Clothoidal Interpolation — A New Tool for High-Speed Continuous Path Control

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SUMMARY

The Clothoid or Cornu Spiral is a planar curve which is frequently used in high-way design. The curvature of the curve is proportional to the length of the curve measured from the origin of the spiral. In this report, methods of connecting double Clothoids between two lines are mainly discussed. By applying this methods to welding, painting, sealing and bonding robots, it is expected to obtain faster continuous path motion. Theoretical analysis is given and some experimental results with the SCARA robot are shown.

KEY WORDS

Clothoid, interpolation, continuous path control, robot, trajectory design.

1. INTRODUCTION

The Clothoid or Cornu Spiral (1) is a planar curve which is frequently used in high-way design (2). To drive a car, it is requested that the angle of the handle, and consequently the curvature of the road, changes continuously. The curvature of the Clothoid is proportional to the ammount of the curve traversed. By using the curve, acceleration of the moving body is kept continuous under constant line speed.

The travelling speed of the robot tool used in arc welding, painting, sealant applying, and especially in adhesive bonding is becoming higher and higher. The path of the tool in such application should have continuous change of curvature to prevent unexpected vibration. The Clothoid is one of the most hopeful curve for such use.

Kanayama (3) introduced Clothoid for mobile robot trajectory control. He proposed to use "Clothoid pair" to interpolate between two straight lines. This idea can be extended to "between points interpolation", by assuming the curvature at the given points are all zero.

In this report, the basic mathematics on Clothoid is introduced and methods for interpolating two lines using Clothoid are discussed. Experimental results with the SCARA robot are also shown.

2. NOMENCLATURE

a	:scale factor
$c\ (= dc_v/ds)$	:curl ratio
$c_v\ (= d\phi/ds)$	:curvature
$j\ (= \sqrt{-1})$	:imaginary unit
l	:chord length , length of the line segment
s	:length of the curve from the spiral origin
t	:time
u	:Clothoid parameter
ϕ	:tangential angle
ϕ_0	:tangential angle at Clothoid origin
$\rho\ (= 1/c_v)$	:radius of curvature
$C_s(u)$	:Fresnel integral (cosine)
$S_n(u)$	:Fresnel integral (sine)
$\mathbf{P}\ (= x + jy)$	:position vector of the Clothoid curve
$\mathbf{P}_u$	:unit Clothoid
$\mathbf{P}_0$	:position vector of the Clothoid origin
$\mathbf{Q}$	:evolute of the Clothoid
$\mathbf{Q}_u$	:evolute of the unit Clothoid

3 THE CLOTHOID CURVE

Fresnel integrals (See Appendix) are given in the following formulae.

$$C_s(u) = \int_0^u \cos\frac{\pi u^2}{2} du = x \tag{1}$$

$$S_n(u) = \int_0^u \sin\frac{\pi u^2}{2} du = y \tag{2}$$

Taking $C_s(u)$ in the abscissa axis and $S_n(u)$ in the ordinate axis, then a planar curve called Cornu Spiral or Clothoid is obtained (Fig. 1). Placing the figure on the complex plane, the curve is denoted by the following simpler form.

$$\mathbf{P}(u) = x \pm jy = \int_0^u (\cos\frac{\pi u^2}{2} \pm j\sin\frac{\pi u^2}{2})du = \int_0^u e^{\pm j\frac{\pi u^2}{2}} du \tag{3}$$

where, $j = \sqrt{-1}$: imaginary unit.

The sign +(plus) and -(minus) correspond to the solid line and dotted line in Fig. 2 respectively. Fig. 1 and 2 are called unit Clothoid*, where the scale factor $a = 1$.

