
PERPETUAL FUTURE CONTRACTS IN CENTRALIZED AND DECENTRALIZED EXCHANGES: MECHANISM AND TRADERS' BEHAVIOR

Erdong Chen

School of Computer Science and Engineering
Nanyang Technological University
50 Nanyang Avenue, Singapore 639798
ed1@e.ntu.edu.sg

Mengzhong Ma

Interdisciplinary Graduate School
Nanyang Technological University
50 Nanyang Avenue, Singapore 639798
mengzhon001@e.ntu.edu.sg

Zixin Nie

School of Computer Science and Engineering
Nanyang Technological University
50 Nanyang Avenue, Singapore 639798
s230135@e.ntu.edu.sg

February 7, 2024

ABSTRACT

This study presents a groundbreaking Systematization of Knowledge (SoK) initiative, focusing on an in-depth exploration of the dynamics and behavior of traders on perpetual future contracts across both centralized exchanges (CEXs), and decentralized exchanges (DEXs). We have refined the existing model for investigating traders' behavior in reaction to price volatility to create a new analytical framework specifically for these contract platforms, while also highlighting the role of blockchain technology in their application. Our research includes a comparative analysis of historical data from CEXs and a more extensive examination of complete transactional data on DEXs. On DEX of Virtual Automated Market Making (VAMM) Model, open interest on short and long positions exert effect on price volatility in opposite direction, attributable to VAMM's price formation mechanism. In the DEXs with Oracle Pricing Model, we observed a distinct asymmetry in trader behavior between buyers and sellers. Such asymmetry might stem from uninformed traders reacting more strongly to positive news than to negative, leading to a tendency to accumulate long positions. This study sheds light on the potential risks and advantages of using perpetual future contracts within the DeFi space while provides mathematical basis and empirical insights based on which future theoretical works can be configurated, offering crucial insights into the rapidly evolving world of blockchain-based financial instruments.

Keywords Blockchain · Perpetual Futures · Volatility · DeFi.

1 Introduction

Over the past decade, blockchain technology has undergone swift and substantial development, garnering an extensive user base exceeding 200 million individuals on a global scale. Among those applications, cryptocurrency derivative tradings are implemented among exchanges organized in both centralized and decentralized manner, within which perpetual futures came to the most prominence with daily trading volume of more than 100 billion¹. Investors prefer

¹At the time of writing, 22 November 2023, the 24H trading volume of perpetual future contracts among all the exchanges is \$109.9 billion (Source: <https://coinalyze.net>).

perpetual future contracts to future contracts, reflected by the far less 24H volume of future contracts (\$1.1 billion), mainly because of the efficiency of perpetuals in hedging and speculation with high leverage while without the need of delivery and rollover. Before 2022, investors trade perpetual futures on centralized exchanges (CEXs), but following the collapse of FTX there emerged many decentralized exchanges (DEXs) carrying perpetual futures, which are operated with more elements on-chain. Since the DEXs are constructed with smart contracts involved, more transparency can be ensured, helping guard against the fraudulence incurred in CEXs.

This study presents a detailed Systematization of Knowledge (SoK) contrasting the operational paradigms of Centralized Exchanges (CEXs) and Decentralized Exchanges (DEXs) in the context of perpetual futures trading. It delves into the distinct mechanisms of order matching, price discovery, and custody of investors' funds in these platforms, elucidating their influence on market microstructure and trader behavior. Unlike CEXs, which utilize an order book system to match orders for price formation, some DEXs match orders with liquidity providers, deriving prices from the relative asset ratios in liquidity pools. This divergence in market making and price discovery mechanisms necessitates a departure from traditional microstructure models, such as Kyle's model (1985) [1], for a comprehensive understanding of DEXs. This study aims to dissect the fundamental design differences between DEXs and CEXs in the realm of perpetual futures, examining their impact on price formation and trader behavior, and to propose empirical insights for potential theoretical models tailored to the microstructure of DEXs. We begin by systematically outlining the structural elements of CEXs and DEXs, followed by an analysis of trader behavior on these platforms, using data spanning from August 2020 to September 2023. This research contributes significantly to the existing literature by offering a detailed exposition of the design elements of DEXs and CEXs, formalizing the trading processes and participant payoffs, thereby laying a foundation for future scholarly work. Our empirical findings validate the design-induced behavioral differences between traders on CEXs and DEXs, marking a pioneering empirical exploration into trader conduct on DEXs and shedding light on potential theoretical frameworks for these platforms.

We categorize CEXs and DEXs based on their Custody Module, with CEXs managing traders' funds off-chain and DEXs recording and managing funds on-chain via smart contracts. DEXs are further classified into three models: Hybrid, Oracle Pricing, and Virtual Market Making (VAMM). The Hybrid Model, straddling CEXs and the other DEX models, still employs limit-order books for trade matching, while the Oracle Pricing and VAMM Models utilize smart contracts. In the latter two, participants are categorized into active traders (Traders) and passive liquidity providers (Liquidity Providers), the latter accepting orders and earning transaction fees for their liquidity provision. The decentralized, transparent nature of DEXs, underpinned by smart contracts, mitigates risks of misconduct and fraud, highlighting their advantages.

We investigate the distinct behaviors of traders on CEXs and DEXs by examining the relationship between the price volatility of the underlying asset, i.e., Bitcoin, and various trading activities, i.e., trading volume, open interest, liquidation, and leverage. Our study encompasses prominent CEXs such as Binance and Bybit, alongside DEXs like GMX, GNS, and Perpetual Protocol V2, with the latter two operating under the Oracle Pricing Model and Perpetual Protocol V2 employing the Virtual Automated Market Making (VAMM) Model.

The empirical evidence suggests a positive correlation between price volatility and trading volumes in CEXs, aligning with traditional futures markets and explainable via Kyle's model [1]. However, the VAMM Pricing Model introduces a differential impact of open interest between long and short positions, attributable to its asset abundance-based price formation mechanism. In contrast, traders in GMX and GNS, adhering to Oracle Pricing, act as price takers, with their trading activity reflecting reactions to underlying asset price changes. This behavior aligns with Shalen's dispersion of beliefs model (1993) [2], highlighting information asymmetry among traders.

Empirical findings indicate a positive correlation between price volatility and trading volumes in CEXs, inversely related to open interests, a pattern consistent with traditional futures markets. This relationship in CEXs, reflective of market depth, aligns with the theoretical underpinnings of Kyle's model (1985) [1]. In contrast, the VAMM Pricing Model introduces a nuanced dynamic in the impact of open interest, varying between long and short positions. This asymmetry is attributable to the VAMM's price formation mechanism, where the rate of change in an asset's relative price inversely correlates with its abundance in the liquidity pool. Consequently, market depth increases with rising open interests in short positions, as the underlying asset accumulates in the liquidity pool, and the reverse holds true for long positions.

In the context of CEXs and Perpetual Protocol V2, traders' activities contribute to price formation. However, in GMX and GNS, traders assume the role of price takers, accepting prices determined by the Price Oracle. Thus, trading activities in GMX and GNS platforms are interpreted as responses to fluctuations in the underlying asset's price. Notably, increased price volatility is associated with heightened trading volumes and reduced open interests, with distinct variations between long and short positions. These empirical observations resonate with the predictions of Shalen's dispersion of beliefs model (1993) [2], which elucidates the asymmetry in information accessibility among

traders. Additionally, our analysis reveals a propensity for uninformed traders to exhibit greater reactivity to positive news compared to negative, as evidenced by more of the accumulated long positions.

This comprehensive examination not only delineates the distinct behavioral patterns of traders across CEXs and DEXs but also provides a deeper understanding of the underlying mechanisms driving these behaviors, offering valuable insights for future research and practical applications in the evolving landscape of digital asset trading. While the primary focus of our research is on Decentralized Exchanges (DEXs) operating with perpetual contracts, the findings and theoretical frameworks developed herein have broader applicability, extending to other trading paradigms within DEXs that exhibit the fundamental bifurcation between Traders and Liquidity Providers. This is exemplified in platforms such as PancakeSwap (<https://pancakeswap.finance>) and Uniswap (<https://uniswap.org>). Furthermore, as the Decentralized Finance (DeFi) ecosystem evolves, the proliferation of synthetic assets is anticipated to enhance the diversification of portfolios managed on DEXs. The insights gleaned from this study are thus of paramount importance to both academic researchers and industry practitioners.

This paper is organized into distinct sections for clarity and depth of exploration. Section 2 provides a discussion of literature related to our work. Section 3 delineates the foundational principles underpinning perpetual trading within both the realms of Decentralized Finance (DeFi) and Centralized Finance (CeFi). In Section 4, we present a systematic and exhaustive survey of the various models associated with perpetual futures, framed within a Systematization of Knowledge (SoK) approach. Section 5 embarks on an empirical investigation into the trading behaviors exhibited on five prominent perpetual futures platforms: GMX, GNS, Perpetual Protocol V2, Binance, and Bybit. The paper culminates with Section 6, where we offer concluding observations and reflections on the implications of our findings for the broader field of blockchain-based financial instruments and their role in the evolving landscape of finance.

2 Related Literature

In broad terms, our study relates to three sorts of literature: those examining DeFi, cryptocurrency future contracts, and Automated Market Making (AMM). The literature on DeFi includes works discussing the fragility and inefficiency of the DeFi lending platforms (e.g., [3, 4, 5]), those examining the liquidation and leverage risks (e.g., [6, 7, 8, 9, 10, 11, 12, 13]), and the ones comparing DeFi and CeFi platforms (e.g., [14, 15]). Qin et al. [10] investigated four major DeFi protocols involving lending (MakerDAO², dYdX³, Aave⁴, and Compound⁵), revealing that liquidators can optimize strategies to maximize profits, potentially increasing borrowers' losses during liquidations. It underscores the inherent risks and instabilities in DeFi markets, particularly through the lens of liquidation mechanics and their impact on market dynamics. Later, Wang et al. [16] analyzed the risks associated with on-chain leverage in the DeFi sector by formalising a model for under-collateralized DeFi lending platforms. They provide evidence that high levels of leverage can lead to significant risks and instabilities. Since liquidation risk is also considered as one of the major risks of perpetual futures, this study evaluates differences in traders' behavior in CEXs and DEXs revealed by liquidation.

Existing literature evaluates cryptocurrency future contracts mainly in terms of their relationship with the spot market. Akyildirim et al. [17] studied the impact of Bitcoin futures on the cryptocurrency market, particularly the introduction of CME and CBOE futures contracts in December 2017. Alexander et al. [18] found that BitMEX derivatives lead the price discovery process over major Bitcoin spot exchanges. Hung et al. [19] identifies substantial pricing effects and breakpoints in market efficiency, indicating the dominant role of Bitcoin futures in price discovery compared to spot markets. As a special case of future contracts, perpetual future contracts are scarcely investigated, although with much higher trading volume. Besides the theoretical discussion on the arbitrage between perpetual future markets and spot markets [20], there lacks enough empirical works examining perpetual future traders' behavior, especially in DEXs. After the pioneering work by Soska et al. [21], which conducted the first analysis on the investor profile for perpetual future contracts in BitMEX, i.e., a CEX, Alexander et al. [22] constructed the optimal hedging strategy with empirical corroboration.

Since some DEXs of perpetual futures adopt VAMM Model, which utilize AMM as the core, our study also shed lights on the effect of traders specifically derived by AMM. There is a emerging sort of research on the economic implication of AMM, most of which, however, focus on the incentive of liquidity provision, driven by the relationship between transaction fee and impermanent loss (e.g., [23, 24, 25, 26]). While the limited existing empirical studies are based on the spot market [25, 27], i.e., Uniswap, our study contribute to the understanding of AMM by conducting the first analysis on the future market supported by AMM from the perspective of traders, enlightening the effect driven by AMM in a new context. As Uniswap and its AMM only consists of a subset of DEXs while other forms of DEXs, i.e.,

²<https://makerdao.com/>.

³<https://dydx.exchange/>.

⁴<https://aave.com/>.

⁵<https://compound.finance/>.

the Hybrid and Oracle Model in this work, have not been reached by the current studies, our study also contribute by clearly defining and examining the implication of decentralization to different extent in exchanges.

3 Background

In this section we elucidate the foundational principles of perpetual trading in both Decentralized Finance (DeFi) and Centralized Finance (CeFi), indispensable for comprehending the innovations discussed in this paper.

3.1 Perpetual Futures

Perpetual futures, a brainchild proposed by Robert J. Shiller in [28], found their foray into the cryptocurrency trading market via BitMEX in 2016 (<https://bitmex.com/>). These futures, a distinctive variant of the traditional futures contract, eschew a fixed delivery date, thereby allowing for an indefinite tenure without necessitating contract rollovers as expiration looms.

In traditional futures contracts, the basis, or the price differential between the futures and the spot price, progressively narrows as the contract nears its expiration date, a phenomenon predominantly influenced by arbitrage activities. In contrast, perpetual futures contracts, which lack a predetermined expiration date, implement a mechanism known as funding fees to align their trading prices with the underlying spot prices. This mechanism operates within a specified interval, wherein the exchange imposes funding fees on either long or short positions, redistributing these fees to the counterparty. The magnitude and direction of the funding rate are contingent upon the prevailing disparity between the prices in the perpetual futures and spot markets. For example, a scenario where the perpetual futures price exceeds the spot market price results in a positive funding rate, necessitating long positions to compensate short positions through funding fees. The extent of the price discrepancy directly influences the funding rate, thereby determining the proportional funding fees imposed on long positions [20].

Another significant characteristic of perpetual futures is the provision of higher leverage. For BTC-USD perpetual contract trading pairs, BitMEX supports a maximum leverage of up to $100\times$, while some smaller exchanges even offer leverage of up to $1001\times$ ⁶. In contrast, for BTC delivery futures, major exchanges typically offer a maximum leverage of only $25\times$.

As of recent, perpetual futures have burgeoned as the most coveted financial derivative in the cryptocurrency market. In the years 2021 and 2022, the nominal trading volume for Bitcoin perpetual futures achieved \$51,989bn and \$39,806bn, respectively [20], significantly surpassing other derivative transactions, including options⁷.

Notwithstanding, a sizable fraction of perpetual futures trading is on centralized platforms, leading many traders to harbor reservations regarding the inherent risks of centralized trading. However, courtesy of DeFi, traders can now participate in perpetual futures trading on decentralized platforms. While CEXs substantially eclipse DEXs in terms of nominal trading volume, the growth rate of DEX trading volume significantly outpaces that of CEX. Prevalent perpetual DEX frameworks encompass the Hybrid Model, Oracle Pricing Model, and VAMM Model.

3.2 Ethereum

Ethereum, conceptualized by Vitalik Buterin[29], epitomizes a blockchain furnished with an integral Turing-complete programming language and has subsequently ascended as the preeminent widely utilized public chain. Ethereum endeavors to instantiate an alternative protocol for the development of decentralized applications, as delineated in its whitepaper [29]. It facilitates users to instantiate and run smart contracts on its decentralized framework. Through the agency of these smart contracts, developers are empowered to devise multifarious programs to articulate varied application logic, thereby proliferating a vast spectrum of internet-based applications.

3.3 Decentralized Finance (DeFi)

Decentralized Finance, abbreviated as DeFi, characterizes a financial paradigm anchored in the robustness and immutability of distributed ledger technology [30]. In terms of capital, a preponderance of DeFi DApps are architected on Ethereum's infrastructure. As of December 2021, the aggregated value locked in DeFi reached a peak of over \$178 billion, primarily concentrated on the Ethereum blockchain. These DeFi DApps capacitate users to fulfill many financial objectives, like Spot trading, Lending, Liquidity Staking, and Derivative trading, simultaneously proffering features not present in CeFi instruments, notably trustless asset custody and complete transparency.

⁶On ApolloX DEX (<https://www.apollox.finance/en>), users can use leverage up to $1001\times$ in Degen Mode.

⁷<https://www.theblock.co/data/crypto-markets/options>.

3.4 Decentralized Exchange (DEX) and Centralized Exchange (CEX)

Two distinct categories of trading platforms facilitate the exchange of blockchain assets: Centralized Exchanges (CEX) and Decentralized Exchanges (DEX). Centralized exchanges, or CEXs, are under the purview of a singular corporate entity, while decentralized exchanges, or DEXs, facilitate trading devoid of intermediaries. A substantial portion of DEXs found their inception within the Ethereum network. Noteworthy is the fact that decentralized exchanges function in a manner that avoids single-entity ownership and operation. Among these DEXs, Uniswap (<https://uniswap.org>) stands out for its pioneering deployment of a novel trading paradigm denoted as an automatic liquidity protocol. Launched in 2018, the Uniswap platform extends support to a comprehensive spectrum of ERC-20 tokens and interfaces seamlessly with decentralized wallet services, including MetaMask (<https://metamask.io>) and WalletConnect (<https://walletconnect.com>).

Conversely, Binance (<https://www.binance.com/en>) operates as a CEX and does not exercise control over or administer PancakeSwap. PancakeSwap closely mirrors the operational framework of Uniswap. It is specifically tailored for BEP-20 tokens operating on the Binance Smart Chain, characterized by superior transaction speed and significantly reduced fees.

3.5 Stablecoins

A stable currency is a digital currency with a relatively stable market price and is legal tender in the digital currency market. Stablecoins were born out of excessive price volatility in the digital currency market, which required a relatively stable digital currency to provide liquidity and serve as a benchmark for other funds. At the same time, a currency in the virtual world is connected to assets in real life, and the first stable currency issued by USDT emerged as a bridge connecting real assets and serving as a safe-haven product in the digital currency market when the market experiences large capital outflows.

USDT is a stable currency issued by the Tether company (<https://tether.to/en/>) and is currently the largest stable currency in circulation. USDC is a stable currency issued by Circle. DAI is a decentralized stable currency issued on the Ether chain. USDT and USDC operate by collateralizing the U.S. dollar and guaranteeing 1:1 payment.

3.6 Price Oracles

Given the constraints of blockchains in accessing off-chain data, DeFi DApps invariably require certain centralized mechanisms, colloquially termed 'Oracles'. Price Oracles predominantly extract recent transactional data from preeminent exchanges, calculate a weighted average price governed by certain criteria, and subsequently dispatch this data to smart contracts, serving as a conduit between DeFi and the external environment.

3.7 Liquidity Provider

Liquidity Providers, pivotal constituents in the DeFi ecosystem, possessing specific assets, contribute them to Liquidity Pools orchestrated by smart contracts. This facilitates catering to the liquidity requisites of diverse traders in scenarios including lending, spot trading, and derivatives trading. Simultaneously, these Liquidity Providers levy various fees from traders.

