

Name (Last, First):

Lopez Sepulveda,
Kevin

Student ID:

U59702827

Assignment 12

METCS544A3A4_F2024

Instructions:

1. For answering programming questions, please use Adobe Acrobat to edit the pdf file in two steps **[See Appendix: Example Question and Answer]**:
 - a. Copy and paste your R code as text in the box provided (so that your teaching team can run your code);
 - b. Screenshot your R console outputs, save them as a .PNG image file, and paste/insert them in the box provided.
 - c. Show all work—credit will not be given for code without showing it in action, including a screenshot of R console outputs.
2. To answer non-programming questions, please type or handwrite your final answers clearly in the boxes. Show all work - credit will not be given for numerical solutions that appear without explanation in the space above the boxes. **You're encouraged to use R to graph/plot the data and produce numerical summaries; please append your code and screenshot of the outputs at the end of your PDF submission.**

[Total 141 pts = 138 pts + 3 Extra Credit pts]

Grading Rubric

Each question is worth 3 points and will be graded as follows:

3 points: Correct answer with work shown

2 points: Incorrect answer but attempt shows some understanding (work shown)

1 point: Incorrect answer but an attempt was made (work shown), or **correct answer without explanation (work not shown)**

0 points: Left blank or made little to no effort/work not shown

Reflective Journal [3 pts]

(Copy and paste the link to your live Google doc in the box below)

https://drive.google.com/drive/folders/1_8qcBjQVMfZggF42UYJuHQzMoBcyAy0Q?usp=drive_link

Part I. Confidence Intervals for Proportions (18 pts)

A live action role-playing game (LARP) is a form of role-playing game where the participants physically portray their characters in a fictional setting, while interacting with each other in character. There are game rules to dictate the interactions between players and to determine the winner of the games.

A local gamemaster would like to determine how many people in the area would be interested in participating in his LARPing event. He sent out a survey to 1500 people in the area and asked if they would be interested in participating. 432 people said that they would be interested in participating in a LARPing event.

(a) State the parameter our confidence interval will estimate.

Answer:

The parameter is the true proportion of people in the area who would be interested in participating in a LARPing event

(b) Identify each of the conditions that must be met to use this procedure, and explain how you know that each one has been satisfied.

Answer:

Random Sampling: The survey responses must come from a random sample of individuals in the area.

Independence: Each response must be independent of others. Since the sample size of 1500 is much smaller than the population of the area, the 10% condition is met (Normality (Large Sample Size): The sample size must be large enough.

(c) Find the appropriate critical value and the standard error of the sample proportion.

Answer:

0.01169

(d) Give the 95% confidence interval.

Answer:

(0.265, 0.311)

(e) Interpret the confidence interval constructed in part (d) in the context of the problem.

Answer:

We are 95% confident that the true proportion of people in the area who are interested in participating in a LARPing event is between 26.5% and 31.1%.

(f) This poll was conducted through email. Explain how undercoverage could lead to a biased estimate in this case, and speculate about the direction of the bias.

Answer:

This could bias the estimate because people who are more engaged with technology (and likely more familiar with or interested in LARPing) might respond at higher rates.

Part II. Significance Test for Proportions (21 pts)

1) (9 pts) Apple runs a quality control check on its products that come off the assembly line. Every 500th product is inspected for defects to make sure that the machines are calibrated correctly and working like they should. Due to machine deviations, they do expect 5% of products to be defective in a single day. The quality control specialist has to determine if a significant number of products is coming off the line defective.

(a) Define the parameter of interest and write the appropriate null and alternative hypotheses for the test that is described.

Answer:

Parameter of interest (p): The true proportion of defective products produced by the assembly line in a single day.
Null hypothesis ($\alpha = 0.05$) (The proportion of defective products is equal to 5% as expected.)

The quality control specialist takes an SRS of products and finds that the sample proportion of defective products is 0.058, which produces a P-value of 0.027.

(b) Interpret the P-value in the context of the problem.

Answer:

The P-value of 0.027 indicates the probability of observing a sample proportion of 0.058 (or more extreme) if the true proportion of defective products is 0.05. Since the P-value is small, there is evidence to suggest that the true proportion of defective products may be higher than expected.

(c) What conclusion would you draw at the $\alpha = 0.05$ level? At the $\alpha = 0.01$ level?