The generalized Clothoid is given by multiplying the unit Clothoid by scale factor a, rotating through angle ϕ_0, and translating by vector $\mathbf{P}_0$(Fig. 3). Then we obtain,

$$\mathbf{P} = \mathbf{P}_0 + ae^{j\phi_0}\int_0^u e^{\pm j\frac{\pi u^2}{2}} du = \mathbf{P}_0 + ae^{j\phi_0}(\mathbf{P}(u))$$

$$= \mathbf{P}_0 + ae^{j\phi_0}(C_s(u) \pm jS_n(u)) \tag{4}$$

Differentiating $\mathbf{P}$ by curve length s the direction or the tangent of the curve ϕ is obtained. The length is measured from the origin of the spiral.

$$\frac{d\mathbf{P}}{ds} = e^{j\phi} = \frac{d\mathbf{P}}{du}\cdot\frac{du}{ds} = ae^{j(\phi_0 \pm \frac{\pi u^2}{2})}\cdot\frac{du}{ds} \tag{5}$$

$$\phi = \phi_0 \pm \frac{\pi u^2}{2} \quad , \quad \frac{du}{ds} = \frac{1}{a} \quad , \quad s = au \tag{6}$$

The curvature c_v and radius of curvature ρ are,

$$c_v = \frac{d\phi}{ds} = \frac{d\phi}{du}\cdot\frac{du}{ds} = \pm\frac{\pi u}{a} = \pm\frac{\pi s}{a^2} \tag{7}$$

$$\rho = \frac{1}{c_v} = \pm\frac{a}{\pi u} = \pm\frac{a^2}{\pi s} \tag{8}$$

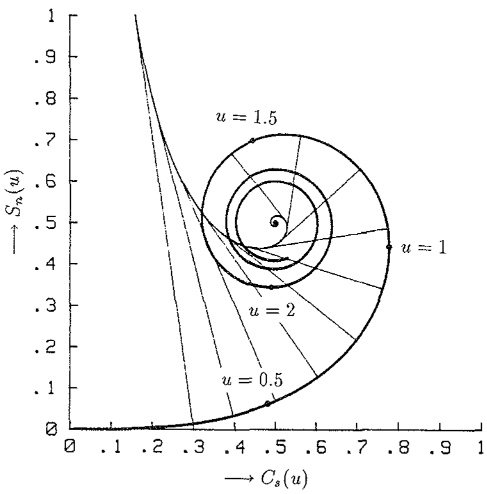

Fig. 1 Clothoid curve with its evolute

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*In civil engineering, the equation of the Clothoid is denoted as $|\rho \cdot s| = A^2$, and when $A = 1$ it is called "unit Clothoid". It correspond to $a = \sqrt{\pi} = 1.772$ by our notation.

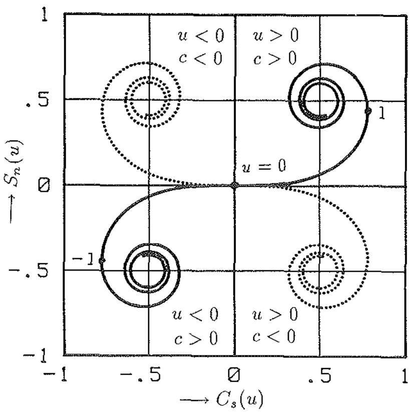


Fig. 2 Unit Clothoid

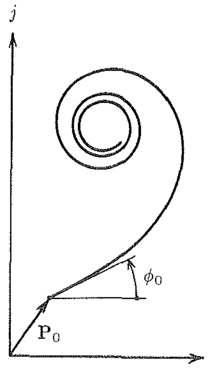
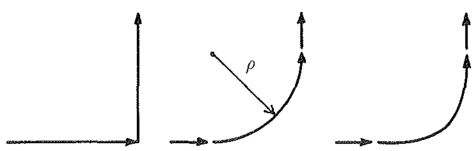


Fig. 3 General Clothoid

The major characteristics of Clothoid is that the curve has linear relation between curvature and length. The changing ratio of curvature vs curve length is called curl ratio. The curl ratio c is constant for a given Clothoid. The sign of the value sgn(c) decides the quadrant of the unit Clothoid (See Fig. 2). The Clothoid parameter u shows the length of the unit Clothoid from the Clothoid origin. The curvature is proportional to this value.



