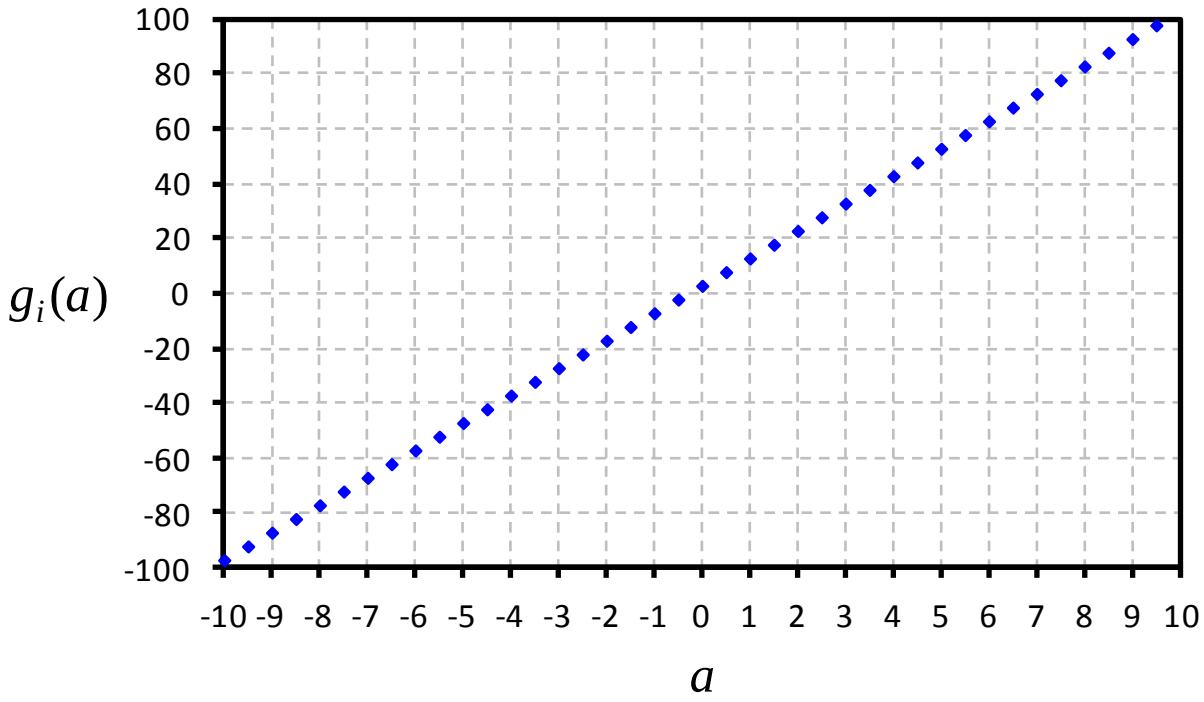

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(Problema linear 1D)

$$g_i(a) = a\,x_i + b$$

$$\begin{bmatrix} g_1(a) \\ g_2(a) \\ g_3(a) \end{bmatrix} = \begin{bmatrix} a\,x_1 + b \\ a\,x_2 + b \\ a\,x_3 + b \end{bmatrix}$$

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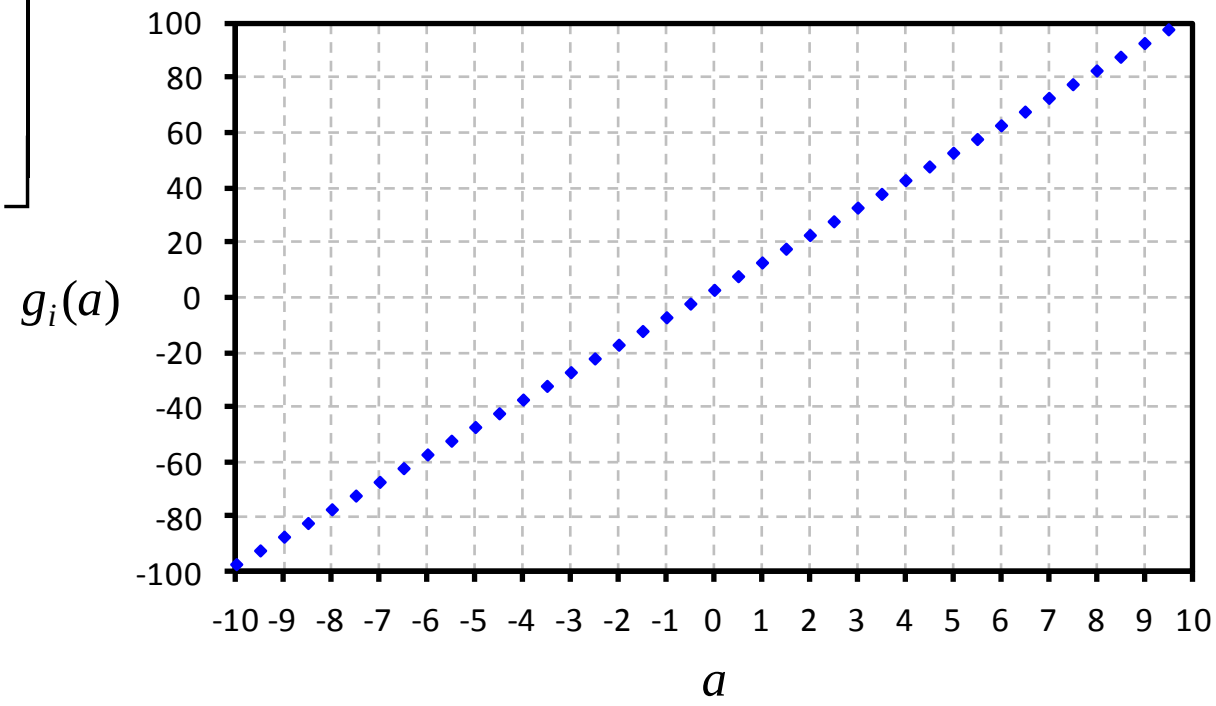

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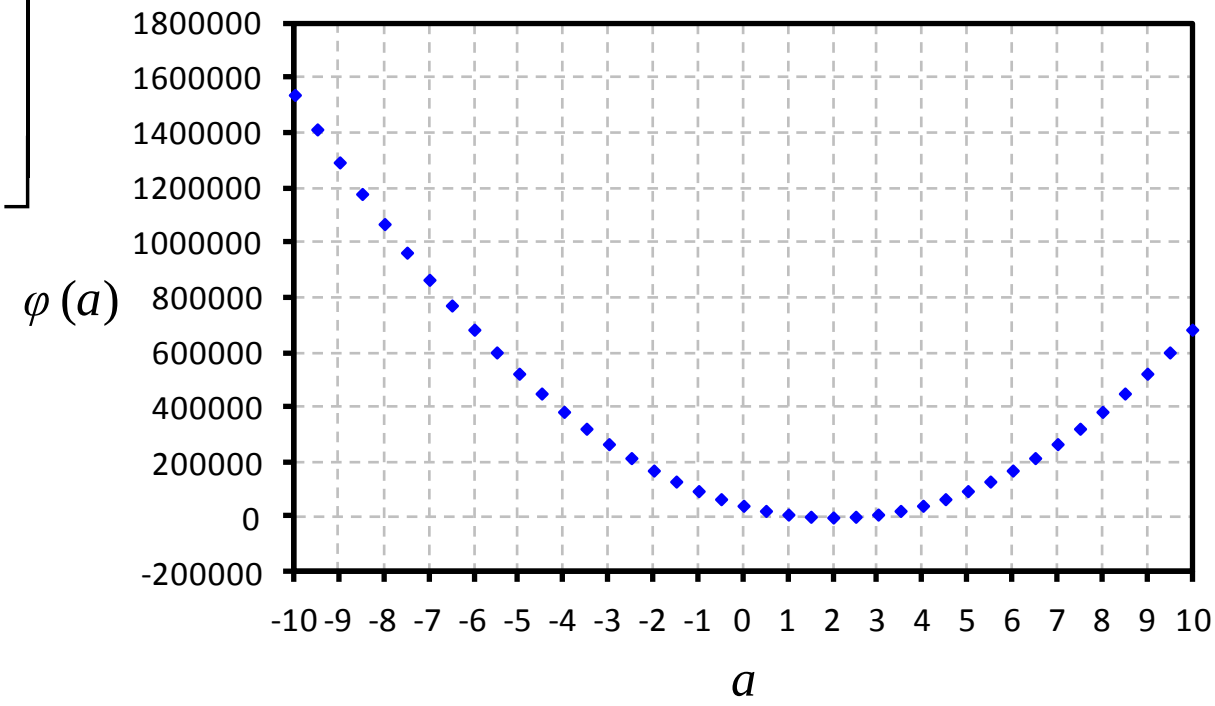

