MATH141: Differential Calculus

Homework #1

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Part I — Properties of elementary functions

Instructions. Unless otherwise stated, all work must be shown clearly and written on separate paper. Answers without sufficient justification may receive reduced credit. Graphs must include labeled axes and intercepts where applicable.

1. Let $f(x) = ax + b$, where a and b are real numbers.

1a. *(easy)* What type of function is f?

1b. *(easy)* What does the constant a control in the function f?

1c. *(easy)* What does the constant b control in the function f?

1d. *(easy)* Let $P_0 = (x_0, y_0)$ and $P_1 = (x_1, y_1)$ be points such that $f(x_0) = y_0$ and $f(x_1) = y_1$. Represent a in terms of the coordinates of P_0 and P_1.

1e. *(medium)* Using the points from part (1d), and including an additional point $P_2 = (x_2, y_2)$ such that $f(x_2) = y_2$, show that the slope of the line between P_0 and P_1 is equal to the slope of the line between P_0 and P_2. Your argument must be algebraic.

1f. *(easy)* If $a = 0$, what type of function is f?

1g. *(easy)* If $a = 1$ and $b = 0$, what type of function is f?

1h. *(easy)* If $a \neq 0$, determine the domain and range of f. Write both in interval notation.

1i. *(easy)* If $a = 0$, what is the range of f?

1j. *(easy)* Let $a < 0$. Choose specific values for a and b, and sketch the graph of f. Your sketch must include all four quadrants, the x- and y-axes, and the corresponding x- and y-intercepts.

1k. *(easy)* Let $a > 0$. Choose specific values for a and b, and sketch the graph of f. Your sketch must include all four quadrants, the x- and y-axes, and the corresponding x- and y-intercepts.

1l. *(medium)* Define a function g that is parallel to f. That is, define g so that the graphs of f and g do not intersect at any point. Clearly justify your construction.

1m. *(medium)* Provide three example applications of functions of the type described in part (1a) in mathematical modeling. You may consult external sources if needed; however, explanations must be written in your own words.

2. Let $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers.

2a. *(easy)* What type of function is f?

2b. *(easy)* In general, define what it means for a number x^* to be a root of a function.

2c. *(easy)* What is a general formula for the root(s) of f? Your answer must be expressed explicitly in terms of the coefficients a, b, and c.

2d. *(easy)* Using the formula from part (2c), what guarantees that the roots of f are real numbers? What is the term in the formula from part (2c) that determines whether the roots are real called?

2e. *(easy)* Rewrite f in standard vertex form. Your final expression must be written entirely in terms of the coefficients of f.

2f. *(easy)* Which constant term in the form obtained in part (2e) controls how the graph of f is scaled?

2g. *(easy)* Which constant term in the form obtained in part (2e) controls how the graph of f is centered or horizontally shifted?

2h. *(easy)* Which constant term in the form obtained in part (2e) controls how the graph of f is vertically shifted?

2i. *(easy)* In general, what is the domain of f? Express your answer in interval notation.

2j. *(easy)* If $a < 0$, determine the range of f in interval notation. Specify whether f has a minimum or maximum value.

2k. *(easy)* If $a > 0$, determine the range of f in interval notation. Specify whether f has a minimum or maximum value.

2l. *(medium)* Let $a > 0$. Choose values of a, b, and c such that $b^2 - 4ac > 0$, and sketch the graph of f. Your sketch must include all four quadrants, the x- and y-axes, and clearly labeled x- and y-intercepts (if applicable).

2m. *(medium)* Let $a < 0$. Choose values of a, b, and c such that $b^2 - 4ac > 0$, and sketch the graph of f. Your sketch must include all four quadrants, the x- and y-axes, and clearly labeled x- and y-intercepts (if applicable).

2n. *(medium)* Provide three example applications of functions of the type described in part (2a) in mathematical modeling. You may consult external sources if needed; however, explanations must be written in your own words.

3. Let

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n,$$

where $a_0, a_1, \ldots, a_n$ are real numbers, $a_n \neq 0$, and n is a positive integer.

3a. *(easy)* What type of function is f? What is the degree of f?

3b. *(easy)* If $n = 2$, then f is still of the type described in part (3a), but more specifically it is a:

3c. *(easy)* If $n = 3$, then f is still of the type described in part (3a), but more specifically it is a:

3d. *(easy)* What is the domain of f in general? Express your answer in interval notation.

3e. *(easy)* If n is even, determine the end behavior of f:

i. as $x \to \infty$,

ii. as $x \to -\infty$.

3f. *(easy)* If n is odd, determine the end behavior of f:

i. as $x \to \infty$,

ii. as $x \to -\infty$.

3g. *(easy)* Up to how many roots can f have, regardless of whether the roots are real or distinct?