3.8 Automated Market Maker (AMM)

Centralized exchanges predominantly utilize the limit order-book model (LOB) for the processes of price discovery and order matching, harmonizing buyers' propositions with sellers' offerings. Conversely, decentralized exchanges have gravitated towards the Automated Market Maker (AMM) owing to its congruence with blockchains of lower throughput. The AMM smart contract, dictated by a prespecified algorithm, ascertains particular trading prices and taps into the reserves of liquidity providers to accommodate traders' transactional needs.

Uniswap, the most prominent spot decentralized exchange on the Ethereum network, employs the constant product market maker (CPMM) [31]. CPMMs are based on the function $x \times y = k$, where x and y denote the quantities of two distinct tokens in the liquidity pool, and their product, k , represents the pool's liquidity depth. In this model, a transaction results in an increase in one token's quantity and a corresponding decrease in the other, maintaining the constancy of their product. This mechanism ensures liquidity equilibrium within the pool during trading activities.

4 Systematization of Knowledge: Perpetual Trading Exchanges and on-chain Protocols

In this section, we illustrate designs of the perpetual future contracts trading system for DEXs and CEXs, with emphasis on their differences and similarities. In Section 4.1, we delineate the key stakeholders and elements of perpetual exchange systems, as illustrated in Fig 1. We formalize the perpetual exchange model using the notations in Table 1. Secondly, we summarized the differences between various models in Section 4.2. Subsequently, in Sections 4.3 to 4.6, we present the distinct architectures of each model through system diagrams. Then we characterize the unique properties of each model using mathematical formulations. Furthermore, utilizing a specific case, we elaborate on the operational processes of each model.

4.1 Fundamental Terminology and Concepts

The key stakeholders and elements are as follows:

- Traders: Individuals or entities engaged in the purchase and sale of perpetual contracts. These traders furnish collateral to maintain and manage their positions through the trading of such contracts.
- Custody Module: This module is responsible for ensuring the security of assets across all trader accounts. It consistently updates and retains the latest balance details and facilitates both deposit and withdrawal operations initiated by traders.
- Matching Module: This module is entrusted with storing, correlating, and executing purchase and sale orders of contracts.
- Risk Control Module: This module is vital for assessing and supervising the position of every trader account, contingent on the orders that have been executed. Its role is pivotal in ascertaining that the provided collateral is sufficient to offset potential deficits. Furthermore, in specific scenarios, it assumes control of a trader's position and proceeds with its liquidation.

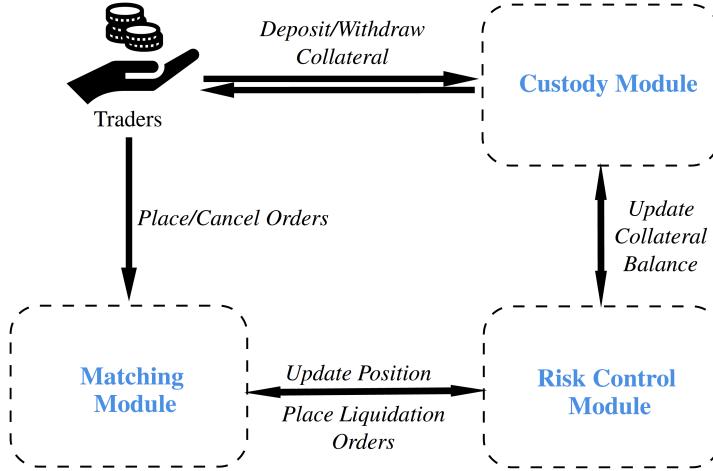


Figure 1: Elements of Exchange Systems

We delineate a formalized model of perpetual contract trading. Additionally, Table 1 is presented to encapsulate and summarize the notations employed throughout this paper.

We denote a perpetual contract trading exchange as $EX = (\mathbb{U}, \mathbb{C}, \mathbb{T}, \mathbb{P})$, where \mathbb{U} denotes the set of underlying assets; \mathbb{C} denotes the set of collateral assets; \mathbb{T} denotes the set of traders; \mathbb{P} denotes the set of positions.

Next, we define the most fundamental element in trading, the order, as $O_i = (U, C)$, where the i -th order is based on U in \mathbb{U} and C in \mathbb{C} . For each order, three key variables are considered: $Und(O_i)$ indicates the underlying asset and quantity of the i -th order, with positive values indicating long and negative values indicating short; $Coll(O_i)$ represents the amount of collateral provided by the trader for the order; $P^{U \rightarrow C}(O_i)$ denotes the execution price of the order. The j -th position, denoted as P_j , is a compilation of a set of k executed orders. Therefore, the net exposure of the j -th

Table 1: Notation Summary

Notations	Definitions
EX	Perpetual contract trading exchange.
$O_i = (U, C)$	An order that provides asset C as collateral to trade the underlying asset U .
$Und(O_i)$	Amount of underlying asset in O_i , positive values indicate long, while negative values indicate short.
$Coll(O_i)$	Amount of collateral asset in O_i .
$P_t^{U \rightarrow C}(O_i)$	Price of U in the unit of C in O_i .
$TF(O_i)$	Transaction fees of O_i .
$P_j = (U, C)$	A position that provides asset C as collateral to hold the exposure on underlying asset U .
$E_t(P_j)$	Amount of net exposure in the unit of U in P_j at time t , positive values indicate long, while negative values indicate short.
$Coll_t(P_j)$	Amount of collateral in P_j at time t .
$\bar{P}_t^{U \rightarrow C}(P_j)$	Average holding price of U in the unit of C in P_j at time t .
$MR_t(P_j)$	Margin ratio of P_j at time t .
$PnL_t(P_j)$	Profit and loss in P_j at time t , positive values indicate profit, while negative values indicate loss.
$LP(P_{Liq_j})$	Liquidation penalty of P_j .
$FF_{t_0 \rightarrow t_1}(P_j)$	Funding fees of P_j from t_0 to t_1 .
$BF_{t_0 \rightarrow t_1}(P_j)$	Borrowing fees of P_j from t_0 to t_1 .
$MR_{min}(U)$	Maintenance margin ratio of underlying asset U .
FR_t	Funding fees rate at time t , positive rate indicates payment from long to short, while negative rate indicates payment from short to long.
BR_t	Borrowing fees rate at time t .
TR_{id}	Transaction fees rate of a trader.
$P_t^{U \rightarrow C}$	Price of underlying asset in the unit of collateral at time t .
$WV_t(T_{id})$	Withdrawable value in USD of a trader.
$VL_t(LP_{id})$	Value in USD of the liquidity that a liquidity provider provided at time t .
$V_{id} = (VA, VD, C)$	A vault.
$VA_t(V_{id})$	Amount of virtual token in asset sheet of V_{id} at time t .
$VD_t(V_{id})$	Amount of virtual token in debt sheet of V_{id} at time t .
$Coll_t(V_{id})$	Amount of collateral in V_{id} at time t .
$LP(V_{id})$	Liquidation penalty of V_{id} .
$IL_t(V_{id}^{LP})$	Impermanent loss from providing liquidity of V_{id} at time t .
$TF_t(V_{id}^{LP})$	Transaction fees earned by providing liquidity of V_{id} at time t .

position at time t can be represented as:

$$E_t(P_j) = \sum_{i=1}^k Und(O_i). \quad (1)$$

The margin ratio $MR_t(P_j)$ of the j -th position at time t can be represented as:

$$MR_t(P_j) = \frac{P_t^{U \rightarrow C} \times |E_t(P_j)|}{Coll_t(P_j)} \times 100\%. \quad (2)$$

When $MR_t(P_j)$ is less than $MR_{min}(U_{P_j})$, which is the maintenance margin ratio required by the exchange, P_j will be taken over by the exchange and liquidated at an appropriate time.

When P_j is liquidated, the collateral is not only used to cover losses but also incurs a liquidation penalty charged by the exchange⁸. This fee is also referred to as the Insurance Clearance fee in some exchanges⁹. As different exchanges have

⁸<https://help.dydx.exchange/en/articles/4797401-perpetual-contract-liquidations>.

⁹<https://www.binance.com/en/support/faq/liquidation-protocols-360033525271>.

Table 2: Comparison of different exchange models

Type	Custody	Matching	Risk Control	Counterparty	Pricing
CEX	Hot & Cold Wallet	Off-chain Server	Off-chain Server	Traders	Traders
Hybrid	Smart contract	Off-chain Server	Off-chain Server	Traders	Traders
Oracle Pricing	Smart contract	Smart contract	Off-chain Keepers	Liquidity Providers	Oracle
VAMM	Smart contract	Smart contract	Off-chain Keepers	Liquidity Providers	AMM Algorithm

diverse methods for calculating the liquidation penalty, we do not provide a unified formula, rather denoting it simply as $\text{LP}(P_{Liq_j})$.

In addition, the average holding price $\bar{P}_t^{U \rightarrow C}$ of the j -th position at time t can be represented as:

$$\bar{P}_t^{U \rightarrow C}(P_j) = \frac{\sum_{i=1}^k (\text{Und}(O_i) \times P_t^{U \rightarrow C}(O_i))}{\sum_{i=1}^k \text{Und}(O_i)}. \quad (3)$$

The profit and loss (P&L) of the j -th position at time t can be represented as:

$$\text{PnL}_t(P_j) = (P_t^{U \rightarrow C} - \bar{P}_t^{U \rightarrow C}(P_j)) \times E_t(P_j). \quad (4)$$

From t_0 to t_1 , the funding fee paid or received by a trader for their held positions P_j can be represented as:

$$\text{FF}_{t_0 \rightarrow t_1}(P_j) = \sum_{t=t_0}^{t_1} (FR_t \times E_t(P_j) \times P_t^{U \rightarrow C}), \quad (5)$$

where FR_t denotes the funding rate.

The trading fees paid by a trader for the orders O_i can be represented as:

$$\text{TF}(O_i) = TR_{id} \times |\text{Und}(O_i)| \times P_t^{U \rightarrow C}(O_i), \quad (6)$$

where TR_{id} is the transaction fee rate for the id -th trader.

At time t , for the id -th Trader who executed k orders to open n positions, where m positions (Liq_1, \dots, Liq_m) were liquidated and $(n - m)$ positions were closed actively, then the withdrawable value he can ultimately access at time t can be represented as:

$$\begin{aligned} \text{WV}_t(T_{id}) = & \sum_{j=1}^n \text{Coll}(P_j) + \sum_{j=1}^n \text{PnL}_t(P_j) - \sum_{j=1}^m \text{LP}(P_{Liq_j}) - \\ & \sum_{j=1}^n \text{FF}_{t_0 \rightarrow t}(P_j) - \sum_{i=1}^k \text{TF}(O_i). \end{aligned} \quad (7)$$

4.2 Model of Exchange Systems

The modules employed by different trading systems (models) vary significantly. In Table 2, we provide a horizontal comparison across five dimensions: Custody, Matching, Risk Control, Counterparty, and Pricing. In Fig. 2 we compare the efficiency and centrality of transaction processing among different models.

The Custody Module assumes a central role in demarcating the distinct classifications within the realm of perpetual futures exchanges. Those that employ centralized custody solutions are classified as Centralized Exchanges (CEX), whereas those that abstain from such practices fall under the rubric of Decentralized Exchanges (DEX). Within the domain of DEX, further subdivisions emerge contingent upon factors encompassing order matching, price determination, and counterpart matching, resulting in the Hybrid Model, Oracle Pricing Model, and Virtual Auto Market Making (VAMM) Model.

The CEX Model closely resembles traditional financial practices. Apart from deposits and withdrawals, it makes limited use of blockchain technology. The technology stack used for custody of client funds, order matching, settlement, risk

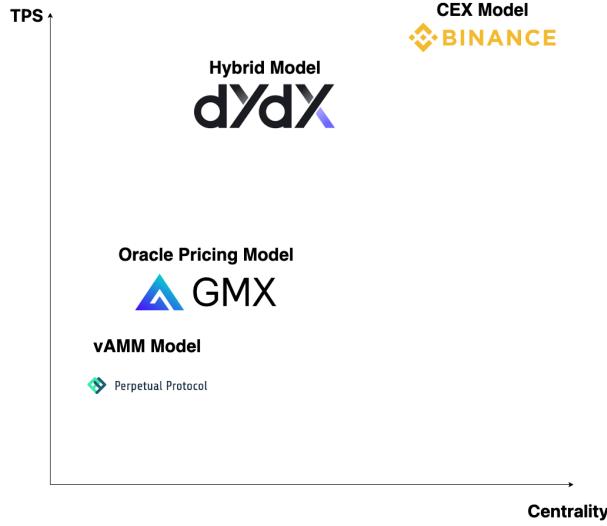


Figure 2: Transaction Per Section (TPS) and Centrality Comparison. In this figure, under each model there exemplifies a corresponding exchange. CEX Model: Binance; Hybrid Model: dYdX (<https://dydx.exchange>); Oracle Pricing Model: GMX (<https://gmx.io>); and VAMM Model: Perpetual Protocol V2 (<https://perp.com>).

assessment, and control closely aligns with traditional financial practices. Due to the imperfect state of infrastructure and regulatory rules in the cryptocurrency trading industry, CEX often combines the functions of custody banks, traditional exchanges, clearinghouses, and brokerages entities.

In contrast, the Hybrid Model harnesses smart contracts for custody and settlement processes, while retaining certain centralized elements. Notably, off-chain servers persist in facilitating order matching and counterparty matching functions.

The Oracle Pricing Model and VAMM Model gravitate toward a greater degree of decentralization. Both models leverage smart contracts for order matching, with Liquidity Providers assuming the role of direct counterparties to traders, engendering an indirect mode of trade execution among traders. Nevertheless, it is imperative to acknowledge that both models still rely upon centralized constituents. The involvement of Oracles and Keepers in the Risk Control Module remains a requisite for effecting the liquidation process. Moreover, the Oracle Pricing Model hinges upon Oracles for the determination of precise trade prices.

The augmentation of decentralization engenders certain trade-offs. While diminishing reliance on centralized components can mitigate specific security vulnerabilities and fortify resistance against censorship, it concurrently imposes constraints on the exchange's transaction processing capacity per second (TPS). As one progresses from the CEX Model to the Hybrid Model, Oracle Pricing Model, and VAMM Model, the degrees of centralization diminish. However, correspondingly, the ability to handle transactions experiences a commensurate decline due to the TPS limitations inherent in the underlying blockchain network, as shown in Fig. 2. Consequently, blockchain developers have embarked upon the development of diverse technologies aimed at enhancing throughput and efficiency. For instance, Scroll¹⁰ constitutes a zero-knowledge scaling solution tailored for Ethereum (zkEVM), characterized by diminished costs, accelerated transaction speeds, and infinite scalability, all without compromising the twin tenets of security and decentralization.

4.3 Centralized Exchange Model

Considering that the CEX Model closely resembles traditional commodity futures exchanges, we will prioritize its introduction. Building upon the CEX Model as a foundation, we will emphasize the differences among CEX Model and other models.

¹⁰<https://scroll.io/>.

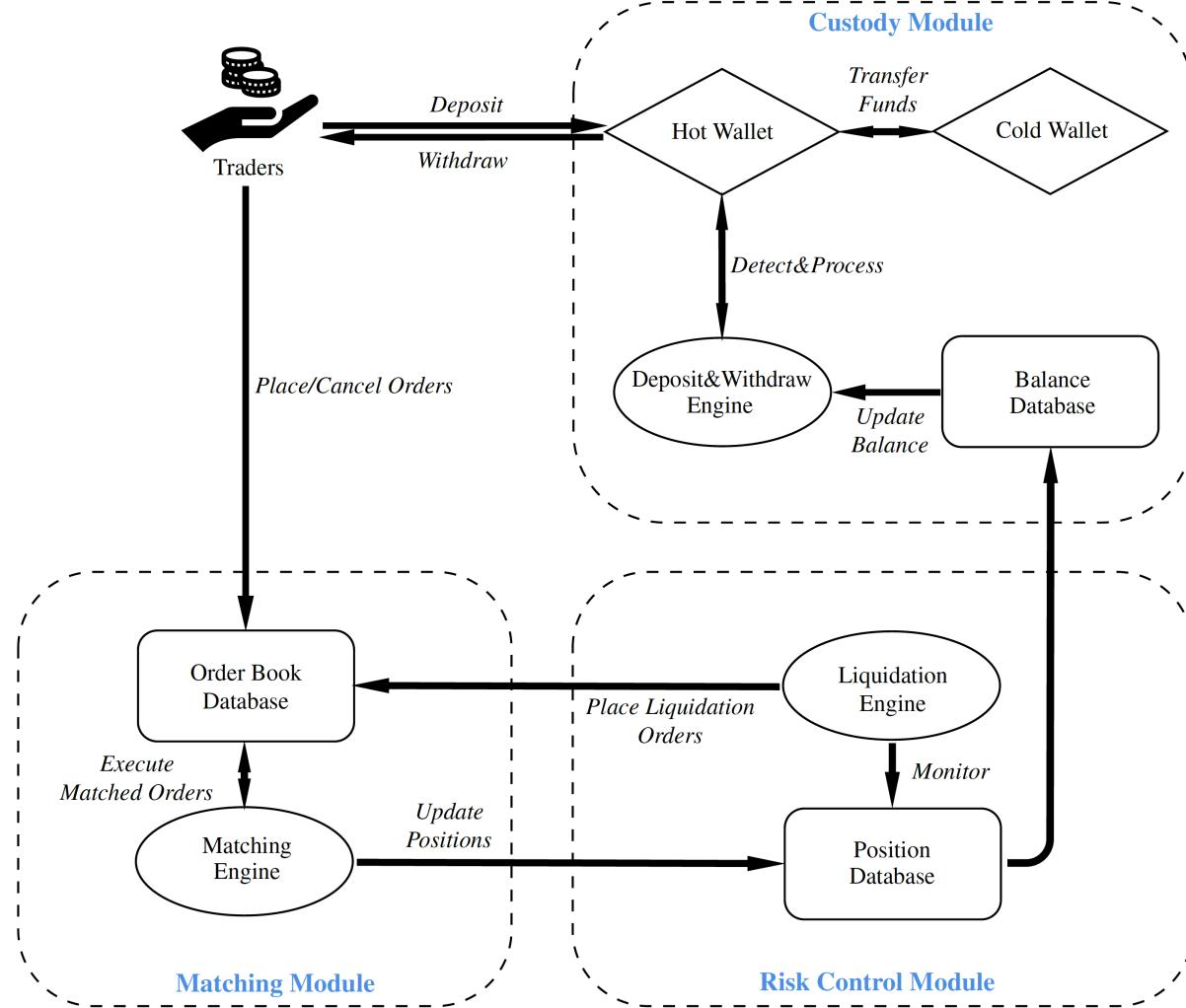


Figure 3: CEX Model. In this diagram, diamonds represent on-chain smart contracts, rectangles represent off-chain databases, and ellipses represent off-chain servers.