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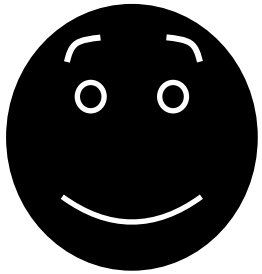

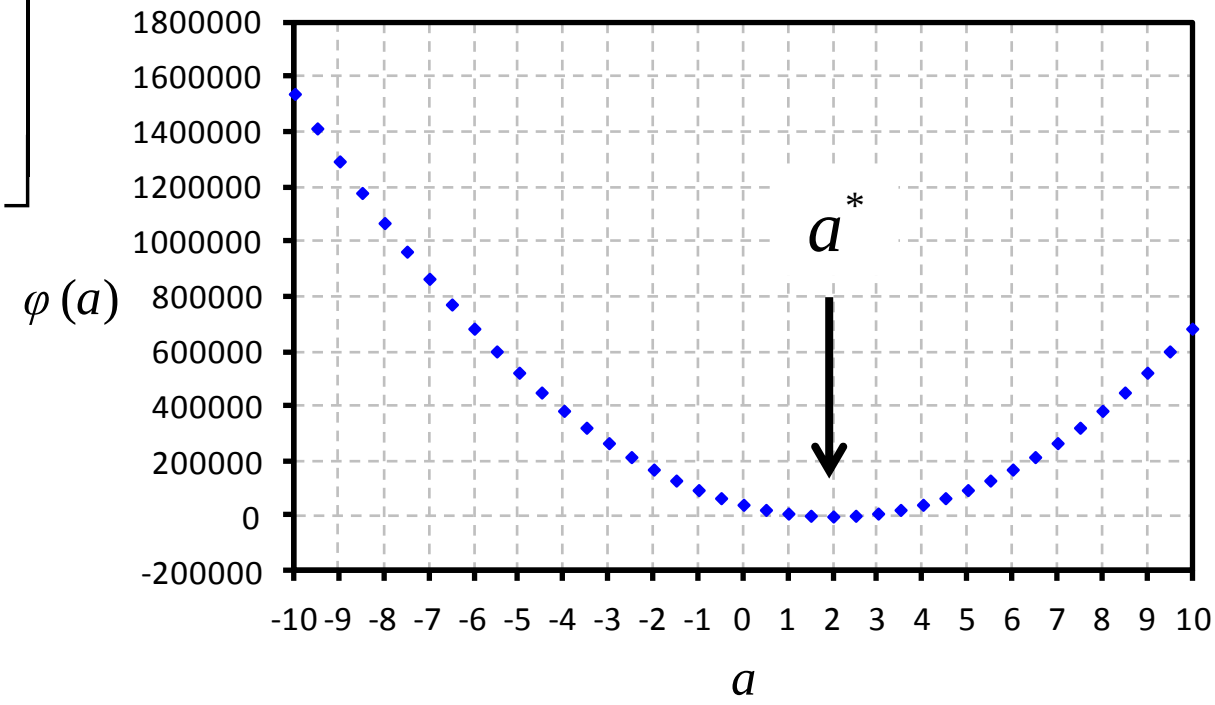

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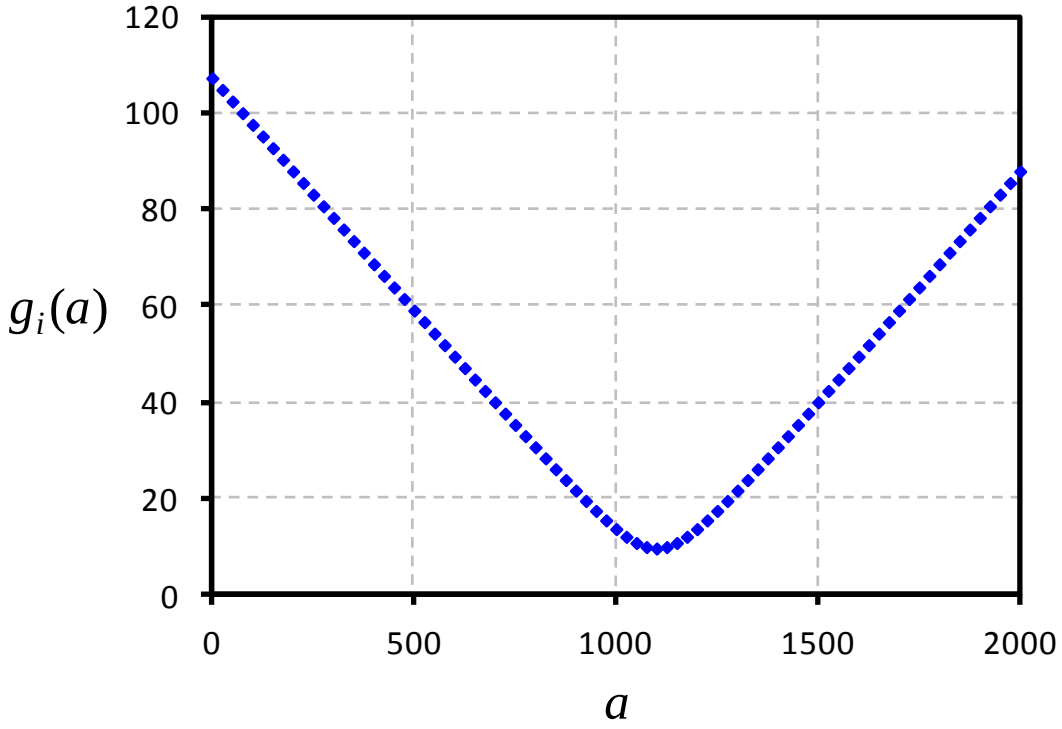

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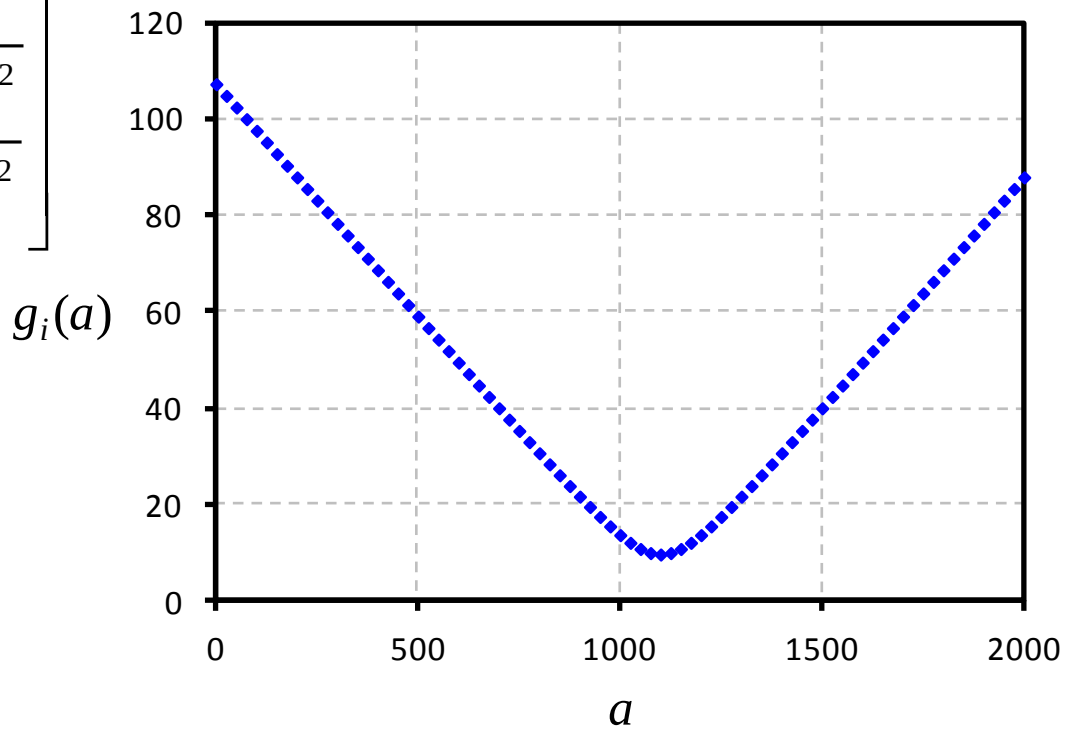

