Leaky Integrator Model & Behavioral/Neuronal Data Agreement

How do we make decisions and balance exploration/exploitation when facing a stochastic environment?

Trying to find the mechanism in how biological organisms make decisions when facing stochasticity choices. Matching belongs to a class of behaviors purported to engage cognitive mechanisms that animals use when **competing for resources in stochastic environments**. Because matching results in an equilibrium state in which returns from competing behaviors are equalized, it represents a **stable and effective foraging strategy** from both an **evolutionary and game theoretic perspective**. Matching behavior in a dynamic context is well described by a simple local reformulation of the classical matching law.

Matching Behaviors

Natural environments are characterized by **uncertainty in both the sources and timing of rewards**. Humans and other animals are sensitive to these variables and adapt the statistics of their foraging behavior to those of the environment. Specifically, animals distribute their time among foraging sites in proportion to their relative value.

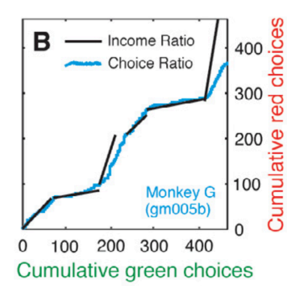
- The matching law asserts that the fraction of choices made to any option will exactly match the fraction of total income (i.e., total rewards) earned from that option:

$$\frac{I_k}{\Sigma I} = \frac{C_k}{\Sigma C}$$

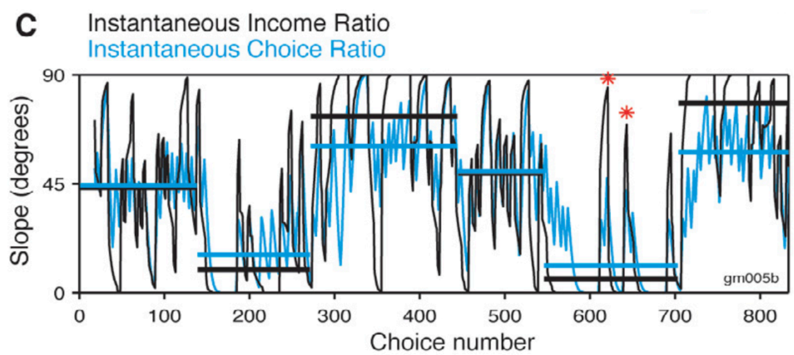
- To match behavior to income, animals must integrate rewards earned from particular behaviors, and the brain, in turn, must **maintain an appropriate representation of the value**. This paper studies the neural mechanism of such a process.

Dynamic Task

A stochastic process of rewarding with a certain rate. For some reason, **biological organisms are able to keep track and handle stochastic processes quite well**.



- Matching behavior effect is present, the reward gain ratio is consistent with the choice ratio.
- Adjustment to unsignaled changes in reward rate is also very rapid as well.
- Relationship between choices and experienced rewards is highly local in time. During transitions between income ratios, behavior lawfully and rapidly adjusts to unsignaled changes in the rates of reward.

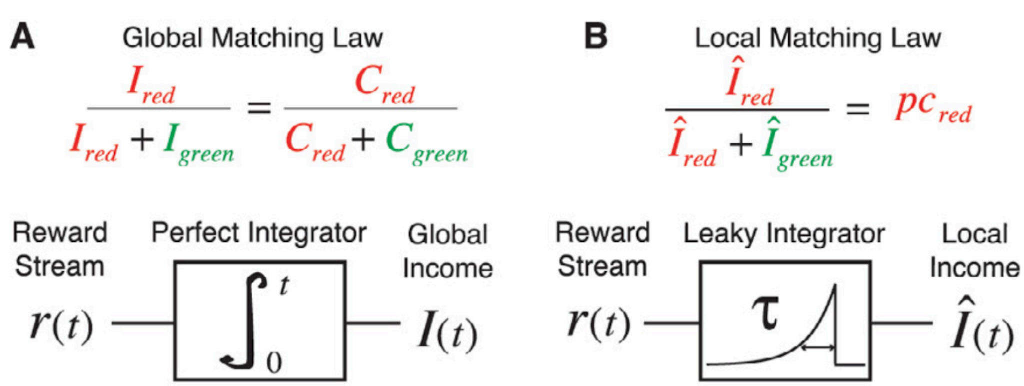


- If behavior were based on a representation of reward history that extended into the distant past, it would be incapable of such rapid adjustment.