4.3.1 Architectures

In accordance with Figure 1, we provide an enhanced delineation and representation of the constituents and stakeholders in the Centralized Exchange Model, as depicted in Figure 3, where diamonds represent on-chain smart contracts, rectangles represent off-chain databases, and ellipses represent off-chain servers.

Within a Centralized Exchange, traders can be bifurcated into two primary categories:

1. Retail Traders: These are users who interact via the front-end interface provisioned by the exchange.
2. API Traders: Engaging through the API interface extended by the exchange, these users are conventionally known as Pro Traders, Institutional Traders, or Market Makers. While the essence of the orders remains invariant regardless of the interaction medium, API Traders possess the capability to initiate or terminate orders at an augmented velocity, thereby enabling sophisticated trading methodologies such as high-frequency quantitative trading. Given that API traders often engage in higher-frequency trading, resulting in significantly higher trading volumes compared to retail traders, their TR_{id} tend to be much lower than those of the latter. This opaque fee structure provides API traders with a significant advantage over their counterpart.

Custody module predominantly functions off-chain, with the sole exception of the on-chain hot wallet. Notably, private keys are stored off-chain.

1. Hot Wallet: This digital wallet is designed to cater to the routine deposit and withdrawal requirements of traders. Functioning under the auspices of the exchange, it maintains a specified percentage of the exchange's total assets, sufficient to accommodate the peak daily withdrawal demands.
2. Cold Wallet: Acting as the primary repository for the exchange's assets, this wallet is entirely offline. This segregation between the Hot and Cold Wallets amalgamates efficient user transactions with fortified security.
3. Deposit & Withdrawal Engine: Comprising both the blockchain's complete nodes and off-chain servers, this engine supervises on-chain deposit maneuvers, operationalizes user withdrawal directives, and refreshes the off-chain Balance Database post on-chain transaction validation.
4. Balance Database: An off-chain server database accountable for documenting the balance of every trader's account.

Matching module consists of two off-chain components.

1. Order Book Database: An off-chain server database responsible for logging various types of orders.
2. Matching Engine: Operating off-chain, this server application is designated for order matching and execution, subsequently conveying the execution outcomes to the Position Database.

Risk control module consists of two off-chain components.

1. Position Database: This off-chain server database is mandated to record the margin balance and P&L (Profit & Loss, aka PNL) for each position, communicating account balance modifications to the Balance Database.
2. Liquidation Engine: An off-chain server program that monitors the Position Database and takes over positions that meet liquidation criteria, closing them under appropriate conditions.

4.3.2 Properties

In the CEX Model, three equivalent relationships exist, as follows:

Firstly, traders act as counterparts to each other. Whenever one trader sells contract, there is another trader matched to buy this contract. Therefore, while it may not always be possible to find two traders with exactly opposite but equally sized positions, the absolute value of the net exposure for all long positions always equals the absolute value of the net exposure for all short positions. So, assuming a CEX has p long positions and q short positions, we can derive:

$$\sum_{j=1}^p E_t^{\text{Long}}(P_j) = - \sum_{j=1}^q E_t^{\text{Short}}(P_j). \quad (8)$$

Furthermore, from t_0 to t_1 , the total funding fees paid by all long positions must equal the total funding fees received by all short positions, and the total profit of all long positions equals the total loss of all short positions. This leads to:

$$\sum_{j=1}^p FF_{t_0 \rightarrow t_1}^{\text{Long}}(P_j) = - \sum_{j=1}^q FF_{t_0 \rightarrow t_1}^{\text{Short}}(P_j), \text{ and} \quad (9)$$

$$\sum_{j=1}^p PnL_{t_0 \rightarrow t_1}^{\text{Long}}(P_j) = - \sum_{j=1}^q PnL_{t_0 \rightarrow t_1}^{\text{Short}}(P_j). \quad (10)$$

4.3.3 Case Study

The life cycle of a position on a CEX can be described as follows:

1. Deposit Funds: Funds are deposited by traders into the exchange's specified on-chain address. Following on-chain transaction validation, traders are enabled to designate a portion or entirety of the funds as collateral for derivative trading.
2. Create and Place Order: Traders opt for order types and input the pertinent parameters to craft an order tailored to their trading objectives. Post order validation, it is transmitted to the exchange's servers.
3. Execute Order and Initiate Position: Successful order matching and its execution influence the trader's position. If the trader did not hold any position in the underlying asset before the trade, the order is considered an opening position operation.

4. Add or Reduce Position: Traders can revisit Steps 2 and 3, executing additional orders to modify the size or direction of their positions.
5. Close Position: Traders can manually close positions or might be forced to close due failing to meet the maintenance margin requirement.
 - (a) Voluntary Closure: Employing Step 2, traders can dispatch orders opposing to their current position, to close their positions and settle actual profits and losses.
 - (b) Forced Closure (Liquidation): During the tenure of a position, if collateral adequacy falters below the maintenance margin threshold, the liquidation engine intercedes. It compulsorily concludes the position, contingent on market liquidity parameters, and imposes a certain percentage of penalty. Residual collateral post loss and penalty settlement is reimbursed to the trader. If there's a deficit, the trader must cover it before continuing to trade.
6. Withdraw Funds: Concurrently with position holding, traders are empowered to retract unutilized funds. Upon position closure, residual collateral can be withdrawn post the reconciliation of profits and losses.

4.4 Hybrid Model

4.4.1 Architectures

The Hybrid Model is a DEX Model that bears the closest resemblance to the CEX Model. Based on Fig. 3, we further delineate and illustrate the actors and components of the Hybrid Model in Fig. 4. In each figure illustrating the DEX models, we use blue background to indicate elements differentiating from the CEX Model, while diamonds represent on-chain smart contracts, rectangles represent off-chain databases, and ellipses represent off-chain servers.

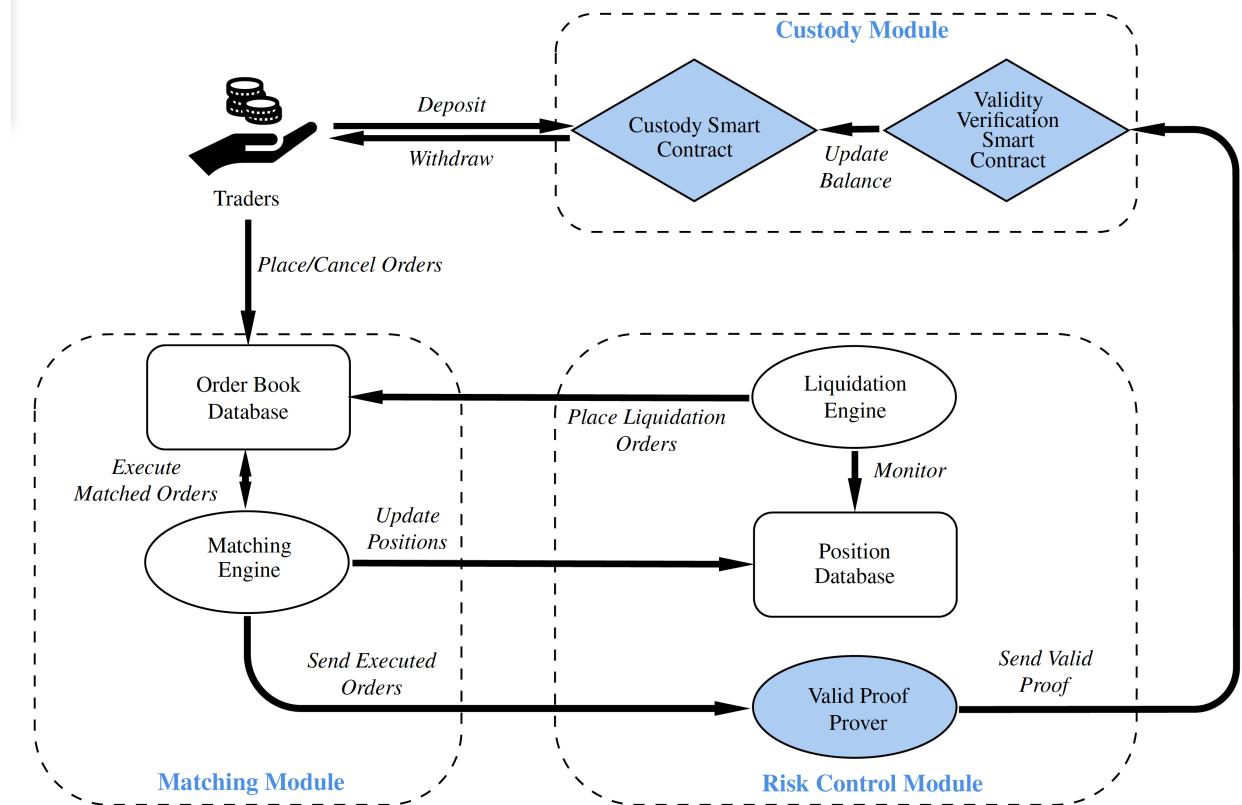


Figure 4: Hybrid Model. In this diagram, diamonds represent on-chain smart contracts, rectangles represent off-chain databases, and ellipses represent off-chain servers. We use blue background to indicate elements differentiating Hybrid Model from the CEX Model.

The hybrid nature of the Hybrid Model lies in its use of off-chain components, running on centralized servers, for trade matching and risk control against liquidation. Simultaneously, it utilizes smart contracts and a valid proof system, e.g.,

Zero Knowledge (ZK) proof¹¹, to manage fund custody and verify that off-chain transaction results are genuine and valid.

This approach addresses the issues of non-transparency and centralization in the CEX Model, ensuring that exchange operators cannot scrutinize trader withdrawals, fabricate trader transactions, or misappropriate trader funds, thus mitigating risks similar to those witnessed in incidents such as FTX¹².

In terms of key stakeholders and elements, the Hybrid Model shares similarities with the CEX Model regarding the composition of traders and the Matching Module. The distinctions lie in the Custody Module and the Risk Control Module.

Custody Module is composed of two on-chain smart contracts.

1. Custody Smart Contract: An on-chain smart contract responsible for safeguarding funds deposited by traders, storing the balance status of each trader account, and handling traders' deposit and withdrawal needs.
2. Validity Verification Smart Contract: An on-chain smart contract tasked with verifying the valid proof corresponding to off-chain transactions to confirm the correctness and reliability of transaction results. It updates the balance status of each trader account in the Custody Smart Contract based on valid transactions.

Risk Control Module is comprised of three off-chain components.

1. Position Database: An off-chain server database responsible for recording the margin balance and P&L of each position, and relaying changes in trader account balances to the Balance Database.
2. Liquidation Engine: An off-chain server program that monitors the Position Database and takes over positions that meet liquidation criteria, closing them under appropriate conditions.
3. Valid Proof Prover: An off-chain server program responsible for packaging successfully executed orders into batches, compressing them, and generating the corresponding valid proof, such as Zero Knowledge Proof. Subsequently, these proofs are transmitted to the on-chain Validity Verification Smart Contract for validation.

4.4.2 Properties

The Hybrid Model shares the same mathematical properties with CEX, as expressed in Eq. (8) to (10), albeit with different technological implementations.

4.4.3 Case Study

The life cycle of a position on a DEX with Hybrid Model can be described as follows:

1. Deposit Funds: Traders deposit funds into the custody smart contract. Once confirmed by the exchange, traders can allocate a portion or the entirety of the funds as collateral for perpetual trading.
2. Create and Place Order: Same as in the Centralized Exchange Model.
3. Execute Order and Initiate Position: When an order is successfully matched and executed, the trader's position is affected. To ensure trading responsiveness, the executed orders are not immediately confirmed on-chain. Instead, the exchange server provides a pre-confirmation, also known as "soft confirm". All pre-confirmed orders are batched and valid proofs are generated offline by the Prover. At predefined intervals, proofs are uploaded to the on-chain Validation Verification Smart Contract. Subsequently, traders' balances are updated, pre-confirmed orders receive their final confirmation, or "hard confirm." In practice, whether the original order data are also stored on-chain is flexible. Not storing them on-chain reduces gas costs, while storing them enhances data availability, thereby increasing security.
4. Add or Reduce Position: Same as in the Centralized Exchange Model.
5. Close Position: Same as the Centralized Exchange Model.
6. Withdraw Funds: Unlike the Centralized Exchange Model, the Hybrid Model includes three mechanisms for withdrawal: Slow Withdrawal, Fast Withdrawal, and Forced Withdrawal^{13 14}.

¹¹<https://ethereum.org/en/zero-knowledge-proofs/>.

¹²<https://www.investopedia.com/what-went-wrong-with-ftx-6828447>.

¹³<https://help.dydx.exchange/en/articles/5108558-withdrawing-funds-from-layer-2>.

¹⁴<https://help.dydx.exchange/en/articles/4797365-what-is-promising-me-that-my-money-can-be-transferred-back-to-l1-what-prevents-dydx-from-ignoring-my-withdrawal-requests>.

- (a) Slow Withdrawal: In Slow Withdrawal, traders' withdrawals may require a longer waiting period compared to the Centralized Exchange Model. Regardless of holding positions, traders need to wait until all pre-confirmed orders under their account are verified before the Custody Smart Contract executes the withdrawal. Generally, the waiting duration is determined by the Valid Proof Prover's upload cycle and the Valid Proof Verifier's verification speed.
- (b) Fast Withdrawal: If traders have an urgent withdrawal requirement, they can pay a fee to access funds in advance from the withdrawal liquidity provider. The withdrawal liquidity provider waits for the entire verification process and eventually receives the funds that would have been returned to the trader.
- (c) Forced Withdrawal: The emergency withdrawal mechanism significantly distinguishes the Hybrid Model from the Centralized Exchange Model. If the exchange's off-chain servers go offline or deliberately ignore traders' withdrawal requests, traders can initiate the *forcedWithdrawal* function in the Custody Smart Contract. When a *forcedWithdrawal* request is made, the Hybrid Exchange must either process the withdrawal or prove that the request is illegitimate. If the Hybrid Exchange fails to do so within a time limit, traders can invoke the "freeze" function. This function prevents any further changes to the contract's state, activating the "escape mode." In this mode, traders can exit their assets based on the total value of their position in the last accepted on-chain batch. This mechanism ensures that traders' funds cannot be frozen or misappropriated by the exchange. They remain under the custody of the smart contract, promoting "self-custody" or "non-custody" principles.

4.5 Oracle Pricing Model

4.5.1 Architectures

Based on Fig. 3, we further refine and illustrate the actors and components of the Oracle Pricing Model in Fig. 5.

Compared to the Hybrid Model, the Oracle Pricing Model relies less on centralized components but mainly on decentralized ones (labeled as blue diamonds in Fig. 5). The custody of funds, the storage, and execution of trades are all undertaken by on-chain smart contracts. Only risk control and trade price confirmation still require some centralized components (labeled as blue ellipses in Fig. 5).

Under the Oracle Pricing Model, traders can be divided into two categories. The first category includes active traders, who are similar to the traders in the Centralized Exchange Model and Hybrid Model. Traders in the second category accept trades passively, acting as the counterparties to traders in the first category, and are therefore referred to as Liquidity Providers.

1. Traders: Users who actively place various orders, either using the exchange's front-end interface or by directly calling the on-chain smart contracts.
2. Liquidity Providers: Users who passively accept orders from Traders. They act as the direct counterparty to all Traders. Liquidity Providers deposit their funds into the on-chain Liquidity Pool, providing liquidity for the Traders' orders.

Custody Module is composed of two on-chain smart contracts.

1. Collateral Custody Smart Contract: An on-chain smart contract responsible for safeguarding the collateral of Traders, storing the balance status of each trader's position, and ultimately settling profits and losses.
2. Liquidity Custody Smart Contract: An on-chain smart contract responsible for safeguarding the funds of Liquidity Providers and calculating and storing the balance of each Liquidity Provider.

Matching Module is composed of one on-chain smart contracts.

1. Trading Smart Contract: An on-chain smart contract responsible for executing trades and updating the status of each position.

Risk Control Module still relies on three off-chain components.

1. Price Oracle Network: An off-chain network responsible for providing the Trading Smart Contract with the prices of underlying assets.
2. Keepers: Off-chain servers responsible for monitoring the status of each position and taking over positions that meet liquidation criteria, closing them under appropriate conditions.
3. Trading Smart Contract: The Trading Smart Contract also holds data for each position, encompassing collateral quantity and position volume. As such, it is also a constituent of the Risk Control Module.

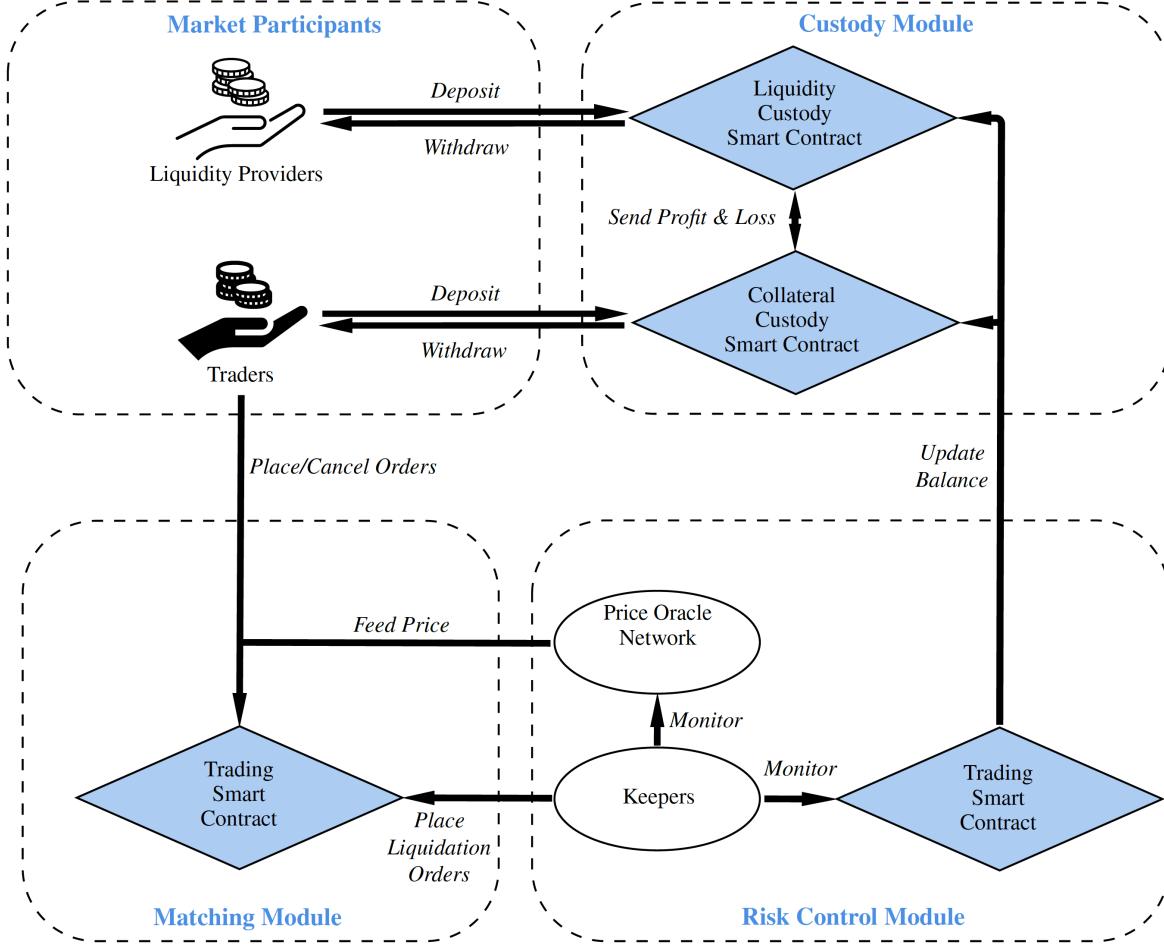


Figure 5: Oracle Pricing Model. In this diagram, diamonds represent on-chain smart contracts, rectangles represent off-chain databases, and ellipses represent off-chain servers. Blue background is used to indicate elements differentiating Oracle Pricing Model from the CEX Model.