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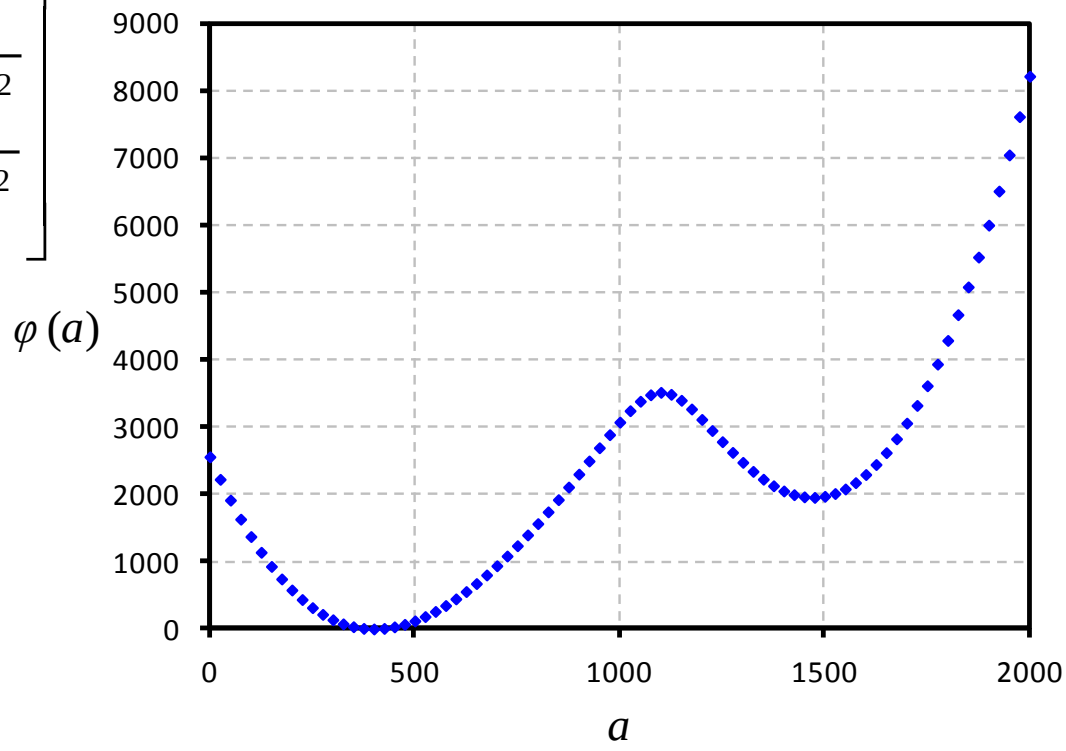

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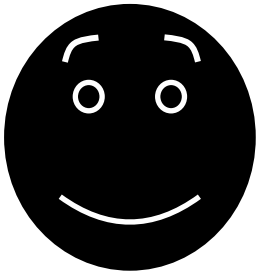

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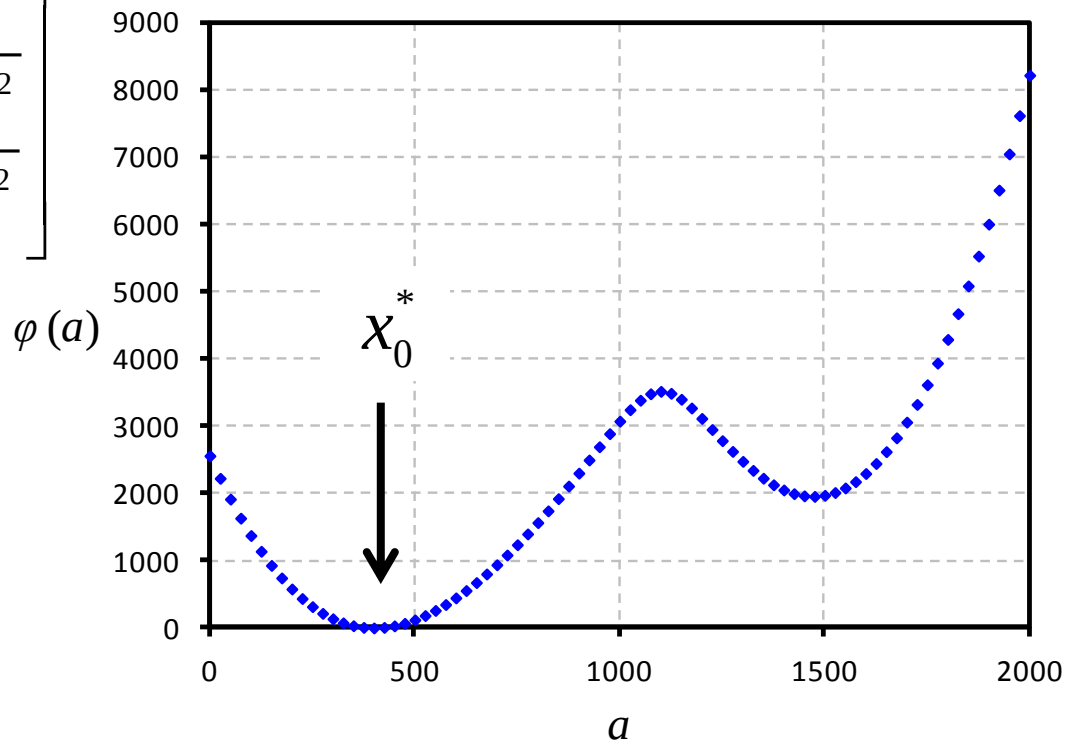

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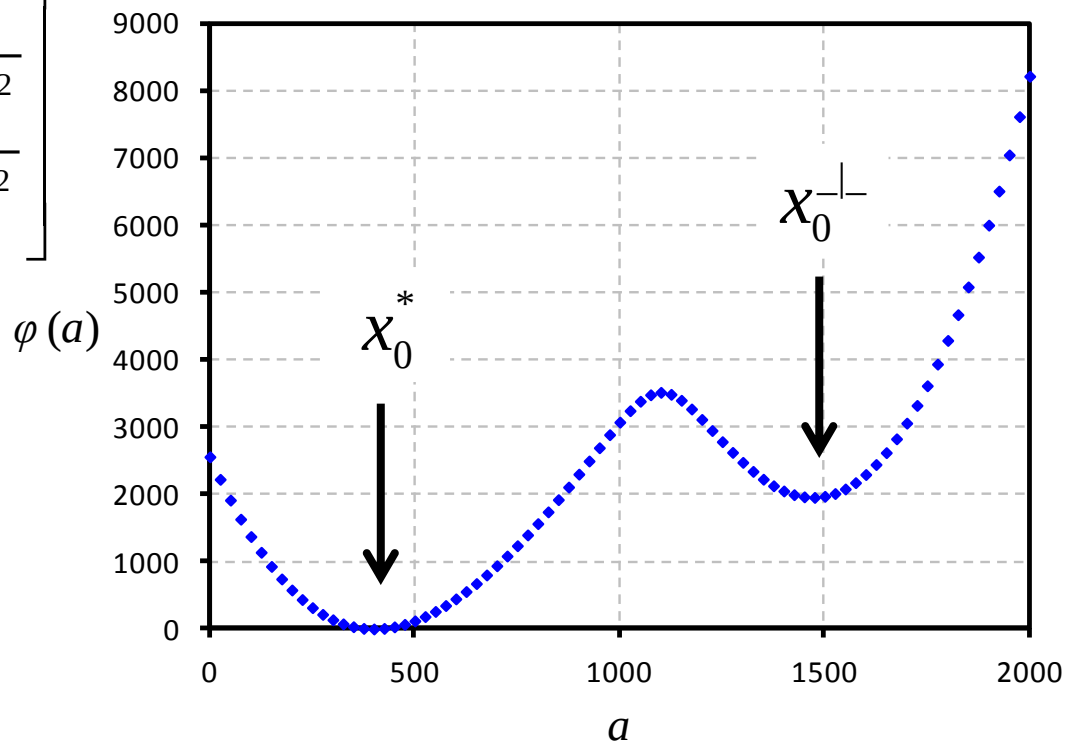