(a)Straight line (b)Circular arc (c)Clothoid

Fig.4 Cornering problem

4. CORNERING PROBLEM

The "cornering problem" is defined as follows (Fig. 4). "Two directed lines exist. Interpolate these lines with a curve or curves so that the direction and the curvature of the curve(s) continuously change throughout the connection including the connecting points with the lines." Such kind of problem often appears in arc welding, painting, sealing, and bonding processes. Circular arc is not sufficient for this problem because at the connecting point with the line there occurs discontinuous change of curvature. The Clothoid has a continuously changing curvature and at the origin it has zero curvature. So the curve is seemed to be the best for connecting with the straight lines.

To connect two directed lines, a Clothoid pair or double Clothoid is used. In Figure 5, two straight lines are given and smooth interpolation between the two is requested. Two lines makes an angle of α. In this case the terminal conditions of the curvatures are both zero, two Clothoids are used symmetrically (Fig.6).

From the given angle α, the maxmum tangent angle of the half Clothoid is obtained, and the minimum curvature is derived as follows.

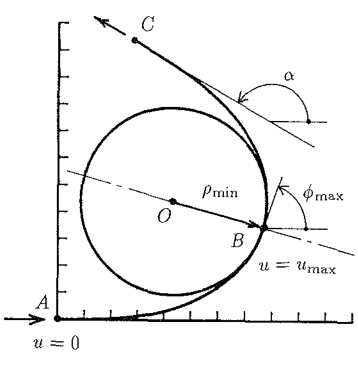


Fig.5 Clothoidal interpolation of angle α

$$\phi_{max}{}^\circ = \frac{\alpha^\circ}{2} \quad , \quad u_{max} = \sqrt{\frac{\phi_{max}{}^\circ}{90^\circ}} \quad , \quad c_{vmax} = \frac{\pi u_{max}}{a} \qquad (9)$$

By cotrolling the scale factor a, the size of the figure changes. By the maximum allowable acceleration, the maximum curvature is limited and the scale factor a is decided.

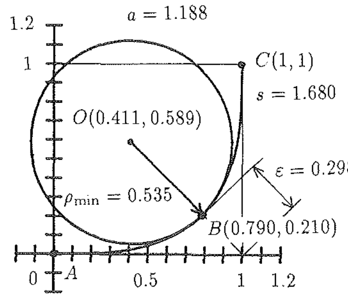


Fig. 6 Clothoidal interpolation of angle 90°

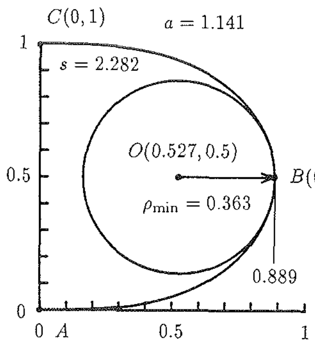


Fig. 7 Clothoidal interpolation of angle 180°

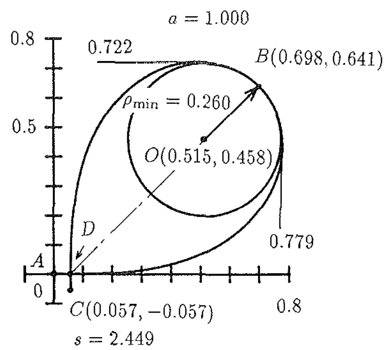


Fig. 8 Clothoidal interpolation of angle 270°

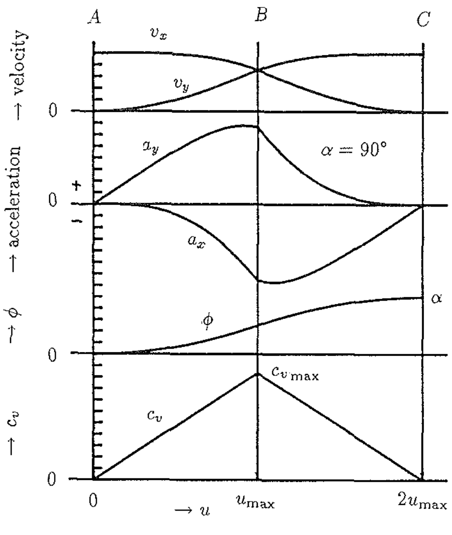


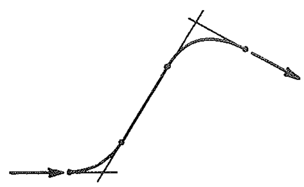
Fig. 9 Continuity of velocity and acceleration

