ECE 302 HW1 Solution

1\. (a) $P_r(A) = P_r(\{1\}) + P_r(\{3\}) + P_r(\{5\})$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$= \frac{1}{2}$

or $P_r(A) = \frac{\text{The number of outcomes in } A}{\text{The total number of outcomes is } S}$

$= \frac{3}{6} = \frac{1}{2}$

(b) $P_r(B) = P_r(\{1\}) + P_r(\{9\}) + P_r(\{11\})$

$= \frac{1}{2}$

(c) $P_r(C) = P_r(\{1\}) + P_r(\{3\}) + P_r(\{9\}) + P_r(\{11\})$

$= \frac{2}{3}$

(d) $A \cup B = \{1, 3, 5, 7, 9, 11\}$

$\Rightarrow P_r(A \cup B) = \frac{6}{6} = 1$

(e) $A \cup C = \{1, 3, 5, 9, 11\}$

$\Rightarrow P_r(A \cup C) = \frac{5}{6}$

(f) $A - C = \{5\}$ $\quad (A-C) \cup B = \{5, 7, 9, 11\}$

$\Rightarrow P_r[(A-C) \cup B] = \frac{4}{6} = \frac{2}{3}$

2\. (a) $S = \{(i, j) : i, j \in \{1, 2, 3, 4, 5, 6\}\}$

the total number of the outcomes in S is 36

i is the outcome of the 1st roll

j - - - - - - - - 2nd roll

(b) i. Actually, there are 18 outcomes statify the requirement that "the sum is even"

$\Rightarrow P_r(\{i+j = \text{even}\}) = \frac{1}{2}$

ii. there are 6 outcomes statisfy the requirment

$\Rightarrow P_r(\{i=j\}) = \frac{6}{36} = \frac{1}{6}$

iii.

i	number of outcomes
1	0
2	1
3	2
4	3
5	4
6	5
total	15

$\Rightarrow Pr(\{i>j\}) = \frac{15}{36} = \frac{5}{12}$

3. Let $W = \{$the selected swith works$\}$

$A = \{$ swith A is selected $\}$

$B = \{ \text{--- B ---} \}$

$C = \{ \text{--- C ---} \}$

We know: $Pr(W|A) = 0.75$

$Pr(W|B) = 0.5$

$Pr(W|C) = 0.25$

$Pr(A) = Pr(B) = Pr(C) = \frac{1}{3}$

(a) $Pr(W) = Pr(W|A)Pr(A) + Pr(W|B)Pr(B) + Pr(W|C)Pr(C)$

$= 0.75 \times \frac{1}{3} + 0.5 \times \frac{1}{3} + 0.25 \times \frac{1}{3}$

$= \frac{1}{2}$

(b) $Pr(\bar{C}|W) = 1 - Pr(C|W)$

$= 1 - \frac{Pr(W|C)Pr(C)}{Pr(W)} = 1 - \frac{\frac{1}{4} \times \frac{1}{3}}{\frac{1}{2}} = \cancel{\frac{5}{6}}$ 2

$= \frac{5}{6}$

4. (a) false

$\overline{A-B} = \overline{A \cap \overline{B}} = \overline{A} \cup B$

~~S~~ counterexample:

$S = \{1,2,3,4,5,6\}$ $A = \{1,2,3\}$ $B = \{1,3,5\}$

$\overline{A-B} = \{1,3,4,5,6\}$ $A \cup \overline{B} = \{1,2,3,4,6\}$

Hense $\overline{A-B} \neq A \cup \overline{B}$

(b) true.

$P_r(A \cup B) = P_r(A) + P_r(B) - P_r(A \cap B)$

$P_r(A \cap B) \geq 0$

Hense $P_r(A \cup B) \leq P_r(A) + P_r(B)$

(c) ~~Pr(A|B) =~~ Pr true.

$P_r(A|B) = \frac{P_r(A \cap B)}{P_r(B)}$

Since $A \subset B$

Hence $A \cap B = A$

$P_r(A|B) = \frac{P_r(A)}{P_r(B)}$

Since $P_r(B) \geq 1$

Hence $P_r(A|B) \geq P_r(A)$

(d) false.

counterexample: let $S = \{1,2,3,4,5,6\}$ $B = \{1,2,3\}$ $A_1 = \{1,3,5\}$

$A_2 = \{4,6\}$

$P_r(B) = \frac{1}{2}$ $P_r(A_1) = \frac{1}{2}$ $P_r(A_2) = \frac{1}{3}$

$P_r(B|A_1) = \frac{P_r(B \cap A_1)}{P_r(A_1)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

$P_r(B|A_2) = \frac{P_r(B \cap A_2)}{P_r(A_2)} = \frac{0}{\frac{1}{3}} = 0$

$P_r(B|A_1)P_r(A_1) + P_r(B|A_2)P_r(A_2) = \frac{2}{3} \times \frac{1}{2} + 0 \times \frac{1}{3} = \frac{1}{3} + 0 = \frac{1}{3} \neq P_r(B)$

(Total probability theorem. requeres. $A_1 \cdots A_n$ are mutually exclusive and collectively exhaustive.)