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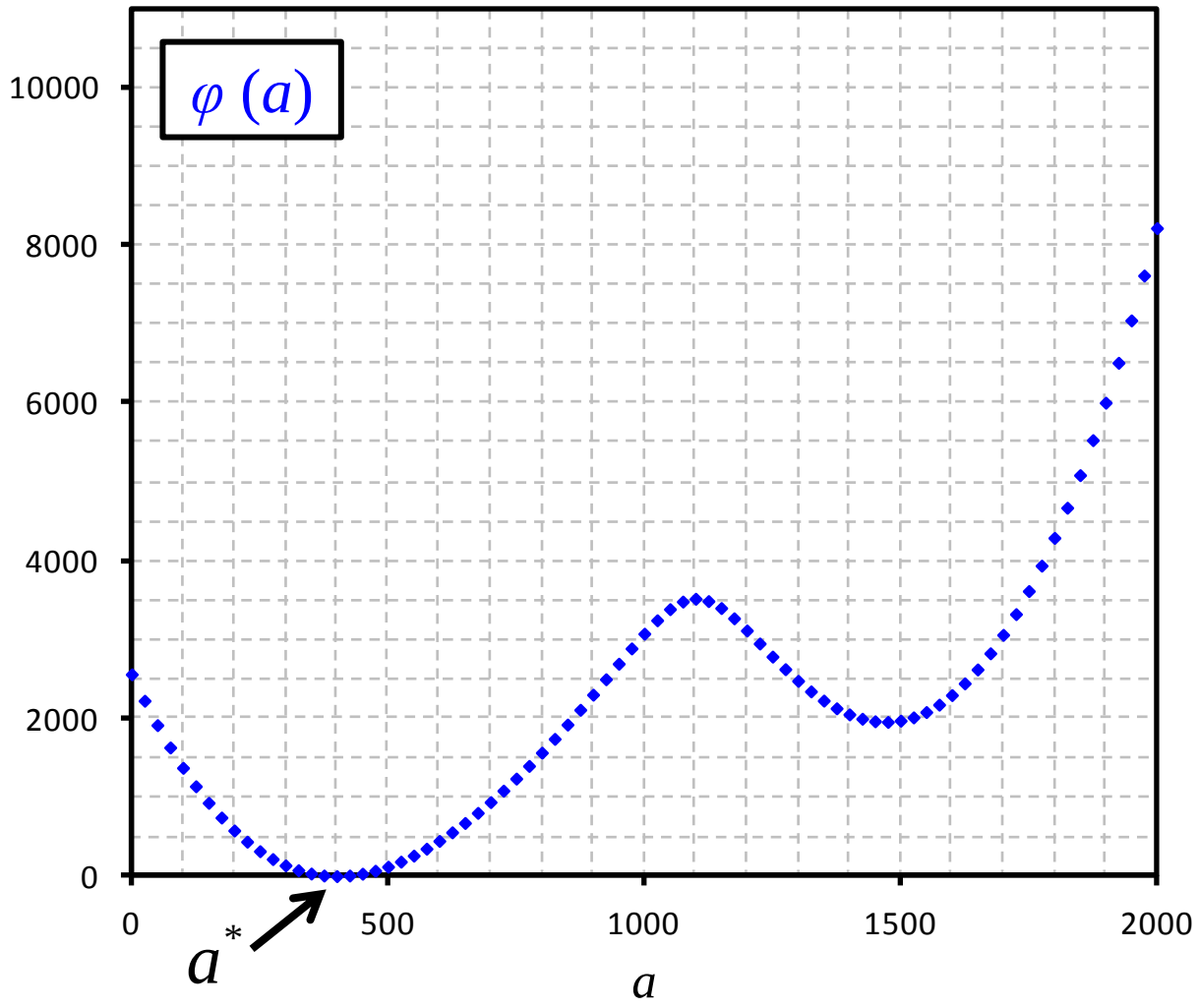

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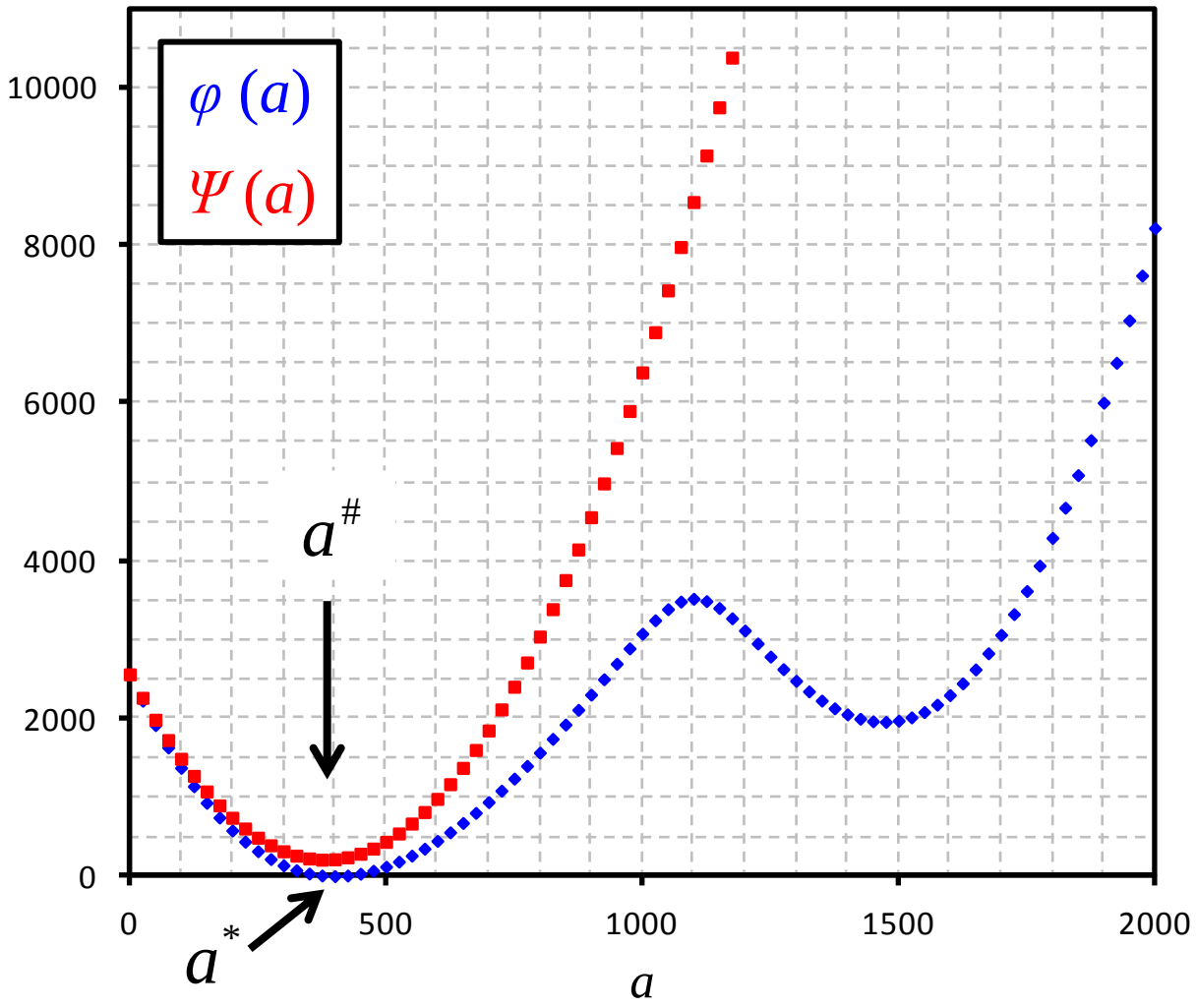

