

① a)

$$S_{1 \rightarrow 2}(S) = \sum_T \phi(F, S) = \begin{array}{cc} \cancel{14} & 6 \\ \checkmark & 14 \\ 0 & 10 \end{array}$$

$$\beta_2(S, T) = \phi(S, T) \cdot S_{12}(S) = \begin{array}{cc|c} S & T & \\ \hline \cancel{14} & \cancel{14} & 6 \\ \cancel{14} & \checkmark & 60 \\ \cancel{14} & 0 & 18 \\ \checkmark & \checkmark & 28 \\ \checkmark & 0 & 14 \\ 0 & \checkmark & 20 \\ 0 & 0 & 30 \\ 0 & 0 & 30 \\ 0 & 0 & 20 \end{array}$$

$$\mu_{1,2}(S) = \sum_T \beta_2(S, T) = \begin{array}{cc} 84 & \cancel{14} \\ 112 & \checkmark \\ 80 & 0 \end{array}$$

b)

$$S_{2 \rightarrow 1}(S) = \begin{array}{cc} \cancel{14} & 14 \\ \checkmark & 8 \\ 0 & 8 \end{array}$$

$$\beta_1(F, S) = \phi(F, S) \cdot S_{12}(S) = \begin{array}{cc|c} F & S & \\ \hline \cancel{14} & \cancel{14} & 14 \\ \cancel{14} & \checkmark & 80 \\ \cancel{14} & 0 & 24 \\ \checkmark & \checkmark & 28 \\ \checkmark & 0 & 8 \\ 0 & \checkmark & 40 \\ 0 & \checkmark & 40 \\ 0 & 0 & 24 \\ 0 & 0 & 16 \end{array}$$

$$\mu_{2,1}(S) = \sum_F \beta_1(F, S) = \begin{array}{cc} 84 & \cancel{14} \\ 112 & \checkmark \\ 80 & 0 \end{array}$$

$$Z = 84 + 112 + 80 = 276$$

c) They are calibrated $\mu_{1,2}(S) = \mu_{2,1}(S)$

②

a)

$$\beta_1(S, T) = \begin{array}{cc|c} S & T & \\ \hline \cancel{14} & \cancel{14} & 6 \\ \cancel{14} & \checkmark & 60 \\ \cancel{14} & 0 & 18 \\ \checkmark & \checkmark & 28 \\ \checkmark & 0 & 14 \\ 0 & \checkmark & 20 \\ 0 & 0 & 30 \\ 0 & 0 & 20 \end{array}$$

Marginalize S
 \sum_S

$$64$$

b) Normalize
 $/Z$

$$0.23$$

$$\beta_2(S, T) = \begin{array}{cc|c} N & N & 6 \\ N & V & 60 \\ N & 0 & 18 \\ V & N & 28 \\ V & V & 14 \\ V & 0 & 30 \\ 0 & N & 30 \\ 0 & V & 30 \\ 0 & 0 & 20 \end{array} = \begin{array}{c} 64 \\ 104 \\ 108 \end{array} = \begin{array}{c} 0,23 \\ 0,37 \\ 0,39 \end{array}$$

$$\textcircled{3} a) \beta_1(F=N, S) = \begin{array}{cc} 14 & N \\ 80 & V \\ 24 & 0 \end{array}$$

$$\mu_{1,2}(S) = \begin{array}{cc} 84 & N \\ 112 & V \\ 80 & 0 \end{array}$$

$$P(T | F=N) = \sum_S \frac{\beta_1(F=N, S) \times \beta_2(S, T)}{\mu_{1,2}(S)}$$

	S	T	$\beta_1 \cdot \beta_2$	$\mu_{1,2}(S)$	Marginalize S
$P(T F=N) =$	N	N	6 \cdot 14 = 84	84	N 30
	N	V	6 \cdot 80 = 480	480	
	N	0	6 \cdot 24 = 144	144	
	V	N	28 \cdot 80 = 2240	2240	V 29
	V	V	28 \cdot 24 = 672	672	
	V	0	28 \cdot 30 = 840	840	
	0	N	30 \cdot 30 = 900	900	0 59
	0	V	30 \cdot 30 = 900	900	
	0	0	20 \cdot 24 = 480	480	

$\mu_{1,2}(S) = \begin{array}{c} 84 \\ 112 \\ 80 \end{array}$
 $\Rightarrow \begin{array}{c} 1 \\ 10 \\ 3 \\ 20 \\ 10 \\ 50 \\ 9 \\ 9 \\ 6 \end{array}$

Normalize with $(30 + 29 + 59) = 118$

$$P(T | F=N) = \begin{array}{cc} 0,2542 & N \\ 0,2458 & V \\ 0,5 & 0 \end{array}$$

b) We can directly read it off the table because knowing $S=V$ disconnects the knowledge gained from $F=N$!

$\Rightarrow \frac{5}{8}$