CIS 477 (Fall 2022) Disclosure Sheet

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HW # 7

Yes No Did you consult with anyone on parts of this assignment, including other students, TAs, or the instructor?

Yes No Did you consult an outside source (such as an Internet forum or a book other than the course textbook) on parts of this assignment?

If you answered Yes to one or more questions, please give the details here:

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CIS477
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HW7

Problem 1: In class, we covered the HAMILTONIAN PATH problem, in which we were given a graph G and have to find a path that touches every node in the graph exactly once (in this version of the problem, assume that the path can begin and end at any node). The HP-TREE problem is the same problem, except that the input graph G must be a tree.

Suppose that I want to prove that HP-TREE is NP-Complete. To do this, I make the following argument: We know that the HAMILTONIAN PATH problem is NP-Complete. HP-TREE reduces to HAMILTONIAN PATH, because if we can solve HAMILTONIAN PATH for graphs in general, then clearly we can also solve it for graphs that are trees. Thus, HP-TREE is also NP-Complete.

Part a: What is wrong with the above argument?

Part b: Show that HP-TREE is *not* NP-Complete. (Hint: what is a tree? Draw an example of one.)

1)

a) The above argument is wrong because Hamiltonian path does not completely reduce to HP-Tree. A tree can have nodes that end such as the binary tree. It can have nodes that lead no where. It automatically assumes the nodes are all connected or not, as they all stem from an original node and do not return back to the same one.
b) 1
c) 2 3
d) 4 5
e) Here we can see 4 and 5 are connected by 3 but can never connect to each other, even though a regular graph can connect these by a single path, a tree will not.

Problem 2: The DOUBLE HAMILTONIAN CYCLE problem is as follows: given a graph, find two non-overlapping cycles (meaning that they don't share any nodes or edges) such that between the two cycles, every node in the graph is touched exactly once. Use HAMILTONIAN PATH to show that DOUBLE HAMILTONIAN CYCLE is NP-Complete. (Note that this problem statement does not have a typo: the first problem is Double Hamiltonian **Cycle**, and the second problem is Hamiltonian **Path**.)

2)

a) Idea: Double Hamiltonian path is a double Hamiltonian cycle.
b) Graph change: None
c) Algorithm change: Run Hamiltonian path on any one of the nodes in the graph. It will return a path. Then on the unexplored nodes, run the Hamiltonian path again. If it returns a path then there is a separate cycle. Keep doing this until there are no more unexplored nodes. If there are only two separate Hamiltonian paths then it is a double Hamiltonian cycle.
d) This can be solved in polynomial time by iterating through each cycle and checking if they visit every node in the graph

Problem 3: In the Max Degree Spanning Tree problem you are given

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a graph and an integer k, and need to find a spanning tree of that graph such that each node in the graph participates in at most k edges in the spanning tree. Prove that Max Degree Spanning Tree is NP-Complete.

3)

a) Idea: Reduce Max Degree Spanning Tree to Hamiltonian path and since Hamiltonian path is NP-Complete, MDST is also NP complete
b) Change in graph: You do not change the graph
c) Change in algorithim: Run Hamiltonian path in a while loop until all paths can be explored. Take the number of paths and check it against k. If it is less than or equal to k then it is a max degree spanning tree
d) This is NP complete because by definition, Hamiltonian path is NP and reduces to max degree spanning tree

Extra Credit: The HITTING SET problem is as follows: Suppose we have a set of elements $V = \{v_1, ..., v_n\}$. Suppose we have a collection of sets $S_1, ..., S_k$, where each S_i is a set containing some of the v_j elements. A hitting set is a set H that is a subset of V, such that H contains at least one element from every S_i. Given some value b, we wish to find a hitting set with b or fewer elements, if it exists.

For example, suppose $S_1 = \{v_1, v_2, v_4\}, S_2 = \{v_2, v_3, v_5\}, S_3 = \{v_1, v_3, v_5\}$, and $b = 3$. Is there a hitting set with 3 or fewer elements? One example of a hitting set is $\{v_2, v_5\}$. This hitting set has size 2, so we have found a solution to the problem.

The VERTEX COVER problem is as follows: Given a graph G and some value c, is there a set of c vertices in the graph such that every edge in the graph is adjacent to at least one of those c vertices (in other words, every edge must have at least one of its two endpoints included in the set of c vertices)? (Note that this problem definition is slightly different from what we did in class.)

Assume VERTEX COVER is NP-Complete. Use that to show that HITTING SET is NP-Complete.

4)

a) Idea: