Name (Last, First):

Student ID:

# Assignment 10

*METCS544A3A4_F2024*

**Instructions:**

1. For answering programming questions, please use Adobe Acrobat to edit the pdf file in two steps **[See Appendix: Example Question and Answer]**:
    a. Copy and paste your R code as text in the box provided (so that your teaching team can run your code);
    b. Screenshot your R console outputs, save them as a .PNG image file, and paste/insert them in the box provided.
    c. Show all work—credit will not be given for code without showing it in action, including a screenshot of R console outputs.
2. To answer non-programming questions, please type or handwrite your final answers clearly in the boxes. Show all work - credit will not be given for numerical solutions that appear without explanation in the space above the boxes. **You're encouraged to use R to graph/plot the data and produce numerical summaries; please append your code and screenshot of the outputs at the end of your PDF submission.**

**[Total 120 pts = 39 + 24 + 30 + 24 pts + 3 Extra Credit pts]**

**Grading Rubric**

Each question is worth 3 points and will be graded as follows:

3 points: Correct answer with work shown

2 points: Incorrect answer but attempt shows some understanding (work shown)

1 point: Incorrect answer but an attempt was made (work shown), or **correct answer without explanation (work not shown)**

0 points: Left blank or made little to no effort/work not shown

**Reflective Journal [3 pts]**

(Copy and paste the link to your live Google doc in the box below)

## Part I. Discrete and Continuous Random Variables (39 pts)

1) A random sample of adults was taken and asked the question "Last week, how many days did you sit down and eat at a restaurant?" X represents the number of days they responded with. Here is the resulting probability model:

| Days (X) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Probability | 0.40 | 0.28 | 0.16 | 0.05 | 0.04 | 0.03 | 0.02 | 0.02 |

(a) Show that the probabilities in this distribution add up to 1.

**Answer:**

(b) Make a histogram of the probability distribution. Describe the shape of the distribution.

(c) Find and interpret $P(X < 4)$.

**Answer:**

(d) Express the event "Ate out at least twice last week" in terms of X. What is the probability of this event?

**Answer:**

(e) Find and interpret the mean.

**Answer:**

(f) Find the standard deviation.

**Answer:**

2) A Denver Ski Resort compiles their profit data yearly and has discovered that if it snows less than 60 inches in a season, they will lose $90,000. If it snows between 60 inches and 80 inches, they will make $70,000, and if it snows above 80 inches, they will make $200,000. Let X represent the profit (any losses would be represented as a negative value). If the probability of less than 60 inches is 40%, the probability of between 60 inches and 80 inches is 45%, create a table to represent the probability distribution. Then comment on if this is a profitable business model or not based on the mean of the distribution. **[6 pts]**

**Answer:**

3) A new type of gambling game uses two dice and the sum of the two dice determines your winnings. It costs $5 to play the game. If the sum is a 2 or a 12, the player will win $50. If the sum is a 7, the player will win $10. Any other sum and the player will lose and not get anything. If X represents the winnings (with -5 representing a loss, since a player will lose that amount of money), create a table to represent the probability distribution. Using the mean of this distribution, do you believe the game is in favor of the player or the "house"? **[6 pts]**

**Answer:**

4) Choose an American household at random and let the random variable X be the number of persons living in the household. The probability of X is:

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 or more |
|---|---|---|---|---|---|---|---|
| P(X) | 0.25 | 0.32 | 0.17 | 0.15 | 0.07 | 0.03 | 0.01 |

Find and interpret the following probabilities:

a) $P(X > 3)$

**Answer:**

b) $P(X \leq 4)$

**Answer:**

c) $P(2 \leq x \leq 5)$

**Answer:**

## Part II. Combining Random Variables (8 x 3 = 24 pts)

1) In the 2021 baseball season, the Brewers runs per game, B, varied from game to game but had $\mu_B = 3.7$ and $\sigma_B = 1.1$.The Cubs runs per game, C, were $\mu_C = 4.9$ and $\sigma_C = 1.6$.

Find the following (show your work):

| | |
|---|---|
| $\mu_{B+C}$ | $\mu_{B-C}$ |
| $\sigma_{B+C}$ | $\sigma_{B-C}$ |

2) At our town's ice cream shop, if you order a cone, you can determine the number of scoops you would like. Let X represent the number of scoops ordered and the probability that a random customer would ask for that number of scoops.

