Direct exponential integration for linear SWE on sphere

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There was a certain interest in using rational approximations for resembling exponential integrators with the SWE on the rotating sphere. Here, it was assumed that rational approximations must be used for solving the linearized SWE. This work will investigate a form of the linearized SWE which doesn't incorporate the Coriolis effect.

1 Linear operator

Using the vorticity-divergence formulation, the linear operator is then given by

$$\begin{bmatrix} \partial_t \Phi' \\ \partial_t \delta \end{bmatrix} = \begin{bmatrix} & -\overline{\Phi} \\ -\nabla^2 & \end{bmatrix} \begin{bmatrix} \Phi' \\ \delta \end{bmatrix}.$$

Using spherical harmonics, the Laplace operator ∇^2 is also diagonal. Therefore, we can rearrange this system in 2x2 blocked matrices along the diagonal.

2 Exp. integrators

With exponential time integrators, we can write

$$U_t = LU$$

with

$$L = \begin{bmatrix} & -\overline{\Phi} \\ -\nabla^2 & \end{bmatrix}$$

and a direct solution

$$U(t) = \exp(tL)U(0).$$

For sake of convenience, we use the letters $D = -\nabla^2$ and $G = -\overline{\Phi}$, giving

$$L = \begin{bmatrix} & G \\ D & \end{bmatrix}$$

2.1 Eigen diagonalization

We can work with an Eigen diagonalization of the matrix, giving

$$U(t) = Q\exp(t\Lambda)Q^{-1}U(0) = \exp(tL)U(0).$$

where $LQ = Q\Lambda$ an Eigenvalue decomposition with

$$Q = \begin{bmatrix} \frac{-G}{\sqrt{DG}} & \frac{G}{\sqrt{DG}} \\ 1 & 1 \end{bmatrix}$$

and

$$\Lambda = \begin{bmatrix} -\sqrt{DG} & \\ & \sqrt{DG} \end{bmatrix}$$

and finally

$$Q^{-1} = \begin{bmatrix} \frac{-\sqrt{DG}}{2G} & \frac{1}{2} \\ \frac{\sqrt{DG}}{2G} & \frac{1}{2} \end{bmatrix}$$