Answer:

At $\alpha = 0.05$, the P-value of 0.027 is less than 0.05. Thus, we reject the null hypothesis
At $\alpha = 0.01$, The P-value of 0.027 is greater than 0.01. Thus, we fail to reject the null hypothesis

2) (12 pts) According to the CDC website, 88% of US adults aged 65 – 74 are fully vaccinated against COVID-19 (as of 11/2021). A researcher suspects that this proportion is lower in their community. She selects an SRS of 125 adults aged 65 – 74 in her community and asks if they are fully vaccinated. 100 of them say that they are fully vaccinated. Does the researcher have convincing evidence that the proportion is lower in her community? Perform a significance test using a 5% level of significance. **Explain using the 4 step process.**

Answer:

Step 1.

Step 1: State the hypotheses and parameter of interest
Parameter of interest: The true proportion of adults aged 65–74 in the researcher's community who are fully vaccinated against COVID-19 (p).
Null hypothesis $= 0.88$
 $p = 0.88$ (The vaccination rate in the community is the same as the national rate.)
Alternative hypothesis $<$
 0.88
 $p < 0.88$ (The vaccination rate in the community is lower than the national rate.)
This is a one-tailed test.

Step 2.

To perform a one-sample z-test for proportions, the following conditions must be met:
Random Sampling: The problem states that the researcher selected a simple random sample (SRS).
Independence: The sample size ($n=125$) is less than 10% of the population of adults aged 65–74 in the community.
Normality is met

Step 3.

P-value: Using a z-table or calculator, the P-value for 2.75
 $z = -2.75$ is approximately: 0.003
 $P \approx 0.003$

Step 4.

Significance level (α) = 0.05
Decision: Since the P-value (0.003) is less than 0.05
 $\alpha = 0.05$, we reject the null hypothesis
Conclusion: There is convincing evidence that the proportion of adults aged 65–74 in the researcher's community who are fully vaccinated is lower than the national rate of 88%.

Part III. Errors and Power (12 pts)

Your local Starbucks is hoping to cut back on the wait time for their customers. The proportion of drive-through customers who wait longer than 3 minutes to get their coffee is $p = 0.76$. In an effort to reduce this, the manager assigns an extra employee to help make the orders in the morning. During the next month, the manager will select an SRS of customer wait times and determine if the proportion who are waiting longer than 3 minutes has decreased.

a) What is the parameter of interest in this problem?

Answer:

b) State the null and alternative hypotheses.

Answer:

c) Describe a Type I and Type II error in this setting and explain the consequences of each.

Answer:

Part IV. Confidence Intervals and Significance Tests (15 pts)

Netflix relies heavily on viewers when it comes to deciding if they are going to make another season of their shows. A recent article stated that Netflix has said they will only continue to make another season of a show if it has been watched by over 20% of its viewers in the first month of the show coming out. Recently, Marvel's Daredevil was canceled. Let p = the true proportion of viewers who watch the show.

(a) State the null and alternative hypotheses for this test.

Answer:

(b) (6 pts) Describe a Type I and Type II error in this context, and explain the consequences of each type of error.

Answer:

(c) The network determines that for a sample size of 2000, the power of this test at a 5% significance level for $H_0: p = 0.20$ is only 0.39. Explain what the power of the test measures in the context of the problem.

Answer:

(d) Based on your answers to (b) and (c), would $\alpha = 0.10$ or $\alpha = 0.01$ be a better significance level for this test? Explain your choice.

Answer:

Part V. Inference for a Difference in Proportions (18 pts)

Use the following situation to answer the questions that follow.

A driving school owner believes that Instructor A is more effective than Instructor B at preparing students to pass the state's driver's license exam. An incoming class of 100 students is randomly assigned to two groups, each of size 50. One group is taught by Instructor A; the other is taught by Instructor B. At the end of the course, 30 of Instructor A's students and 22 of Instructor B's students pass the state exam. Let p_A be the proportion of Instructor A's students who pass the exam, and p_B be the proportion of Instructor B's students who pass the exam.

1) Which of the following statements is FALSE?

