# Intro to FitRec Mathamatical Formulation

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Refer to Fitness Recommendation Algorithms.

In the original paper, FitRec combines static user embeddings (derived from attributes) and temporal embeddings (derived from historical workout data) to support three key applications:

1. **Quantitative Tasks**: Predict workout metrics (e.g., heart rate) across a session, either beforehand or in real time.
2. **Qualitative Tasks**: Identify important factors influencing performance and cluster users based on similar patterns.
3. **Recommendation Tasks**: Suggest alternate routes to meet target workout goals, such as achieving a specific heart rate profile. Based on the heart rate needed (target), give workout information (attributes).

Why do we use this model:

1. **Context-aware**: Workouts don't occur in isolation. A user's performance today is influenced by their previous workouts (e.g., fatigue, recovery, adaptation). The embeddings learn from historical sequences and build personalized, context-aware, and temporal data understandable machine.
2. **Temporal relationships**: Model temporal relationship in the target, understand that the data is temporally connected.

## Temporal Embedding Construction

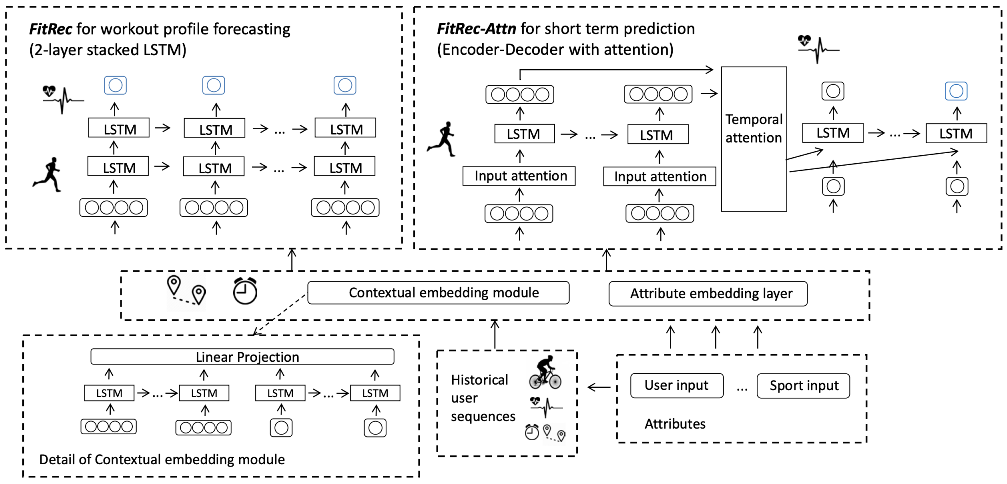
### Data Representation and Notation

The TBR model takes sequences and embeddings as input, essentially building representations across times:

$X$ is combination of various contextual sequences for the current workout, each $x_i$ is a vector of contextual information (altitude, type of workout, ...).

$$X = (x_1, \ldots, x_T) \in \mathbb{R}^{N \times T}$$

- Altitude ($x_t^1$): Elevation at time $t$ (e.g., 200 meters above sea level).
- Speed ($x_t^2$): Speed at time $t$ (e.g., 12 km/h).



FitRec for workout profile forecasting
(2-layer stacked LSTM)
LSTM
FitRec-Attn for short term prediction
(Encoder-Decoder with attention)
Input attention
Temporal attention
Contextual embedding module
Attribute embedding layer
Linear Projection
Detail of Contextual embedding module
Historical user sequences
User input
Sport input
Attributes

Figure 1: Structure Overview

- Distance ($x_t^3$): Cumulative distance covered up to time $t$ (e.g., 3.5 km).
- Type of Workout ($x_t^4$): Encoded categorical information about the workout (e.g., running = 0, biking = 1).
- For $T = 10$ and $N = 4$, $X$ might look like:

$$X = \begin{bmatrix} 200 & 201 & \dots & 210 \\ 12 & 11.8 & \dots & 10.5 \\ 0.1 & 0.2 & \dots & 1.0 \\ 1 & 1 & \dots & 1 \end{bmatrix}.$$

$y$ is the target sequence (heart rate, speed).

$$y = (y_1, \dots, y_T) \in \mathbb{R}^T$$

- Heart Rate ($y_t$): The user's heart rate at time $t$ (e.g., 140 bpm, 145 bpm).
- $y$ might be: $y = [140, 142, 145, 148, \dots, 155]$ (bpm).

