



N-body simulations

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1. C++ code

General principles

- Maximize readability by hiding `for` loops



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General principles

- Maximize readability by hiding `for` loops
- Optimize for speed keeping the first rule in mind
- Gragg-Bulirsch-Stür should adjust timestep if necessary
- Provide a nice user interface



1. C++ code

Optimalizations

- Pass classes by reference if it doesn't obstruct readability
- valarray and array instead of vector



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 - speed record: Burrau problem with RK in **0.2s**



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Optimalizations

- Pass classes by reference if it doesn't obstruct readability
- valarray and array instead of vector
 - speed record: Burrau problem with RK in **0.2s**
- [] operator instead of .at()
- `size_t valarray.size()`
- Loops over all possible pairs are symmetric



```
Johannes@Maria-Elisabeth ~/Documents/UGent/1e Master Fysica
master*) $ ./simulation presentation_list.txt
GBS integration of input/burrau.txt
Progress
100 %
Integration finished in:
1.06138s
Average number of driver-evaluations per unit time:
20599.7
RK integration of input/burrau.txt
Progress
100 %
Integration finished in:
3.16072s
```

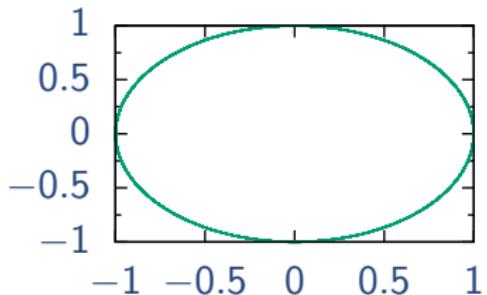


2. Are the integrators correct?

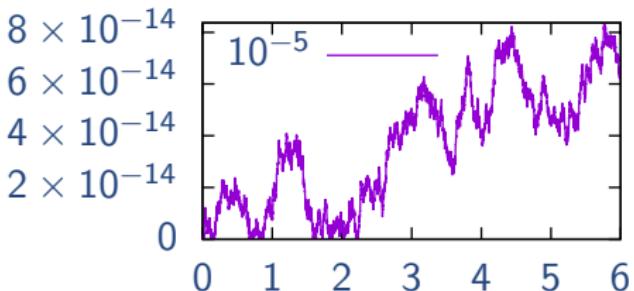
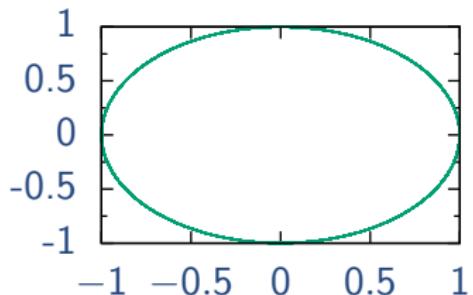
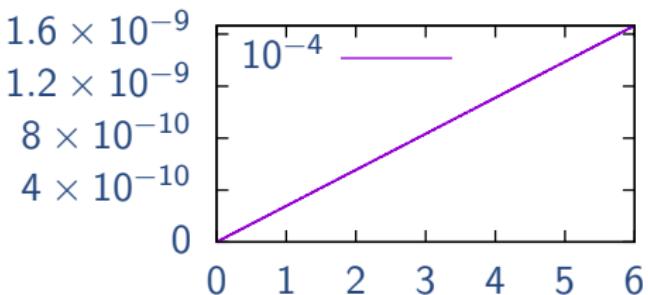
- Correct h scaling
- Test with analytic solvable differential equations