4.5.2 Properties

In the Oracle Pricing Model, all traders directly trade to liquidity providers (LP) rather than other traders. Consequently, whenever a trader holds a long position, there is an LP holding an equally sized short position, and vice versa. Thus, the sum of the net exposure of all traders equals the sum of the net exposure of all LPs in the opposite direction. Assuming a total of p long positions and q short positions held by all traders in an exchange, we can derive the net exposure of all LPs as follows:

$$E_t(P_{total}^{LP}) = -\left(\sum_{j=1}^p E_t^{Long}(P_j^T) + \sum_{j=1}^q E_t^{Short}(P_j^T)\right), \quad (11)$$

where P_{total}^{LP} denotes the aggregated position passively held by all LPs while P_j^T denotes the j -th position held by a Trader. The positions passively held by LP are referred to as *impermanent positions*. In Oracle Pricing model, LP proportionally share the size of the *impermanent positions* based on the provided liquidity. The exposure of the *impermanent positions* assumed by the id -th LP can be determined as follows:

$$E_t^{Net}(LP_{id}) = \frac{E_t^{Net}(P_{total}^{LP}) \times VL_t(LP_{id})}{\sum_{i=1}^l VL_t(LP_i)}, \quad (12)$$

where $VL_t(LP_{id})$ denotes the dollar value of liquidity that the id -th liquidity provider provides at time t . Furthermore, we can calculate the profit and loss incurred by all l LPs due to the *impermanent positions* and the profit and loss borne by id -th individual LP:

$$PnL_t^{\text{Net}}(P_{\text{total}}^{LP}) = -\left(\sum_{j=1}^p PnL_t^{\text{Long}}(P_j) + \sum_{j=1}^q PnL_t^{\text{Short}}(P_j)\right), \quad (13)$$

$$PnL_t^{\text{Net}}(LP_{id}) = \frac{PnL_t^{\text{Net}}(P_{\text{total}}^{LP}) \times VL_t(LP_{id})}{\sum_{i=1}^l VL_t(LP_i)}. \quad (14)$$

In addition to *funding fees* and *transaction fees*, traders in the Oracle Pricing Model also incur two other specific costs, namely, *borrowing fees* and *gas fees*.

As Traders' profits and losses are ultimately redeemed by LP, the exchange locks funds in the liquidity pool for possible redemption during the holding period. Consequently, Traders are obliged to pay interest, referred to as *borrowing fees*. The value of *borrowing feesrate* at time t , denoted as BR_t , is typically determined by the proportion of funds occupied in the liquidity pool, with higher proportions resulting in increased BR_t .

From t_0 to t_1 , the *borrowing fees* paid by a Trader for his/her held position P_j can be represented as:

$$BF_{t_0 \rightarrow t_1}(P_j) = \sum_{t=t_0}^{t_1} (BR_t \times |E_t(P_j)| \times P_t^{U \rightarrow C}), \quad (15)$$

The distinct feature of the Oracle Pricing Model, setting it apart from the CEX Model and Hybrid Model, is the inclusion of *gas fees*. This divergence arises from the fact that the Oracle Pricing Model executes each order within smart contracts on the blockchain, while the CEX Model and Hybrid Model execute orders exclusively within off-chain servers. Consequently, traders in the Oracle Pricing Model bear the responsibility of covering *gas fees* for every order, a obligation not shared by traders in the CEX Model and Hybrid Model. The specific amount of *gas fees* paid by a trade depends on the mechanism set by the blockchain network. This paper refrains from presenting specific formulas but simply denotes the *gas fees* for the i -th order as $GF(O_i)$.

As a result, for the id -th Trader who opened and closed n positions with k orders, where m positions (Liq_1, \dots, Liq_m) were liquidated, the withdrawable value he can ultimately access at time t can be represented as:

$$\begin{aligned} WV_t(T_{id}) = & \sum_{j=1}^n Coll(P_j) + \sum_{j=1}^n PnL_t(P_j) - \sum_{j=1}^m LP(P_{Liq_j}) - \\ & \sum_{j=1}^n BF_{t_0 \rightarrow t}(P_j) - \sum_{j=1}^n FF_{t_0 \rightarrow t}(P_j) - \sum_{i=1}^k TF(O_i) - \sum_{i=1}^k GF(O_i). \end{aligned} \quad (16)$$

4.5.3 Case Study

The life cycle of a position on a oracle pricing exchange can be described as follows:

1. Provide Liquidity: Before Traders can engage in trading, Liquidity Providers must offer liquidity to the Liquidity Pool. Liquidity Providers deposit funds into the Liquidity Custody Smart Contract and receive corresponding deposit certificates (LP Tokens).
2. Create and Place Order: Unlike the previous two models, Traders on DEX with Oracle Pricing Model do not need to deposit funds into the Custody Smart Contract before initiating trades. In the Oracle Pricing Model, traders encapsulate orders as Ethereum transactions off-chain, sign them with their private keys, and then broadcast them to the blockchain. This transaction also includes an "Approve" action, authorizing the Custody Smart Contract to debit the corresponding collateral from their Ethereum address.
3. Execute Order and Initiate Position: When the price offered by the Oracle matches the conditions of an order, the order is executed, resulting in a change to the trader's position. As the order's execution occurs entirely on-chain, there are no pre-confirmations as in the Hybrid Model. The execution is always a hard confirmation.
4. Add or Reduce Position: Traders can repeat step 3 by executing additional orders to adjust the size or direction of their positions.
5. Close Position: Traders can initiate a voluntary position closure, or their position might be forcibly closed if they fail to meet the maintenance margin requirements.

- (a) Voluntary Position Closure: Traders can repeat step 3 and 4 by executing orders in the opposite direction of their existing position to close it and settle realized gains or losses.
 - (b) Forced Position Closure: While holding a position, if a Trader's provided collateral fails to meet the maintenance margin requirement, their position becomes liquidatable. Off-chain Keepers continuously monitor all Traders' positions. When a liquidatable position is detected, a liquidation transaction is sent to the on-chain contract by keepers. Similarly, the contract imposes a penalty on the liquidated position, and any surplus collateral is returned to the liquidated Trader.
6. Withdraw Funds: After reducing or closing a position, Traders automatically receive released collateral from settled gains and losses. There is no need for manual extraction of collateral. As for Liquidity Providers, they can burn their LP Tokens and exchange them for funds as long as their capital is not tied up by Traders' positions.

4.6 Virtual Auto Market Making (VAMM) Model

4.6.1 Architectures

Based on Fig. 3, we further refine and illustrate the actors and components of the Virtual Auto Market Making (VAMM) Model in Fig. 6.

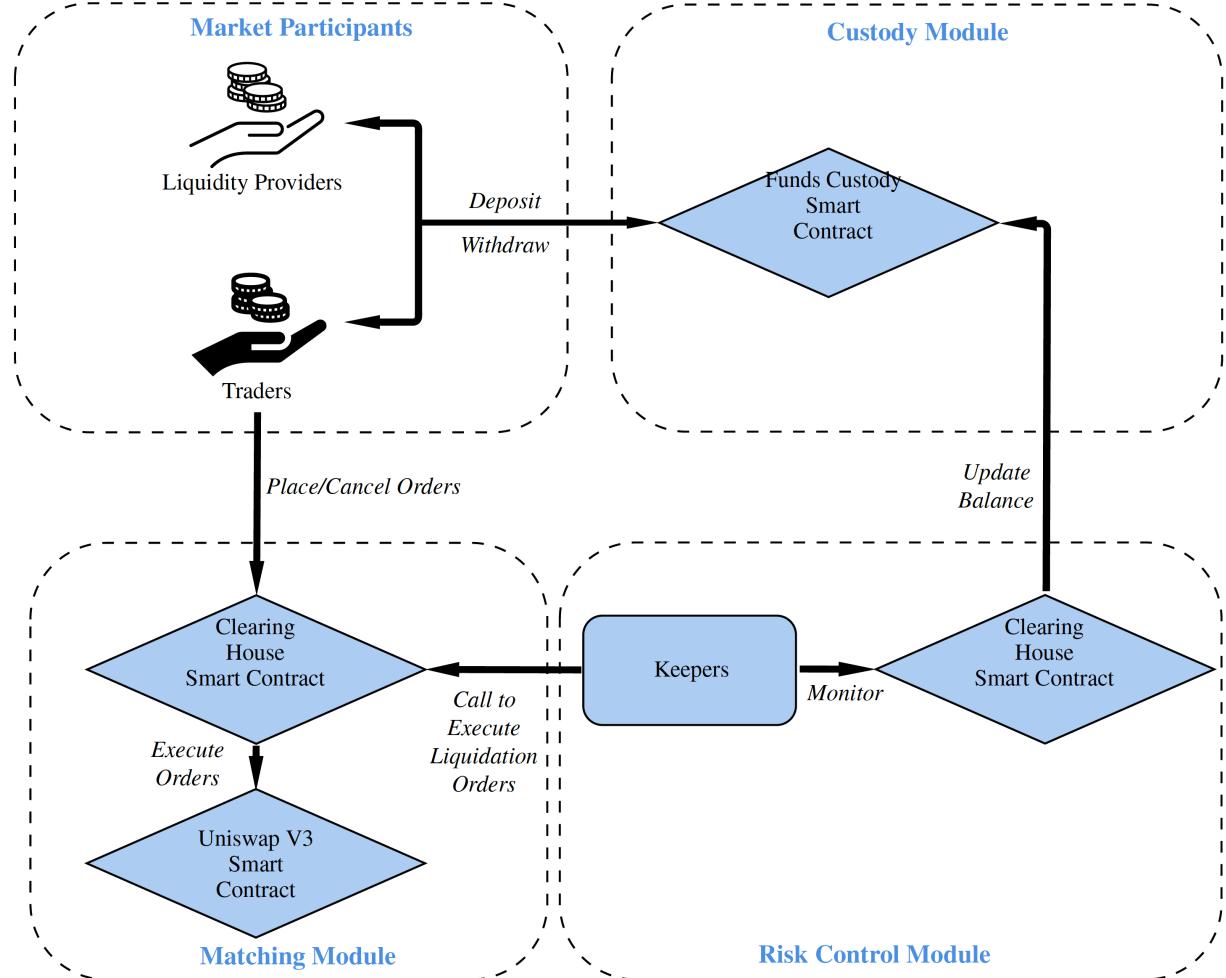


Figure 6: VAMM Model. In this diagram, diamonds represent on-chain smart contracts, rectangles represent off-chain databases, and ellipses represent off-chain servers. Blue background is used to indicate elements differentiating VAMM Model from the CEX Model.

The VAMM Model shares many similarities with the Oracle Pricing Model: traders can also be categorized into liquidity providers and active traders. However, apart from this, even though both employ smart contracts for user fund custody, trade order management, and off-chain Keepers to control risk, there are differences in the specific implementation details.

Unlike the Oracle Pricing Model, where the collateral provided by Traders and the funds offered by Liquidity Providers are kept separately, in this model, both Trader's collateral and Liquidity Providers' funds are stored within the same smart contract:

1. Funds Custody Smart Contract: This on-chain smart contract is responsible for safeguarding both the collateral of Traders and the funds of Liquidity Providers.

The distinctive characteristic of the VAMM Model resides within its Matching Module. Under this model, Liquidity Providers opt to allocate funds for specific trading pairs, prompting the Clearing House Smart Contract to generate virtual tokens that serve the purpose of augmenting leverage. Each virtual token is endowed with a nominal valuation equivalent to that of an underlying token, denoted as follows: 1 vUSDC = 1 USDC, 1 vETH = 1 ETH, and so forth. These virtual tokens are subsequently injected into the AMM Smart Contract, where they undergo trading activities akin to those of actual underlying tokens. However, it is imperative to underscore that exclusive authority for the minting of vTokens, management of Add/Remove Liquidity operations, and facilitation of vToken exchanges resides solely within the purview of the Clearing House Smart Contract. In essence, vTokens assume a role confined to accounting tools exclusively operational within the ecosystem of the VAMM Exchange, without circulation beyond its ecosystem.

1. Clearing House Smart Contract: The Clearing House Smart Contract serves as the central component within the VAMM Model, responsible for executing all transactions and maintaining records of all Traders' and Liquidity Providers' positions.
2. Spot DEX Smart Contract: For the VAMM Model, the AMM Smart Contract functions as an external contract, acting as a repository for holding and swapping virtual tokens.

Risk Control Module still relies on off-chain components.

1. Keepers: These are off-chain servers tasked with monitoring position statuses and intervening in positions that meet liquidation criteria. They execute closure actions under appropriate circumstances.
2. Clearing House Smart Contract: Given its comprehensive record-keeping of all Traders' and Liquidity Providers' positions, the Clearing House Smart Contract also functions akin to a "Position Database." Thus, it constitutes an integral part of the Risk Control Module.

4.6.2 Properties

In the Virtual Auto Market Making (VAMM) Model, the exchange utilizes virtual tokens as trading intermediaries and accounting tools, while actual trading occurs within a Spot DEX.

The specific process involves LP's providing funds as collateral, allowing the ClearingHouse contract to mint two types of virtual tokens for a specific trading pair. These tokens are then injected into the liquidity pool to provide liquidity for traders. E.g. by offering USDC as collateral, LP mint vBTC and vUSDC, depositing them into the AMM liquidity pool.

Subsequently, Traders provide funds as collateral to the ClearingHouse contract, and mints a specific virtual token. Traders then swap this token from the liquidity pool for another virtual token. E.g. by using USDC as collateral, traders mint vUSDC and swap it for vBTC from the vBTC-vUSDC liquidity pool, effectively going long on vBTC.

In essence, the VAMM Model employs virtual tokens as leverage tools, amplifying the purchasing power of traders and LPs. It utilizes the ClearingHouse contract and other on-chain Spot AMM DEX to create a perpetual contract trading DEX.

In practice, each user's address corresponds to a Vault in the ClearingHouse contract, documenting the user's debts and assets. The virtual tokens minted by users represent their debts, while the collateral provided and the purchased virtual tokens or liquidity provided constitute their assets. The user's net position is equal to the actual virtual tokens held minus the virtual tokens minted. The deposited assets serve as the collateral for this position. This conclusion holds true whether the user acts as a trader, an LP, or both simultaneously.

For any user, their net exposure is equal to the quantity of virtual tokens held in the Vault minus the accumulated quantity of virtual tokens in debt. Positive values represent a long position in the corresponding virtual token, while negative values indicate a short position. An example vault balance sheet is illustrated in Fig. 7. Thus, we can express

Vault Balance Sheet	
Debt	Asset
$a_0 vUSDC$	$x USDC$
$b_0 vBTC$	$a_1 vUSDC$
	$b_1 vBTC$

Net Position = $(a_1 - a_0) vUSDC + (b_1 - b_0) vBTC$
Collateral = $x USDC$

Figure 7: VAMM Vault Balance Sheet

this relationship as follows:

$$E_t^{Net}(V_{id}) = VA_t(V_{id}) - VD_t(V_{id}), \quad (17)$$

where V_{id} denotes the Vault corresponding to the address of id -th user, $E_t^{Net}(V_{id})$ represents the aggregated net exposure of the vault for the id -th user that might hold multiple positions at time t , $VA_t(V_{id})$ represents the amount of virtual token as asset in the balance sheet of the vault for the id -th user at time t , and $VD_t(V_{id})$ denotes the the amount of virtual token as debt in the balance sheet of the vault for the id -th user at time t .

Therefore, the PnL of the vault belonging to the id -th user is equal to the current USD value of its net exposure:

$$PnL_t^{Net}(V_{id}) = E_t^{Net}(V_{id}) \times P^{VT \rightarrow USD}, \quad (18)$$

where $PnL_t^{Net}(V_{id})$ represents the aggregated profit and loss of the vault for the id -th user at time t and $P^{VT \rightarrow USD}$ denotes the the dollar value of virtual tokens.

Similar to the Oracle Pricing Model, the sum of the net positions of all traders equals the sum of the net positions of all LP in the opposite direction. However, unlike the Oracle Pricing Model, the size of the *impermanent position* passively assumed by each LP is not evenly distributed based on the quantity of liquidity provided. Instead, it is determined by the quantity of virtual tokens provided, the price range at which liquidity was provided, the initial liquidity-providing price, and the current price. Thus, the properties represented by Eq. (11) and Eq. (13) still hold and can be expressed as follows:

$$\sum_{id=1}^l E_t^{Net}(V_{id}^{LP}) = -\left(\sum_{id=1}^p E_t^{Long}(V_{id}^T) + \sum_{id=1}^q E_t^{Short}(V_{id}^T)\right), \quad (19)$$

$$\sum_{id=1}^l PnL_t^{Net}(V_{id}^{LP}) = -\left(\sum_{id=1}^p PnL_t^{Long}(V_{id}^T) + \sum_{id=1}^q PnL_t^{Short}(V_{id}^T)\right). \quad (20)$$

However, the property represented by Eq. (12) does not hold in the VAMM Model. In VAMM, to calculate the *impermanent position* borne by a LP, it is necessary to isolate the virtual tokens related to providing liquidity in their Vault and apply Eq. (17). This can be expressed as:

$$E_t^{Net}(V_{id}^{LP}) = VA_t(V_{id}^{LP}) - VD_t(V_{id}^{LP}), \quad (21)$$

where V_{id}^{LP} denotes the Vault corresponding to the address of id -th Liquidity Provider.

