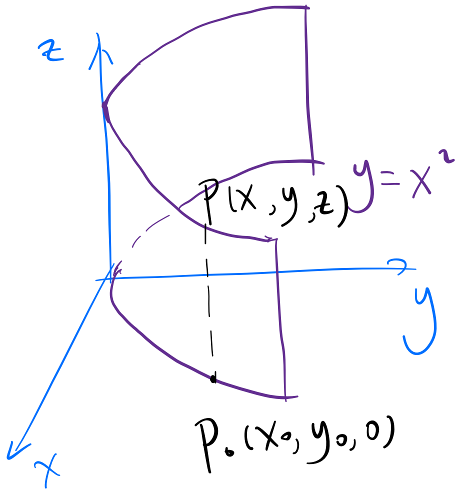
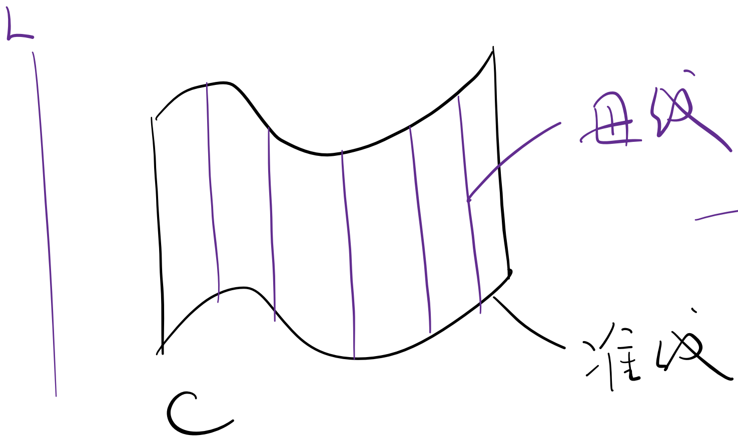


抛物柱面

$x=x_0,\ y=y_0$

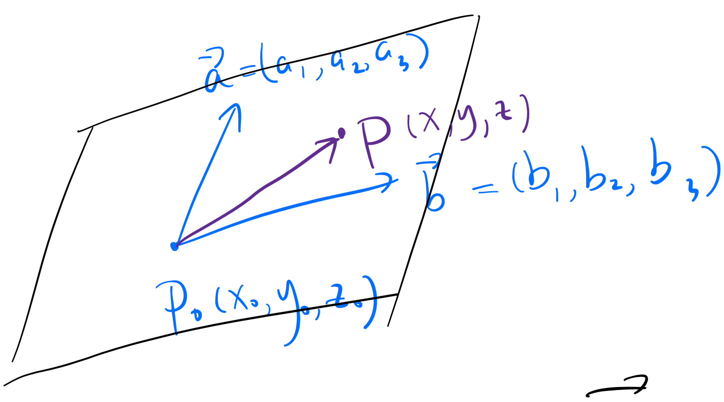
$y_0=x_0^2,\quad y=x^2$



柱面

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$：椭圆柱面.　　$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. 双曲柱面

空间曲面的参数方程



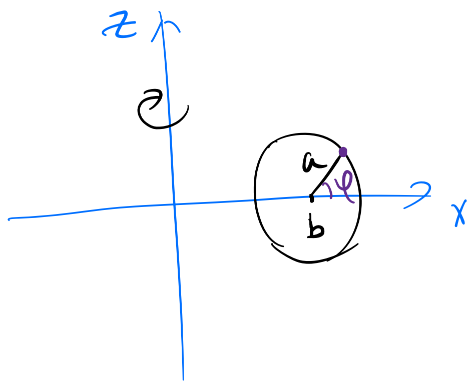
$$\overrightarrow{P_0P}=u\vec{a}+v\vec{b}$$

$$(x-x_0,\ y-y_0,\ z-z_0)=u(a_1,a_2,a_3)+v(b_1,b_2,b_3)$$

$$\begin{cases} x = x_0 + u a_1 + v b_1 \\ y = y_0 + u a_2 + v b_2 \\ z = z_0 + u a_3 + v b_3 \end{cases} \qquad u, v \in \mathbb{R}$$

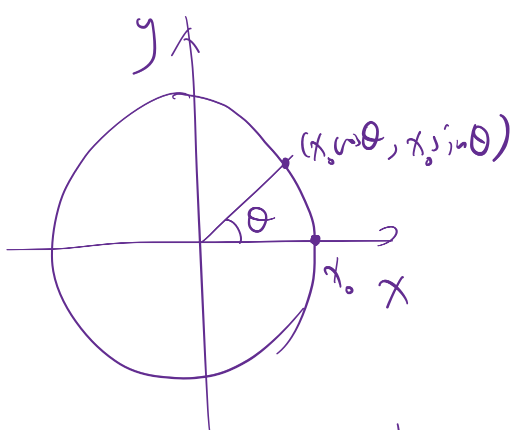
$$\begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases}, \quad (u,v) \in D: \quad \text{空间曲面}$$