Two-body problem with RK timestep 10^{-4} and 10^{-5}

Positions

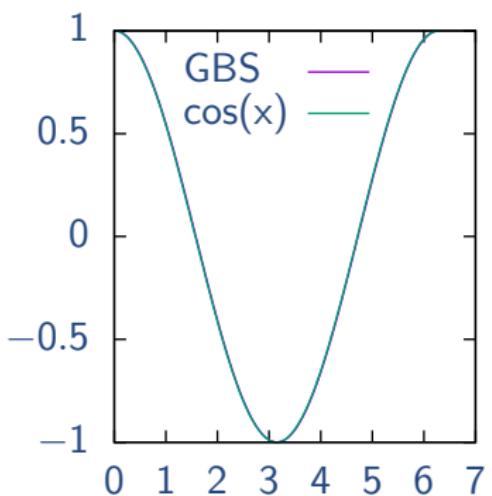


Energy error

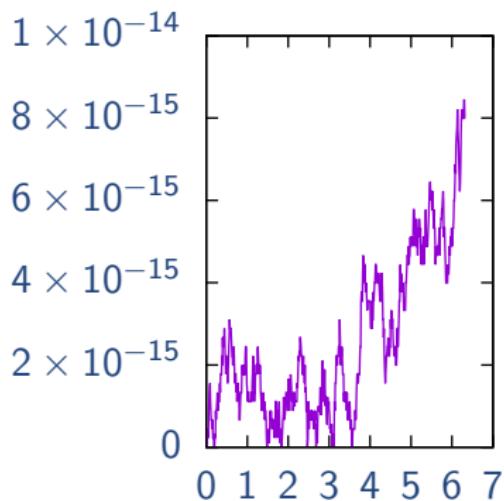


Harmonic oscillator with GBS and timestep 10^{-2}

Position

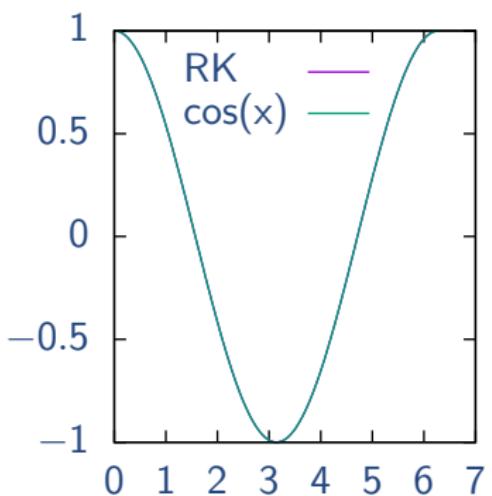


Energy error

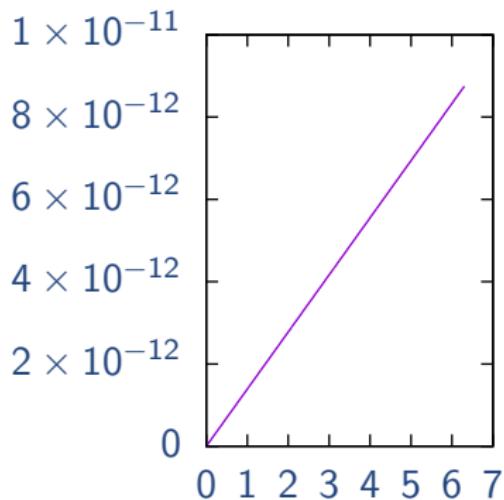


Harmonic oscillator with RK and timestep 10^{-2}

Position



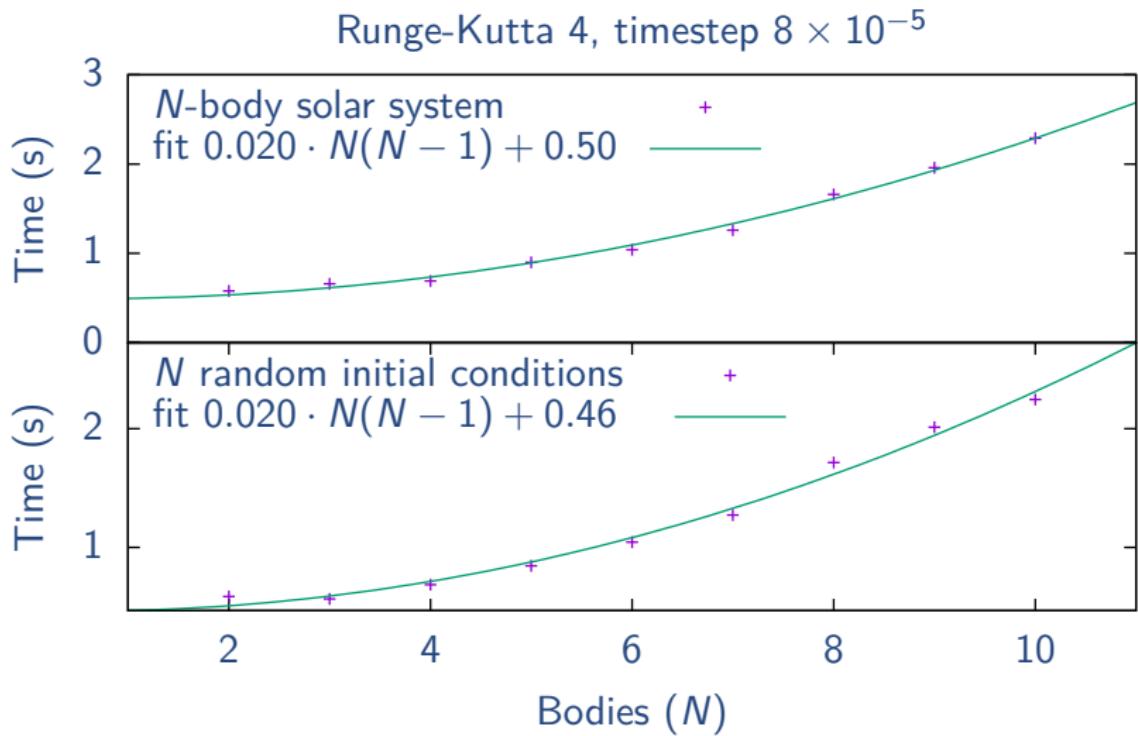
Energy error





3. Scaling behaviour

- All functions that loop over all the bodies are symmetric