Moreover, the *impermanent loss* resulting from the price change on the *impermanent position* can be calculated using Eq. (18), expressed as:

$$IL_t(V_{id}^{LP}) = E_t^{Net}(V_{id}^{LP}) \times P^{VT \rightarrow USD}. \quad (22)$$

The transaction fees earned by the id -th LP for providing liquidity from t_0 to t_1 can be expressed as:

$$\sum_{t=t_0}^{t_1} TF_t(V_{id}^{LP}) = \sum_{t=t_0}^{t_1} \frac{\sum_{i=1}^k TF(O_i) \times VL_t(LP_{id})}{\sum_{id=1}^l VL_t(LP_{id})}. \quad (23)$$

Unlike the Oracle Pricing Model, in the VAMM Model, traders are essentially trading spot virtual tokens. During the holding period, there is no need to lock funds in the liquidity pool, and as a result, no borrowing fees are incurred. However, since transactions are still executed through smart contracts, traders are still required to pay *gas fees*.

The value of assets that each vault can withdraw is represented as:

$$\begin{aligned} \text{WV}_t(V_{id}) = & \text{Coll}(V_{id}) + \text{PnL}_t^{\text{Net}}(V_{id}) - \text{LP}(V_{id}) - \text{FF}_{t_0 \rightarrow t}(V_{id}) - \\ & \sum_{i=1}^k \text{TF}(O_i) + \text{TF}_t(V_{id}^{LP}) - \sum_{i=1}^k \text{GF}(O_i). \end{aligned} \quad (24)$$

4.6.3 Case Study

The life cycle of a position on a VAMM exchange can be described as follows:

1. Provide Liquidity: Similar to the Oracle Pricing Model, before traders can engage in trading, Liquidity Providers must offer liquidity to the Liquidity Pool. However, unlike the Oracle Pricing Model, Liquidity Providers have the option to use leverage to amplify the liquidity they provide. The process involves Liquidity Providers depositing funds into the Funds Custody Smart Contract, selecting the trading pairs for which they wish to provide liquidity and the desired leverage ratio. They then encapsulate this operation into an Ethereum transaction, sign it with their private key, and broadcast it to the blockchain. When the blockchain executes this transaction, the Clearing House Smart Contract creates corresponding virtual tokens and injects them into the AMM Liquidity Pool. Liquidity Providers' positions are calculated based on the virtual tokens they hold, using their deposited funds as collateral. If the collateral provided by Liquidity Providers fails to meet the required maintenance margin for their positions, they may also face liquidation.
2. Deposit Funds: Similarly, Traders need to deposit funds into the Funds Custody Smart Contract to initiate trading.
3. Create and Place Order: Similar to the Oracle Pricing Model, traders encapsulate orders into Ethereum transactions off-chain, sign them with their private keys, and then broadcast them to the blockchain.
4. Execute Order and Initiate Position: When the blockchain executes the aforementioned transaction, the Clearing House Smart Contract mints the virtual tokens that traders wish to sell and performs a Swap operation within the AMM Liquidity Pool, converting them into the virtual tokens that traders wish to buy.
5. Add or Reduce Position: Traders can repeat step 3 by executing additional orders to adjust the size or direction of their positions.
6. Close Position: Traders can voluntarily close positions or might be forcibly closed if they fail to meet the maintenance margin requirement.
 - (a) Voluntary Position Closure: Traders can repeat steps 3 and 4 by executing orders in the opposite direction of their existing position to close it and settle realized gains or losses.
 - (b) Forced Position Closure: The liquidation process is similar to the Oracle Pricing Model, and will not be reiterated. Notably, unlike the Oracle Pricing Model, not only traders' positions but also Liquidity Providers' positions subject to liquidation.
7. Withdraw Funds: Traders and Liquidity Providers can withdraw funds that have not been used to mint virtual tokens at any time. For funds used as collateral, they need to close positions and settle gains and losses before withdrawal is possible.

5 Empirical Analysis of Traders' Behavior

Soska et al.[21] presents the initial foray into the examination of liquidation processes within the realm of perpetual futures, with BitMEX serving as the focal point of investigation. The researchers observed a correlation between the escalation in the daily volume of liquidated contracts and the heightened price volatility of the underlying asset, specifically Bitcoin, within this particular context. However, it is imperative to acknowledge a notable constraint inherent to BitMEX's operational framework, namely its status as a centralized exchange (CEX). In consequence, the transactions executed on BitMEX are not subject to on-chain recording, thereby engendering a notable degree of skepticism concerning the paper's data accuracy and precision. In contrast, transactions involving perpetual contracts within decentralized exchanges (DEXes) are on-chain, ensuring a verifiable level of data accuracy. This inherent characteristic affords a heightened level of confidence when analyzing trader behavior within the DEX milieu.

Diverse configurations of the liquidation process engender varying degrees of credit risk exposure for investors holding different categories of perpetual futures contracts. The consequent divergence in credit risk profiles, stemming from the disparate liquidation mechanisms, significantly influences the decision-making process of investors when confronted with a choice among multiple types of perpetual futures contracts linked to the same underlying asset. Theoretical assessment of credit risk, often synonymous with default risk, within the context of futures contracts presents inherent complexities, primarily attributed to the mutual risk-sharing dynamics between the two contracting parties [32]. However, in the case of perpetual futures contracts operational within decentralized exchanges (DEXs), exemplified by platforms such as GMX and GNS, orders are meticulously matched between individual traders and the liquidity pool. Consequently, credit risk is predominantly contingent upon the risk of traders facing liquidation, given that liquidity providers can always cover their loss.

This section of the study undertakes an examination of trader behavior through an analysis of their activities and the corresponding volumes subject to liquidation in response to fluctuations in price volatility. The scope of our analysis encompasses perpetual futures contracts linked to Bitcoin, spanning five exchanges, categorized into two distinct classifications: (1) GMX¹⁵, GNS¹⁶, and Perpetual Protocol V2¹⁷, representing decentralized exchanges (DEXs), and (2) Binance¹⁸ and Bybit¹⁹, constituting centralized exchanges (CEXs). In addition, GMX and GNS adopt oracel pricing model, which is elaborated in Section 4.5, while Perpetual Protocol V2 adopts VAMM pricing model, which is discussed in Section 4.6. We compare traders' behavior implied by different designs of the exchange with different pricing mechanisms.

5.1 Data

In this section, as a case study, we select GMX, GNS, and Perpetual Protocol V2 as the representatives for DEXs, while Binance and Bybit as the representatives for CEXs. Specifically, we focus on perpetual futures on Bitcoin listed on the five exchanges.

GMX constructs its trading system on Arbitrum and Avalanche²⁰, and users can trade perpetual futures on both of the two layer-2 networks. In this study, we focus on tradings on Arbitrum from 31 August 2021 to 26 September 2023, the data of which are retrieved from Dune (<https://dune.com/queries/3110823>). Similarly, GNS is based on Arbitrum and Polygon²¹, and we choose the perpetual futures on Polygon from 20 December 2021 to 26 September 2023 to examine, the data of which are retrieved by our constructed queries on Dune (<https://dune.com/queries/2829105> and <https://dune.com/queries/2825251>).

We also retrieve data for Perpetual Protocol V2 (based on Etherum mainnet) from 27 November 2021 to 26 September 2023 by constructing the query on Dune (<https://dune.com/queries/3039245>). All the above data for DEXs are at transaction level and with length from launches of the instruments to the time of writing this work.

We retrieve data on Binance and Bybit at the daily-level frequency using API provided by Coinalyze (<https://coinalyze.net/>). Data for these two CEXs are at market-level rather than transaction-level. BTC Perpetual data for Binance and Bybit are available from 05 August 2020. Data of BTC pricing at the daily-level are collected from CryptoCompare with API (<https://cryptocompare.com/>). The time frames of data vary across exchanges because they are launched on different dates. However, this would not harm the validity of our analysis, as the time frames largely overlap and we examine traders' behavior in different exchanges separately.

5.2 Empirical Approaches

Drawing inspiration from previous research conducted in the realm of traditional futures markets [33, 34], this study introduces methodological approaches tailored to the investigation of trader behavior within the perpetual futures market. These methodologies, subsequently applied in Section 5.3, facilitate an in-depth exploration of leverage dynamics and liquidation processes.

It is paramount to recognize that the pricing mechanisms governing perpetual futures contracts are intentionally designed to approximate the spot price of the underlying asset. Consequently, traders engaged in the trading of perpetual futures

¹⁵<https://gmx.io>.

¹⁶<https://gains.trade>.

¹⁷<https://perp.com>.

¹⁸<https://www.binance.com/en>.

¹⁹<https://www.bybit.com>.

²⁰Arbitrum and Avalanche are two chains to address scalability of Ethereum (<https://vitalik.eth.limo/general/2023/10/31/l2types.html>), while detailed explanation can be found at: arbitrum.io and www.avax.network.

²¹Information about Polygon can be found at: polygon.technology.

are confronted with prices inherently tethered to the underlying asset's market valuation. This inherent convergence between the price of perpetual futures and the spot price signifies that perpetual futures contracts endow traders with an efficient means of exposure to fluctuations in the spot price of the underlying asset [20]. For the purpose of our analysis, and without loss of generality, we direct our focus toward an examination of how traders operating within the perpetual futures market respond to variations in the price of the underlying asset, specifically Bitcoin.

5.2.1 Speculation metrics.

Perpetual futures contracts, characterized by their provision of exceedingly high leverage, create an environment conducive to speculative activities. The presence of speculators assumes considerable significance within the market ecosystem, as they assume the role of risk bearers, thereby effectively absorbing risk transferred by hedgers. Conversely, arbitragers capitalize on market inefficiencies to generate gains with very limited risk exposure.

Hedgers, in contrast, adopt a risk-mitigation strategy by retaining their futures positions for relatively longer durations. Their decision-making process remains relatively impervious to short-term price fluctuations, as it predominantly hinges upon non-pricing determinants, encompassing factors such as the quantity of spot goods at their disposal and the designated delivery timelines. Consequently, transient shifts in prices are frequently instigated by speculators, who exhibit responsiveness to short-term price oscillations [35].

In light of the substantial risk assumed by speculators within this framework, this study takes an initial step by proposing a set of speculation metrics. These metrics, encompassing trading volumes and liquidations standardized by open interests, are instrumental in shedding light on speculative activities within the market.

The standard speculative index (\mathcal{SI}) proposed by [36] is defined as:

$$\mathcal{SI} := \frac{\text{Trading Volume}}{\text{Open Interest}}, \quad (25)$$

in which speculators drive up trading volume while having little impact on open interest in the short-run. So, large \mathcal{SI} indicates plenty of speculation.

Another measure of speculation is proposed by [35], defined as the ratio of liquidation volume to open interest. The only difference in this study is that, since traders trade with liquidity pools rather than other matched traders in DEXs, the aggregate position of sellers does not necessarily equal to that of the buyers, different from the case of CEXs where there are always a buyer and a seller for any contract. Therefore, we modify the definition of *liquidation index* (\mathcal{LIQ}) to fit the features of perpetual in DEXs:

$$\begin{aligned} \mathcal{LIQ}_{\text{short}} &= \frac{\text{Short Liquidation Volume}}{\text{Open Interest for Short}}, \quad \text{and} \\ \mathcal{LIQ}_{\text{long}} &= \frac{\text{Long Liquidation Volume}}{\text{Open Interest for Long}}. \end{aligned} \quad (26)$$

As Eq. (26) measures the liquidation as a proportion of open interest, the expected value of \mathcal{LIQ} is the probability of being liquidated in a certain day. In addition to measuring speculation, the standardization of trading volumes and liquidations by dividing them with open interests makes those metrics comparable among different exchanges, as exchanges vary considerably in size.

5.2.2 Traders' behavior examination.

To analyze the response of perpetual traders to Bitcoin price fluctuations, it is imperative to first establish an estimation for the daily volatility of Bitcoin's price. This study employs the extreme-value volatility estimator, as proposed by Garman and Klass (1980) [37] (on P.74), which incorporates intra-day price volatility. The simplified expression of this estimator is represented as follows:

$$\hat{\sigma}_t = \left\{ 0.5 \times (\ln(P_{t,H}/P_{t,L}))^2 - (2 \ln(2) - 1) (\ln(P_{t,O}/P_{t,C}))^2 \right\}^{1/2}, \quad (27)$$

where $P_{t,H}$, $P_{t,L}$, $P_{t,O}$, and $P_{t,C}$ are the high, low, opening, and closing prices of Bitcoin on date t , respectively.

Previous research has consistently shown that variables related to traders' activities, such as trading volumes and open interests, exhibit significant correlation [33], suggesting their predictability based on the previous values. Consequently, these variables are often analyzed by dividing them into expected and unexpected components. The expected component, constructed from lagged variables, encompasses information about current trends, while the unexpected component captures unforeseen changes in traders' behavior. This concept is illustrated in the following equation:

$$\hat{\varepsilon}_t = \text{Activity}_t - E \{ \text{Activity}_t | \text{Activity}_{t-\tau}, \tau = 1, \dots, 10 \}, \quad (28)$$

traders' activity at date t can be predicted using their activities in previous 10 days, i.e., $E \{ \text{Activity}_t \mid \text{Activity}_{t-\tau} \}$, while the unexpected portion $\hat{\varepsilon}_t$ equals to the difference between the realized value Activity_t . Hedgers, who adjust positions infrequently with a long-term focus, exhibit predictable behavior captured in the expected component. In contrast, speculators, who frequently adjust positions to manage risk, are represented by the unpredictable, unexpected component of activity.

Beyond traditional trading activity variables like trading volumes and open interests, this study also assesses the conditional effects of leverage and liquidation on price volatility. We decompose these four types of variables into expected and unexpected components to differentiate the effects of hedgers and speculators, who vary in their trading willingness. Following the methodology of Bessembinder and Seguin (1993) [33], we partition trading volume, open interest, volume of liquidated positions, and daily average leverage using the Autoregressive Integrated Moving Average (ARIMA(p, k, q)) model. The ARIMA(p, k, q) model, which predicts future values based on past data smoothed by lagged moving averages, allows us to designate the model's fitted values as the expected component and its residuals as the unexpected component. For the ARIMA(p, k, q) model, the existence of a unit root determines k , in which case a variable without a unit root is decomposed using ARIMA($p, 0, q$), while a non-stationary variable is decomposed using ARIMA($p, 1, q$)²². The corresponding series is differenced k times as an Autoregressive Moving Average (ARMA(p, q)) process. The values of p and q are selected based on the Akaike Information Criterion. The existence of unit roots in each time series is tested using the Augmented Dickey–Fuller (ADF) test, with results detailed in Appendix.

To explore how investors' behavior vary with volatility, we adapt the model proposed by Wang and Yau (2002) [34] to include daily volumes of liquidated short and long positions separately:

$$\begin{aligned} \hat{\sigma}_t = & \mu + \sum_{i=1}^m \phi_i \hat{\sigma}_{t-i} + \sum_{j=1}^3 \alpha_j EA_{j,t} + \sum_{j=1}^3 \beta_j UA_{j,t} + \\ & \sum_{k=1}^2 \gamma_k EL_{k,t} + \sum_{k=1}^2 \lambda_k UL_{k,t} + \varepsilon_t, \end{aligned} \quad (29)$$

where $\hat{\sigma}_t$ is the estimated volatility on day t . $EA_{j,t}$ and $UA_{j,t}$ denote the expected and unexpected trading activity respectively, with j represents different trading activity when assigned different values: $j = 1$ for open interest on long positions, $j = 2$ for open interest on short positions, and $j = 3$ for trading volumes. $EL_{k,t}$ and $UL_{k,t}$ denote represent the expected and unexpected daily liquidated volumes for long ($k = 1$) and short ($k = 2$) positions. The model accounts for volatility persistence through lagged volatility estimates, with the lag structure (m) determined by the Akaike Information Criterion. Given the established correlation between trading activities and volatility [34], these variables are included as controls. This analysis using Eq. (29) is applied to perpetual futures from all three DEXs and two CEXs, given the availability of trading volume, open interest, and liquidation data.

In addition to assessing how traders' activities and liquidations respond to price volatility, we also examine speculation, as indicated by Eq. (25) and Eq. (26), using the following specification:

$$\begin{aligned} \hat{\sigma}_t = & \mu + \sum_{i=1}^m \phi_i \hat{\sigma}_{t-i} + \sum_{j=1}^3 \alpha_j TA_{j,t} + \beta_{SI} SI + \\ & \beta_{LIQS} LIQS_{\text{short}, t} + \beta_{LIQL} LIQL_{\text{long}, t} + \varepsilon_t, \end{aligned} \quad (30)$$

where β_{SI} , β_{LIQS} , and β_{LIQL} measure whether speculation is stronger in respond to the more volatile change in price. Among those, β_{LIQS} and β_{LIQL} indicate whether a short or long position is more likely to be liquidated than usual when the market become more volatile. $TA_{j,t}$ represent trading activities, i.e., j represents trading volume, open interest on short positions, and open interest on long positions. Considering the considerable correlation between trading activities and price volatility, we include trading activities in Eq. (30) but do not decompose them as those are not the variables of interest.

Anticipating that traders are inclined to modify their leverage strategies in response to heightened liquidation risks under increased volatility of the underlying asset, this study incorporates an analysis of leverage adjustments. Research focusing on centralized exchanges (CEXs) often omits leverage considerations due to the general unavailability of such data [35, 21]. However, leveraging transaction-level data from GMX and GNS, this investigation calculates and integrates the daily average leverages for both long and short positions into the analytical framework. This is achieved

²²Readers can refer to [38] for details about ARIMA(p, k, q) model.

by adapting the existing volatility estimation model, as delineated in Equation (29):

$$\begin{aligned}\hat{\sigma}_t = & \mu + \sum_{i=1}^m \phi_i \hat{\sigma}_{t-i} + \sum_{j=1}^3 \alpha_j EA_{j,t} + \sum_{j=1}^3 \beta_j UA_{j,t} + \sum_{k=1}^2 \gamma_k EL_{k,t} + \\ & \sum_{j=k}^2 \lambda_j UL_{k,t} + \sum_{k=1}^2 \rho_k ELV_{k,t} + \sum_{k=1}^2 \omega_k ULV_{k,t} + \varepsilon_t,\end{aligned}\quad (31)$$

where $ELV_{k,t}$ and $ULV_{k,t}$ denote expected and unexpected average leverage for long ($k = 1$) and short positions ($k = 2$), respectively, on day t . Notably, due to the cross-margining feature in Perpetual Protocol v2, where a collective pool of funds backs each position²³, the specific leverage level for each position remains undefined. Consequently, the leverage analysis using Equation (31) is confined to perpetual futures from GMX and GNS.