Global (Perfect) & Local (Leaky) Matching Rule

It is somewhat like doing a Monte-Carlo sampling or a Temporal-Differences sampling.

- Traditionally applied to foraging in **stationary environments**, the matching law relates cumulative choice to total experienced income and is intrinsically a **global description** of behavior averaged over **long periods** of time.
- **Perfect Integrator:**
    - Rewards earned during a behavioral session is simply the integrated reward stream that an animal has experienced. In the traditional matching law, **each new reward contributes equally to the income attributed to a particular option without discount or decay**.
    - The fraction reward of a particular option is directly the integral or fraction of that reward divided by the total reward.
- **Leaky Integrator:**
    - Given **a finite effective memory on estimates of income**, making them **local rather than global**. The local fractional income translates directly into the **instantaneous probability of choice for a given option.**
    - When given large data sets and stationary environments, the predictions of our local matching rule approximate those of the classical global matching law.



Matching law: relation of fractional of income (I) to fraction of choices (C)

- **Leaky integrators give more weight to recent rewards** and the instantaneous probability would be described as:

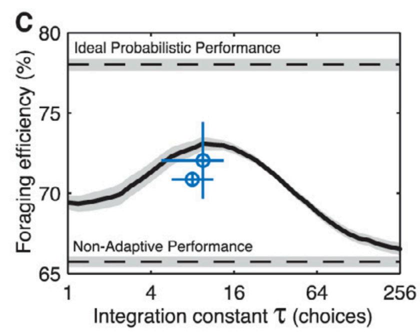
$$\hat{I}(t) = \int_{-\infty}^{t} \exp\left(-\frac{t-\tau}{\tau}\right) I(\tau)\, d\tau \qquad p_c(t) = \frac{\hat{I}_k(t)}{\sum \hat{I}}$$

Math Model: Accuracy & Adaptability Tradeoff

Postulating a process of leaky integration marks a **conceptual shift** from the parameter- less matching law to a **one-parameter model** (leaky time constant tau) of matching behavior appropriate for dynamic conditions.

- Higher tau means slower leaks and would give rise to more stable and accurate estimates of income, but respond sluggishly to changes in the environment. Conversely
- Lower tau would produce estimates of income that respond quickly to change, but are substantially noisier during periods of stability.

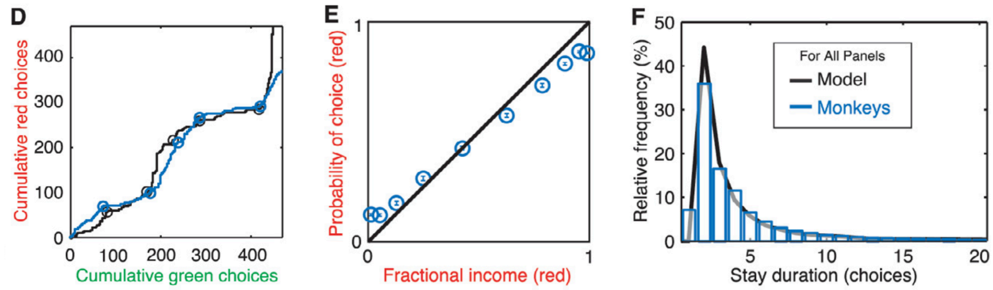
- The **upper bound** demarcates the average performance of an ideal probabilistic forager. This hypothetical **ideal strategy "knows" the reward rate** of each option, thereby dispensing with the estimation process, and uses this information to make choices that maximize its expected rate of reward.
- The **lower bound** shows the average performance of a completely **random foraging strategy** and represents chance performance in our task.
- Despite its simplicity, the best-performing leaky integrator model does well relative to these bounds, collecting 93% of the rewards attained by the ideal clairvoyant strategy.
- **This is a pure mathematical model performance!!!**