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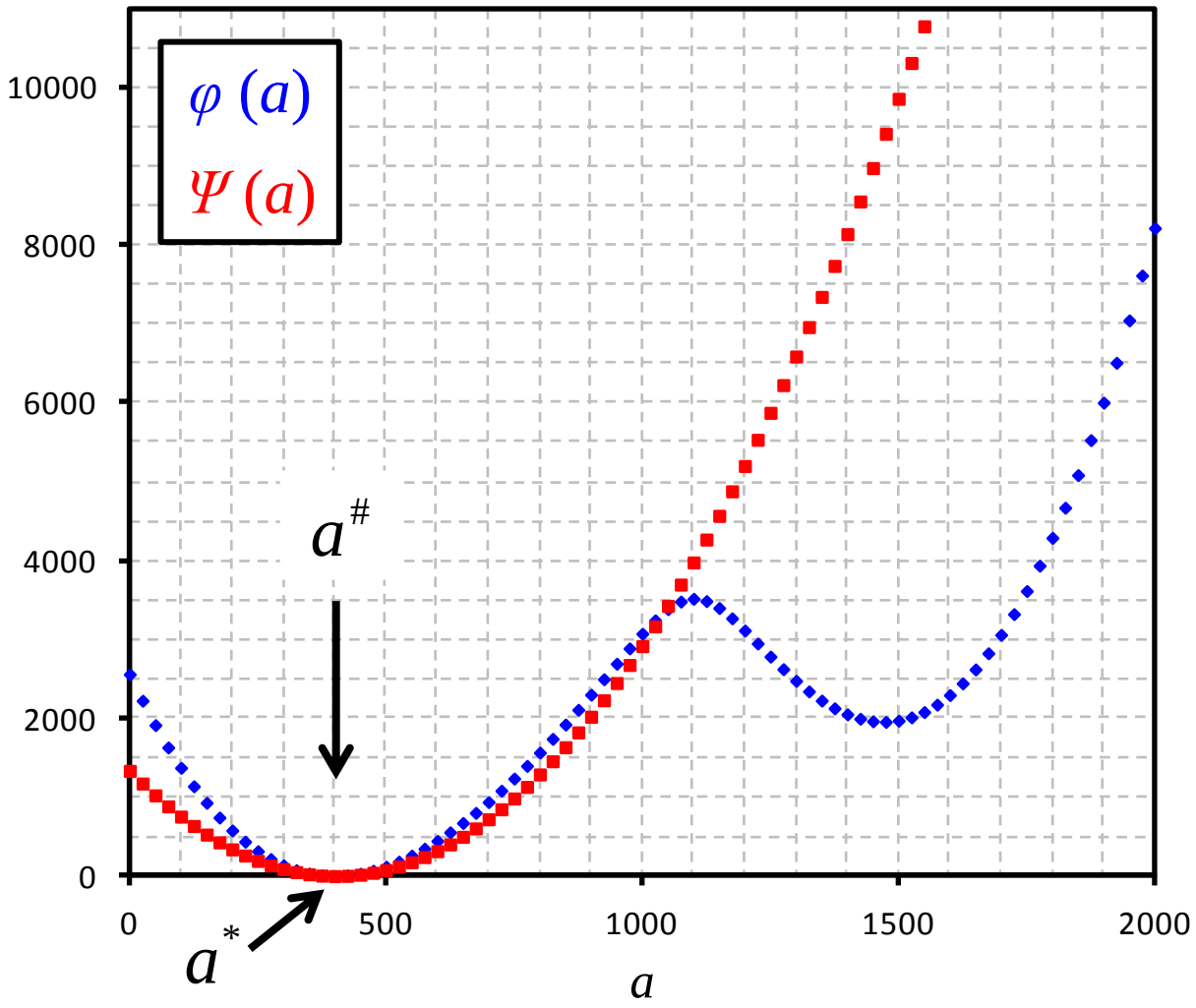

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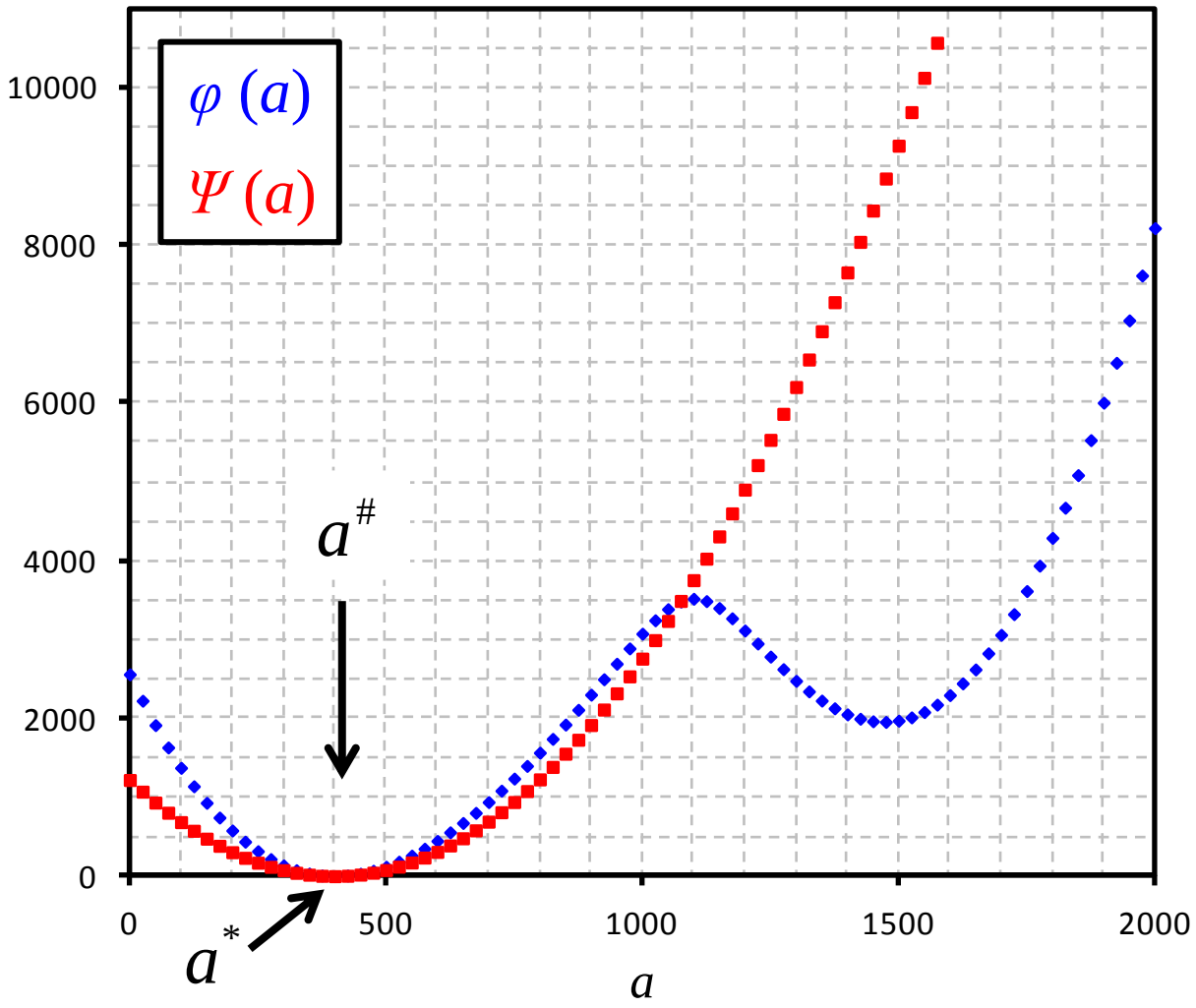

