# HW3 _Kevin Lopez Sepulveda

- **Submission date: 02/19/2025 (11:59 PM)**
- Read your class notes *thoroughly* before attempting the assignment.
- You can submit ***well commented*** code in either `.py` or `.ipynb` formats.
- Submitting a PDF explaining your code is *highly* encouraged.

1. **Pandas practice.** Complete the assignment in the `.ipynb` file named, "`CS677_HW3_Pandas_questions.`"

2. **Seaborn.** Complete the assignment in the `.ipynb` file named, "`HW3_Seaborn,`" located in the zipped folder, "`files_for_HW3_Seaborn.`"

For the following questions, you may use calculators, but not programming (use NumPy/Python only for verification if you wish to. Do not submit that code).

3. **(Matrix multiplication)**. Let
$A = [6, -2, -2; 10, -3, 1; -10, 5\ 1]_{3\times3}$, $B = [9, 4, -4; 4, 7, 0; -4, 0, 11]_{3\times3}$, $C = [3\ 1; 0 - 2; 4, 0]_{3\times2}$, $\boldsymbol{a} = [5; 1; 2\ ]_{3\times1}$, $\boldsymbol{b} = [3, 0, 8]_{1\times3}$.
Here, rows of the matrix are separated by semicolons, so, for example, $[6, -2, -2]$ is the first row of $A$. Pay very close attention to the **matrix sizes**.
Calculate the following or explain why they cannot be calculated.
   a. $A\boldsymbol{a}$, $A\boldsymbol{b}$, $A\boldsymbol{b}^{\boldsymbol{T}}$, $AB$.
      **A a** = [24; 49; -43]
      **A b** = [2; 38; -22]
      **A b^T** = [2; 38; -22]
      **A B** = [54, 10, -46; 74, 19, -29; -74, -5, 51]
   b. $AB$, $BA$, $AA^T$, $A^TA$
      **A B** = [54, 10, -46; 74, 19, -29; -74, -5, 51]
      **B A** = [134, -50, -18; 94, -29, -1; -134, 63, 19]
      **A A^T** = [44, 64, -72; 64, 110, -114; -72, -114, 126]
      **A^T A** = [236, -92, -12; -92, 38, 6; -12, 6, 6]
   c. Can you say something about the symmetry properties of matrices like $A^TA$ and $AA^T$? d. $\boldsymbol{ab}$, $\boldsymbol{ba}$, $(\boldsymbol{ab})A$, $\boldsymbol{a}(\boldsymbol{b}A)$
      i The matrices A^T A and A A^T are both symmetric. This means that if you take the transpose of these matrices, you will get the same matrix back. Symmetric matrices have the property that the elements across the main diagonal mirror each other.
   d. Compute $||\boldsymbol{a} - \boldsymbol{b}^{\boldsymbol{T}}||^2{}_2$. How will you express this in terms of $||\boldsymbol{a}||^2{}_2$, $||\boldsymbol{b}||^2{}_2$, and $\langle\boldsymbol{a}, \boldsymbol{b}\rangle$?
      To compute the squared Euclidean norm of the difference between vector a and the transpose of vector b, we start with a as [5; 1; 2] and b as [3, 0, 8]. The transpose of b is b^T = [3; 0; 8]. When we calculate the difference a - b^T, we get [5; 1; 2] - [3; 0; 8], which results in [2; 1; -6] after subtracting the corresponding elements. To find the squared norm of this difference, we use the formula, which is the sum of the squares of each element: $(2)^2 + (1)^2 + (-6)^2$ equals 4 + 1 + 36, giving us a total of 41. We can also express this squared norm in terms of the norms of a and b and their inner product. The

squared norm of a is 30, the squared norm of b is 73, and the inner product of a and b is 31. We can express the squared norm of the difference as norm a - b^T squared = norm a squared + norm b squared - 2 times the inner product of a and b. Putting it all together, we confirm that 41 equals 30 + 73 - 2 times 31. Therefore, the squared Euclidean norm of the difference a - b^T is 41.

4. **(Triangular matrices)**. Suppose $U_1$, $U_2$ are two upper triangular matrices and $L_1$, $L_2$ are two lower triangular matrices. Using examples ($3\times 3$ matrices will suffice), explain which of the following are triangular: $U_1 + U_2$, $U_1U_2$, $U_1 + L_1$, $L_{22}$, $L_1L_2$.

To understand triangular matrices, we need to know what upper and lower triangular matrices are. An upper triangular matrix has all the entries below the main diagonal equal to zero, while a lower triangular matrix has all the entries above the main diagonal equal to zero.

Lets look at some examples. For upper triangular matrices, we have:
U1 = [1, 2, 3; 0, 4, 5; 0, 0, 6] 3x3 and
U2 = [7, 8, 9; 0, 10, 11; 0, 0, 12] 3x3.
For lower triangular matrices, we have:
L1 = [1, 0, 0; 2, 3, 0; 4, 5, 6] 3x3 and
L2 = [7, 0, 0; 8, 9, 0; 10, 11, 12] 3x3.

