

- (10) 1. Translate each of the following sentences into the language of the sentential calculus using the letters indicated for **atomic** sentences.

(a) A necessary condition for students to be happy is that the exam is neither long nor difficult. (H,L,D)

$$H \rightarrow (\neg L \wedge \neg D)$$

(b) A sufficient condition for the students to be happy is that the exam is not difficult. (H, D)

$$\neg D \rightarrow H$$

- (10) 2. Translate the following sentences into the language of the Predicate Calculus using the following relation symbols.

Sx : x is a student
 Ex : x is an exam
 Dxy : y is difficult for x .

a) Some student finds all exams to be difficult.

$$(\exists x)(Sx \wedge (\forall y)(Ey \rightarrow Dyx))$$

b) Some exam is not difficult for some student.

$$(\exists x)(\exists y)(Ey \wedge Sx \wedge \neg Dyx)$$

- (10) 3. For each of the following expressions state whether it is a term (t), an atomic formula (a), a formula (f), a sentence (s) or none of these (n). (Circle *all* that apply).

(a) $(x \geq 0) \rightarrow (\exists y)(y^2 = x)$	t	a	f	s	n
(b) $(\exists x)(x > 1 \wedge x < 0)$	t	a	f	s	n
(c) $[1 - (x^2 + y^2)]^3$	t	a	f	s	n
(d) $(1 < 2) \wedge (2 < 3)$	t	a	f	s	n
(e) $(\exists x)(x = 0 \vee 1)$	t	a	f	s	n

- (a) f
 (b) f, s
 (c) t
 (d) f, s
 (e) n

- (15) 4. Find a formula in **conjunctive** normal form logically equivalent to

$$[(X \vee Y) \rightarrow Z] \leftrightarrow (Y \vee Z).$$

Call the formula ϕ .

Construct the truthtable for ϕ .

Write down a DNF logically equivalent to $\neg\phi$:

$$\neg\phi \iff (X \wedge Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z)$$

Take the negation of both sides use de Morgan to get

$$\phi \iff (\neg X \vee \neg Y \vee Z) \wedge (X \vee \neg Y \vee Z) \wedge (X \vee Y \vee Z)$$

- (10) 5. Show that the following argument is not valid.
 Premises: $\neg A \rightarrow (B \vee C)$, $\neg C \rightarrow (\neg A \wedge D)$, $D \rightarrow (B \vee C)$
 Conclusion: C

Find a truth assignment that makes the conclusion False and the premises True:

$$C = F, \quad A = F, \quad D = T, \quad B = T$$

- (15) 6. Give a formal proof of the conclusion D from the following premises.

{1}	1.	$C \rightarrow (\neg E \rightarrow D)$	P
{2}	2.	$C \rightarrow (E \rightarrow A)$	P
{3}	3.	$\neg C \rightarrow B$	P
{4}	4.	$\neg(A \vee B)$	P
{4}	5.	$\neg A$	4 T
{4}	6.	$\neg B$	4T
{3, 4}	7.	C	3, 6T
{1, 3, 4}	8.	$\neg E \rightarrow D$	1, 7T
{2, 3, 4}	9.	$E \rightarrow A$	2, 7T
{2, 3, 4}	10.	$\neg E$	5, 9, T
{1, 2, 3, 4}	11.	D	8, 10T

(15) 7. The following set of premises is inconsistent. Give a formal proof of a contradiction.

{1}	1.	$A \leftrightarrow (B \vee \neg C)$	P
{2}	2.	$B \leftrightarrow \neg A$	P
{3}	3.	$B \vee D$	P
{4}	4.	$C \vee \neg D$	P
{5}	5.	B	P (for IP)
{4,2,5}	6.	$\neg A$	2, 5 T
{1,2,5}	7.	$\neg(B \vee \neg C)$	1, 6 T
{1,2,5}	8.	$\neg B \wedge C$	7 T
{1,2,5}	9.	$B \wedge \neg B$	5,8 T
{1,2}	10.	$\neg B$	5, 9 IP
{1,2,3}	11.	D	3,10 T
{1,2,3,4}	12.	C	4,11 T
{1,2}	13.	A	2,10 T
{1,2}	14.	$B \vee \neg C$	1,13 T
{1,2,3,4}	15.	B	12,14 T
{1,2,3,4}	16.	$B \wedge \neg B$	10,15 T

(15) 8. On the island of TuFa are two kinds of people: Tus who always tell the truth and Fas who always lie. Three inhabitants, A,B and C are interviewed. A and B make the following statements:
A: if B is a Fa, so is C.
B: A is a Fa
Can one determine which kind of person each of A,B and C is? Give clear reasons.

If B is a Tu then A is a Fa. But then what A said is true, contradicting that A is a Fa.
Hence B is a Fa and A is a Tu. So what A said must be true and so C is a Fa.