- Profiling shows most time is spent on calculating gravity
- Expected scaling is $\frac{N(N - 1)}{2} + C$
 - constant term C is mainly (valarray) constructors

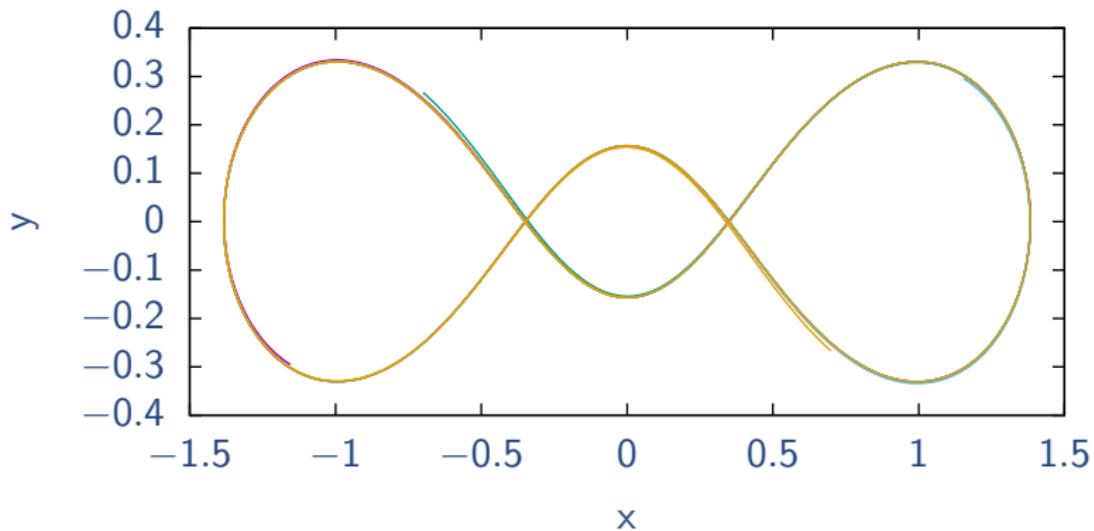


4. ΔE and driver evaluations

- ΔE is a good measure for accuracy
- Number of driver evaluations is linear for Runge-Kutta 4
 - driver evaluations = $4 \cdot \frac{\text{unit time}}{\text{timestep}}$
- Number of driver evaluations for GBS depends on convergence criterium
- Only lower bound can be set for systems with close encounters

