(10) 1. Translate each of the following sentences into the language of the sentential calculus using the letters indicated for **atomic** sentences.

(a) A sufficient condition for students to be happy is that the exam is neither long nor difficult. (H,L,D)

$$(\neg L \wedge \neg D) \rightarrow H$$

(b) A necessary condition for the students to be happy is that the exam is not difficult. (H, D)

$$H \rightarrow \neg D$$

(10) 2. Translate the following sentences into the language of the Predicate Calculus using the following relation symbols.

Sx:	x is a student
Ex:	x is an exam
Dxy:	x is difficult for y.

a) Some student finds all exams to be difficult.

$$(\exists x)(Sx \wedge (\forall y)(Ey \rightarrow Dyx))$$

b) Some exam is not difficult for some student.

$$(\exists x)(\exists y)(Sx \wedge Ey \wedge Dyx)$$

(10) 3. For each of the following expressions state whether it is a term (t), an atomic formula (a), a formula (f), a sentence (s) or none of these (n). (Circle *all* that apply).

(a) $(\exists x)(x > 1 \wedge x < 0)$	t	a	f	s	n
(b) $[1 - (x^2 + y^2)]^3$	t	a	f	s	n
(c) $(1 < 2) \wedge (2 < 3)$	t	a	f	s	n
(d) $(\exists x)(x = 0 \vee 1)$	t	a	f	s	n
(e) $(x \geq 0) \rightarrow (\exists y)(y^2 = x)$	t	a	f	s	n

(a) f, s
(b) t
(c) f, s
(d) n
(e) f

(15) 4. Find a formula in **conjunctive** normal form logically equivalent to

$$[(X \vee Y) \to Z] \leftrightarrow (Y \vee Z).$$

Call the formula ϕ.
Construct the truthtable for ϕ.
Write down a DNF logically equivalent to $\neg\phi$:

$$\neg\phi \iff (\neg X \wedge Y \wedge \neg Z) \vee (X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge \neg Z)$$

Take the negation of both sides use de Morgan to get

$$\phi \iff (X \vee \neg Y \vee Z) \wedge (\neg X \vee \neg Y \vee Z) \wedge (\neg X \vee Y \vee Z)$$

(10) 5. Show that the following argument is not valid.
Premises: $\neg A \to (B \vee C)$, $\neg B \to (\neg A \wedge D)$, $D \to (B \vee C)$
Conclusion: B

Find a truth assignment that makes the conclusion False and the premises True:

$$B = F, \quad A = F, \quad D = T, \quad C = T$$

(15) 6. Give a formal proof of the conclusion D from the following premises.

$\{1\}$	1.	$C \to (E \to D)$	P
$\{2\}$	2.	$C \to (\neg E \to A)$	P
$\{3\}$	3.	$\neg C \to B$	P
$\{4\}$	4.	$\neg(A \vee B)$	P
$\{4\}$	5.	$\neg A$	4 T
$\{4\}$	6.	$\neg B$	$4T$
$\{3,4\}$	7.	C	$3,6T$
$\{1,3,4\}$	8.	$E \to D$	$1,7T$
$\{2,3,4\}$	9.	$\neg E \to A$	$2,7T$
$\{2,3,4\}$	10.	E	$5,9,T$
$\{1,2,3,4\}$	11.	D	$8,10T$

(15) 7. The following set of premises is inconsistent. Give a formal proof of a contradiction.

{1}	1.	$A \leftrightarrow (B \vee C)$	P
{2}	2.	$B \leftrightarrow \neg A$	P
{3}	3.	$B \vee D$	P
{4}	4.	$\neg C \vee \neg D$	P
{5}	5.	B	P (for IP)
{42,5}	6.	$\neg A$	2, 5 T
{1,2,5}	7.	$\neg(B \vee C)$	1, 6 T
{1,2,5}	8.	$\neg B \wedge \neg C$	7 T
{1,2,5}	9.	$B \wedge \neg B$	5,8 T
{1,2}	10.	$\neg B$	5, 9 IP
{1,2,3}	11.	D	3,10 T
{1,2,3,4}	12.	$\neg C$	4,11 T
{1,2}	13.	A	2,10 T
{1,2}	14.	$B \vee C$	1,13 T
{1,2,3,4}	15.	B	12,14 T
{1,2,3,4}	16.	$B \wedge \neg B$	10,15 T

(15) 8. On the island of TuFa are two kinds of people: Tus who always tell the truth and Fas who always lie. Three inhabitants, A,B and C are interviewed. A and B make the following statements:
A: B is a Fa
B: if A is a Fa, so is C.
Can one determine which kind of person each of A,B and C is? Give clear reasons.

If A is a Tu then B is a Fa. But then what B said is true, contradicting that B is a Fa.

Hence A is a Fa and B is a Tu. So what B said must be true and so C is a Fa.