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Assignment 12

METCS544A3A4_F2024

Instructions:

1. For answering programming questions, please use Adobe Acrobat to edit the pdf file in two steps **[See Appendix: Example Question and Answer]**:
    a. Copy and paste your R code as text in the box provided (so that your teaching team can run your code);
    b. Screenshot your R console outputs, save them as a .PNG image file, and paste/insert them in the box provided.
    c. Show all work—credit will not be given for code without showing it in action, including a screenshot of R console outputs.
2. To answer non-programming questions, please type or handwrite your final answers clearly in the boxes. Show all work - credit will not be given for numerical solutions that appear without explanation in the space above the boxes. **You're encouraged to use R to graph/plot the data and produce numerical summaries; please append your code and screenshot of the outputs at the end of your PDF submission.**

[Total 141 pts = 138 pts + 3 Extra Credit pts]

Grading Rubric

Each question is worth 3 points and will be graded as follows:
3 points: Correct answer with work shown
2 points: Incorrect answer but attempt shows some understanding (work shown)
1 point: Incorrect answer but an attempt was made (work shown), or **correct answer without explanation (work not shown)**
0 points: Left blank or made little to no effort/work not shown

Reflective Journal [3 pts]
(Copy and paste the link to your live Google doc in the box below)

https://drive.google.com/drive/folders/1_8qcBjQVMfZggF42UYJuHQzMoBcyAy0Q?usp=drive_link

Part I. Confidence Intervals for Proportions (18 pts)

A live action role-playing game (LARP) is a form of role-playing game where the participants physically portray their characters in a fictional setting, while interacting with each other in character. There are game rules to dictate the interactions between players and to determine the winner of the games.

A local gamemaster would like to determine how many people in the area would be interested in participating in his LARPing event. He sent out a survey to 1500 people in the area and asked if they would be interested in participating. 432 people said that they would be interested in participating in a LARPing event.

(a) State the parameter our confidence interval will estimate.

Answer:

The parameter is the true proportion of people in the area who would be interested in participating in a LARPing event

(b) Identify each of the conditions that must be met to use this procedure, and explain how you know that each one has been satisfied.

Answer:

Random Sampling: The survey responses must come from a random sample of individuals in the area.
Independence: Each response must be independent of others. Since the sample size of 1500 is much smaller than the population of the area, the 10% condition is met (
Normality (Large Sample Size): The sample size must be large enough.

(c) Find the appropriate critical value and the standard error of the sample proportion.

Answer:

0.01169

(d) Give the 95% confidence interval.

Answer:

(0.265,0.311)

(e) Interpret the confidence interval constructed in part (d) in the context of the problem.

Answer:

We are 95% confident that the true proportion of people in the area who are interested in participating in a LARPing event is between 26.5% and 31.1%.

(f) This poll was conducted through email. Explain how undercoverage could lead to a biased estimate in this case, and speculate about the direction of the bias.

Answer:

> This could bias the estimate because people who are more engaged with technology (and likely more familiar with or interested in LARPing) might respond at higher rates.

Part II. Significance Test for Proportions (21 pts)

1) **(9 pts)** Apple runs a quality control check on its products that come off the assembly line. Every 500th product is inspected for defects to make sure that the machines are calibrated correctly and working like they should. Due to machine deviations, they do expect 5% of products to be defective in a single day. The quality control specialist has to determine if a significant number of products is coming off the line defective.

(a) Define the parameter of interest and write the appropriate null and alternative hypotheses for the test that is described.

Answer:

Parameter of interest (p): The true proportion of defective products produced by the assembly line in a single day.
Null hypothesis (alpha =0.05(The proportion of defective products is equal to 5% as expected.)

The quality control specialist takes an SRS of products and finds that the sample proportion of defective products is 0.058, which produces a P-value of 0.027.

(b) Interpret the P-value in the context of the problem.

