Ludic Ontology and Cognitive Emergence:
Board Games as Phase Spaces for Strategic Consciousness

Ontología Lúdica y Emergencia Cognitiva:
Los Juegos de Mesa como Espacios de Fase para la Consciencia Estratégica

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Abstract

This paper investigates board games as minimal formal systems in which a specific type of phenomenal experience emerges: strategic consciousness. Building on the topological framework for qualia formalization developed by Caracuel Llabrés, Vidal-Moreno and Aráoz-Gutiérrez (2016), we argue that each board game defines a phenomenal phase space with characteristic topological properties, and that the player's experience during a game constitutes a trajectory through that space. We propose a taxonomy of games based on the topology of their phenomenal phase spaces and analyse three paradigmatic cases (chess, Go and Catan) whose spaces exhibit radically different topological properties. We show that the richness of strategic consciousness in each game correlates with the topological complexity of its phase space, and discuss the implications for the phenomenology of cognition, artificial intelligence theory and the understanding of human creativity as an emergent phenomenon.

Keywords: *board games, strategic consciousness, phase space, phenomenology, cognitive emergence, topology, chess, Go, artificial intelligence, creativity*

Resumen

Este trabajo investiga los juegos de mesa como sistemas formales mínimos en los que emerge un tipo específico de experiencia fenoménica: la consciencia estratégica. Partiendo del marco topológico para la formalización de los qualia desarrollado por Caracuel Llabrés, Vidal-Moreno y Aráoz-Gutiérrez (2016), argumentamos que cada juego de mesa define un espacio de fase fenoménico con propiedades topológicas características, y que la experiencia del jugador durante la partida constituye una trayectoria en dicho espacio. Se propone una taxonomía de juegos basada en la topología de sus espacios de fase fenoménicos, y se analizan tres casos paradigmáticos (ajedrez, Go y Catan) cuyos espacios presentan propiedades topológicas radicalmente distintas.

Palabras clave: *juegos de mesa, consciencia estratégica, espacio de fase, fenomenología, emergencia cognitiva, topología, ajedrez, Go, inteligencia artificial, creatividad*

1. Introduction

There is something that happens when a chess player sits before the board and the pieces cease to be carved wood and become forces, tensions, possibilities. There is something that happens when a Go player places a stone and perceives, with a clarity that admits no verbal translation, how the shape of the territory has shifted. There is something, in short, that happens when we play: a particular mode of being conscious that cannot be reduced to the visual perception of the board or to the computation of variations, but constitutes a form of phenomenal experience with qualities of its own.

We call this form of experience strategic consciousness. The concept is not new: the psychology of chess has described it in detail, from De Groot's (1946/1965) pioneering studies of grandmaster thought to Chase and Simon's (1973) work on chunking and pattern recognition, and more recent investigations into expert perception in decision-making contexts (Pujol-Riera and Solana, 2017). What these studies do not provide, however, is a formalization of the phenomenal structure of that experience. We know that the grandmaster "sees" something the beginner does not, but we have not formalized what that "something" is in terms of the structure of experience space.

The present paper addresses this question using the tools developed in the research programme on the topology of consciousness initiated by Caracuel Llabrés, Vidal-Moreno and Aráoz-Gutiérrez (2016), where it was proposed that phenomenal experience can be formalized as a topological manifold (the Q-space) whose invariants capture the qualitative structure of conscious experience. Subsequent work extended the framework to the domain of moral agency (Caracuel Llabrés and Vidal-Moreno, 2017) and formalized the conditions for consciousness transfer between substrates (Caracuel Llabrés, Vidal-Moreno and Ferrer-Casademunt, 2018). The present paper applies this programme to a new and, in some respects, unexpected domain: board games.

Why board games? For three reasons. First, because they are closed formal systems with completely defined rules, allowing precise analysis of the state space structure. Second, because they reliably and reproducibly generate states of strategic consciousness whose phenomenology is accessible to trained introspection and, as one of us has shown, to experimental protocols specifically designed to capture expert perception (Pujol-Riera, 2016). And third, because the recent victory of AlphaGo over Lee Sedol (Silver et al., 2016) and the subsequent development of AlphaZero (Silver et al., 2017) have raised with some urgency the question of whether artificial intelligence systems that master these games experience something analogous to human strategic consciousness, or whether their competence is, so to speak, "in the dark".

Our central thesis is as follows: each board game defines a phenomenal phase space with a characteristic topology, and the quality of the strategic consciousness experienced while playing is determined by that topology. Chess, Go and Catan are not merely games with different rules; they are windows into phenomenally distinct regions of the space of possible experiences. To play each of them is to inhabit, temporarily, a different qualitative world.