In addition to exploring how average leverage changes with price volatility, we are also wondering how trading volume in different leverage level contribute to the change in average leverage level. To this end, we propose the following specification:

$$\begin{aligned}\hat{\sigma}_t = & \mu + \sum_{i=1}^m \phi_i \hat{\sigma}_{t-i} + \sum_{j=1}^2 \alpha_j EA_{j,t} + \sum_{j=1}^2 \beta_j UA_{j,t} + \sum_{k=1}^2 \gamma_k EL_{k,t} + \\ & \sum_{j=1}^2 \lambda_j UL_{k,t} + \sum_{k=1}^{10} \psi_k ELC_{k,t} + \sum_{j=k}^{10} \kappa_j ULC_{k,t} + \varepsilon_t,\end{aligned}\quad (32)$$

In this equation, trading volumes are categorized into ten distinct groups based on their leverage levels: less than 1X, 1X to 5X, 5X to 10X, 10X to 15X, 15X to 20X, 20X to 25X, 25X to 30X, 30X to 35, 35X to 40X, 40X to 45X, 45X to 50X, and more than 50X. $ELC_{k,t}$ and $ULC_{k,t}$ denote the expected and unexpected portions of trading volume within each leverage category k . To avoid issues of multicollinearity, total trading volume is not included as a separate variable in $EA_{j,t}$ and $UA_{j,t}$, given that the aggregated trading volume across all leverage categories equates to the total trading volume.

5.3 Empirical Results and Discussion

5.3.1 Descriptive statistics and dynamics of metrics.

Table 3 delineates the descriptive statistics for trading volumes, open interests, liquidations, and leverage of the five perpetual future platforms under examination, while Figure 8 graphically depicts these metrics on a weekly basis. Centralized exchanges (CEXs) have historically exhibited preeminence in trading volume and open interest, yet decentralized exchanges (DEXs) have demonstrated a notable ascension since 2023, as detailed in Appendix²⁴. An analysis of average daily liquidation volumes reveals a predilection for higher liquidations in long positions over short positions, with the exception of Perpetual Protocol V2. In the cases of GMX and GNS, the open interest for long positions surpasses that for short positions, and the leverages for both orientations (long & short) are relatively equivalent (15.57 & 15.80 for GMX and 72.33 & 75.28 for GNS), logically inferring a higher liquidation volume for long positions. With liquidation standardized, the close proximate values of \mathcal{LQ}_{short} and \mathcal{LQ}_{long} for GMX (0.016 and 0.016) and GNS (0.035 and 0.032) suggest a comparable probability of liquidation for both long and short positions. A symmetrical pattern in liquidation is anticipated between long and short positions with similar leverage levels and volumes. This symmetry is observed for Perpetual Protocol V2 and Bybit, but not for Binance. A two-sample t-test indicates that the discrepancy between \mathcal{LQ}_{short} and \mathcal{LQ}_{long} is statistically significant exclusively for Binance (p-value = 0.001), whereas it remains statistically insignificant for the other platforms, namely GMX (p-value = 0.997), GNS (p-value = 0.813), Perpetual Protocol V2 (p-value = 0.883), and Bybit (p-value = 0.184). Despite the absence of leverage data for Binance, the order book matching mechanism posits an equivalent leverage distribution between sellers and buyers, yet empirical observations contradict this equilibrium. This discrepancy remains inconclusive pending further data.

Additionally, Figure 8 reveals an anomaly for Binance, where liquidation volumes for both long and short positions were concentrated during the bullish period in the first half of 2021, peaking in March and declining post-June, yet maintaining a consistently low level throughout the cryptocurrency market downturn in 2022. This trend may be

²³Documents of Leverage and Margin Ratio for Perpetual Protocol v2: support.perp.com/hc/en-us/articles/5257393633945-Leverage-Margin-Ratio.

²⁴Refer to Appendix for a visual representation of trading volume, open interest, and liquidation exclusive to DEXs.

Table 3: Descriptive Statistics of Trading Volumes, Open Interest, Liquidation, and Leverage of Perpetual Futures

Statistics		Perpetual Contracts on Bitcoin				
		GMX	GNS	Perpetual Protocol V2	Binance	Bybit
Mean	Trading Volume (M)	40.46	7.62	2.65	411.55	92.18
	OI on Short (M)	10.43	20.29	6.51	2235.61	831.39
	OI on Long (M)	18.31	26.38	4.13	2235.61	831.39
	Liquidation on Short (K)	62.68	197.30	4.53	11875.03	4527.90
	Liquidation on Long (K)	124.09	248.76	3.54	17128.46	4753.27
	Leverage on Short (K)	15.57	72.33	-	-	-
	Leverage on Long (K)	15.80	75.28	-	-	-
	\mathcal{SI}	5.74	0.93	1.40	0.24	0.13
	\mathcal{LIQ}_{short}	0.016	0.035	0.002	0.009	0.012
	\mathcal{LIQ}_{long}	0.016	0.032	0.002	0.013	0.013
Std. Dev.		50.82	12.40	2.75	249.56	84.57
		10.88	11.45	3.31	898.54	519.86
		19.37	16.87	1.36	898.54	519.86
		459.08	1406.25	27.11	24231.71	7933.40
		947.45	609.53	18.11	49440.90	7263.63
		6.89	14.32	-	-	-
		6.89	21.34	-	-	-
		14.71	2.28	5.51	0.19	0.10
		0.173	0.359	0.017	0.022	0.024
		0.173	0.101	0.029	0.040	0.026
No. of observations		747	628	668	1147	1147

Note. This table reports the daily mean and standard deviation of trading volumes (in million USD), open interest (O.I. in million USD), short liquidations (in thousand USD), long liquidations (in thousand USD), \mathcal{SI} , \mathcal{LIQ}_{short} , \mathcal{LIQ}_{long} of USD or USDT bitcoin perpetuals across GMX, GNS, Perpetual Protocol V2, Binance, and Bybit. Daily mean and standard deviation of average leverage on short and long positions (in thousands USD) are reported for perpetuals on GMX and GNS. The measure of daily average leverage is calculated by dividing the sum of daily transaction volume by the total amount of collateral supporting those transactions, which is the weighted average leverage of transactions in a day. We also conduct ADF test for each of the series, the results of which can be found in Appendix.

attributable to Binance's historical allowance of up to $125\times$ leverage, which was reduced to a maximum of $20\times$ on July 26, 2021²⁵ [39], significantly diminishing the likelihood of liquidation events.

Upon examination of the mean values for \mathcal{SI} and \mathcal{LIQ} delineated in Table 3 across various trading platforms, it is discernible that GMX exhibits the highest mean value for \mathcal{SI} at 5.74, while GNS registers the maximal values for both \mathcal{LIQ}_{short} (0.035) and \mathcal{LIQ}_{long} (0.032) respectively. This suggests a relatively heightened level of speculative trading activity on GMX and GNS compared to other exchanges. Such a trend aligns with the operational parameters of GMX and GNS, which afford traders the opportunity to engage with higher leverage options and impose a lower threshold for maintenance margins relative to their counterparts. Specifically, GMX and GNS extend maximum leverage ratios of $100\times$ and $150\times$ respectively, in contrast to centralized exchanges (CEXs) like Binance and Bybit, which implement a tiered leverage system that inversely correlates the maximum allowable leverage with the size of the position²⁶. The maintenance margin required by GMX is 1%, and the liquidation price on GNS is calculated by:

²⁵For further details, refer to Binance's announcement on July 27, 2021: <https://www.binance.com/en/support/announcement/updates-on-rules-of-binance-futures-leverage-for-new-accounts-2021-07-27-d6457e23eb2e42f2b9c3ce44f46f9a6d>.

²⁶For maximum leverages allowed in each tier of position size on Binance and Bybit, readers can refer to <https://www.binance.com/en-IN/futures/trading-rules/perpetual/leverage-margin> and <https://www.bybit.com/en-US/help-center/article/Risk-Limit-USDC-Contract>. Since Perpetual Protocol V2 adopts cross-margining, the leverage for individual position is undefined.

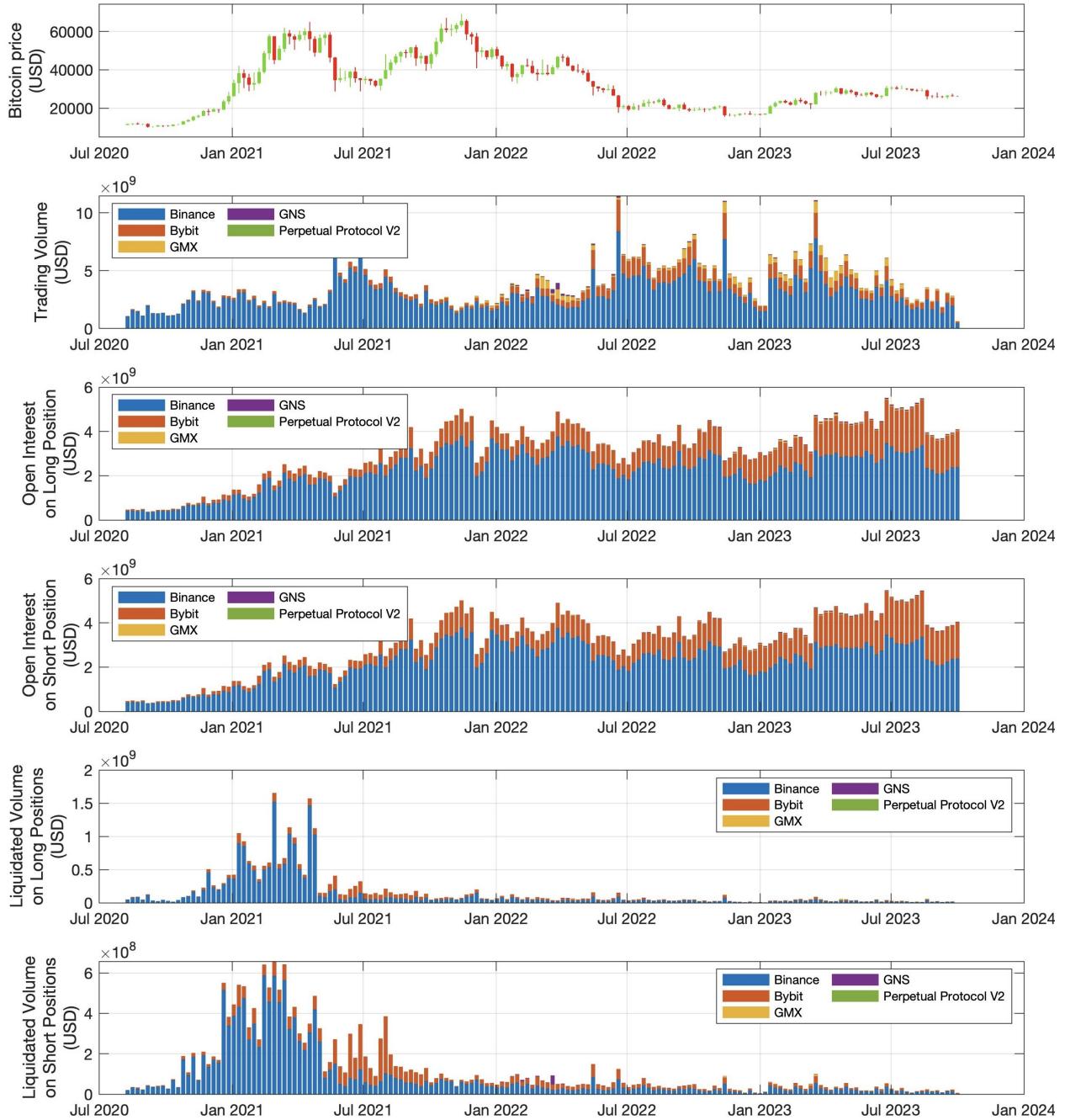


Figure 8: Trading Volumes, Open Interest, and Liquidated Volumes of Perpetual Futures on all the Five DEXs and CEXs. As CEXs dominate the market, we also create the figure solely illustrating DEXs, which is in Appendix.

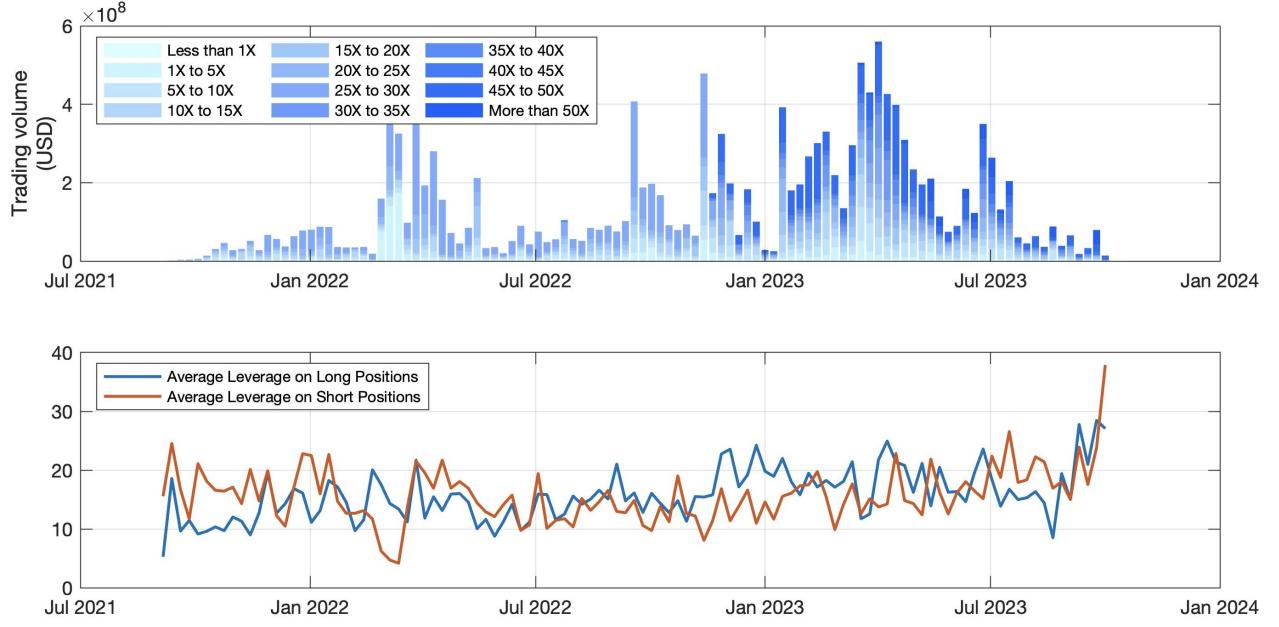


Figure 9: Trading Volume in Leverage Categories and Average Leverage of Daily Transactions (GMX). The upper panel shows the dynamics of the daily trading volume in each of the 12 leverage categories defined in Section 5.2.2, while the lower panel shows the daily average leverage for long and short positions.

$$\begin{aligned} \text{Liquidation Price} &= \text{Open Price} - \text{Liquidation Price Distance}, \text{if long, and} \\ \text{Liquidation Price} &= \text{Open Price} + \text{Liquidation Price Distance}, \text{if short, where} \end{aligned}$$

$$\text{Liquidation Price Distance} = \frac{\text{Open Price} \times \text{Leverage}}{\text{Collateral}} \times (\text{Collateral} \times 0.9 - \text{Borrowing Fees}). \quad (33)$$

Effectively, traders on GNS trade with the lowest maintenance margin ratio among the five exchanges, where Perpetual Protocol V2 adopts the maintenance margin ratio of 6.25% and 0.4% for both Binance and Bybit in the tier of the smallest position²⁷.

Fig. 9 and Fig. 10 illustrate the dynamics of the daily trading volume in different leverage categories defined in Section 5.2.2 as well as the daily average leverage level for long and short positions respectively. GMX announced to rise its maximum leverage on 8 November 2021 [40], followed with significant trading volumes in leverages more than 35× after November 2021, as shown in the upper panel of Fig. 9. It shows that although trading volumes with different leverages vary across time, the level of daily average leverage oscillates within a narrow range. The two-sample t-test indicates the insignificant difference of leverage between long and short positions for GMX, with p-value of 0.515, while the difference is significant for GNS, with p-value of 0.004, in which case long positions are with higher leverages than short positions averagely for GNS. This finding is consistent with the record by BitMEX CEO Arthur Hayes [41], in which the average effective leverage for long positions is on average higher than that for short positions. Around April 2023, there was a sudden drop in average leverage on long positions for GNS, shown in Fig. 10. This change could be attributed to the huge drop in the daily value of collateral used for transactions on long positions during this period, as the average leverage is calculated with dividing the aggregated value of collateral by the aggregated transaction value each day.

Fig. 11 shows the fraction of liquidations over total trading volume across time for DEXs, which is the liquidations standardized by trading volumes and also a proxy for evaluating the risk of an instrument [21]. As expected, GNS acts as the most risky DEX across time, for its liquidations standardized by trading volumes for both long and short positions consistently higher than that for GMX and Perpetual Protocol V2 overtime. The standardized liquidation of long (short) positions increases even when the Bitcoin price trended up (down), however, short after July 2022,

²⁷With Eq. (33), if a trader opens a long position with collateral of 50 USD and open price of 20,000 USD, he/she would be liquidated when the price drops to 19,824 USD, liquidated with a maintenance margin ratio of 0.1% equivalently.