Answer:

The P-value of 0.027 indicates the probability of observing a sample proportion of 0.058 (or more extreme) if the true proportion of defective products is 0.05. Since the P-value is small, there is evidence to suggest that the true proportion of defective products may be higher than expected.

(c) What conclusion would you draw at the $\alpha = 0.05$ level? At the $\alpha = 0.01$ level?

Answer:

At alpha=-.5, the P-value of 0.027 is less than 0.05. Thus, we reject the null hypothesis
At alpha =0.1, The P-value of 0.027 is greater than 0.01. Thus, we fail to reject the null hypothesis

2) **(12 pts)** According to the CD website, 88% of US adults aged 65 – 74 are fully vaccinated against COVID-19 (as of 11/2021). A researcher suspects that this proportion is lower in their community. She selects an SRS of 125 adults aged 65 – 74 in her community and asks if they are fully vaccinated. 100 of them say that they are fully vaccinated. Does the researcher have convincing evidence that the proportion is lower in her community? Perform a significance test using a 5% level of significance. **Explain using the 4 step process**.

Answer:

Step 1.

Step 1: State the hypotheses and parameter of interest
Parameter of interest: The true proportion of adults aged 65–74 in the researcher's community who are fully vaccinated against COVID-19 (p).
Null hypothesis =0.88
$p=0.88$ (The vaccination rate in the community is the same as the national rate.)
Alternative hypothesis <
0.88
$p<0.88$ (The vaccination rate in the community is lower than the national rate.)
This is a one-tailed test.

Step 2.

To perform a one-sample z-test for proportions, the following conditions must be met:
Random Sampling: The problem states that the researcher selected a simple random sample (SRS).
Independence: The sample size ($n=125$) is less than 10% of the population of adults aged 65–74 in the community.
Normality is met

Step 3.

P-value: Using a z-table or calculator, the P-value for 2.75
$z=-2.75$ is approximately:0.003
$P\approx 0.003$

Step 4.

Significance level (
Decision: Since the P-value (0.003 is less than 0.05
$\alpha=0.05$, we reject the null hypothes
Conclusion: There is convincing evidence that the proportion of adults aged 65–74 in the researcher's community who are fully vaccinated is lower than the national rate of 88%.

Part III. Errors and Power (12 pts)

Your local Starbucks is hoping to cut back on the wait time for their customers. The proportion of drive-through customers who wait longer than 3 minutes to get their coffee is $p = 0.76$. In an effort to reduce this, the manager assigns an extra employee to help make the orders in the morning. During the next month, the manager will select an SRS of customer wait times and determine if the proportion who are waiting longer than 3 minutes has decreased.

a) What is the parameter of interest in this problem?

Answer:

The parameter of interest in this problem is the true proportion of drive-through customers who wait longer than 3 minutes for their coffee after the manager assigns an extra employee to help make the orders. Denote this parameter as p.

b) State the null and alternative hypotheses.

Answer:

Null Hypothesis: $p=0.76$ (The proportion of customers who wait longer than 3 minutes has not decreased.)
Alternative hypothesis :$p<0.76$(The proportion of customers who wait longer than 3 minutes has decreased.)

c) Describe a Type I and Type II error in this setting and explain the consequences of each.

Answer:

Type I Error:

Definition: Rejecting the null hypothesis when it is actually true.
In this context: Concluding that the extra employee reduced the proportion of customers waiting longer than 3 minutes when, in reality, the wait time has not decreased.
Consequences: The manager might keep assigning an extra employee unnecessarily, incurring additional labor costs without any actual improvement in service times.
Type II Error:

Definition: Failing to reject the null hypothesis when the alternative hypothesis is true.
In this context: Concluding that the extra employee did not reduce the proportion of customers waiting longer than 3 minutes when, in reality, the wait time has decreased.
Consequences: The manager might decide to stop assigning an extra employee, missing an opportunity to improve customer satisfaction and reduce wait times.