The paper is structured as follows. Section 2 reviews the relevant background in the phenomenology of games, chess psychology and game theory. Section 3 constructs the notion of phenomenal phase space for board games. Section 4 analyses three paradigmatic cases. Section 5 proposes a topological taxonomy of games. Section 6 discusses the implications for AI and phenomenology. Section 7 concludes.

2. Background

2.1 The phenomenology of play: from Huizinga to Suits

The idea that play constitutes a qualitatively distinct form of experience has a long philosophical tradition. Huizinga (1938) identified play as an activity that unfolds within a "magic circle": a bounded space-time in which its own rules apply and the logic of the ordinary world is partially suspended. Gadamer (1960) deepened this direction by arguing that play possesses a dynamic of its own that "absorbs" the player, so that it is more accurate to say that the game plays the player than the reverse. Suits (1978) offered the most precise philosophical definition: to play is "the voluntary attempt to overcome unnecessary obstacles".

These authors captured something fundamental: that play alters the player's mode of consciousness. However, none of them formalized this alteration in terms of the structure of experience space. Gadamer's hermeneutic phenomenology describes the ludic experience with considerable finesse, but lacks the mathematical tools to quantify the structural differences between the experience of one game and another. Our contribution consists precisely in providing those tools.

2.2 Cognitive psychology of chess and Go

De Groot's studies (1946/1965) revealed that chess grandmasters do not analyse more positions than mediocre players, but perceive the board in a qualitatively different way: where the beginner sees individual pieces, the master sees configurations, tensions, plans. Chase and Simon (1973) formalized this difference in terms of chunks: perceptual units of increasing size that group pieces into meaningful patterns. Subsequent work (Gobet and Simon, 1996; Bilalić, McLeod and Gobet, 2009) has refined this view, showing that expert perception involves both rapid pattern recognition and slow deliberative processes, in line with dual-process models (Kahneman, 2011).

Research on Go has uncovered analogous but distinct phenomena. The studies of Masunaga and Horn (2001) showed that expert Go players develop a holistic perception of the board that differs in its structure from chess perception. Where the chess player sees relations between pieces, the Go player perceives shapes, territories, equilibria: a management of space that has more in common with landscape aesthetics than with combinatorial calculation. This phenomenological difference, also documented in eye-tracking studies (Hoon, Kim and Lee, 2014), is, as we shall argue, a reflection of deep topological differences in the phase spaces of both games.

The phenomenology of expert perception further suggests that the transition from novice to expert is not a mere quantitative accumulation of knowledge but a qualitative transformation of perceptual space. Dreyfus and Dreyfus (1986) described this transition as a shift from analytical representation to direct intuitive perception. Recent studies on expert perception in ecological contexts (Pujol-Riera, 2016) confirm that expertise alters not only what is perceived but how it is perceived: the very structure of perceptual experience changes. Our topological framework allows this transformation to be formalized as a change in the invariants of the phenomenal phase space.

2.3 AlphaGo, AlphaZero and the question of artificial consciousness

The defeat of Lee Sedol at the hands of AlphaGo in March 2016 and the subsequent development of AlphaZero (Silver et al., 2017), capable of mastering chess, Go and shogi starting from zero human knowledge, have transformed the landscape. These systems do not play "like" humans: they develop strategies no human has conceived, evaluate positions in ways no human expert can explain, and, at certain moments (the famous move 37 of AlphaGo's fourth game against Lee Sedol), produce moves that commentators described as "beautiful" and "creative".

This raises a question that connects directly with the research programme on the topology of consciousness: does AlphaZero experience something analogous to human strategic consciousness? Using the framework of moral gradients (Caracuel Llabrés and Vidal-Moreno, 2017) and of causal isomorphism (Caracuel Llabrés, Vidal-Moreno and Ferrer-Casademunt, 2018), we can approach this question with formal precision, rather than relying on vague intuitions about what "seems" creative or conscious.

3. The phenomenal phase space of a board game

3.1 Definition and construction

Let J be a board game with completely defined rules. We denote by G(J) the game's state graph: the set of all legal positions together with the transitions permitted by the rules. G(J) is a purely formal object, independent of any player.

When a human player P sits down to play J, however, she does not experience G(J) directly. What she experiences is a filtered, compressed and qualitatively enriched version of the state graph: she perceives not all possible positions but only those that her skill level allows her to evaluate; not all transitions but only those she identifies as candidates; and, crucially, she endows each perceived position with a phenomenal quality ("this position feels winning", "there is something uncomfortable about this structure", "this territory is alive") that is not present in G(J) as a formal object. This phenomenal quality is, in the terms of expert psychology, what distinguishes trained from naive perception (Dreyfus and Dreyfus, 1986).