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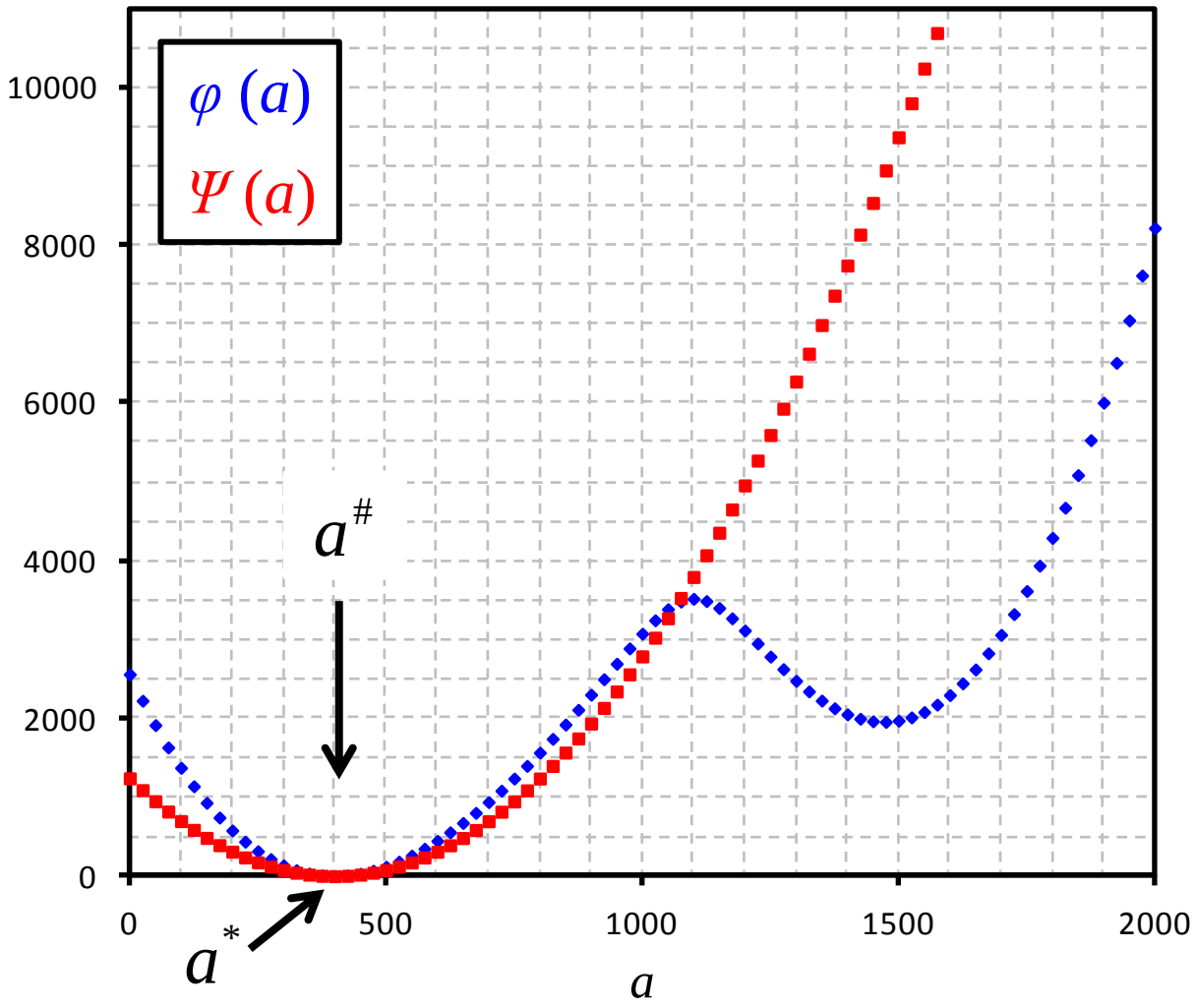

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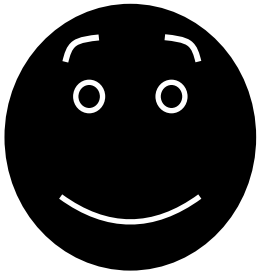