Part IV. Confidence Intervals and Significance Tests (15 pts)

Netflix relies heavily on viewers when it comes to deciding if they are going to make another season of their shows. A recent article stated that Netflix has said they will only continue to make another season of a show if it has been watched by over 20% of its viewers in the first month of the show coming out. Recently, Marvel's Daredevil was canceled. Let p = the true proportion of viewers who watch the show.

(a) State the null and alternative hypotheses for this test.

Answer:

Null hypothesis: $p \leq 0.20$
(The true proportion of viewers who watch the show is 20% or less.)

Alternative hypothesis: $p > 0.20$
(The true proportion of viewers who watch the show is greater than 20%.)

(b) **(6 pts)** Describe a Type I and Type II error in this context, and explain the consequences of each type of error.

Answer:

Type I Error:

Definition: Rejecting the null hypothesis when it is actually true.
In this context: Concluding that more than 20% of viewers watched the show in the first month when, in reality, 20% or fewer did.
Consequences: Netflix might continue producing additional seasons of Daredevil unnecessarily, incurring significant costs for a show with insufficient viewer interest.
Type II Error:

Definition: Failing to reject the null hypothesis when the alternative hypothesis is true.
In this context: Concluding that 20% or fewer viewers watched the show in the first month when, in reality, more than 20% did.
Consequences: Netflix might cancel a show like Daredevil that actually has strong enough viewer interest to justify another season, potentially missing revenue and viewer satisfaction opportunities.

(c) The network determines that for a sample size of 2000, the power of this test at a 5% significance level for H_0: $p = 0.20$ is only 0.39. Explain what the power of the test measures in the context of the problem.

Answer:

(d) Based on your answers to (b) and (c), would α = 0.10 or α = 0.01 be a better significance level for this test? Explain your choice.

Answer:

The power of a test is the probability of correctly rejecting the null hypothesis when the alternative hypothesis is true.
In this context: A power of 0.39 means that if the true proportion of viewers who watch Daredevil in the first month is greater than 20% , the test has only a 39% chance of detecting this and rejecting the null hypothesis.
This low power indicates a high likelihood of a Type II error, suggesting that the test may not reliably detect when the show's viewership exceeds the 20% threshold. To increase power, Netflix could consider increasing the sample size or choosing a higher significance level, depending on the context.

Part V. Inference for a Difference in Proportions (18 pts)

Use the following situation to answer the questions that follow.

A driving school owner believes that Instructor A is more effective than Instructor B at preparing students to pass the state's driver's license exam. An incoming class of 100 students in randomly assigned to two groups, each of size 50. One group is taught by Instructor A; the other is taught by Instructor B. At the end of the course, 30 of Instructor A's students and 22 of Instructor B's students pass the state exam. Let p_A be the proportion of Instructor A's students who pass the exam, and p_B be the proportion of Instructor B's students who pass the exam.

C

1) Which of the following statements is FALSE?

(A) We use the pooled sample proportion because we are assuming the null hypothesis is true.
(B) We do not have to check that the sample size is less than 10% of the population size because we are performing an experiment.
(C) Because we are not randomly selecting from the population, we should use the pooled population proportion.
(D) The null hypothesis for this test would be H_0: $p_A - p_B = 0$
(E) The alternative hypothesis for this test would be H_a: $p_A - p_B > 0$

E

2) What is the z test statistic?

(A) $\dfrac{0.6-0.44}{\sqrt{\frac{.52(.48)}{100}+\frac{.52(.48)}{100}}}$

(B) $\dfrac{0.6-0.44}{\sqrt{\frac{.6(.4)}{100}+\frac{.44(.56)}{100}}}$

(C) $\dfrac{0.6-0.44}{\sqrt{\frac{.6(.4)}{50}+\frac{.44(.56)}{50}}}$

(D) $\dfrac{0.6-0.44}{\sqrt{\frac{(.52)(.48)}{100}}}$

(E) $\dfrac{0.6-0.44}{\sqrt{(.52)(.48)\left(\frac{1}{50}+\frac{1}{50}\right)}}$

B

3) What is the p-value?