$Z$ is the historical contextual sequences (recent workout contextual sequences).

$$Z = (z_1, \dots, z_T) \in \mathbb{R}^{N \times T}$$

- This $Z$ comes from the most recent workout, sort of a Markov property idea? Can we find a equivalent form in our data set?
- All $N$ dimension category need to be matching with $X$'s $N$ dimension.

$y'$ is the historical target sequence (recent workout heart rate or speed).

$$y' = (y'_1, \dots, y'_T) \in \mathbb{R}^T$$

- Same as in the decription for $Z$.

Each workout has associated metadata or static attributes:

$$a = (a_1, \dots, a_m),$$

- $|a_i|$ is the number of distinct values of attribute $a_i$.
- User ID ($a_1$): Encoded identifier for the user (e.g., User 1, User 2).
- Sport Type ($a_2$): Encoded categorical type of workout (e.g., 0 = running, 1 = biking).
- Gender ($a_3$): Encoded value (e.g., 0 = male, 1 = female).
- $a$ for a workout could be:

$$a = (\text{User ID: } 42, \text{Sport Type: } 1, \text{Gender: } 0).$$

## Static Meta-data Embedding

Attributes are embedded into a latent space (embeds user specific information such as fitness level and gender, into latent features that summarize the user's characteristics.):

$$e_{a_i} = E_{a_i}(a_i), \quad \forall i \in [1, m],$$

where $e_{a_i} \in \mathbb{R}^{D_1}$ is the embedding of the $i$-th attribute, and $E_{a_i} \in \mathbb{R}^{|a_i| \times D_1}$ is the learned embedding matrix.

## Contextual Sequential Consistency Embedding

Encodes the historical workout data to capture trends and patterns, like how a user's heart rate responds to certain activities. Two LSTMs encode historical sequences for 1 historical contextual sequence $z_t$ and 1 historical target sequence $y'_t$ into hidden states (as $t$ increases, we would use previous historical state to process, so the hiddeb state we use for embedding later is still in a sequential consistency manner):

$$h_{1,t} = \text{LSTM}_1(z_t, h_{1,t-1}), \quad h_{2,t} = \text{LSTM}_2(y'_t, h_{2,t-1}),$$

where $h_{1,t}$ and $h_{2,t}$ are the hidden states for historical contextual and target sequences, respectively. The embeddings are combined via a linear projection:

$$e_t = W_e[h_{1,t}; h_{2,t}] + b_e,$$

where $W_e$ and $b_e$ are learned parameters.

## Input Construction

For each time step $t$, inputs are constructed from using current contextual information, embeddings of attributes and embeddings of historical contextual information:

$$u_t = [x_t; e_t; e_{a_1}; \ldots; e_{a_m}],$$

where $u_t \in \mathbb{R}^K$, with $K$ as the concatenated dimension.

## Trajectory Modeling

### Trajectory Prediction (FitRec)

Given the contextual sequences $X$, attributes $a$ for a candidate workout, historical sequences $Z$ and $y'$, as well as the total time of the workout, predict the entire target sequence $y = (y_1, \ldots, y_T)$.

This can be sued for commanding types of workout (attributes) based on the target heart rate that the user specify (prediction matches).

Practically, a 2-layer LSTM predicts the entire target sequence (this would captures how the relationship of $y_t$ and $y_{t-1}$ varies overtime, establishing connections between them):

$$h_t = \text{LSTM}(u_t, h_{t-1}),$$

$$\hat{y}_t = \text{SELU}(W_{NAT} h_t + b_{NAT}),$$

where $W_{NAT}$ and $b_{NAT}$ are learned parameters, and SELU is the Scaled Exponential Linear Unit activation function.