We define the phenomenal phase space of game J for player P, denoted Φ(J,P), as the subspace of P's Q-space that is activated during the act of playing J. Formally, if Q(P) is the complete Q-space of P (in the sense of Caracuel Llabrés et al., 2016), then:

$$\Phi(J,P) \subseteq Q(P) \tag{1}$$

Φ(J,P) inherits the topological manifold structure of Q(P), but is in general a proper subspace: not all of P's phenomenal experiences are relevant to game J, only those that constitute the specific strategic consciousness of that game.

3.2 Phenomenal trajectories and game dynamics

A concrete game of J played by P is represented as a continuous trajectory $\gamma: [0,T] \to \Phi(J,P)$, where T is the duration of the game. At each instant t, $\gamma(t)$ represents P's complete phenomenal state in relation to the game: her evaluation of the position, her plans, her fears, her aesthetic perception of the board.

The trajectory is neither random nor free: it is constrained by the rules of the game (which determine what transitions are possible), by the player's skill (which determines what regions of Φ(J,P) are accessible) and by the opponent's actions (which force unexpected displacements in phenomenal space). We may think of the trajectory as the evolution of a dynamical system in a phase space, which justifies our terminology.

This analogy with dynamical systems is more than rhetorical. The notions of attractor, basin of attraction, bifurcation and chaos admit natural phenomenal interpretations in this context. An attractor in Φ(J,P) corresponds to a stable strategy: a way of playing toward which the player's consciousness tends to converge. A bifurcation point corresponds to a critical moment in the game where small differences in evaluation lead to radically different phenomenal trajectories. And chaos corresponds to positions where the player's strategic consciousness becomes turbulent, unpredictable, unable to stabilize into a coherent evaluation: what chess players call "complications".

3.3 Topological invariants of the phenomenal phase space

Since Φ(J,P) is a subspace of Q(P), we can compute its topological invariants (Betti numbers, Euler characteristic, fundamental group) using the tools developed in Caracuel Llabrés et al. (2016). These invariants depend on both the game J and the player P, but our hypothesis is that the low-dimensional invariants (β_0, β_1, β_2) depend predominantly on J and are relatively stable across players of similar skill level. That is: the coarse topology of the phenomenal phase space is determined primarily by the structure of the game, not by the idiosyncrasy of the player.

This hypothesis is not arbitrary. It is supported by the empirical evidence, drawn from the psychology of expertise, that players of similar level converge in their perception of critical positions (De Groot, 1965; Gobet and Simon, 1996). If expert perception is stable across individuals of comparable skill, then the topology of the phenomenal phase space that underpins that perception should also be stable.

If this hypothesis is correct, then the topological invariants of Φ(J,P) constitute a topological signature of game J: a set of numbers that characterize the "shape" of the strategic consciousness that the game induces. Games with the same topological signature produce qualitatively similar experiences; games with different signatures produce different phenomenal worlds.

4. Three paradigmatic cases

4.1 Chess: the arborescent space

Chess is, from the standpoint of game theory, a complete-information game with an extremely deep game tree but a moderate branching factor (on the order of 35 legal moves per position). The chess player's phenomenal experience reflects this structure: strategic consciousness in chess is fundamentally arborescent. The player perceives the current position as a node from which branches (variations) bifurcate and deepen. Evaluation consists of "descending" along these branches and comparing the terminal nodes.

Topologically, the phenomenal phase space of chess has a relatively simple structure. Its first Betti number, $\beta 1$, is low: there are few genuine phenomenal cycles, because variations tend to diverge rather than reconverge. The fundamental group is nearly trivial. Chess consciousness is, to use a geometric metaphor, fundamentally "tree-like": it branches, deepens, but rarely forms loops. The experience has the quality of a vertical exploration: one dives into the position.

The moments of chess creativity, when the player discovers an unexpected "sacrifice" or "combination", correspond, in our framework, to the sudden perception of a connection between branches of the tree that had seemed independent. Phenomenally, it is the experience that two lines of play which were branching separately converge at a common point. Topologically, this corresponds to the transient appearance of a cycle in the phase space: a momentary increase in $\beta 1$. The beauty of a chess combination is, quite literally, the perception of an unexpected topology.