(A) 1.6012
(B) 0.0547
(C) 0.6
(D) 0.1094
(E) 0.52

A

________4) Do these results give convincing evidence at the 5% significance level that Instructor A is more effective?

(A) Because our p-value is lower than 5%, we reject H_0. We have convincing evidence that Instructor A is more effective.
(B) Because our p-value is higher than 5%, we fail to reject H_0. We do not have convincing evidence that Instructor A is more effective.
(C) Because our p-value is lower than 5%, we reject H_0. We have convincing evidence that Instructor B's students do not pass the state exam.
(D) Because our p-value is higher than 5%, we fail to reject H_0. We do not have convincing evidence that there is a difference between Instructor A and Instructor B.
(E) Because our p-value is lower than 5%, we reject H_0. We have convincing evidence that there is a difference between Instructor A and Instructor B.

A

________ 5) Which of the following errors could you have made from your conclusion above?

(A) Type I; saying that Instructor A was more effective when they were really not.
(B) Type II; saying that Instructor A was more effective when they were really not.
(C) Type I; saying that we do not have convincing evidence that Instructor A was more effective when they really were.
(D) Type II; saying that we do not have convincing evidence that Instructor A was more effective when they really were.
(E) None of the above errors are correct.

6) Create a 95% confidence interval for the difference in the proportion of students who pass the exam between Instructor A and Instructor B. Do we come to the same conclusion as we did in (4)? Why or why not?

Answer:

The 95% confidence interval for the difference in proportions is $(-0.033, 0.353)$.

The confidence interval contains 0, meaning there is no statistically significant difference between Instructor A and Instructor B at the 95% confidence level.

This contradicts the conclusion in (4), where we rejected the null hypothesis because the p-value was less than 0.05.

Part VI. Confidence Interval for Means (12 pts)

A nutritionist wants to estimate the average amount of sugar in a certain brand of yogurt. She takes an SRS of 20 containers and measures the sugar content, in grams. She creates a 90% confidence interval for the true amount of sugar in this brand of yogurt: (11.42, 12.58).

(a) What is the point estimate from this sample?

Answer:

Point Estimate
(11.42+12.58)/2 =12.00
The point estimate is 12.00 grams.

(b) What is the margin of error?

Answer:

ME=(12.58−12.00)=0.58
The margin of error is 0.58 gram

(c) Interpret the 90% confidence interval in the context of the problem.

Answer:

We are 90% confident that the true average sugar content in this brand of yogurt is between 11.42 grams and 12.58 grams.

(d) Interpret the confidence level of 90% in the context of the problem.

Answer:

If we were to take many random samples of 20 containers and compute a confidence interval for each sample, approximately 90% of those confidence intervals would capture the true average sugar content for this brand of yogurt.

Part VII. Significant Test for Means (12 pts)

Functional Fitness claims that its new exercise program helps their members lose 5 pounds in the first month. To test this claim, you select a random sample of 13 members who completed the program and record their weight loss (in pounds) as follows:

5.5	5.2	6	5.5	6.8	4	4.2	5.5	6.2	5.1	4.8	6.6	3.9

Is there evidence, at the 10% significance level, that this new exercise program helps members lose more than 5 pounds in their first month?

Answer:

Null Hypothesis (H0): The mean weight loss of the members is 5 pounds (mu = 5).
Alternative Hypothesis (Ha): The mean weight loss of the members is greater than 5 pounds (mu > 5).

From the sample data (n = 13), we calculated the sample mean (x = 5.33) and the sample standard deviation (s = 0.936). Using these values, we computed the test statistic using the formula:

t = (x - mu0) / (s / sqrt(n))

Substituting the values:
t = (5.33 - 5) / (0.936 / sqrt(13)) = 1.27

The degrees of freedom (df) for this test is n - 1 = 13 - 1 = 12. The p-value was calculated using a one-tailed t-test, resulting in p = 0.113.