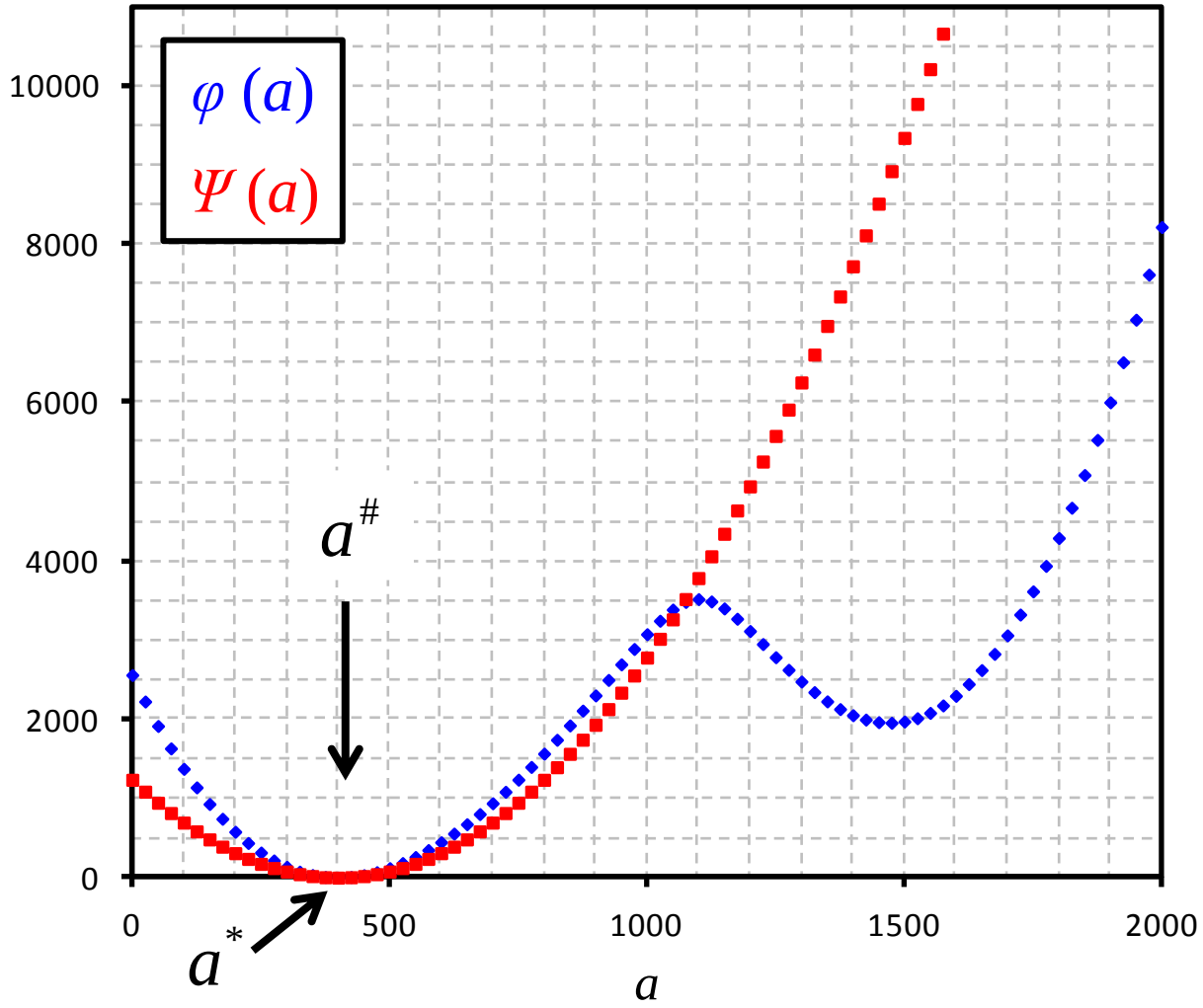

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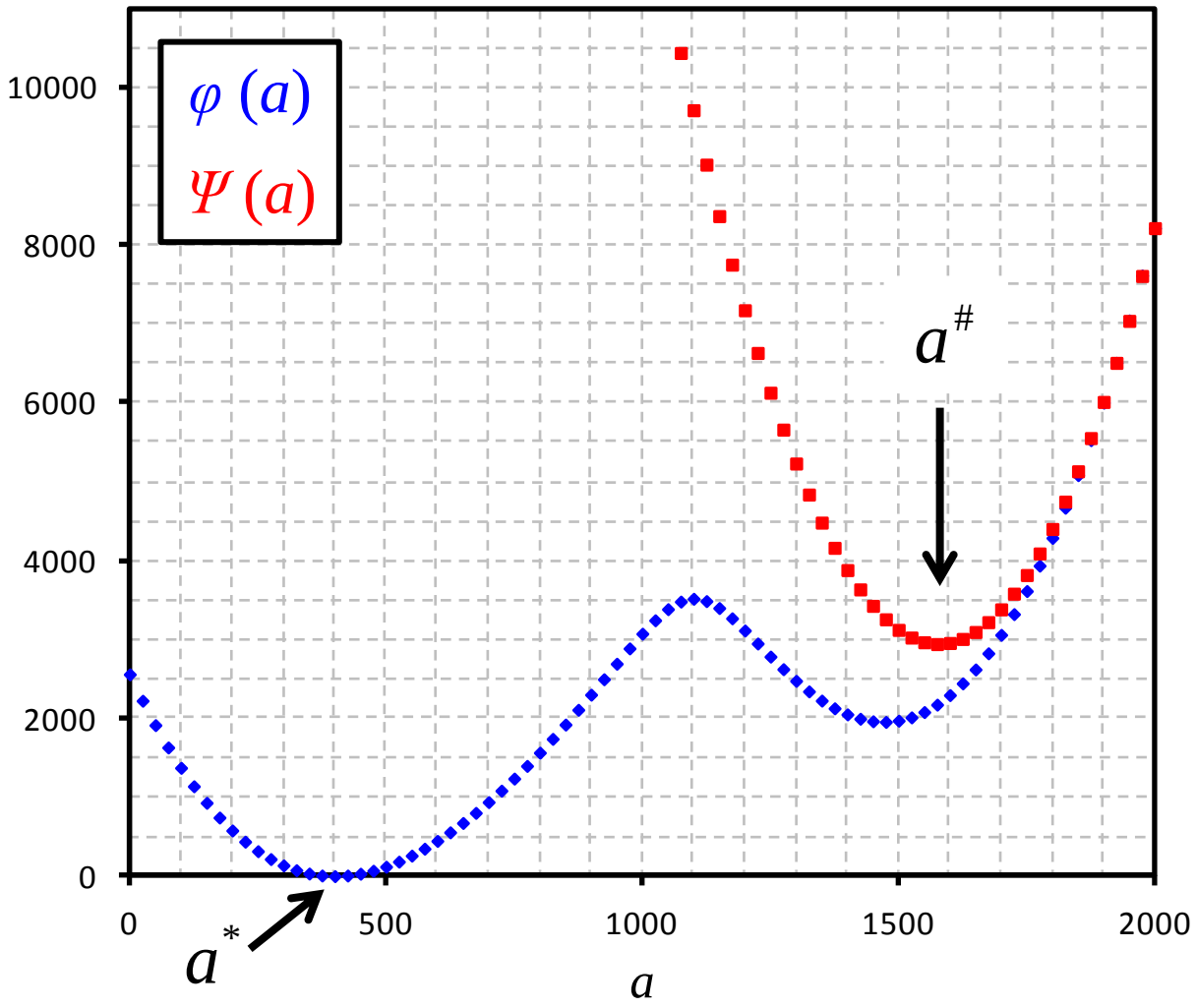

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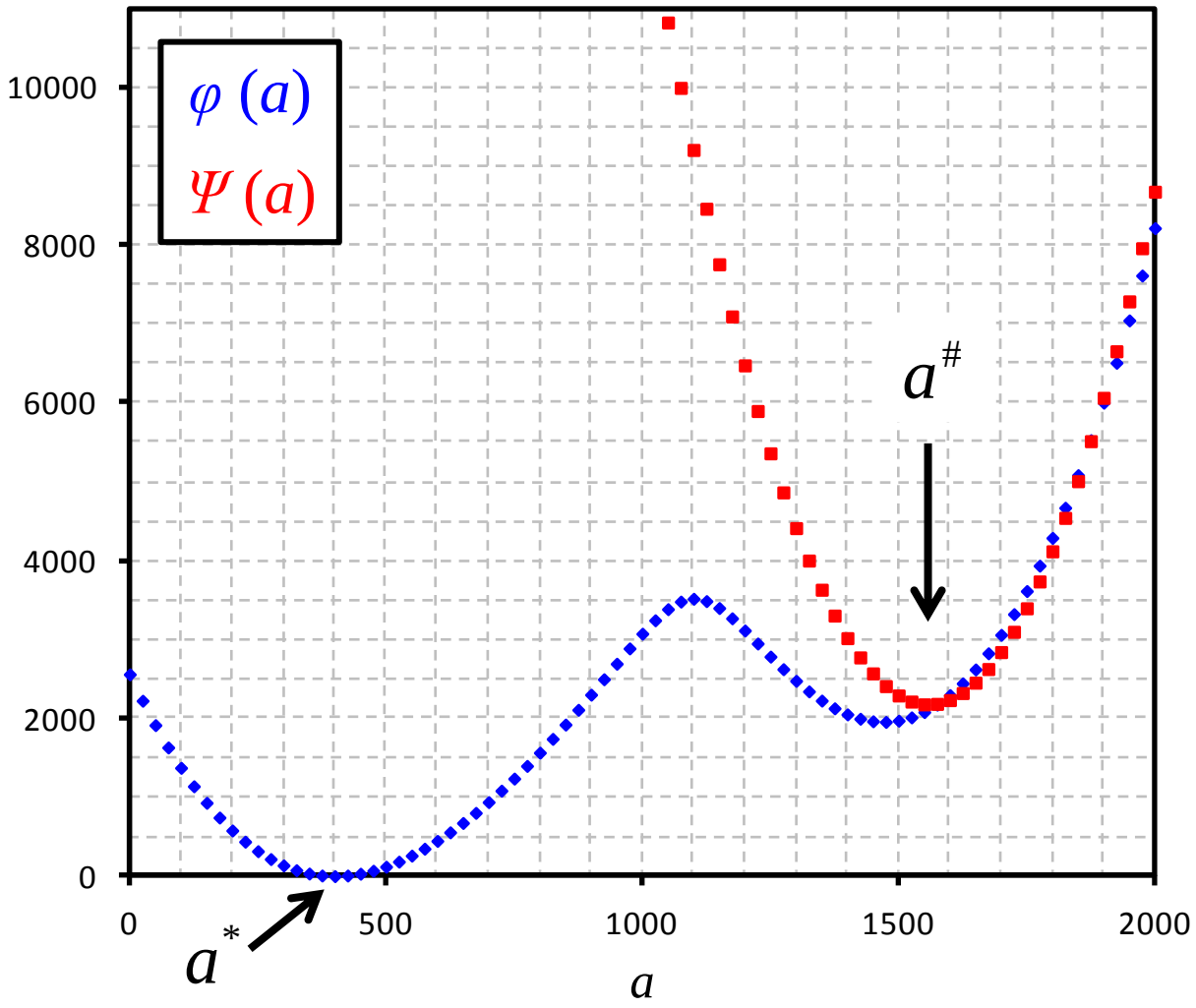