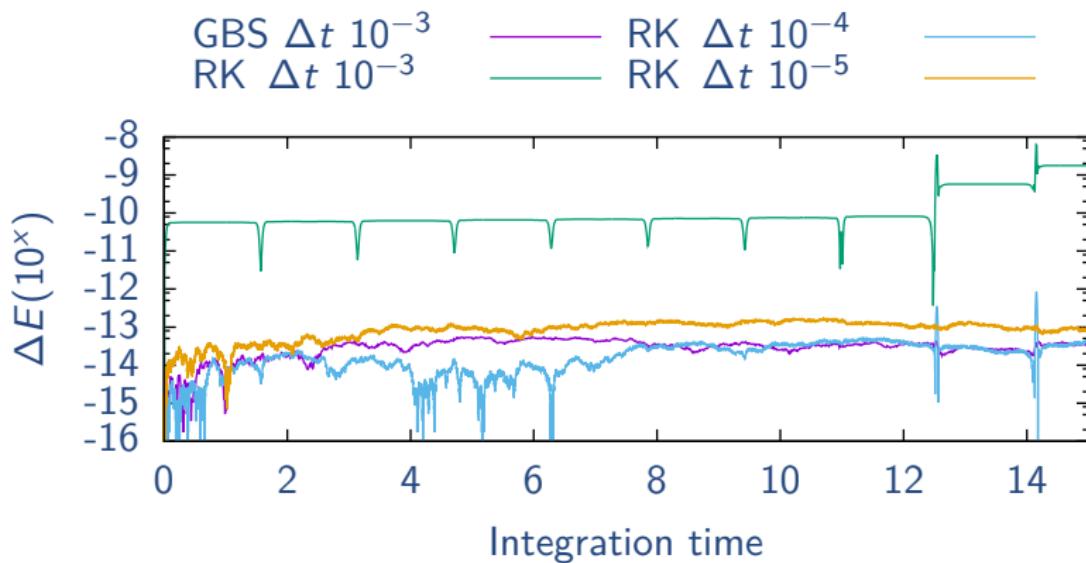


Supereight choreography



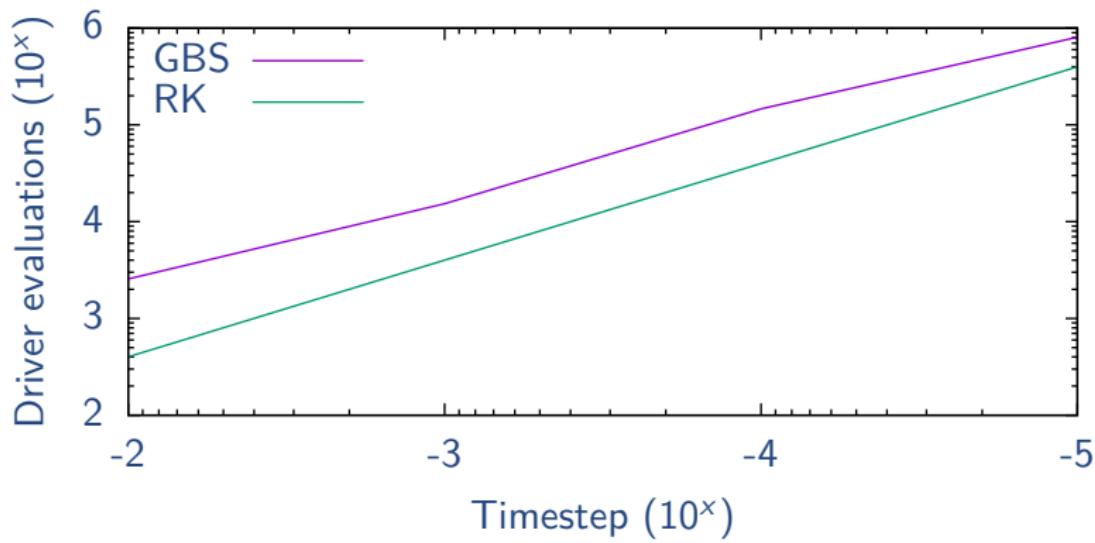


ΔE as function of the integration time





Number of driver evaluations as function of the timestep





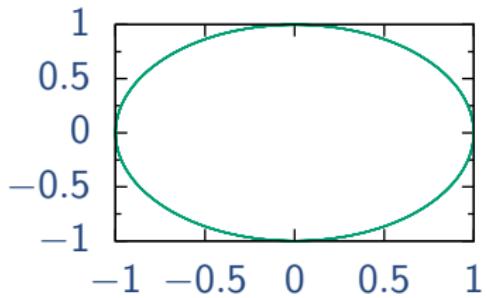
5. Long term accuracy

- Simple two-body system
- Period 0.06 unit time
- Motion of sun eliminated by giving it negative initial speed
- Runge-Kutta with timestep 8×10^{-5}
- Gragg-Bulirsch-Stür with timestep 1×10^{-3}