例:



以 $(b, 0)$ 为圆心, a 为半径的圆 $(b > a > 0)$

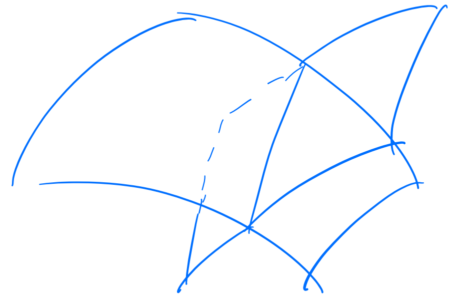
圆: $\begin{cases} x = b + a\cos\varphi \\ z = a\sin\varphi \end{cases} \qquad \varphi \in [0, 2\pi)$



$$\begin{cases} x = (b + a\cos\varphi)\cos\theta \\ y = (b + a\cos\varphi)\sin\theta \\ z = a\sin\varphi \end{cases} \qquad \begin{aligned} &\varphi \in [0, 2\pi) \\ &\theta \in [0, 2\pi) \end{aligned}$$

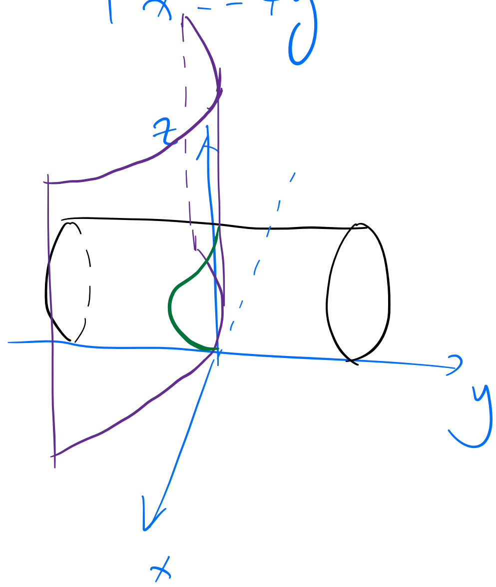
$x_0 = b + a\cos\varphi > 0$

$$\begin{cases} F(x,y,z)=0 \\ G(x,y,z)=0 \end{cases} \text{：曲线}$$



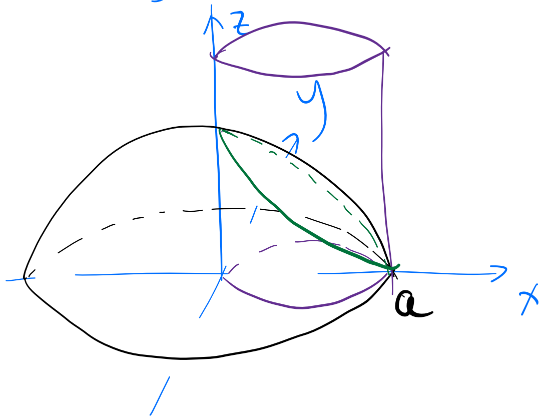
例：

$$\begin{cases} x^2+z^2=4z \\ x^2=-4y \end{cases}$$

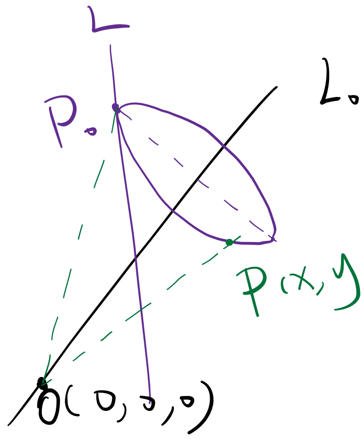


例：

$$\begin{cases} x^2+y^2+z^2=a^2 \quad (z\geq 0) \\ x^2+y^2=ax \end{cases}$$



例：求直线 $L: \frac{x-1}{1}=\frac{y}{3}=\frac{z}{3}$ 绕直线 $L_0: \frac{x}{2}=\frac{y}{1}=\frac{z}{-2}$ 旋转一周所得的曲面 Σ 的方程



$P_0(x_0,y_0,z_0) \in L$

$$\frac{x_0-1}{1}=\frac{y_0}{3}=\frac{z_0}{3}$$

$\overrightarrow{P_0P} \perp L_0$

$\|\overrightarrow{OP}\|=\|\overrightarrow{OP_0}\|$

$$\begin{cases} x^2+y^2+z^2=x_0^2+y_0^2+z_0^2 \\ 2(x-x_0)+(y-y_0)-2(z-z_0)=0 \end{cases}$$