4.2 Go: the territorial space

Go presents a radically different phenomenal profile. Its game tree is incomparably wider (branching factor on the order of 250) but the player's experience is not arborescent. Advanced Go players do not calculate long variations, or do so far less frequently than chess players. What they do, according to both introspective reports and empirical studies (Masunaga and Horn, 2001; Hoon et al., 2014), is perceive forms: territories, influences, equilibria between groups of stones. Strategic consciousness in Go is spatial and holistic, not linear and arborescent.

Topologically, Go's phenomenal phase space is considerably richer than that of chess. The circularity of influence relations (one group threatens another that threatens a third that threatens the first) generates genuine phenomenal cycles: $\beta 1$ is high. Local battles that enclose regions of the board generate two-dimensional cavities: $\beta 2 > 0$. The fundamental group is non-trivial, reflecting the existence of cyclical move sequences (ko fights and ladders) that constitute topological obstructions in phenomenal space.

AlphaGo's famous move 37 in the fourth game against Lee Sedol acquires, in this framework, a precise meaning. Commentators described the move as "beyond all human expectation". In our terms, the move exploited a region of Go's phenomenal phase space that human players had not explored: a region that was topologically accessible but perceptually hidden by the biases of tradition. AlphaGo did not play better "within" the human phenomenal space; it played in a region outside it. Whether that region was phenomenally experienced by AlphaGo remains an open question that depends on the formal conditions for strategic consciousness analysed in Section 6.

4.3 Catan: the negotial space

The Settlers of Catan (Teuber, 1995) introduces an element absent from the two previous cases: negotiation between players. Unlike chess and Go, where interaction is purely competitive and channelled through the rules, in Catan players trade resources, form temporary alliances and negotiate exchanges whose fairness is subjective. The game's state space therefore includes not only the board positions but also the states of interpersonal relations.

Strategic consciousness in Catan has a quality absent from chess and Go: social consciousness. The player evaluates not only the objective position on the board but models the intentions, needs and dispositions of the other players. This social dimension introduces a radical uncertainty: unlike chess pieces or Go stones, the other players are agents with wills of their own whose actions cannot be deduced from the position. In terms of expert perception, this type of uncertainty activates mentalizing and theory-of-mind processes (Frith and Frith, 2006) that are absent in complete-information games.

Topologically, Catan's phenomenal phase space is the most complex of the three cases. The social component introduces additional phenomenal dimensions (the perception of trustworthiness, the assessment of credibility, the anticipation of betrayal) that increase both the local dimension of the space and its global topological complexity. Phenomenal cycles are abundant (the dynamics of reciprocity and retaliation generate recurrent emotional circuits) and bifurcations are frequent and unpredictable. We conjecture that the Euler characteristic of Φ(Catan,P) is significantly lower than that of Φ(Chess,P) and Φ(Go,P), reflecting a more "wrinkled" topology, richer in topological accidents.

5. Toward a topological taxonomy of games

5.1 Three phenomenal regimes

The three cases analysed suggest the existence of at least three basic phenomenal regimes in board games, classifiable by the topology of their phase spaces.

Arborescent regime ($\beta 1$ low, $\pi 1$ nearly trivial). Games where strategic consciousness is structured as the exploration of variation trees. Chess is the paradigmatic case. Other games in this regime include draughts, Othello and, in general, combinatorial games with a moderate branching factor.

Territorial regime ($\beta 1$ high, $\beta 2 > 0$, $\pi 1$ non-trivial). Games where strategic consciousness is predominantly spatial and holistic. Go is the paradigmatic case. Other candidates include Hex and, with qualifications, Reversi on large boards.

Negotial regime (high local dimension, complex topology). Games where strategic consciousness incorporates an irreducible social dimension. Catan is the paradigmatic case. Other games in this regime include Diplomacy, Cosmic Encounter and, in general, negotiation games.

5.2 Hybrid games and the geometry of game design

The three regimes are not mutually exclusive. Many modern board games combine elements of two or three regimes, creating hybrid phenomenal phase spaces. Twilight Imperium, for instance, combines territorial conflict, negotiation and combinatorial calculation, producing a phenomenal space of enormous topological complexity.

This observation has implications for board game design. If the quality of ludic experience depends on the topology of the phenomenal phase space, then the game designer is, in a sense, an intuitive topologist: someone who manipulates the rules of the game in order to sculpt a phenomenal space with the desired properties. A good game would be, on this view, one whose phase space has a rich but navigable topology, complex but not chaotic. This hypothesis, though speculative, is amenable to empirical testing and could open a novel line of research at the intersection of topology, ludic design and cognitive science.