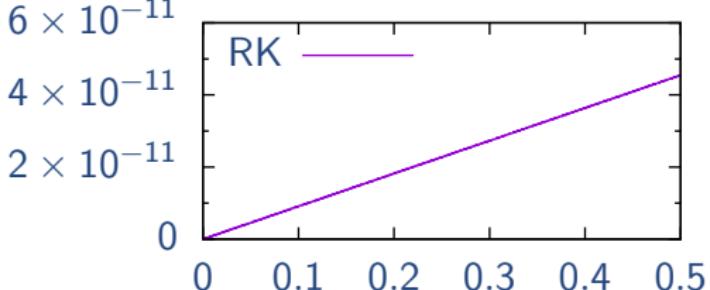
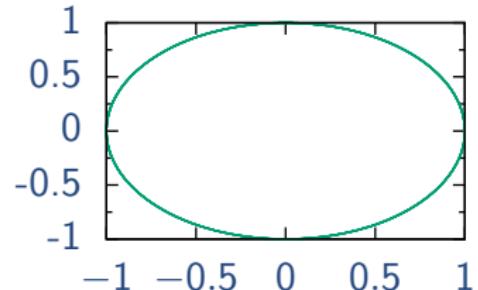
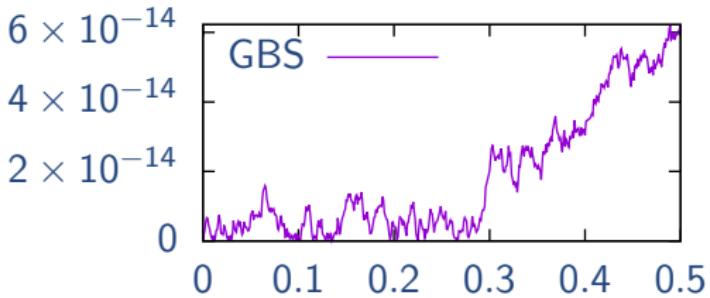


10 periods of a simple two-body system

Positions

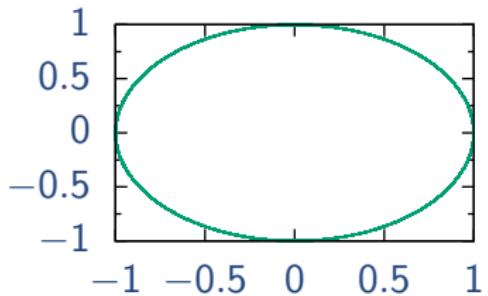


Energy error

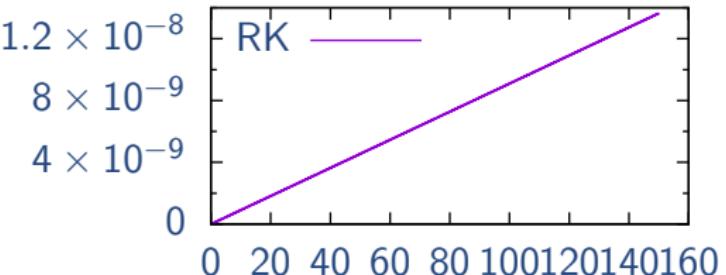
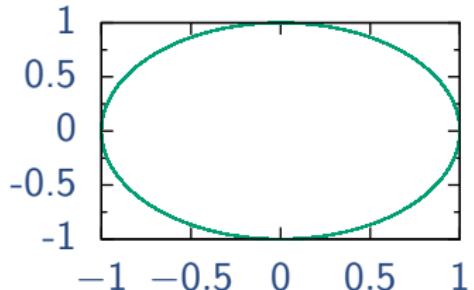
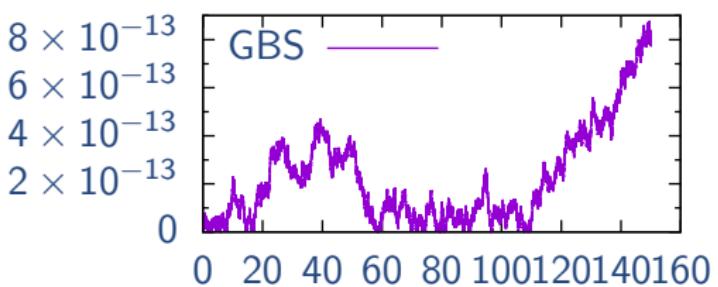