令 $$\frac{x_0-1}{1}=\frac{y_0}{3}=\frac{z_0}{3}=t$$

$$\begin{cases} x_0=1+t, \\ y_0=3t \\ z_0=3t \end{cases}$$

$$2x+y-2z=2x_0+y_0-2z_0$$
$$=2+2t+3t-6t$$
$$=2-t$$

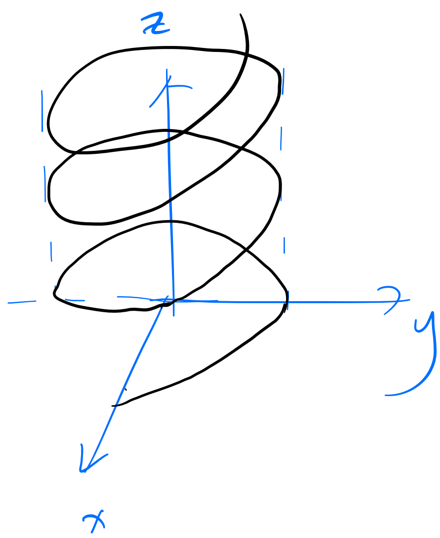
$$t=-(2x+y-2z)+2$$
$$1+t=-(2x+y-2z)+3$$

$$x^2+y^2+z^2 = (1+t)^2 + (3t)^2 + (3t)^2$$
$$= (2x+y-2z-3)^2 + 18(2x+y-2z-2)^2$$

$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}, \quad t \in I$$

空间曲线的参数方程

例：
$$\begin{cases} x = a\cos t \\ y = a\sin t \\ z = ct \end{cases} \quad t \geq 0, \quad (a, c > 0)$$



曲面 Σ：
$$\begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases} \quad (u,v) \in D$$

取定 $v = v_0$，
$$\begin{cases} x = x(u,v_0) \\ y = y(u,v_0) \\ z = z(u,v_0) \end{cases} \quad : \ u\text{-曲线}$$

取定 $u=u_0$

$$\begin{cases} x=x(u_0,v) \\ y=y(u_0,v) \\ z=z(u_0,v) \end{cases} \quad : \quad v-\text{曲线}$$

平面上:

$$ax^2+bxy+cy^2+dx+ey+f=0 \quad : \text{二次曲线}$$

空间中:

$$a_{11}x^2+a_{22}y^2+a_{33}z^2+2a_{12}xy+2a_{13}xz+2a_{23}yz$$
$$+b_1x+b_2y+b_3z+c=0$$

: 二次曲面

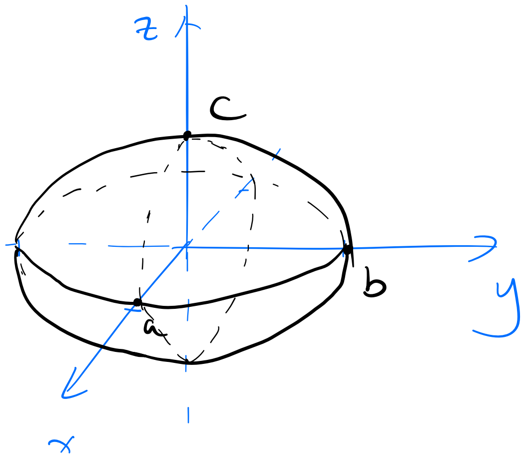
标准形式:

$$Ax^2+By^2+Cz^2+J=0$$
$$Ax^2+By^2+Iz=0$$

$$x^2+2z^2-6x-y+10=0 \quad \Rightarrow \quad (x-3)^2+2z^2-(y-1)=0$$

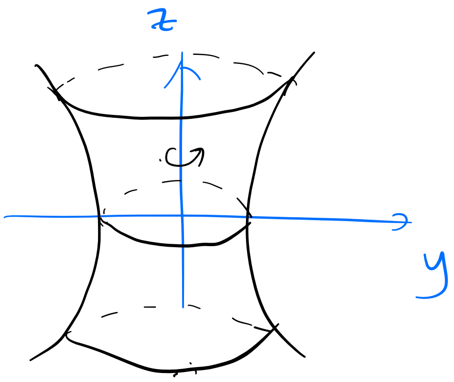
一. 椭球面

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



二、双曲面

1、单叶双曲面

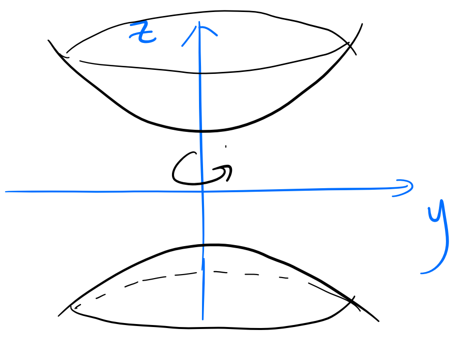


$$\frac{y^2}{a^2} - \frac{z^2}{b^2} = 1$$

$$\frac{x^2+y^2}{a^2} - \frac{z^2}{b^2} = 1$$

一般地 $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

2、双叶双曲面



$$-\frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$$

$$-\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1$$