Formulations of the shallow-water equations

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Equations (2.3a) and (2.3b) relate the Rossby R and Burgers B number to the non-dimensional SWE. We have to identify parameters H, f and g in order to run this in SWEET.

1 Mass conservation term

The mass conservation term is given by

$$\partial_t h = -(hu)_x - (hv)_y$$

with h the total surface height.

Polvani's paper states

$$RB^{-1}\left[\partial_t h' + \nabla.(h'\boldsymbol{V})\right] + \nabla.\boldsymbol{V} = 0$$

which we reformulate to

$$\partial_t h' + \nabla.(h'\boldsymbol{V}) + (R^{-1}B)\nabla.\boldsymbol{V} = 0$$

where we can directly see that

$$H = R^{-1}B$$

with

2 Advective terms

We can use the advective formulation which is given by

$$\begin{aligned}
\partial_t u &= -g\partial_x u + fv - uu_x - vu_y \\
\partial_t v &= -g\partial_y v - fu - uv_x - vv_y.
\end{aligned}$$

For the advective part and using $V = (u, v)^T$, we can write

$$\partial_t \boldsymbol{V} = -g\nabla h + \boldsymbol{k} \times \boldsymbol{V} - (\boldsymbol{V}.\nabla)\boldsymbol{V}$$

The Polvani paper uses the formulation

$$\begin{aligned} R\left[\partial_t \boldsymbol{V} + (\boldsymbol{V}.\nabla)\boldsymbol{V}\right] + \boldsymbol{k} \times \boldsymbol{V} &= -\nabla h \\ \partial_t \boldsymbol{V} + (\boldsymbol{V}.\nabla)\boldsymbol{V} + R^{-1}\boldsymbol{k} \times \boldsymbol{V} &= -R^{-1}\nabla h \end{aligned}$$

and we can get this formulation by setting

$$f = R^{-1}$$

and

$$g = R^{-1}$$