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P(X) | 0.20 | 0.38 | 0.28 | 0.11 | 0.03 |

It costs $2.50 for the cone and $0.75 for each scoop of ice cream. Find the amount of money the store expects to make on a randomly selected customer. **[6 pts]**

**Answer:**

3) Suppose the owners of the ice cream shop want to keep track of their profits. The cost of a cone is $0.10 for the shop and the cost of a scoop of ice cream is $0.45. What is the expected profit on a randomly selected customer? Use the information from problem #2 to help you answer this question. Show your work. **[6pts]**

**Answer:**

## Part III. The Binomial Distribution (10 x 3 = 30 pts)

Directions: For the situations below, define a random variable X for the situation and then decide if they follow a binomial distribution model by commenting on the four requirements.

1) You roll a DnD dice (20-sided), 20 times, and record the number that shows on the dice.

2) A basketball player can make 60% of their free throws. The coach plans on having a free throw shooting competition and the player will be shooting 100 shots.

3) From a standard deck of cards, you pull out a card, record the suit, put it back, and reshuffle. You continue until you get two spades in a row.

4) You are conducting a survey at your school to see how many students own smartphones. The probability that a student will own a smartphone is 0.85. You plan on having 200 students participate in your survey.

5) Zach is an 85% field goal kicker. Each goal he kicks is independent of the next kick. At practice today, he will be doing 30 kicks. For the following table, fill in the missing pieces.

| *Situation* | *Probability Notation* | *Formula* | *Value* |
|---|---|---|---|
| What is the probability Zach will make exactly 25 kicks? | | | |
| | P(X = 20) | | |
| What is the probability Zach will make no more than 20 kicks? | | | |
| | P(X ≤ 24) | | |
| | | $(30\ 28\ )0.85^{28}0.15^{2}$ | |
| | | P(X = 27) + P(X = 28) + P(X = 29) + P(X = 30) | |

## Part IV. The Geometric Distribution (8 x 3 = 24 pts)

1) In a recent Pew study, it was found that 28% of all homes have at least one dog. Let X = the number of houses you look at before finding a house with a dog. Find and interpret the following probabilities.

a) $P(X = 3)$

b) $P(X < 5)$

c) $P(X > 4)$

d) $E(X)$

2) Zach's teammate, Taylor, is the quarterback for the football team. She can complete 65% of her passes.

For the following table, fill in the missing pieces.

| ***Situation*** | ***Probability Notation*** | ***Formula*** | ***Value*** |
|---|---|---|---|
| What is the probability that it takes Taylor 3 incomplete passes before she has a completion? | | | |
| | P(X < 3) | | |
| | | $(0.35)^4(0.65)$ | |
| | | 1 – (P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)) | |

## Appendix: Example Question and Answer for R programming questions:

Calculate the sum $\sum_{j=0}^{n} r^j$, where $r$ has been assigned the value 1.08, and compare with $(1-r^{n+1})/(1-r)$, for $n = 10, 20, 30, 40$.

**Answer: Copy and paste your R code in the box below (not an image but the text).**

```
r <- 1.08
n <- c(10, 20, 30, 40)
sum1 <- c()
for(i in n){
  x <- 0:i
  sum1 <- c(sum1, sum(r^x))
}
sum1   # This gives the calculated sums for n = 10, 20, 30, 40.

sum2 <- (1 - r^(n + 1)) / (1 - r)
sum2

sum2 - sum1   # The formula works.
```

**Screenshot of your R console outputs and paste the image in the box below**

```
> r <- 1.08
> n <- c(10, 20, 30, 40)
> sum1 <- c()
> for(i in n){
+   x <- 0:i
+   sum1 <- c(sum1, sum(r^x))
+ }
> sum1   # This gives the calculated sums for n = 10, 20, 30, 40.
[1]  16.64549  50.42292 123.34587 280.78104
> sum2 <- (1 - r^(n + 1)) / (1 - r)
> sum2
[1]  16.64549  50.42292 123.34587 280.78104
> sum2 - sum1   # The formula works.
[1] 0 0 0 0
```

**THE END**