- (A) We use the pooled sample proportion because we are assuming the null hypothesis is true.
- (B) We do not have to check that the sample size is less than 10% of the population size because we are performing an experiment.
- (C) Because we are not randomly selecting from the population, we should use the pooled population proportion.
- (D) The null hypothesis for this test would be $H_0: p_A - p_B = 0$
- (E) The alternative hypothesis for this test would be $H_a: p_A - p_B > 0$

2) What is the z test statistic?

(A)
$$\frac{0.6 - 0.44}{\sqrt{\frac{.52(.48)}{100} + \frac{.52(.48)}{100}}}$$

(B)
$$\frac{0.6 - 0.44}{\sqrt{\frac{.6(.4)}{100} + \frac{.44(.56)}{100}}}$$

(C)
$$\frac{0.6 - 0.44}{\sqrt{\frac{.6(.4)}{50} + \frac{.44(.56)}{50}}}$$

(D)
$$\frac{0.6 - 0.44}{\sqrt{\frac{(.52)(.48)}{100}}}$$

(E)
$$\frac{0.6 - 0.44}{\sqrt{(.52)(.48)\left(\frac{1}{50} + \frac{1}{50}\right)}}$$

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3) What is the p-value?

- (A) 1.6012
- (B) 0.0547
- (C) 0.6
- (D) 0.1094
- (E) 0.52

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4) Do these results give convincing evidence at the 5% significance level that Instructor A is more effective?

- (A) Because our p-value is lower than 5%, we reject H_0 . We have convincing evidence that Instructor A is more effective.
- (B) Because our p-value is higher than 5%, we fail to reject H_0 . We do not have convincing evidence that Instructor A is more effective.
- (C) Because our p-value is lower than 5%, we reject H_0 . We have convincing evidence that Instructor B's students do not pass the state exam.
- (D) Because our p-value is higher than 5%, we fail to reject H_0 . We do not have convincing evidence that there is a difference between Instructor A and Instructor B.
- (E) Because our p-value is lower than 5%, we reject H_0 . We have convincing evidence that there is a difference between Instructor A and Instructor B.

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5) Which of the following errors could you have made from your conclusion above?

- (A) Type I; saying that Instructor A was more effective when they were really not.
- (B) Type II; saying that Instructor A was more effective when they were really not.
- (C) Type I; saying that we do not have convincing evidence that Instructor A was more effective when they really were.
- (D) Type II; saying that we do not have convincing evidence that Instructor A was more effective when they really were.
- (E) None of the above errors are correct.

6) Create a 95% confidence interval for the difference in the proportion of students who pass the exam between Instructor A and Instructor B. Do we come to the same conclusion as we did in (4)? Why or why not?

Answer:

Part VI. Confidence Interval for Means (12 pts)

A nutritionist wants to estimate the average amount of sugar in a certain brand of yogurt. She takes an SRS of 20 containers and measures the sugar content, in grams. She creates a 90% confidence interval for the true amount of sugar in this brand of yogurt: (11.42, 12.58).

(a) What is the point estimate from this sample?

Answer:

(b) What is the margin of error?

Answer:

(c) Interpret the 90% confidence interval in the context of the problem.

Answer:

(d) Interpret the confidence level of 90% in the context of the problem.

Answer:

Part VII. Significant Test for Means (12 pts)

Functional Fitness claims that its new exercise program helps their members lose 5 pounds in the first month. To test this claim, you select a random sample of 13 members who completed the program and record their weight loss (in pounds) as follows:

| | | | | | | | | | | | | |
|-----|-----|---|-----|-----|---|-----|-----|-----|-----|-----|-----|-----|
| 5.5 | 5.2 | 6 | 5.5 | 6.8 | 4 | 4.2 | 5.5 | 6.2 | 5.1 | 4.8 | 6.6 | 3.9 |
|-----|-----|---|-----|-----|---|-----|-----|-----|-----|-----|-----|-----|

Is there evidence, at the 10% significance level, that this new exercise program helps members lose more than 5 pounds in their first month?

Answer:

Part VIII. Margin of Error and Matched Pairs (15 pts)

1) Your local gym wants to estimate the mean amount of time (in hours) that their members spend each week at their center. They would like a 95% confidence interval with a margin of error of 30 minutes. Based on previous data, they know that the population standard deviation is 2.2 hours. What is the smallest sample size that will allow this confidence interval to be built?

