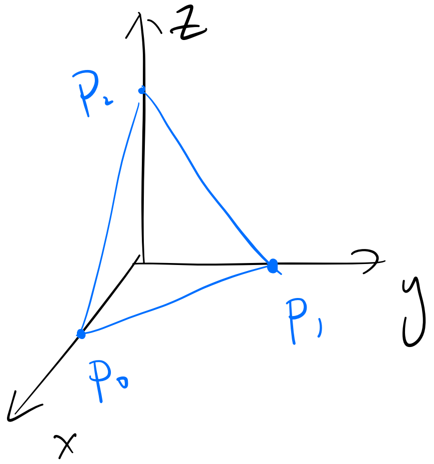
$P_0(a,0,0)$　　$P_1(0,b,0)$　　$P_2(0,0,c)$



$$Aa+D=0$$
$$Bb+D=0$$
$$Cc+D=0$$

$a,b,c\neq 0$：　令 $D=-1$

$$A=\frac{1}{a},\ B=\frac{1}{b},\ C=\frac{1}{c}$$

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\vec{n}=\left(\frac{1}{a},\frac{1}{b},\frac{1}{c}\right)$$

$a=0,\ b,c\neq 0,\ A=1,\ B=C=D=0$：　$x=0$

例：$(3,2,1)$，$(0,1,0)$，$(-1,0,2)$

$$Ax+By+Cz+D=0$$

$$\begin{cases} 3A+2B+C+D=0 \\ B+D=0 \\ -A+2C+D=0 \end{cases}$$

$D = 1, \quad B = -1$

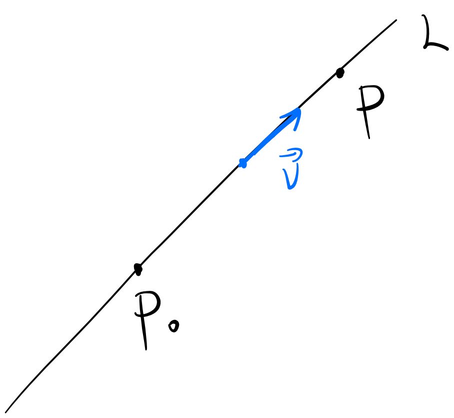
$$\begin{cases} 3A + C = 1 \\ -A + 2C = -1 \end{cases}$$

$7A = 3. \qquad A = \frac{3}{7}$

$C = -\frac{2}{7}$

$\frac{3}{7}x - y - \frac{2}{7}z + 1 = 0$

$3x - 7y - 2z + 7 = 0$



$P_0(x_0, y_0, z_0) \qquad P(x, y, z)$

$\vec{v} = (a, b, c) \qquad a^2 + b^2 + c^2 \neq 0$: 方向向量

$\overrightarrow{P_0P} \parallel \vec{v}$

$\overrightarrow{P_0P} = t\vec{v}$

$(x - x_0, y - y_0, z - z_0) = t(a, b, c)$

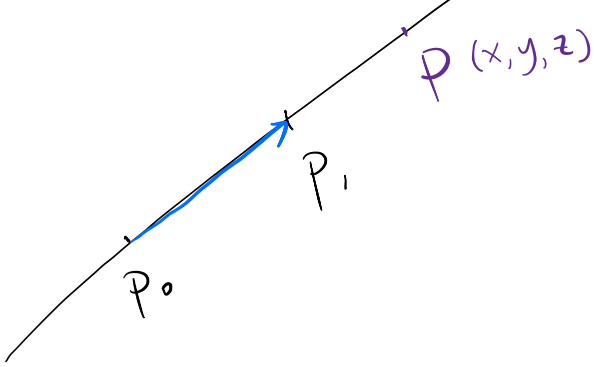
$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$

参数方程

$a, b, c \neq 0$:
$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$
(对称式方程
点向式方程)

$a = 0,\ b, c \neq 0$:
$$\begin{cases} x = x_0 \\ \dfrac{y-y_0}{b} = \dfrac{z-z_0}{c} \end{cases}$$

$a = b = 0,\ c \neq 0,$
$$\begin{cases} x = x_0 \\ y = y_0 \end{cases}$$



$P_0(x_0, y_0, z_0)$

$P_1(x_1, y_1, z_1)$

$$\overrightarrow{P_0P_1} = (x_1-x_0,\ y_1-y_0,\ z_1-z_0)$$

$$\frac{x-x_0}{x_1-x_0} = \frac{y-y_0}{y_1-y_0} = \frac{z-z_0}{z_1-z_0}$$ （两点式方程）

例：求直线 $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{2}$ 与平面 $2x+y+z-6=0$ 的交点

$$\begin{cases} x = 2+t \\ y = 3+t \\ z = 4+2t \end{cases}$$

$$2(2+t) + (3+t) + (4+2t) - 6 = 0$$

$$5t + 5 = 0$$

$t=-1$. $x=1$, $y=2$, $z=2$

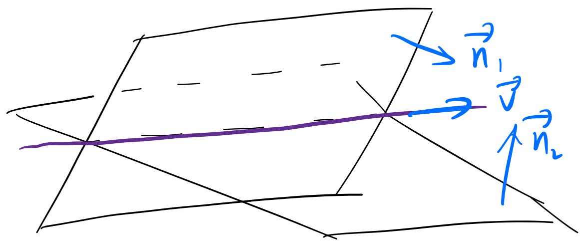
$(1, 2, 2)$

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases}$$

$(A_1, B_1, C_1) \nparallel (A_2, B_2, C_2)$

直线（一般方程）

$\vec{n}_1 = (A_1, B_1, C_1) \qquad \vec{n}_2 = (A_2, B_2, C_2)$



$\vec{v} \perp \vec{n}_1, \quad \vec{v} \perp \vec{n}_2$

$$\vec{v} = \vec{n}_1 \times \vec{n}_2$$

例

$$\begin{cases} 2x + y - z + 1 = 0 \\ x + 2z + 4 = 0 \end{cases}$$

$\vec{n}_1 = (2, 1, -1). \qquad \vec{n}_2 = (1, 0, 2)$

$$\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ 1 & 0 & 2 \end{vmatrix} = 2\vec{i} - 5\vec{j} - \vec{k}$$