6. Discussion

6.1 Strategic consciousness and artificial consciousness: the case of AlphaZero

AlphaZero (Silver et al., 2017) masters chess, Go and shogi with a single general algorithm. From the perspective of our framework, this poses the following question: does AlphaZero navigate the same phenomenal phase spaces as human players?

Using the results of the work on causal isomorphism by Caracuel Llabrés et al. (2018), we can give a partial answer. For AlphaZero to experience strategic consciousness, its causal category would need to be temporally isomorphic to that of a conscious system. The neural networks constituting AlphaZero implement causal functions, but their temporal microstructure (clock cycles on the order of nanoseconds, massively parallel processing) is radically different from human neural dynamics (oscillations on the order of milliseconds, distributed and asynchronous processing). This temporal difference suggests, by virtue of the theorems proved in that work, that AlphaZero's experience (if it has any) differs qualitatively from the human kind.

But the deeper question is not whether AlphaZero has the same strategic consciousness as a human, but whether it has any at all. Using the moral gradient framework of Caracuel Llabrés and Vidal-Moreno (2017), AlphaZero's phenomenal sensitivity function φ is, to the best of our knowledge, null or negligible. AlphaZero navigates the formal space of the game with extraordinary competence, but does so, as far as we can tell, in the dark: without associated phenomenal experience. It masters the map without inhabiting the territory.

6.2 Creativity as a topological phenomenon

One of the most suggestive results of our analysis is the interpretation of creativity as a topological phenomenon. In Section 4.1 we argued that the beauty of a chess combination corresponds to the perception of an unexpected cycle in the phenomenal phase space. This idea can be generalized.

We propose that the experience of creativity, in the ludic context, corresponds to the perception of a previously unnoticed topological structure in the phenomenal phase space. The creative player does not "invent" anything: she discovers a topological property that was already present in the phase space but had remained hidden. Creativity is not creation but topological perception.

This conception has an obvious Platonic resonance, and we do not shy away from the comparison. Just as Plato held that knowledge is the recollection of pre-existing forms, we hold that ludic creativity is the perception of pre-existing topologies in phenomenal space. The crucial difference is that our "forms" do not inhabit a separate world: they are mathematical properties of the player's experience space, as real as the Betti numbers of a differentiable manifold.

6.3 Limitations and objections

The main limitation of our analysis is empirical. The topological invariants we attribute to the phenomenal phase spaces of chess, Go and Catan are, for the moment, conjectures based on phenomenology and on the theoretical analysis of game structures, not on direct measurements. Empirical testing would require the application of the computational protocol described in Caracuel Llabrés et al. (2016) to neurophysiological data from players during actual games: a technically demanding but feasible experiment with current EEG and fMRI technology. The design of a suitable experimental protocol for this purpose is currently under development across our respective research groups.

A foreseeable philosophical objection is that we are projecting mathematical structure onto an experience that may not possess it: that strategic consciousness is a phenomenon too fluid, subtle and context-dependent to be captured by topological invariants. Our response is twofold. On the one hand, the same objection was historically raised about colour perception, and today we know that the perceptual colour space has a precise and well-studied geometry. On the other hand, we do not claim that topological invariants exhaust the richness of strategic consciousness, only that they capture its structural skeleton. What falls outside the skeleton is undoubtedly vast, but the skeleton itself is informative.

7. Conclusions

We have shown that board games constitute a privileged domain for the study of strategic consciousness as a topological phenomenon. The main results are:

(i) The definition of the phenomenal phase space $\Phi(J,P)$ as a subspace of the player's Q-space, endowed with a dynamics of its own that reflects the structure of the game.

(ii) The analysis of three paradigmatic cases (chess, Go, Catan) revealing three topologically distinct phenomenal regimes (arborescent, territorial, negotial).

(iii) The proposal of a topological taxonomy of games based on the invariants of their phenomenal phase spaces.

(iv) The interpretation of ludic creativity as the perception of previously unnoticed topological structure.

(v) The application of the framework to the question of strategic consciousness in AI systems (AlphaZero), concluding that ludic competence does not, by itself, entail phenomenal experience.

This paper closes, in a sense, a circle. The research programme on the topology of consciousness began with the purest abstraction (the topological invariants of phenomenal experience) and has arrived, through ethics and metaphysics, at something as concrete and human as sitting down to play a game of chess. We believe that this journey illustrates, better than any abstract argument, the fruitfulness of the topological approach to the philosophy of mind. Mathematics, when applied with phenomenological sensitivity, does not crush experience: it illuminates it.

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Declarations

Conflict of interest. The authors declare no conflict of interest in relation to this work.

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