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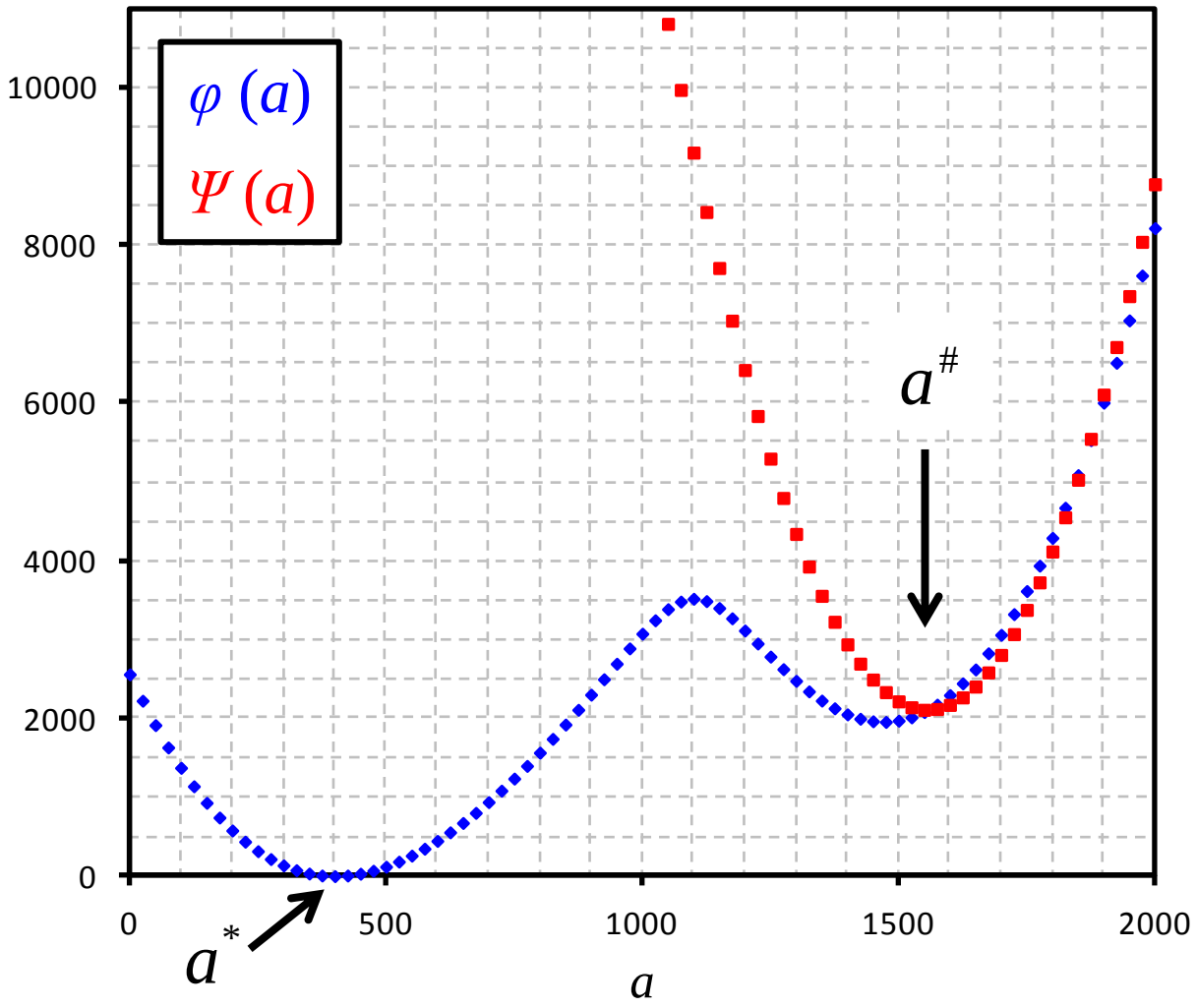
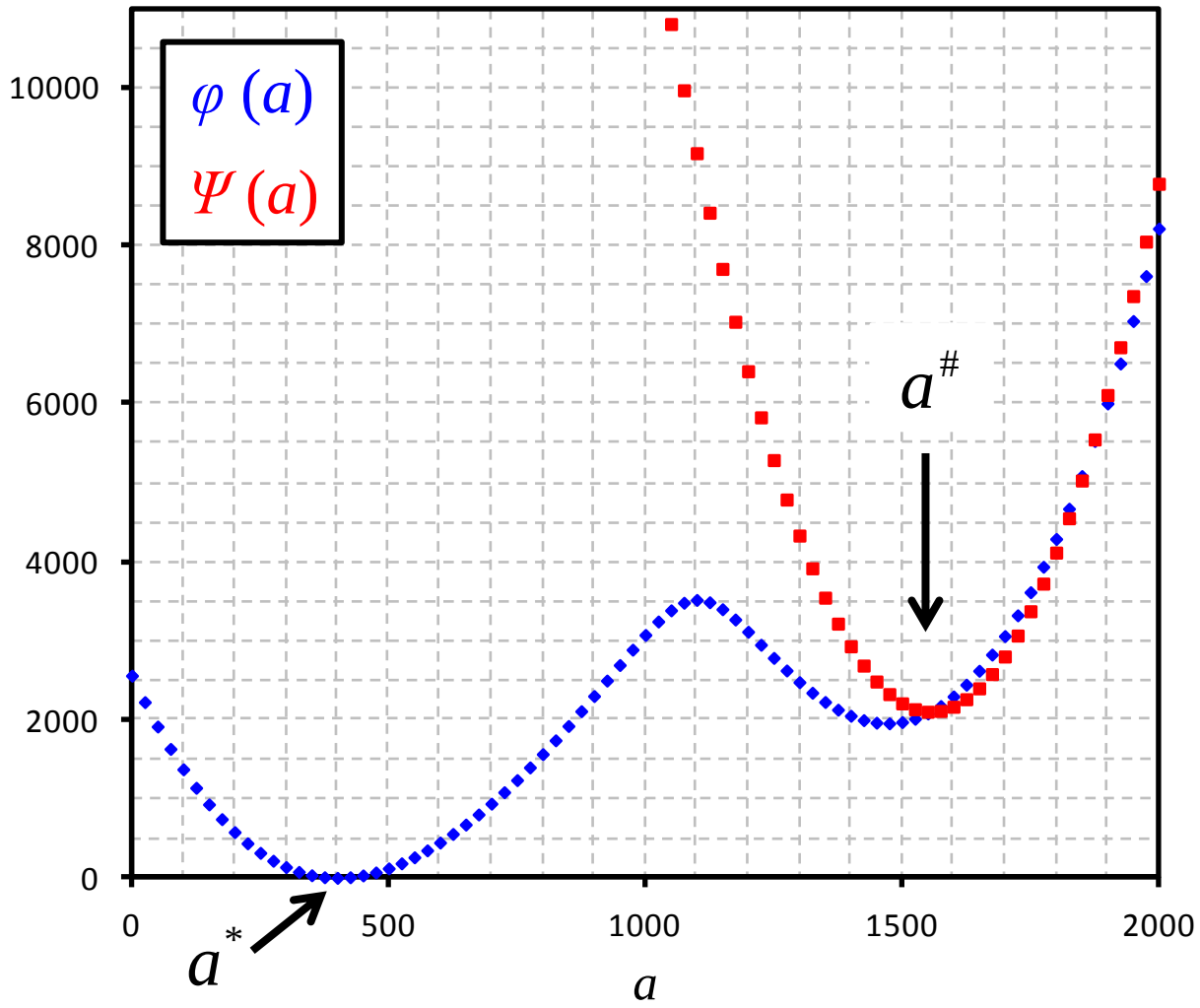


Aspectos geométricos
(Problema não-linear 1D)



Exercícios

- Formular o Problema Inverso linear 1D
  - Equação de Mínimos Quadrados
- Formular o Problema Inverso não-linear 1D
  - Equação do método de Gauss-Newton