$z_0 = 0, \quad x_0 = -4. \quad y_0 = 7 \qquad (-4, 7, 0)$

$$\frac{x+4}{2} = \frac{y-7}{-5} = \frac{z}{-1}$$

例：求过点 $M_0(0,0,-2)$，与直线 $L_1: \frac{x-1}{4}=\frac{y-3}{-2}=\frac{z}{1}$ 相交，平行于平面 $\pi_1: 3x-y+2z-1=0$ 的直线的方程

$M_1(x_1,y_1,z_1)\in L_1$，$\overrightarrow{M_0M_1}=(x_1,y_1,z_1+2)$

$(x_1,y_1,z_1+2)\cdot(3,-1,2)=0$

$x_1=1+4t$，$y_1=3-2t$，$z_1=t$

$3(1+4t)-(3-2t)+2(t+2)=0$

$16t+4=0 \qquad t=-\frac{1}{4}$

$x_1=0$，$y_1=\frac{7}{2}$，$z_1=-\frac{1}{4}$

$$\frac{x-0}{0-0}=\frac{y-0}{\frac{7}{2}-0}=\frac{z+2}{-\frac{1}{4}+2}$$

$$\begin{cases} x=0 \\ y=2(z+2) \end{cases}$$

$$\begin{cases} x=0 \\ y=\frac{7}{2}s \\ z=-2+\frac{7}{4}s \end{cases} \qquad \begin{cases} x=0 \\ y=2t \\ z-2=t \end{cases} \qquad \left(t=\frac{7}{4}s\right)$$

平面束

$$L: \begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$$

(A_1, B_1, C_1) 和 (A_2, B_2, C_2) 不成比例

λ, μ 不同时为 0

$$\lambda(A_1x+B_1y+C_1z+D_1)+\mu(A_2x+B_2y+C_2z+D_2)=0$$

:给过 L 的平面全体：通过 L 的平面束

$$A_1x+B_1y+C_1z+D_1+k(A_2x+B_2y+C_2z+D_2)=0$$

(不包含 平面 $A_2x+B_2y+C_2z+D_2=0$)

$$k(A_1x+B_1y+C_1z+D_1)+A_2x+B_2y+C_2z+D_2=0$$

(不包含 平面 $A_1x+B_1y+C_1z+D_1=0$)

$Ax+By+Cz+\lambda=0$: 相互平行的平面

平行平面束方程

例: 求过点(1,1,1)和直线 L: $\begin{cases} 3x-y+2z+2=0 \\ x-2y+3z-5=0 \end{cases}$ 的平面方程

平面束 $3x-y+2z+2+k(x-2y+3z-5)=0$

$$3-1+2+2+k(1-2+3-5)=0$$

$$6-3k=0$$

$$k=2$$

$$3x-y+2z+2+2(x-2y+3z-5)=0$$

$$5x-5y+8z-8=0$$

例: 求直线 $\begin{cases} x+y-z-1=0 \\ x-y+z+1=0 \end{cases}$, 在平面 $x+y+z=0$ 上的投影直线的方程

平面束 $(x+y-z-1)+k(x-y+z+1)=0$

$$(1+k,\ 1-k,\ -1+k)\cdot(1,1,1)=0$$

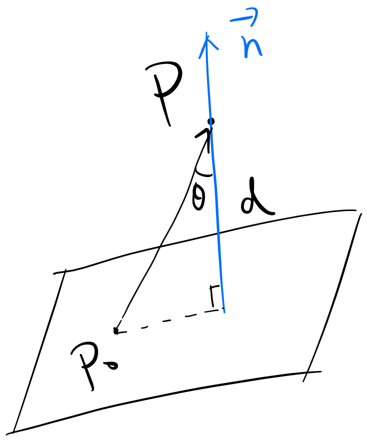
$(1+k)+(1-k)+(-1+k)=0 \qquad k=-1,$

平面：$(x+y-z-1)-(x-y+z+1)=0$

$2y-2z-2=0$

$y-z-1=0$

直线：$\begin{cases} y-z-1=0 \\ x+y+z=0 \end{cases}$



$d=\|\overrightarrow{P_0P}\||\cos\theta| = \dfrac{\|\overrightarrow{P_0P}\|\|\vec{n}\||\cos\theta|}{\|\vec{n}\|}$

$= \dfrac{|\overrightarrow{P_0P}\cdot\vec{n}|}{\|\vec{n}\|}$

$P_0=(x_0,y_0,z_0)$ ， $P(x,y,z)$

$\vec{n}=(A,B,C)$

$$d=\frac{1}{\sqrt{A^2+B^2+C^2}}\left|A(x-x_0)+B(y-y_0)+C(z-z_0)\right|$$