### Training Objective

The model minimizes the mean squared error (MSE) between predicted ($\hat{y}_t$) and actual ($y_t$) values:

$$L = \frac{1}{|T_{train}|} \sum_{y \in T_{train}} \sum_{t=1}^{L} (\hat{y}_t - y_t)^2,$$

where $|T_{train}|$ is the total number of sequences in the training set.

### Evaluation Metrics

Performance is assessed using:

- **Root Mean Squared Error (RMSE):**

$$\text{RMSE} = \sqrt{\frac{1}{|T_{test}|} \sum_{y \in T_{test}} \sum_{t=1}^{L} (\hat{y}_t - y_t)^2}.$$

- **Mean Absolute Error (MAE):**

$$\text{MAE} = \frac{1}{|T_{test}|} \sum_{y \in T_{test}} \sum_{t=1}^{L} |\hat{y}_t - y_t|.$$

## LSTM Attention Prediction

FitRec-Attn introduces attention mechanisms to focus on critical features and time steps, making it more effective for real-time, short-term predictions.

## Encoder

The encoder processes the contextual sequences $u = (u_1, \ldots, u_T)$ using input attention and an LSTM.

**Input Attention:** The input attention mechanism determines the importance of each input feature:

1. Compute attention scores for each input feature:

$$s_t^k = v_s^\top (\mathbf{W}_s[h_{t-1}; u^k] + b_s), \quad 1 \leq k \leq K,$$

   where:

   - $h_{t-1}$: Previous encoder hidden state.
   - $u^k$: The $k$-th input dimension.
   - $\mathbf{W}_s, b_s, v_s$: Learnable parameters.

2. Normalize the scores using softmax:

$$\alpha_t^k = \frac{\exp(s_t^k)}{\sum_{j=1}^K \exp(s_t^j)}.$$

3. Weight the input features to create a new input:

$$\tilde{u}_t = (\alpha_t^1 u_t^1, \ldots, \alpha_t^K u_t^K).$$

**LSTM Encoder:** The encoder LSTM processes the attention-weighted input:

$$h_t = \text{LSTM}_e(h_{t-1}, \tilde{u}_t),$$

where $h_t$ encodes the contextual information across time steps.

## Decoder

The decoder generates predictions one step at a time using temporal attention and an LSTM.

**Temporal Attention:** Temporal attention determines how much each encoder hidden state contributes to the decoder's prediction:

1. Compute attention scores for all encoder hidden states:

$$\ell_{i,t} = v_\ell^\top (\mathbf{W}_\ell[d_{t-1}; h_i] + b_\ell), \quad 1 \leq i \leq T,$$

   where:

   - $d_{t-1}$: Previous decoder hidden state.
   - $h_i$: Encoder hidden state at time $i$.
   - $\mathbf{W}_\ell, b_\ell, v_\ell$: Learnable parameters.

2. Normalize the scores using softmax:

$$\beta_{i,t} = \frac{\exp(\ell_{i,t})}{\sum_{j=1}^{T} \exp(\ell_{j,t})}.$$

3. Compute the context vector as a weighted sum of encoder hidden states:

$$c_t = \sum_{i=1}^{T} \beta_{i,t} h_i.$$

**LSTM Decoder:** The decoder LSTM updates its hidden state based on the context vector and previous target value:

$$d_t = \text{LSTM}_d(d_{t-1}, [y_{t-1}; c_t]).$$

**Final Prediction:** The decoder produces the target prediction $\hat{y}_t$ through a linear transformation:

$$\hat{y}_t = W_{AT}[d_t; c_t] + b_{AT}.$$

## Objective Function

The model minimizes the Mean Squared Error (MSE) between the predicted and actual target values:

$$L = \frac{1}{|T_{\text{train}}|} \sum_{y \in T_{\text{train}}} \sum_{t=1}^{T} (\hat{y}_t - y_t)^2.$$