At the 0.10 significance level, the p-value (p = 0.113) is greater than alpha = 0.10, so we fail to reject the null hypothesis (H0).

Conclusion: There is not enough evidence at the 10% significance level to conclude that the new exercise program helps members lose more than 5 pounds in their first month.

Part VIII. Margin of Error and Matched Pairs (15 pts)

1) Your local gym wants to estimate the mean amount of time (in hours) that their members spend each week at their center. They would like a 95% confidence interval with a margin of error of 30 minutes. Based on previous data, they know that the population standard deviation is 2.2 hours. What is the smallest sample size that will allow this confidence interval to be built?

Answer:

To determine the smallest sample size required for a local gym to achieve a margin of error of 30 minutes (0.5 hours) with a 95% confidence interval, we use the formula for the margin of error (E) in estimating a population mean: E = z * (σ / sqrt(n)). Here, E is 0.5 hours, the z-value for a 95% confidence level is approximately 1.96, and the population standard deviation (σ) is 2.2 hours. Rearranging the formula to solve for n gives $n = (z * σ / E)^2$. Substituting the known values results in $n = (1.96 * 2.2 / 0.5)^2$, which calculates to $n = (4.312 / 0.5)^2 = (8.624)^2 \approx 74.34$. Rounding up to the next whole number, the smallest sample size needed is 75.

2) The number of hours college students spend studying per week follows a normal distribution. If you go three standard deviations out on either side of the mean, you get an interval of values from 10 hours to 34 hours. We want to estimate the mean study hours of a group of college students with 90% confidence and a margin of error of 2 hours. What is the sample size needed to get this confidence interval?

Answer:

we want to estimate the mean study hours of college students, where the distribution ranges from 10 hours to 34 hours. The mean (μ) is calculated as (10 + 34) / 2 = 22 hours, and since the interval from 10 to 34 hours represents 6 standard deviations, the standard deviation (σ) is (34 - 10) / 6 = 4 hours. For a 90% confidence level with a margin of error of 2 hours, we again use the margin of error formula: E = z * (σ / sqrt(n)). The z-value for a 90% confidence level is approximately 1.645. Rearranging for n gives $n = (z * σ / E)^2$. Substituting the values results in $n = (1.645 * 4 / 2)^2$, which calculates to $n = (6.58 / 2)^2 = (3.29)^2 \approx 10.82$. Rounding up to the next whole number, the sample size needed is 11.

Use the following scenario for questions 3 – 5 ***[Mark your choice in the boxes provided]***.

Does listening to positive affirmations tapes (for example, "I am in charge of my life") while sleeping decrease depression? Dr. Goodyear wants to test this theory! Dr. Goodyear randomly selected 7 of his patients with mild depression. He gives them a "test" to score their depression on a 100 point scale, where the higher the score, the more clinically depressed you are. He then assigns the 7 patients to listen to an affirmation tape for one month while sleeping. After one month, the patients are given the test again.

	Test Scores						
Person	A	B	C	D	E	F	G
Before	54	57	68	43	42	49	66
After	52	50	67	44	30	38	61

B

3) Which of the following conditions must be met in order to use a *t*-procedure on these paired data?

(A) The distribution of both before times and after times must be approximately Normal.
(B) The distribution of before times and the distribution of differences (before – after) must be approximately Normal.
(C) Only the distribution of before times must be approximately Normal.
(D) Only the distribution of differences (before – after) must be approximately Normal.
(E) All three distributions—before, after, and the difference—must be approximately Normal.

C

4) If μ_d represents the mean different in before – after, which of the following would be the appropriate hypothesis to test if positive affirmations make a student complete a puzzle faster?