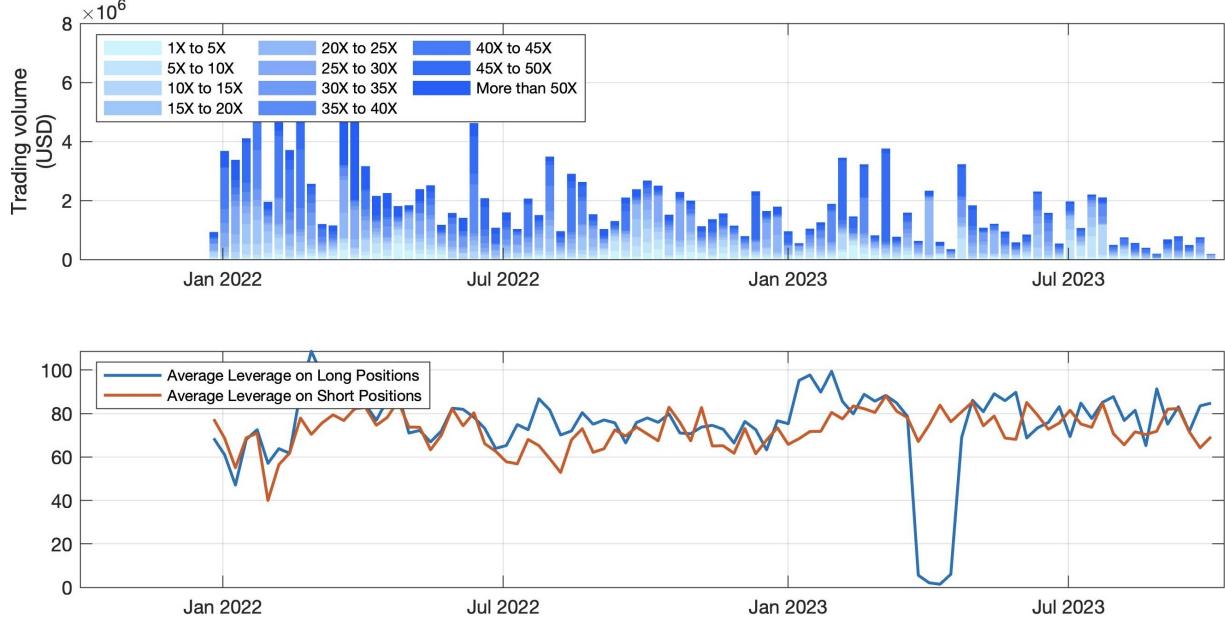


Figure 10: Trading Volume in Leverage Categories and Average Leverage of Daily Transactions (GNS). The upper panel shows the dynamics of the daily trading volume in each of the 11 leverage categories defined in Section 5.2.2, i.e., there does not exist transaction with less than 1× leverage in our dataset on GNS, while the lower panel shows the daily average leverage for long and short positions.

a counter-intuitive pattern also noted by [21]. [21] mentions the increased volatility accompanied by the uptrend of Bitcoin price, which presents as high intra-day price volatility due to price expansion, as a potential explanation but without empirical evidence. In the rest of this section, we systematically examine the association between liquidation the price volatility of Bitcoin, explaining the pattern of liquidation as well as the asymmetry between long and short positions.

5.3.2 Empirical results and analysis for CEXs.

Table 4 aggregates regression results on Eq. (29) and Eq. (30). Coefficient estimates for expected and unexpected trading volume are positive across all the five exchanges, and are statistically significant except the one for expected trading volume on GNS. Estimated coefficients for the unexpected volume (the shocks) are uniformly higher than those for the unexpected volume, suggesting that one unit change in the unexpected portion of trading volumes has higher effect on the price volatility of Bitcoin than one unit change in the expected volume, consistent with the findings in the previous studies on traditional futures market [33, 34].

For CEXs, i.e., Binance and Bybit, all the estimated coefficients measuring the marginal effects of expected and unexpected open interest are negative and significant, while the estimated coefficients for the unexpected portion are also larger in magnitude than those for the expected portion. Since in CEXs each seller is matched with a buyer of the perpetual futures, aggregated value of long positions equals to that of short positions, so do the open interests. We only include open interests on short in Eq. (29), as open interests on long are of the same value. Since CEXs adopt the mechanism with order books, it relates to the determinants of market depth, which [1] defines as the volume of unanticipated order flow required to move price by one unit. When capital inflows into the market, the liquidity increases, which is reflected by higher level of open interests, making the price less volatile in response to new orders. Consequently, it is expected that when open interest is large the price volatility conditional on contemporaneous trading volume, which proxies market depth, would be lower. Therefore, those negative estimated coefficients imply that an increase in unexpected open interest, e.g., inflow of speculative capital, lessens the impact of trading volume shock (unexpected volume) on price volatility. In our case, for Binance, when a trader increase his/her position by 1 USD, the marginal effect of the 1 USD increase in trading volume on price volatility is 5.943×10^{-8} , while mitigated by the effect of 1 USD increase in open interest (-2.365×10^{-11}), resulting in a smaller aggregated effect ($5.943 \times 10^{-8} - 2.365 \times 10^{-11}$). On the one hand, estimated coefficients for trading volumes and open interests have opposite signs, while on the other hand, whether the marginal effect of volume on price volatility is enlarged or mitigated depends on whether open interest is reduced or increased. Trades closing, decreasing, or liquidating positions



Figure 11: Volume-normalized Liquidations for DEXs. The middle and bottom panel depict the fraction of liquidations on long and short positions over total trading volume across time respectively.

actually reduce open interest, enlarging the effect of trading volume on price volatility²⁸. Price would move with larger distance, as less liquidity reacts to absorb a trade. This is reflected by that seven out of eight coefficients for liquidated volume in Binance and Bybit are significant and positive, with expected liquidated volume on short positions for Bybit as the exception. Thus, ceteris paribus, an unexpected rise in liquidation reduces market depth, via lowering open interest and liquidity. Considering the abnormality in the dynamics of liquidation data for Binance, we replicate the regressions in Eq. (29) and Eq. (30) for two periods dichotomized by 1 May 2021, the date around which liquidations on Binance suddenly dropped and became consistently low. The estimating results and analysis for the two periods separately are included in Appendix. Traders' behavior reflected in the two periods is almost the same with that reflected in Table 4.

Table 4: Regression Results on Eq. (29) and Eq. (30)

Variables	Perpetual Contracts on Bitcoin				
	GMX	GNS	Perpetual Protocol V2	Binance	Bybit
Panel a: Regression Results on Eq. (29):					
Intercept	1.438e-12 (16.82)***	0.017 (8.82)***	0.009 (3.48)***	0.011 (6.27)***	2.145 (0.15)
Lagged volatility	0.256 (8.10)***	0.371 (10.57)***	0.252 (8.19)***	0.200 (10.16)***	0.210 (17.19)***
Trading Activity:					
Expected trading volume	1.186e-10 (6.30)***	1.222e-11 (0.19)	2.59e-09 (8.76)***	3.06e-08 (12.87)***	6.041e-08 (8.91)***
Unexpected trading volume	1.29e-10 (9.83)***	4.53e-10 (6.65)***	3.62e-09 (14.42)***	5.943e-08 (24.07)***	1.409e-07 (15.62)***
Expected OI on short	-4.765e-10 (-7.36)***	-2.059e-10 (-1.99)**	-1.678e-09 (-4.40)***	-1.11e-12 (-2.24)***	-8.55e-12 (-8.69)***
Unexpected OI on short	-2.808e-10 (-3.27)***	-4.654e-10 (-3.08)***	1.125e-08 (4.53)***	-2.365e-11 (-8.93)***	-3.138e-11 (-4.65)***

(continued on next page)

²⁸Trades opening positions or rising positions increases open interests.

Table 4 (continued)

Variables	GMX	GNS	Perpetual Protocol V2	Binance	Bybit
Expected OI on long	-1.88e-10 (-5.99)***	2.98e-11 (0.44)	3.569e-09 (3.98)***	-	-
Unexpected OI on long	-1.75e-10 (-2.16)**	-4.261e-11 (-0.29)	1.481e-09 (0.161)	-	-
Liquidation:					
Expected liquidated volume on short positions	2.449e-09 (1.09)	-1.483e-09 (-1.26)	-5.983e-07 (-2.09)**	2.864e-10 (5.46)***	-2.832e-10 (-2.32)**
Unexpected liquidated volume on short positions	2.007e-09 (1.61)	3.386e-10 (0.835)	-3.032e-08 (-1.61)	1.655e-10 (6.95)***	5.098e-10 (7.22)***
Expected liquidated volume on long positions	1.82e-07 (17.71)***	7.295e-09 (3.26)***	5.008e-07 (4.16)***	8.534e-11 (3.00)***	1.672e-09 (11.96)**
Unexpected liquidated volume on long positions	2.593e-09 (5.08)***	1.448e-09 (1.46)	1.482e-07 (5.39)***	7.143e-11 (6.82)***	1.278e-09 (16.62)***
Adjusted R^2	0.375	0.310	0.507	0.647	0.649
AIC	-4381	-3670	-4067	-6580	-6664
No. of obs.	747	628	668	1147	1147
Panel b: Regression Results on Eq. (30):					
Intercept	0.021 (18.02)***	0.017 (9.88)***	0.001 (0.51)	-0.003 (-1.46)	0.015 (10.00)***
Lagged volatility	0.260 (8.40)***	0.382 (10.69)***	0.256 (8.18)***	0.224 (11.12)***	0.189 (8.95)***
Trading Activity:					
Trading volume	1.368e-10 (10.58)***	3.007e-10 (4.72)***	3.335e-09 (15.79)***	3.66e-08 (13.28)***	1.096e-07 (9.17)***
OI on short	-4.31e-10 (-8.66)***	-2.802e-10 (-3.33)***	-1.098e-09 (-2.89)***	2e-12 (2.38)**	-7.609e-12 (-5.60)***
OI on long	-1.966e-10 (-6.22)***	6.244e-11 (1.10)	3.74e-09 (3.95)***	-	-
\mathcal{SI} and \mathcal{LIQ} :					
\mathcal{SI}	-2.597e-05 (-0.63)	-0.001 (-1.79)*	3.000e-04 (3.19)***	0.020 (4.07)***	0.004 (0.40)
$\mathcal{LIQ}_{\text{short}}$	0.004 (1.48)	-0.001 (-0.816)	-0.065 (-2.29)**	0.184 (7.30)***	0.169 (7.58)***
$\mathcal{LIQ}_{\text{long}}$	0.015 (5.78)***	0.017 (2.75)***	0.019 (1.19)	0.156 (11.73)***	0.34 (0.40)
Adjusted R^2	0.380	0.282	0.466	0.600	0.600
AIC	-4389	-3649	-4018	-6435	-6511
No. of obs.	747	628	668	1147	1147

Notes: Panel a of this table reports the regression results of Eq. (29) while Panel b reports the regression results of Eq. (30). For perpetual futures on both DEXs and CEXs, contract size is undefined and traders trade perpetual futures with arbitrary units of Bitcoins, so trading volumes and open interests are measured in USD. Since for DEXs each future contract is matched with a buyer and a seller, resulting in the same dollar value of open interests for long and short, we regress price volatility only on open interest on short. This is only for notation while does not mean CEXs are only with short positions, because the dollar value of open interest on short is always equals to that on long. In each cell, the t-statistics are in the parentheses. *, **, and *** denote significance at 0.1, 0.05, and 0.01 level, respectively.

5.3.3 Empirical results and analysis for Perpetual Protocol V2 (with VAMM Model).

In CEXs, traders are matched by the market maker and provide liquidity. Different from the case of CEXs, traders in DEXs are matched with liquidity pools, in which liquidity providers act passively as the counterparty. Traders' open interest thus does not necessarily influence the market depth in the same way as in CEXs and the theory of market depth by [1] for traditional futures market is inapplicable in this context. For Perpetual Protocol V2, which adopts VAMM as the pricing mechanism, liquidity providers deposit stable coins in the vault based on which the clearing house mints

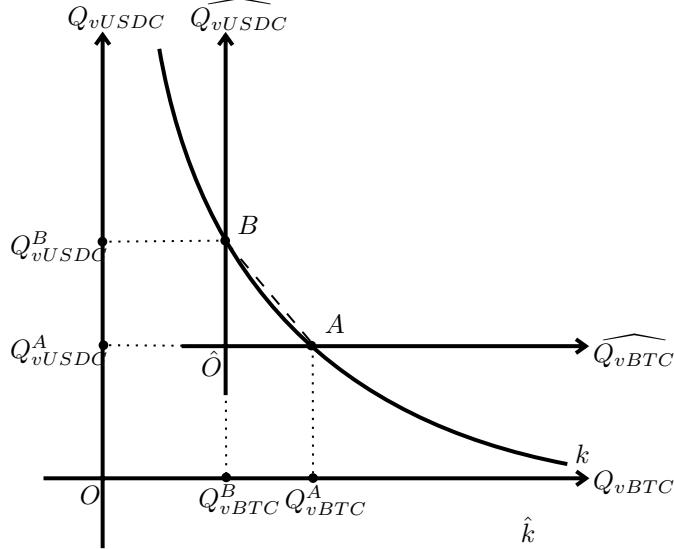


Figure 12: A Uniswap v3 pool with a single liquidity provider.

virtual tokens and places those virtual tokens into the Uniswap v3 AMM²⁹. For illustration, assume the price of the Bitcoin perpetual is determined by the exchange function defined in Section 3.8:

$$Q_{vUSDC} \times Q_{vBTC} = k \quad (34)$$

where Q_{vUSDC} and Q_{vBTC} denote the quantities of virtual USDC (vUSDC) and virtual Bitcoin (vBTC) in the liquidity pool, and k reflects the depth of liquidity, which is determined by liquidity providers. With constant k , the price of virtual Bitcoin is determined by the ratio of Q_{vUSDC} and Q_{vBTC} :

$$P_{vBTC} = \frac{Q_{vUSDC}}{Q_{vBTC}} = k \times Q_{vBTC}^{-2}. \quad (35)$$

The absolute value of the first derivative of P_{vBTC} with respect to Q_{vBTC} is monotonously decreasing, which can be derived as:

$$\left| \frac{\partial P_{vBTC}}{\partial Q_{vBTC}} \right| = \left| -2 \times k \times Q_{vBTC}^{-3} \right| = 2 \times k \times Q_{vBTC}^{-3}, \quad (36)$$

indicating the decreasing rate of change in pricing as vBTC goes abundant in the liquidity pool. The more vBTC there are in the liquidity pool, the lower volatile its price is. When a trader opens a long position, the clearing house swaps vUSDC for vBTC from the liquidity pool, adding vUSDC to and withdrawing vBTC from the pool. Q_{vBTC} decreases while P_{vBTC} becomes more volatile.

Besides, Uniswap v3 AMM implements concentrated liquidity to enhance capital efficiency, where liquidity providers specify a price range, $[\underline{P}, \bar{P}]$, within which to add liquidity³⁰. If there is only a single liquidity provider specifying the price interval $[P_{vBTC}^A, P_{vBTC}^B]$ to add liquidity, where $P_{vBTC}^A = \frac{Q_{vUSDC}^A}{Q_{vBTC}^A}$ and $P_{vBTC}^B = \frac{Q_{vUSDC}^B}{Q_{vBTC}^B}$, this concentrated liquidity can be characterized by drawing a new set of axes, i.e., \widehat{Q}_{vBTC} and \widehat{Q}_{vUSDC} with the origin \hat{O} as shown in Fig. 12, to track the real reserve offered in range AB . While the exchange function is defined in the $Q_{vBTC} - Q_{vUSDC}$ space (*virtual* reserve termed by Unisawp v3), the actual reserve specified in the price range is called *real*. Any point (Q_{vBTC}, Q_{vUSDC}) in the *virtual* space can be transformed to a point $(\widehat{Q}_{vBTC}, \widehat{Q}_{vUSDC}) = (Q_{vBTC} - Q_{vBTC}^B, Q_{vUSDC} - Q_{vUSDC}^A)$ in the *real* space. The liquidity provider actually defines the coordinate of the *real* origin \hat{O} , which is a function of the interval $[P_{vBTC}^A, P_{vBTC}^B]$.

In practice, liquidity offered by multiple liquidity providers cumulates around the current price level, as illustrated in Fig. 13. Trades at the tails of the distribution lead to larger price move, i.e., thus larger price volatility, due to lower liquidity than at the center. Therefore, while the design of concentrated liquidity determines market depth locally,

²⁹Details of how VAMM works in Perpetual Protocol V2 can be found in <https://support.perp.com/hc/en-us/articles/9594157347481-How-It-Works>.

³⁰For details of geometric explanation on concentrated liquidity, readers can refer to [42].

the exchange function Eq. (34) forms price globally. In the long-run, (Q_{vBTC}, Q_{vUSDC}) moves along the exchange function and the change in slope determines the price volatility. In the short-run, the liquidity distribution centered at the current average price determines price volatility.



Figure 13: Example of Liquidity Distribution on Uniswap v3 AMM. This figure shows the liquidity distribution for the pool of the pair WBTC/USDC on Uniswap V3. This snapshot is made on 7 Nov 2023. Source: <https://dune.com/queries/65034/129883>.

The implication of Uniswap v3 AMM on pricing volatility is consistent with our empirical findings. In Table 4, the coefficient for expected open interest on short is estimated to be significantly negative (-1.678×10^{-9}) while that on long is estimated to be significantly positive (3.569×10^{-9}), indicating a negative association between expected open interest on short and price volatility while a positive association for long. Since the expected portion of open interest reflects traders' long-run behavior, e.g., hedgers, in cope with the trend, the change could be mainly be described by Eq. (34). When the expected open interest on short rises, the clearing house retrieve vBTC from traders' account and credit to the balance of the liquidity pool. The increased vBTC balance in the liquidity pool helps decrease the rate of pricing change and thus reducing volatility according to Eq. (36). Besides, when expected open interest on long increases, the clearing house withdraw vBTC from the pool and credit to traders', decreasing vBTC balance in the liquidity pool and thus increasing price volatility.

While the expected portion of open interest captures how market status moves along the exchange function specified by Eq. (34), the unexpected portion reflects traders' unanticipated behavior, e.g., speculation, on the basis of liquidity distribution. In the short-run, liquidity providers are lagged to adjust their bound of price within which to add liquidity, i.e., $[P, \bar{P}]$, especially when facing unanticipated change. Therefore, open interest on both long and short positions would drive price to the tails of the liquidity distribution, enlarging pricing volatility due to lower liquidity. In Table 4, the estimated coefficients for the association between price volatility and unexpected open interest are all positive, i.e., 1.125×10^{-8} for short and 1.481×10^{-9} for long, substantiating our prediction based on the Uniswap v3 AMM pricing mechanism.

In the context of Perpetual Protocol V2, liquidity providers have the capability to contribute liquidity utilizing leverage, thereby incurring a risk of passive liquidation when counterparties are liquidated. Transactions that liquidate liquidity providers' positions during a price increase are categorized as *liquidation on short*, whereas those during a price decline are classified as *liquidation on long*. The liquidation of liquidity providers' positions results in a reduction of market depth, which is hypothesized to have a direct correlation with increased price volatility and the frequency of liquidations among liquidity providers.

Regarding the expected liquidation volume of traders' positions, its impact on price volatility is anticipated to be inversely proportional to the effect of increasing expected open interest. Specifically, an expected increase in liquidation on short positions, which reduces open interest in these positions, is posited to have a positive correlation with price volatility, and the converse holds true for long positions. In the case of the unexpected component of liquidation, its influence on price volatility aligns with that of unexpected open interest, with all unforeseen trading activities driving price movements towards the extremities of the liquidity distribution, thereby exacerbating price volatility.