2500 periods of a simple two-body system

Positions



Energy error





6. Adaptive timestep

Burrau problem with *no* adaptive timestep





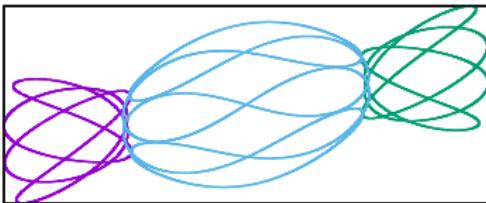
Burrau problem *with* adaptive timestep



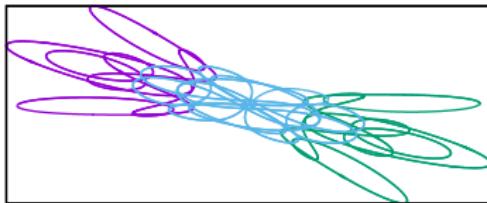
7. Choreographies

Three-body choreographies with RK timestep 10^{-4}

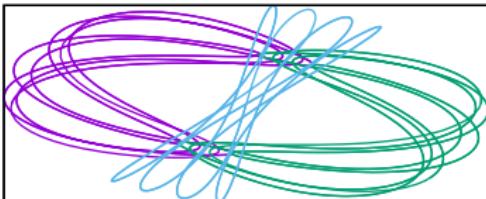
Butterfly



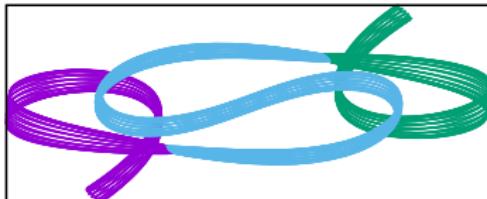
Goggles



Dragonfly

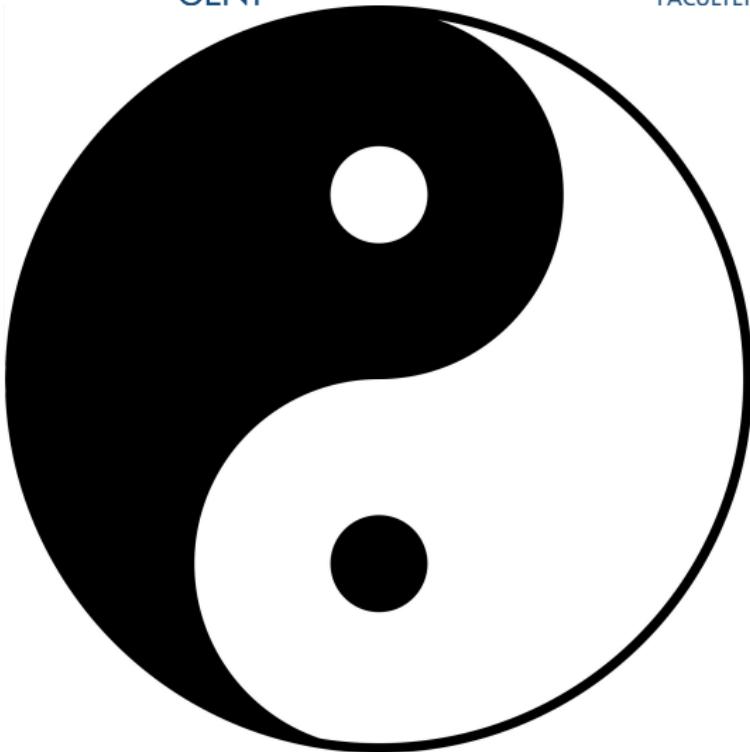


Yin-Yang





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