(A) H_0: $\mu_d > 0$ and H_a: $\mu_d = 0$
(B) H_0: $\mu_d \neq 0$ and H_a: $\mu_d = 0$
(C) H_0: $\mu_d = 0$ and H_a: $\mu_d < 0$
(D) H_0: $\mu_d = 0$ and H_a: $\mu_d \neq 0$
(E) H_0: $\mu_d = 0$ and H_a: $\mu_d > 0$

B

5) You run a one sample t test on the mean difference in before time – after time and obtain a p-value of 0.0155. If you significance level is 0.05, which of the following is correct?

I. We are 95% confident that we can reject the null hypothesis.
II. We have convincing evidence there is a positive difference in test scores.
III. The probability of making a Type I error is 0.0155

(A) I and III only (B) I and II only (C) II and III only (D) II only (E) I, II, and III

Part IX. Difference in Means (15 pts)

Do high school girls and boys differ in how many hours they get per night? At a large high school, 35 girls were randomly selected and they got an average of 6.7 hours of sleep the night before, with a standard deviation of 0.65 hours. 40 boys were randomly selected and they got an average of 6.4 hours of sleep the night before, with a standard deviation of 0.45 hours. Construct a 90% confidence interval and conduct a two-sided significance test at the 10% significance level. Do these two results support each other? Explain.

Answer:

o compare the hours of sleep between high school girls and boys, we first constructed a 90% confidence interval for the difference in means and then conducted a two-sided significance test. The mean hours of sleep for girls is 6.7 hours with a standard deviation of 0.65 hours from a sample of 35, while boys averaged 6.4 hours with a standard deviation of 0.45 hours from a sample of 40. The formula for the confidence interval for the difference between two means is (x_G - x_B) ± z* × √(s_G²/n_G + s_B²/n_B). For a 90% confidence level, the z-value is approximately 1.645. The difference in means is 0.3 hours. Calculating the standard error gives us approximately 0.1307. The margin of error is about 0.215, leading to a confidence interval of (0.085, 0.515). For the significance test, the null hypothesis is H0: μ_G - μ_B = 0 and the alternative hypothesis is Ha: μ_G - μ_B ≠ 0. The test statistic is calculated as t = (x_G - x_B) / SE, which gives approximately 2.295. The degrees of freedom are about 66. Since the calculated t value of 2.295 is greater than the critical value of approximately 1.645, we reject the null hypothesis. The confidence interval of (0.085, 0.515) indicates that we are 90% confident that the true difference in average sleep hours between girls and boys lies between 0.085 hours and 0.515 hours, suggesting that girls tend to sleep more than boys. Both analyses support the conclusion that there is a significant difference in sleep hours between high school girls and boys, with girls getting more sleep on average.

Appendix: Example Question and Answer for R programming questions:

Calculate the sum $\sum_{j=0}^{n} r^j$, where r has been assigned the value 1.08, and compare with $(1-r^{n+1})/(1-r)$, for $n = 10, 20, 30, 40$.

Answer: Copy and paste your R code in the box below (not an image but the text).

```
r <- 1.08
n <- c(10, 20, 30, 40)
sum1 <- c()
for(i in n){
  x <- 0:i
  sum1 <- c(sum1, sum(r^x))
}
sum1   # This gives the calculated sums for n = 10, 20, 30, 40.

sum2 <- (1 - r^(n + 1)) / (1 - r)
sum2

sum2 - sum1   # The formula works.
```

Screenshot of your R console outputs and paste the image in the box below

```
> r <- 1.08
> n <- c(10, 20, 30, 40)
> sum1 <- c()
> for(i in n){
+   x <- 0:i
+   sum1 <- c(sum1, sum(r^x))
+ }
> sum1   # This gives the calculated sums for n = 10, 20, 30, 40.
[1]  16.64549  50.42292 123.34587 280.78104
> sum2 <- (1 - r^(n + 1)) / (1 - r)
> sum2
[1]  16.64549  50.42292 123.34587 280.78104
> sum2 - sum1   # The formula works.
[1] 0 0 0 0
```

THE END