Figures 6, 7, and 8 show the dimentional data for the Clothoidal interpolation for the corner of 90, 180, and 270 degree, respectively. The scale factor is adjusted to ease practical design. In Figure 8, the cross point of the two Clothoid (shown "D") is not at the point A. From point A to D, and from point D to C are not exact straight line, however the difference seems to be neglegible small in practice.

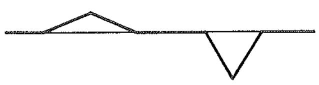
Fig. 9 shows the velocity and acceleration components of each axis together with tangential angle ϕ and curvature c_v. It is known from the figure that the acceleration is changing smoothly.

5. MOBILE ROBOT CONTROL

Kanayama and Miyake (3)introduced Clothoid curves for trajectory generation for mobile robot. They try to introduce Clothoid pair or pairs to interpolate two directed half lines (Fig. 10). The gradient of the curvature, e.g. the curl ratio, (although they call it "sharpness"), is chosen so as to fit the optimum control strategy. These Clothoids are obtained naturally with the moving robot by changing the rotational accelerations of both right and left wheels.



(a)trajectory



(b)curvature

Fig.10 Clothoid pair trajectory (Kanayama)

6. POINT SERIES INTERPOLATION

In the case of welding, painting, sealing, and bonding robots, it is needed to obtain a free curve passing all given series of points, rather than to connect given straight lines. There are many methods or algorisms considered for this problem. However, the easier method is to connect each points by Clothoid pairs. Fig. 11 shows an example of Clothoidal interpolation of five given points (shown in single circles). To limit the change of the direction to some value (in the figure, 45°), several points (shown in dual circles) are added.

If a Clothoid pair has a cross point in their evolutes (the paths of the center of curvature), then a circular arc can be inserted into the two Clothoids (Fig. 12 and 13). Then the curvature diagram becomes to be trapezoidal, and less maximum curvature and less curve length are obtained.

These two methods are, however, not regarded as the best. Because the curvatures at the given points are always zero, the path looks like a little dedicated. Better solutions are being studied.