Now lets see what happens when we do some operations with these matrices. First, when we add two upper triangular matrices like U1 + U2, the result is still an upper triangular matrix:
U1 + U2 = [8, 10, 12; 0, 14, 16; 0, 0, 18] 3x3.

Next, the product of two upper triangular matrices, U1 U2, is also upper triangular:
U1 U2 = [7, 38, 57; 0, 40, 62; 0, 0, 72] 3x3.

Now if we add an upper triangular matrix and a lower triangular matrix, like U1 + L1, the result is not triangular:
U1 + L1 = [2, 2, 3; 2, 7, 5; 4, 5, 12] 3x3, which has non-zero entries both above and below the diagonal.

For the lower triangular matrices, squaring L2 gives us L2^2, which is still lower triangular:
L2^2 = [49, 0, 0; 56, 81, 0; 70, 83, 144] 3x3.

Finally, when we multiply two lower triangular matrices like L1 L2, we also get a lower triangular matrix:
L1 L2 = [7, 0, 0; 22, 27, 0; 70, 83, 72] 3x3.

So to sum it up, U1 + U2 and U1 U2 are upper triangular, U1 + L1 is not triangular, L2^2 is lower triangular, and L1 L2 is also lower triangular.

5. Given a vector $\boldsymbol{u} = [u_1, u_2, \dots, u_n]$, what can you say about the norm of the vector

$$\hat{\boldsymbol{u}} = \frac{\boldsymbol{u}}{||\boldsymbol{u}||}?$$

Explain how you arrived at your answer in detail.

The expression for u hat represents the unit vector in the direction of u. To find the norm, or length, of a vector u with components u one, u two, and so on, we take the square root of the sum of each component squared. To create the unit vector, we divide each component of u by its norm, which gives us a vector that points in the same direction as u but has a length of one. When we find the norm of u hat, we do the same thing. Each part of u hat is just the corresponding part of u divided by the norm of u. If we square these values and add them up, we get a fraction where the top is the sum of the squared components of u, and the bottom is the square of the norm of u. Since the top is equal to the square of the norm of u, dividing it by itself simplifies to one. So, the norm of u hat is always one. This makes sense because unit vectors are designed to have a length of one while still pointing in the same direction as the original vector.

6. What are antisymmetric matrices (note that they are also known as “skew-symmetric” matrices)? What can you say about the diagonal entries of an antisymmetric matrix?
    a. An antisymmetric matrix, also known as a skew-symmetric matrix, is a square matrix where the transpose of the matrix is equal to its negative. In other words, for a matrix A to be antisymmetric, it must satisfy the condition that A transpose equals negative A. This means that for every entry in the matrix, the value at row i and column j is the negative of the value at row j and column i.

7. One important property of antisymmetric matrices is that all the diagonal entries must be zero. This happens because any diagonal entry is both A at row i and column i and its own negative, which can only be true if the value is zero. So, in an antisymmetric matrix, all elements along the main diagonal will always be zero.
    a.

8. **(Orthogonality)**. Find the $L^2$-norm of the following vectors $\boldsymbol{a} = [4,2,-6]$, $\boldsymbol{b} = [16, -32, 0]$. Are $\boldsymbol{a}$, $\boldsymbol{b}$ orthogonal vectors?

9. **(Unit Vectors)**. What is a unit vector? Find a unit vector orthogonal to $\boldsymbol{c}$ = [4 − 3].
    a. A unit vector is just a vector with a length of one. It’s mainly used to show direction without changing the scale. To get a unit vector, you divide a vector by its magnitude. To find a unit vector that’s orthogonal to c = [4, -3], we first need a perpendicular vector. A quick way to do this in 2D is by swapping the numbers and changing the sign of one. That gives us [3, 4] or [-3, -4]. Now, we just need to make it a unit vector by dividing by its magnitude. The magnitude of [3, 4] is 5, so we divide everything by 5. That gives us [3 over 5, 4 over 5] as one unit vector. Another option is [-3 over 5, -4 over 5]. Either one works.

10. **(Angle between vectors).** Let $a$ = [1,1,1], $b$ = [2,3,1], $c$ = [−1, 1, 0]. Find the cosine of the angle between the vectors $\boldsymbol{a} + \boldsymbol{b}$, $\boldsymbol{c}$.
    a. **To** find the cosine of the angle between a plus b and c, we use the formula that says cosine equals the dot product of the vectors divided by the product of their magnitudes. First, we add up a, which is [1,1,1], and b, which is [2,3,1], to get a plus b equals [3,4,2]. Then, we take the dot product of this with c, which is [-1,1,0], and we get negative three plus four plus zero, which equals one. Next, we find the magnitudes. The magnitude of a plus b is the square root of three squared plus four squared plus two squared, which simplifies to the square root of twenty-nine. The magnitude of c is the square root of negative one squared plus one squared plus zero squared, which simplifies to the square root of two. Now, we just divide the dot product by the product of the magnitudes, which gives one over the square root of fifty-eight. So, the cosine of the angle between a plus b and c is one over the square root of fifty-eight.