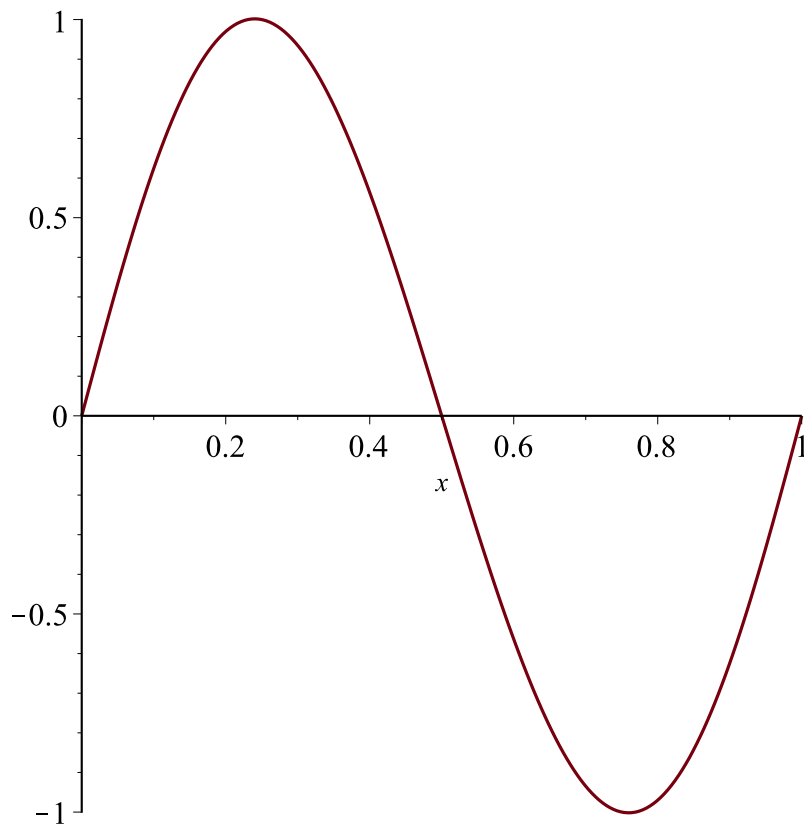


```

> restart;
# Generic form
> fun := (x) → a + b·x + c·x2 + d·x3 + e·x4 + f·x5;
      fun := x → a + b x + c x2 + d x3 + e x4 + f x5 (1)
# Diff of fun
> dfun := (x) → diff(fun(x), x);
      dfun := x →  $\frac{d}{dx}$  fun(x) (2)
> d2fun := (x) → diff(fun(x), x, x);
      d2fun := x →  $\frac{d^2}{dx^2}$  fun(x) (3)
# s = slope at 0 and 1 should be the same
> eqa := unapply(dfun(x), x)(0) = unapply(dfun(x), x)(1);
      eqa := b = 5 f + 4 e + 3 d + 2 c + b (4)
> eqb := fun(0) = 0;
      eqb := a = 0 (5)
> eqc := fun(1) = 0;
      eqc := a + b + c + d + e + f = 0 (6)
> eqd := fun( $\frac{1}{2}$ ) = 0;
      eqd := a +  $\frac{1}{2}$  b +  $\frac{1}{4}$  c +  $\frac{1}{8}$  d +  $\frac{1}{16}$  e +  $\frac{1}{32}$  f = 0 (7)
> eqe := fun( $\frac{1}{4}$ ) = 1;
      eqe := a +  $\frac{1}{4}$  b +  $\frac{1}{16}$  c +  $\frac{1}{64}$  d +  $\frac{1}{256}$  e +  $\frac{1}{1024}$  f = 1 (8)
> eqf := unapply(d2fun(x), x)(0) = unapply(d2fun(x), x)(1);
      eqf := 2 c = 20 f + 12 e + 6 d + 2 c (9)
> sol := solve([eqa, eqb, eqc, eqd, eqe, eqf], [a, b, c, d, e, f]);
      sol :=  $\left[ \left[ a = 0, b = \frac{512}{75}, c = 0, d = -\frac{1024}{15}, e = \frac{512}{5}, f = -\frac{1024}{25} \right] \right]$  (10)
> fun(x);
      f x5 + e x4 + d x3 + c x2 + b x + a (11)
> afun := applyrule(sol[1], fun(x));
      afun :=  $-\frac{1024}{25} x^5 + \frac{512}{5} x^4 - \frac{1024}{15} x^3 + \frac{512}{75} x$  (12)
> realfun := (x) → afun;
      realfun := x → afun (13)
> plot(realfun(x), x = 0 .. 1);

```



So finally, we get the function

> *realfun*(x);

$$-\frac{1024}{25}x^5 + \frac{512}{5}x^4 - \frac{1024}{15}x^3 + \frac{512}{75}x \quad (14)$$

Its 1st derivative

> *diff*(*realfun*(x), x);

$$-\frac{1024}{5}x^4 + \frac{2048}{5}x^3 - \frac{1024}{5}x^2 + \frac{512}{75} \quad (15)$$

Its 2nd derivative

> *diff*(*realfun*(x), x, x);

$$-\frac{4096}{5}x^3 + \frac{6144}{5}x^2 - \frac{2048}{5}x \quad (16)$$

>