

HW3

- **Submission date: 02/19/2025 (11:59 PM)**
- Read your class notes *thoroughly* before attempting the assignment.
- You can submit *well commented* code in either `.py` or `.ipynb` formats.
- Submitting a PDF explaining your code is *highly* encouraged.

1. **Pandas practice.** Complete the assignment in the `.ipynb` file named, “CS677_HW3_Pandas_questions.”
2. **Seaborn.** Complete the assignment in the `.ipynb` file named, “HW3_Seaborn,” located in the zipped folder, “files_for_HW3_Seaborn.”

For the following questions, you may use calculators, but not programming (use NumPy/Python only for verification if you wish to. Do not submit that code).

3. **(Matrix multiplication).** Let

$$A = [6, -2, -2; 10, -3, 1; -10, 5, 1]_{3 \times 3}, \quad B = [9, 4, -4; 4, 7, 0; -4, 0, 11]_{3 \times 3}, \quad C = [3, 1; 0, -2; 4, 0]_{3 \times 2},$$

$$\mathbf{a} = [5; 1; 2]_{3 \times 1}, \mathbf{b} = [3, 0, 8]_{1 \times 3}.$$
 Here, rows of the matrix are separated by semicolons, so, for example, $[6, -2, -2]$ is the first row of A . Pay very close attention to the **matrix sizes**.
 Calculate the following or explain why they cannot be calculated.
 - a. $A\mathbf{a}, A\mathbf{b}, A\mathbf{b}^T, AB$.
 - b. $AB, BA, AA^T, A^T A$
 - c. Can you say something about the symmetry properties of matrices like $A^T A$ and AA^T ?
 - d. $\mathbf{ab}, \mathbf{ba}, (\mathbf{ab})A, \mathbf{a}(\mathbf{b}A)$
 - e. Compute $\|\mathbf{a} - \mathbf{b}^T\|_2^2$. How will you express this in terms of $\|\mathbf{a}\|_2^2, \|\mathbf{b}\|_2^2$, and $\langle \mathbf{a}, \mathbf{b} \rangle$?
4. **(Triangular matrices).** Suppose U_1, U_2 are two upper triangular matrices and L_1, L_2 are two lower triangular matrices. Using examples (3×3 matrices will suffice), explain which of the following are triangular: $U_1 + U_2, U_1 U_2, U_1 + L_1, L_2^2, L_1 L_2$.
5. Given a vector $\mathbf{u} = [u_1, u_2, \dots, u_n]$, what can you say about the norm of the vector

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{\|\mathbf{u}\|}?$$
 Explain how you arrived at your answer in detail.
6. What are antisymmetric matrices (note that they are also known as “skew-symmetric” matrices)? What can you say about the diagonal entries of an antisymmetric matrix?
7. **(Orthogonality).** Find the L^2 -norm of the following vectors $\mathbf{a} = [4, 2, -6]$, $\mathbf{b} = [16, -32, 0]$. Are \mathbf{a}, \mathbf{b} orthogonal vectors?
8. **(Unit Vectors).** What is a unit vector? Find a unit vector orthogonal to $\mathbf{c} = [4, -3]$.
9. **(Angle between vectors).** Let $\mathbf{a} = [1, 1, 1]$, $\mathbf{b} = [2, 3, 1]$, $\mathbf{c} = [-1, 1, 0]$. Find the cosine of the angle between the vectors $\mathbf{a} + \mathbf{b}$, \mathbf{c} .