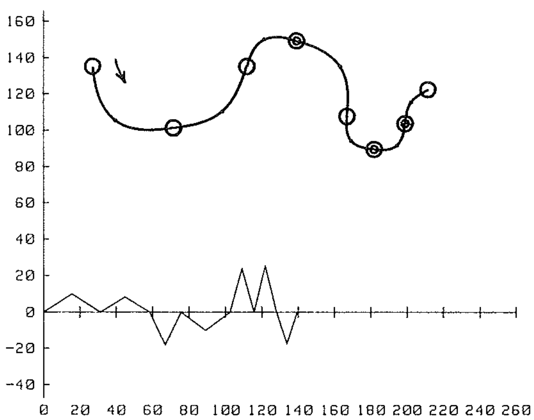


Fig.11 Interpolation by Clothoid pairs

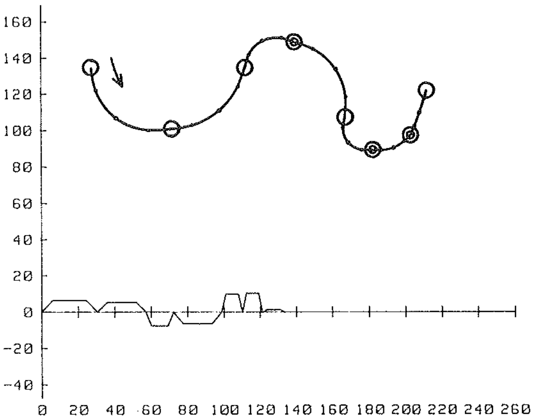


Fig.12 Interpolation by two Clothoid and one arc

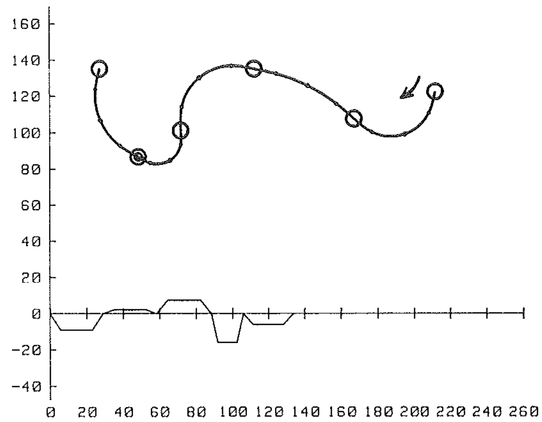


Fig.13 Interpolation by two Clothoid and one arc (direction reversed)

7. EXPERIMENTAL RESULTS

Experiments were done by using the SCARA prototype II robot (Fig.14). Corner interpolation of 90 degree (Fig. 6) of 25 X 25mm was chosen as the given curve and continuous path (CP) control of the robot was made to write the curve on a paper by a ball point pen. A leading 40mm straight line and a following 30mm straight line were connected to the Clothoid Fig. 15).

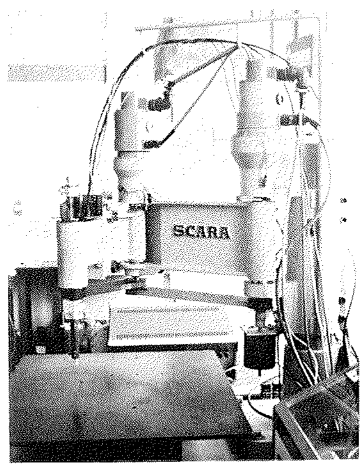


Fig. 14 SCARA prototype II robot

For the sake of comparison, other two curves were added: one was circular arc of 25mm in radius, and the other was circular arc of the same radius as the Clothoide's minimum radius of curvature.

The control algorism of the robot for CP motion is shown in Fig. 16. It consists of two phases: teaching or calculating phase, and playback or executing phase.

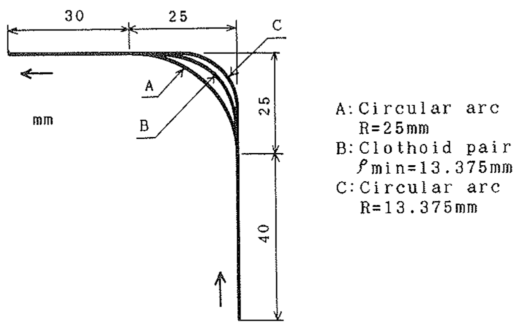


Fig. 15 Given paths for experiment

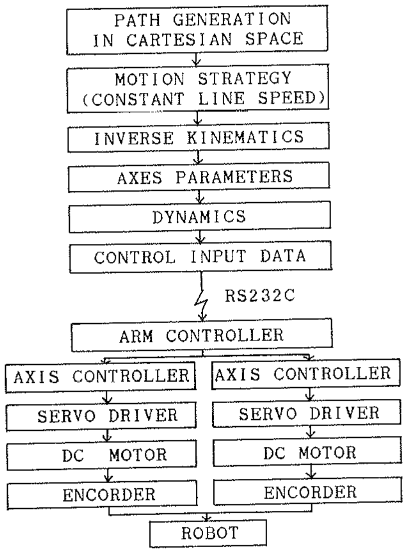


Fig. 16 Control algorism

In the teaching phase, a 16 bit computer (IBM Japan PC 5550) is used. First the path including Clothoid is planned in the Cartesian coordinates, and then the curve is divided into a number of the same pitch points. The velocity, i.e. the passing time for each line segment is given. Next, by the Inverse Kinematics subroutines, the angular position, velocity, and acceleration of each axes are calculated. By using these values, the necessary torques of axes motors are obtaied by dynamic formulae. Thus the input torque values at each points are obtained and with the data of position and velocity, are sent to the robot controller memory through RS232C communication.