Matching Behavioral Counterparts

Qualitatively, the model **exhibits dynamic matching behavior that is very similar to that of the animal**. This local model may be an adequate descriptive model of real choice behaviors in the foraging tasks. **By minimizing MSE, the best tau** for each of the monkeys can be found and get their corresponding mathematical models of decision making.

1. Easiest comparison is that their prediction of each of the colors is roughly the same.
2. The model predicts that the probability of choosing red will vary linearly with the local fractional income from red, which is approximately true for the behavioral data.
3. Because the model is strictly probabilistic, the number of successive trials on which it will choose a given color before switching (stay duration) will be distributed as the average of a family of exponentials, which is the same with the Monkeys.

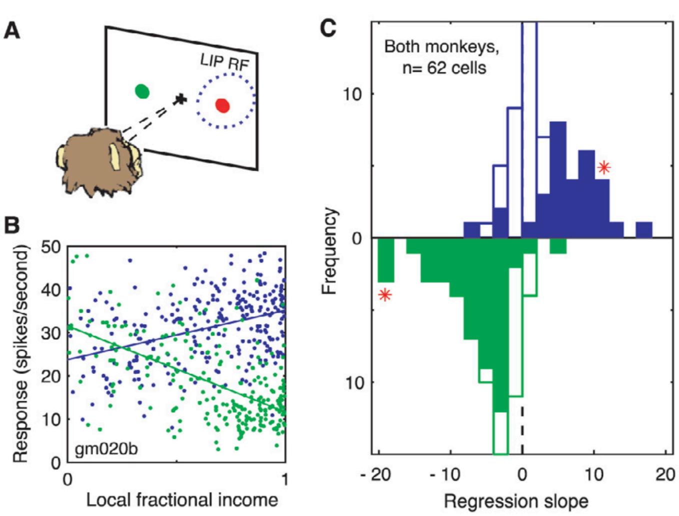


The model provides us a window into the animal's **internal valuation** of available options and gives us a metric—local fractional income—that **allows us to estimate how the monkey values each of the two colors on every trial, even before it renders a decision**.

Neural Representation Matching in LIP

Questions whether, within each category of motor response, activity in LIP is influenced by the local fractional income of the chosen target (since LIP has been found to be related to a certain decision making process).

1. Positive correlation between **LIP firing rate** and **fractional income** for choices **into the RF** and a negative correlation for **choices out of the RF**. The solid lines are regressions fit to these two sets of data by the method of least squares and are characterized by positive and negative slopes for choices into and out of the RF, respectively.
    - When in RF, LIP firing and local fractional income plays a positive correlation role
    - When the fractional income of the chosen color is low, the clouds of blue and green points overlap, indicating that the activity of this particular cell is **no longer a reliable indicator of the direction of the monkey's saccade at the end of the trial**.
        - When the fraction income is low, LIP fires uniformly for both colors
2. Same with regression analysis
    - The upper blue distribution is for detection inside RF, this distribution is centered to the right of zero, indicating positive regressions of activity on fractional income.



It is pretty unlikely that the so-called fraction-reward value is calculated in the LIP, but rather that area LIP plays a critical role in **remapping abstract valuation to concrete action**.

- LIP may **contribute to this transformation and directly influence the probability** that a particular region of space will serve as the endpoint of the next saccade.
- This aligns with the unifying proposal that area LIP functions as a **saliency map of visual space** of the type invoked in visual psychophysics or computational vision, capable of flexibly combining and representing a variety of information for the purpose of guiding eye movements or shifts in visual attention.