3h. *(hard)* Let $n = 4$, and suppose f has roots

$$x_1^* = 1, \quad x_2^* = -2, \quad x_3^* = 3, \quad x_4^* = -4.$$

Define a function f satisfying these constraints, and determine the coefficients a_0, a_1, a_2, a_3, a_4 in the expanded form

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4.$$

3i. *(medium)* Sketch the graph of the function defined in part (3h) to the best of your ability. Your sketch must include the x- and y-axes, all relevant intercepts, and all four quadrants. You may use graphing technology as an aid.

3j. *(medium)* Provide three example applications of functions of the type described in part (3a) in mathematical modeling. You may consult external sources if needed; however, explanations must be written in your own words.

4. Let

$$f(x) = \frac{p(x)}{q(x)},$$

where

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n, \quad q(x) = b_0 + b_1 x + b_2 x^2 + \cdots + b_m x^m.$$

Assume $a_0, \ldots, a_n, b_0, \ldots, b_m$ are real numbers, $q(x) \neq 0$, $a_n \neq 0$, $b_m \neq 0$, and n, m are distinct positive integers.

4a. *(easy)* What type of function is f?

4b. *(medium)* Let $p(x) = 1$ and

$$q(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3,$$

and let x_1^*, x_2^*, x_3^* denote three distinct real roots of $q(x)$ satisfying

$$0 < x_1^* < x_2^* < x_3^*.$$

Determine the domain of $f(x)$ and express it using interval notation.

4c. *(easy)* Let $p(x) = 1$ and $q(x) = x$. What is the name of the shape of the graph of $f(x)$?

4d. *(medium)* Sketch the graph of f in the form described in part (4c). Your sketch must include all four quadrants, the x- and y-axes, and any asymptotic behavior.

4e. *(easy)* Let $p(x) = a$ with $a \neq 0$ and $q(x) = x - b$. Describe how the constants a and b transform the graph of $f(x)$. What is the vertical line $x = b$ called?

5. Let

$$f(x) = A\sin\big(B(x-C)\big) + D, \qquad g(x) = A\cos\big(B(x-C)\big) + D,$$

where A, B, C, D are real constants.

5a. *(easy)* What type of functions are f and g?

5b. *(easy)* Describe what each constant controls in the graphs of f or g:

i. A:

ii. B:

iii. C:

iv. D:

5c. *(easy)* What is the line $y = D$ called?

5d. *(medium)* Let $A = 1$, $B = 1$, $C = 0$, and $D = 0$. Determine the roots of f.

5e. *(medium)* Under the same coefficients as in part (5d), determine the roots of g.

5f. *(medium)* Under the coefficients of part (5d), sketch the graph of f over the domain

$$[-2\pi,\, 2\pi].$$

Your sketch must include all four quadrants, the x- and y-axes, all x-intercepts, and clearly labeled crest and trough points.

5g. *(medium)* Under the coefficients of part (5d), sketch the graph of g over the domain

$$[-2\pi,\, 2\pi].$$

Your sketch must include all four quadrants, the x- and y-axes, all x-intercepts, and clearly labeled crest and trough points.

5h. *(easy)* Functions that satisfy

$$h(x+\omega) = h(x)$$

for a constant ω are called:

5i. *(easy)* What does ω equal if $h = f$? What does ω equal if $h = g$?

5j. *(medium)* Provide three example applications of functions of the type described in part (5a) in mathematical modeling. You may consult external sources if needed; however, explanations must be written in your own words.

6. The following questions concern trigonometric identities. Let

$$f(\theta) = \sin\theta, \qquad g(\theta) = \cos\theta.$$

6a. *(easy)* $\dfrac{f(\theta)}{g(\theta)} =$

6b. *(easy)* $\dfrac{1}{f(\theta)} =$

6c. *(easy)* $\dfrac{1}{g(\theta)} =$

6d. *(easy)* $[f(\theta)]^2 + [g(\theta)]^2 =$

6e. *(easy)* Let $\alpha, \beta \in \mathbb{R}$.

i. $\sin(\alpha+\beta) =$

ii. $\cos(\alpha+\beta) =$

iii. $\tan(\alpha+\beta) =$

6f. *(medium)* $f(2\theta) =$

6g. *(medium)* $g(2\theta) =$

6h. *(medium)*

$$\frac{f(2\theta)}{g(2\theta)} =$$

Your final answer must be expressed entirely in terms of tangent functions.

6i. *(easy)*

$$f\left(\theta + \frac{\pi}{2}\right) =$$

Express your answer in terms of a trigonometric function other than $f(\theta)$.

6j. *(hard)* Using your results from Problem 6, prove the identity

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}.$$

Hint: Use the identity $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ with $\alpha = \beta = \theta/2$.