The robot controller is built using three Z-80 8bit CPU: one is for arm (robot) control and the other two are for each axis control. Giving the start command, the controller bigins to read data and the robot moves. Position feedback as well as velocity feedback is made in real time. The feedback gain can be adjusted.

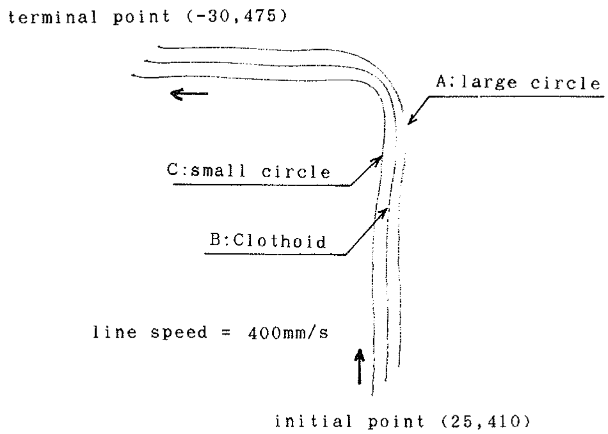


Fig. 17 Experimental results for CP motion

One of the experimental results is shown in Fig. 17. The three curves are shown slightly shifted. The moving speed of the pen was set to 400mm/s. Although it is not so clear, the Clothoid curve is the best precise curve among the three. The amounts of the error pulses are shown in Fig. 18. From these results it is known that the Clothoid interpolation is smoother in curvature change and cause less vibration than circular arcs.

These experimental values are affected by the characteristics of the servo controller. If there exists much delay in it, the difference between curves cannot be distinguished, in the contrary, if the servo is ideal, then the difference of the input curves become problem.

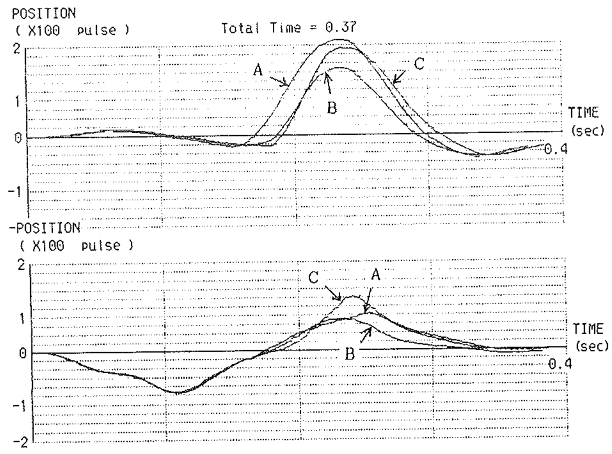


Fig. 18 Positional error pulses

7. CONCLUSION

The Clothoid curve is introduced to interpolate lines and point series. By using the curve, direction and curvature of the interpolated curve is kept continuous, and higher speed continuous path control of the robot is expected. Experimental results showed that it is better than the circular arc interpolation.

In this report the comparison between other free curve interpolation method such as three powered polynomial was not discussed but one of the advantage of the Clothoid is that the curve is mathematically simple, and easy to calculate characteristic values such as direction and curvature.

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APPENDIX

The Fresnel integrals are usually calculated by the following Gilbert's formulae. This expansion is moderate while parameter u is not so large.

$$C_s(u) = \sum_{n=0}^{n}(-1)^{n+1}(\frac{\pi}{2})^{2n+2}\frac{u^{4n+5}}{(2n+2)!(4n+5)}$$

$$S_n(u) = \sum_{n=0}^{n}(-1)^{n+1}(\frac{\pi}{2})^{2n+3}\frac{u^{4n+7}}{(2n+3)!(4n+7)}$$