Answer:

2) The number of hours college students spend studying per week follows a normal distribution. If you go three standard deviations out on either side of the mean, you get an interval of values from 10 hours to 34 hours. We want to estimate the mean study hours of a group of college students with 90% confidence and a margin of error of 2 hours. What is the sample size needed to get this confidence interval?

Answer:

Use the following scenario for questions 3 – 5 [Mark your choice in the boxes provided].

Does listening to positive affirmations tapes (for example, "I am in charge of my life") while sleeping decrease depression? Dr. Goodyear wants to test this theory! Dr. Goodyear randomly selected 7 of his patients with mild depression. He gives them a "test" to score their depression on a 100 point scale, where the higher the score, the more clinically depressed you are. He then assigns the 7 patients to listen to an affirmation tape for one month while sleeping. After one month, the patients are given the test again.

| | Test Scores | | | | | | |
|--------|-------------|----|----|----|----|----|----|
| Person | A | B | C | D | E | F | G |
| Before | 54 | 57 | 68 | 43 | 42 | 49 | 66 |
| After | 52 | 50 | 67 | 44 | 30 | 38 | 61 |

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3) Which of the following conditions must be met in order to use a t -procedure on these paired data?

- (A) The distribution of both before times and after times must be approximately Normal.
- (B) The distribution of before times and the distribution of differences (before – after) must be approximately Normal.
- (C) Only the distribution of before times must be approximately Normal.
- (D) Only the distribution of differences (before – after) must be approximately Normal.
- (E) All three distributions—before, after, and the difference—must be approximately Normal.

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4) If μ_d represents the mean different in before – after, which of the following would be the appropriate hypothesis to test if positive affirmations make a student complete a puzzle faster?

- (A) $H_0: \mu_d > 0$ and $H_a: \mu_d = 0$
- (B) $H_0: \mu_d \neq 0$ and $H_a: \mu_d = 0$
- (C) $H_0: \mu_d = 0$ and $H_a: \mu_d < 0$
- (D) $H_0: \mu_d = 0$ and $H_a: \mu_d \neq 0$
- (E) $H_0: \mu_d = 0$ and $H_a: \mu_d > 0$

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5) You run a one sample t test on the mean difference in before time – after time and obtain a p -value of 0.0155. If you significance level is 0.05, which of the following is correct?

- I. We are 95% confident that we can reject the null hypothesis.
- II. We have convincing evidence there is a positive difference in test scores.
- III. The probability of making a Type I error is 0.0155

- (A) I and III only
- (B) I and II only
- (C) II and III only
- (D) II only
- (E) I, II, and III

Part IX. Difference in Means (15 pts)

Do high school girls and boys differ in how many hours they get per night? At a large high school, 35 girls were randomly selected and they got an average of 6.7 hours of sleep the night before, with a standard deviation of 0.65 hours. 40 boys were randomly selected and they got an average of 6.4 hours of sleep the night before, with a standard deviation of 0.45 hours. Construct a 90% confidence interval and conduct a two-sided significance test at the 10% significance level. Do these two results support each other? Explain.

Answer:

Appendix: Example Question and Answer for R programming questions:

Calculate the sum $\sum_{j=0}^n r^j$, where r has been assigned the value 1.08, and compare with $(1 - r^{n+1})/(1 - r)$, for $n = 10, 20, 30, 40$.

Answer: Copy and paste your R code in the box below (not an image but the text).

```
r <- 1.08
n <- c(10, 20, 30, 40)
sum1 <- c()
for(i in n){
  x <- 0:i
  sum1 <- c(sum1, sum(r^x))
}
sum1      # This gives the calculated sums for n = 10, 20, 30, 40.

sum2 <- (1 - r^(n + 1)) / (1 - r)
sum2

sum2 - sum1      # The formula works.
```

Screenshot of your R console outputs and paste the image in the box below

```
> r <- 1.08
> n <- c(10, 20, 30, 40)
> sum1 <- c()
> for(i in n){
+   x <- 0:i
+   sum1 <- c(sum1, sum(r^x))
+ }
> sum1      # This gives the calculated sums for n = 10, 20, 30, 40.
[1] 16.64549 50.42292 123.34587 280.78104
> sum2 <- (1 - r^(n + 1)) / (1 - r)
> sum2
[1] 16.64549 50.42292 123.34587 280.78104
> sum2 - sum1      # The formula works.
[1] 0 0 0 0
```

THE END