Overall, an increase in unexpected liquidation in both long and short positions is expected to elevate price volatility. This is empirically corroborated in Table 4, where the estimated coefficient for unexpected liquidated volume on long positions is significantly positive (5.008×10^{-7}). Conversely, the aggregated effect of an increase in expected liquidation on short positions should theoretically be positive. However, this is contradicted by the negative sign of the estimated coefficient for expected liquidated volume on short positions (-5.983×10^{-7}). A plausible explanation for this anomaly is that a price rise, typically accompanying an increase in short position liquidations, may attract additional liquidity providers. This influx of liquidity providers potentially mitigates price volatility, with their dampening effect on volatility surpassing the amplifying impact of liquidations. We have conducted the Granger Causality Test to examine the relationship between the return on Bitcoin and the net change in liquidity in the vBTC-vUSDC pool on Perpetual Protocol V2, which you can find in Appendix. It is shown that return on Bitcoin Granger-causes the net change in liquidity in the vBTC-vUSDC pool, while the reversed relationship does not hold.

The asymmetrical effect of open interest on long and short positions is also predicted in the theoretical work by [43], which suggests that consuming liquidity involves a larger price impact than adding it, resulting in the larger marginal cost of a ETH buy order than the marginal benefit of a ETH sell order. The model proposed in [43] implies an ambiguous reaction of AMM liquidity to asset volatility, driven by the migration of both traders and liquidity providers between CEXs and DEXs. In our test, since liquidation explicitly includes behavior of both traders and liquidity providers, the ambiguity mentioned in [43] may help explain the inconsistency between our predicted effect of expected liquidation on short positions and the estimated coefficient.

5.3.4 Empirical results and analysis for GMX and GNS (with Oracle Pricing Model).

In the realm of centralized exchanges (CEXs) and Perpetual Protocol V2, traders' behaviors and informational inputs actively shape market prices. Conversely, in decentralized exchanges (DEXs) such as GMX and GNS, traders accept prices as a given, lacking the capacity to exert direct influence on price movements. The pricing mechanism in these DEXs is governed by a Price Oracle, as elaborated in Section 3.6, which assimilates prices from various exchanges, thereby rendering transactions on GMX and GNS incapable of impacting the price directly³¹. As a result, trading activities on these platforms, encompassing trading volume and open interest, merely reflect traders' interpretations and reactions to the information encapsulated within the Price Oracle's determined prices.

Drawing upon Shalen's dispersion of beliefs model (1993) [2], which posits a market dichotomy between asymmetrically informed traders, market participants can be categorized into informed and uninformed traders³². Uninformed traders, lacking access to private information, often engage in irrational trading based on market noise and tend to overreact to information. They strive, albeit unsuccessfully, to discern private information and market trends from current price fluctuations, leading to heightened position-taking during periods of increased price volatility. In contrast, informed traders, operating on the basis of their private information, exhibit relatively consistent beliefs over time and typically engage in trading within a constrained price range, resulting in a negative correlation between their positions and price volatility [34].

In Panel a of Table 4, the estimated coefficients that measure the relationship between open interest and price volatility are predominantly negative, except the case of long open interests in GNS. This indicates a more pronounced influence of informed traders' behavior, leading to the negative signs in these estimations. The impact of uninformed traders on open interest is effectively counterbalanced by the actions of informed traders. Notably, traders' behaviors exhibit asymmetry between buyers and sellers, as evidenced by less negative (in the case of GMX) or even insignificant (for GNS) coefficients for long positions. This asymmetry could be attributed to uninformed traders' tendency to overreact more to positive news than negative, leading to an increased propensity for long position accumulation³³.

The asymmetry in long and short positions is further evidenced by variations in liquidated volume in relation to price volatility. As indicated in Table 4, for GMX and GNS, the coefficients linking liquidated volume on long positions are significantly positive, whereas those for short positions are not statistically significant. Additionally, Panel b of Table 4 reveals that long positions are more likely to liquidation than short positions, as denoted by the significantly positive coefficients for $\mathcal{L}\mathcal{T}Q_{\text{long}}$ compared to the insignificant coefficients for $\mathcal{L}\mathcal{T}Q_{\text{short}}$. This suggests that liquidation on long positions escalates both in frequency and magnitude when price volatility intensifies, corroborating the notion of traders' asymmetric overreaction to good news. Traders tend to overreact by not only increasing long positions but also by employing higher leverage for long positions compared to short, as evidenced in Table 3, resulting in a higher rate of liquidation for long positions in both volume and probability.

Aligned with Shalen's model, it is anticipated that uninformed traders are more likely to increase leverage during periods of heightened price volatility to maximize capital efficiency, thereby enlarging their positions at a lower cost. In contrast, informed traders are less inclined to risk capital under volatile conditions, leading to a negative correlation between leverage and price volatility. Table 5 delineates the estimated results for Eq.(31) and Eq.(32). All eight coefficients relating price volatility and average leverage are estimated to be negative (as shown in the two columns under *Regression on Eq.(31)*), indicating a more pronounced change in leverage as influenced by informed traders compared to uninformed traders. This inference is further substantiated by the estimation results for Eq.(32), which demonstrate that trading volumes with lower leverages (less than 50×) increase, while those with higher leverages (more than 50×) remain statistically unchanged in response to rising price volatility.

³¹The code for GMX's price feeding smart contract is publicly accessible at: <https://github.com/gmx-io/gmx-contracts/blob/master/contracts/oracle/FastPriceFeed.sol>.

³²Private information refers to the advantageous knowledge held predominantly by major market players such as fund companies and institutions, who have superior information gathering capabilities and insights into customer positions, distinguishing them from smaller retail investors.

³³Existing literature on traditional spot and future markets provides evidence of traders' asymmetric reactions to positive and negative news [44, 34].

In summation, the analysis reveals that informed traders exhibit a higher level of activity compared to their uninformed counterparts on GMX and GNS, a conclusion substantiated by empirical data pertaining to the dynamics of open interests and leverages. Additionally, the observed asymmetry in open interests and liquidations between long and short positions suggests a tendency among traders to disproportionately overreact to good news. However, it is imperative to acknowledge a critical distinction for analytical purposes: unlike GMX and GNS, centralized exchanges (CEXs) possess a price discovery function, leading to a reciprocal influence between traders' behavior and market prices. Under Efficient-Market Hypothesis [45], the price aggregate and reflect all the current private information and information contained in the past pricing history. Consequently, it is methodologically unsound to interpret changes in trading activities, e.g., volume, open interest, liquidation, and leverage, solely as reactions to price fluctuations. This perspective challenges the conclusions drawn by [21], who posited that changes in liquidation are merely passive responses to price movements, thereby overlooking the role of liquidation as a contributory factor in price formation within BitMEX, a CEX. This oversight underscores the necessity for a nuanced understanding of the interplay between trader behavior and price dynamics, particularly in the context of CEXs where the price discovery process is inherently more complex.

Table 5: Regression results on Eq. (31) and Eq. (32)

Variables	Perpetual Contracts on Bitcoin			
	Regression on Eq. (31)		Regression on Eq. (32)	
	GMX	GNS	GMX	GNS
Intercept	2.287e-12 (9.47)***	0.028 (3.84)***	1.609e-12 (15.55)***	9.729e-12 (3.89)***
Lagged volatility	0.229 (7.26)***	0.369 (10.66)***	0.200 (6.37)***	0.374 (10.05)***
Trading Activity:				
Expected trading volume	1.051e-10 (5.60)***	5.201e-11 (0.83)	-	-
Unexpected trading volume	1.359e-10 (10.41)***	4.593e-10 (6.86)***	-	-
Expected OI on short	-4.405e-10 (-6.52)***	-1.626e-10 (-1.58)	-5.415e-10 (-7.32)***	-1.893e-10 (-1.67)*
Unexpected OI on short	-2.485e-10 (-2.89)***	-4.423e-10 (-2.98)***	-2.767e-10 (-3.01)***	-4.355e-10 (-2.71)***
Expected OI on long	-1.373e-10 (-4.11)***	3.964e-11 (0.58)	-2.318e-10 (-4.76)***	2.426e-11 (0.33)
Unexpected OI on long	-1.091e-10 (-1.36)	-5.505e-11 (-0.38)	-2.97e-10 (-3.61)***	4.033e-11 (0.26)
Liquidation:				
Expected liquidated volume on short positions	2.538e-09 (1.15)	-1.467e-09 (-1.27)	4.888e-10 (0.21)	5.541e-10 (0.40)
Unexpected liquidated volume on short positions	1.891e-09 (1.55)	3.177e-10 (0.80)	2.289e-09 (1.86)*	1.925e-10 (0.42)
Expected liquidated volume on long positions	2.77e-07 (9.14)***	7.573e-09 (3.45)***	1.965e-07 (14.98)***	5.472e-09 (2.27)**
Unexpected liquidated volume on long positions	2.539e-09 (5.06)***	1.384e-09 (1.42)	2.455e-09 (4.78)***	3.984e-09 (3.70)***
Leverage (average level):				
Expected average leverage on short positions	-0.0004 (-2.21)**	-0.0001 (-1.40)	-	-
Unexpected average leverage on short positions	-0.0002 (-3.00)***	-0.0002 (-4.37)***	-	-
Expected average leverage on long positions	-0.0004 (-2.15)**	-1.193e-05 (-0.36)	-	-
Unexpected average leverage on long positions	-0.0003 (-3.65)***	-7.237e-05 (-1.93)**	-	-
Leverage (in categories):				
Expected: less than 1×	-	-	-2.112e-08 (-1.21)	-
Unexpected: less than 1×	-	-	1.688e-08 (3.46)***	-
Expected: 1× to 5×	-	-	1.829e-10 (1.08)	1.684e-08 (0.23)

(continued on next page)

Table 5 (continued)

Variables	Regression on Eq. (31)		Regression on Eq. (32)	
	GMX	GNS	GMX	GNS
Unexpected: 1× to 5×	-	-	9.772e-10 (5.51)***	-1.289e-08 (-0.54)
Expected: 5× to 10×	-	-	1.847e-09 (3.44)***	-2.534e-08 (-0.55)
Unexpected: 5× to 10×	-	-	1.012e-09 (5.27)***	1.736e-08 (1.08)
Expected: 10× to 15×	-	-	-6.697e-10 (-1.50)	5.39e-08 (0.96)
Unexpected: 10× to 15×	-	-	8.724e-11 (0.63)	6.408e-09 (0.44)
Expected: 15× to 20×	-	-	1.998e-09 (3.75)***	9.314e-08 (1.84)*
Unexpected: 15× to 20×	-	-	2.023e-10 (1.28)	5.102e-09 (0.33)
Expected: 20× to 25×	-	-	-1.06e-10 (-0.12)	2.698e-07 (3.89)***
Unexpected: 20× to 25×	-	-	2.66e-10 (2.15)**	6.261e-09 (0.92)
Expected: 25× to 30×	-	-	-9.034e-11 (-0.76)	-4.597e-08 (-1.31)
Unexpected: 25× to 30×	-	-	9.099e-11 (1.70)**	-7.329e-09 (-0.91)
Expected: 30× to 35×	-	-	5.761e-10 (1.91)*	8.934e-08 (1.34)
Unexpected: 30× to 35×	-	-	-3.618e-12 (-0.02)	-3.778e-09 (-0.26)
Expected: 35× to 40×	-	-	2.882e-11 (0.03)	2.465e-07 (3.10)***
Unexpected: 35× to 40×	-	-	2.266e-10 (0.93)	-1.913e-09 (-0.32)
Expected: 40× to 45×	-	-	1.289e-09 (0.51)	-2.922e-10 (-0.01)
Unexpected: 40× to 45×	-	-	3.24e-10 (1.12)	-2.72e-09 (-0.61)
Expected: 45× to 50×	-	-	2.626e-11 (0.15)	1.235e-08 (0.74)
Unexpected: 45× to 50×	-	-	1.692e-10 (1.73)*	9.194e-10 (0.30)
Expected: more than 50×	-	-	-1.529e-10 (-0.29)	-1.466e-09 (-0.13)
Unexpected: more than 50×	-	-	-3.74e-11 (-0.20)	-1.21e-08 (-0.96)
Adjusted R^2	0.398	0.339	0.427	0.260
AIC	-4405	-3693	-4424	-3608
No. of obs.	747	628	747	628

Notes. This table reports regression results on Eq. (31) and Eq. (32). For the regression with Eq. (32), *Leverage (in categories)* means trading volumes in leverage within different ranges, which are grouped into categories. For GNS, our dataset does not include any transaction with leverage less than 1×, so we drop the variables of trading volumes with the leverage less than 1×. In each cell, the t-statistics are in the parentheses. *, **, and *** denote significance at 0.1, 0.05, and 0.01 level, respectively.

6 Conclusion

In conclusion, this study presents an in-depth Systematic Survey of Knowledge (SoK) that effectively contrasts the operational dynamics of Centralized Exchanges (CEXs) and Decentralized Exchanges (DEXs) in the realm of perpetual futures trading. It significantly enriches the academic discourse by meticulously delineating the design elements, order matching mechanisms, price discovery processes, and custody of funds in both CEXs and DEXs, thereby offering a

robust foundation for future research in this field. The study underscores the necessity of developing new theoretical models, moving beyond conventional frameworks like Kyle's model[1], to aptly comprehend the intricacies of DEXs. Through an empirical analysis of transaction data from August 2020 to September 2023, it reveals a positive correlation between price volatility and trading volumes in CEXs, in line with traditional futures markets. However, it also uncovers distinct variations in DEXs, particularly under the VAMM Pricing Model, where open interest impacts long and short positions differently. The behavior of traders in GMX and GNS, who operate under Oracle Pricing and act as price takers, is found to be consistent with Shalen's dispersion of beliefs model[2]. Furthermore, the study highlights the propensity of uninformed traders to disproportionately react to positive news than negative news, thereby deepening our understanding of trader behavior in DEXs and offering valuable insights for the development of future theoretical models in this evolving domain.

In terms of future work, firstly, the observed asymmetry in trading behaviors between buyers and sellers within the Oracle Price Model DEX warrants a deeper exploration into the psychological underpinnings of this phenomenon. Future research could delve into whether uninformed traders disproportionately react to positive news compared to negative news, and examine the extent to which cognitive biases or prevailing social norms may influence such behaviors. This line of inquiry could provide valuable insights into the psychological mechanisms at play in decentralized exchange environments.

Secondly, the role of liquidity providers, a novel concept in the decentralized finance (DeFi) ecosystem, presents a rich area for academic investigation. While this paper primarily analyzes the behaviors of traders in Perpetual Future Contracts, a comparative study focusing on their counterparts, the liquidity providers, could yield significant findings. Future research should consider the impact of variables such as interest rates and the availability of alternative DeFi products on the behaviors of liquidity providers, particularly in the context of Perpetual Future Contracts. Understanding how these factors interact to influence liquidity provider behaviors could offer a more comprehensive view of market dynamics in DeFi.

Lastly, the potential of blockchain-based financial instruments to enhance transparency and accountability in micro financial markets is an important avenue for future research. The decentralized and transparent nature of blockchain technology suggests that these instruments could be instrumental in promoting accountability and transparency in micro financial markets, potentially encouraging more ethical investment practices. Future studies should explore the capacity of blockchain-based instruments to achieve these outcomes, examining the factors that may affect their effectiveness in fostering responsible financial practices.

7 Acknowledgement

ChatGPT has been used in the writing for the purpose of spelling, syntax and grammar checks. The authors take full responsibility of the output.

Appendix

Table 6 reports the results of the Augmented Dickey–Fuller (ADF) test, which is used to check whether the time series we are examining are stationary. According to Section 5.2, when decomposing the series into expected and unexpected portions, the series for which we fail to reject the hypotheses of the existence of a unit root are differenced, i.e., the series with insignificant statistics in Table 6.

Fig. 14 shows the dynamics of trading volumes, open interests, and liquidations solely for DEXs. GMX dominated across time with the highest trading volumes. Although the trading volumes on GNS are far less than those on GMX, it has open interest comparable to GMX. In addition, liquidations on long positions are more frequent and higher in value than those on short positions along the time.

Table 6: Augmented Dickey–Fuller (ADF) test statistics

Variables	Perpetual Contracts on Bitcoin				
	GMX	GNS	Perpetual Protocol V2	Binance	Bybit
Trading Volume	-4.850***	-4.178***	-3.680***	-3.725***	-2.609*
OI on Short	-2.896**	-1.868	-1.621	-2.930*	-1.428
OI on Long	-3.556***	-1.559	-2.179	-	-
Liquidation on Short	-13.712***	-16.172***	-24.228***	-2.115	-2.886**

(continued on next page)

Table 7: Granger Causality Test of Return on Bitcoin and the Net Change in Liquidity in the vBTC-vUSDC Pool on Perpetual Protocol V2

H_0	Max-lag	F-statistics	p-value ($Prob > F$)
Return on BTC does not Granger-cause net change in liquidity pool	15	1.624	0.079
Net change in liquidity pool does not Granger-cause Return on BTC	15	1.164	0.311

Notes. This table reports the results of Granger Causality Test in detecting the relationship between return on Bitcoin and the net change in the liquidity contained in vBTC-vUSDC pool on Perpetual Protocol V2.

Table 6 (continued)

Variables	Perpetual Contracts on Bitcoin				
	GMX	GNS	Perpetual Protocol V2	Binance	Bybit
Liquidation on Long	-26.746***	-8.175***	-8.540***	-2.787*	-3.430***
Leverage on Short	-6.769***	-5.844***	-	-	-
Leverage on Long	-4.110***	-4.204***	-	-	-
$\mathcal{L}\mathcal{T}$	-3.388**	-2.542	-4.375***	-3.044**	-3.836***
$\mathcal{L}\mathcal{T}\mathcal{Q}_{short}$	-26.480***	-8.453***	-23.066***	-1.799	-2.982**
$\mathcal{L}\mathcal{T}\mathcal{Q}_{long}$	-27.120***	-2.872**	-15.664***	-2.764*	-2.260
Leverage (in categories):					
less than 1×	-6.106***	-	-	-	-
1× to 5×	-5.149***	-6.247***	-	-	-
5× to 10×	-5.194***	-4.790***	-	-	-
10× to 15×	-3.846***	-8.132***	-	-	-
15× to 20×	-9.448***	-8.472***	-	-	-
20× to 25×	-4.339***	-24.221***	-	-	-
25× to 30×	-4.614***	-7.505***	-	-	-
35× to 40×	-2.338	-15.250***	-	-	-
40× to 45×	-3.139**	-3.196***	-	-	-
45× to 50×	-2.245	-14.365***	-	-	-
More than 50×	-4.361***	-4.145***	-	-	-

Notes. This table reports the statistics for Augmented Dickey–Fuller (ADF) test, which is used to test whether a time series is stationary. ADF test statistics are for the hypothesis that a series contains a unit root. *, **, and *** denote significance at 0.1, 0.05, and 0.01 level, respectively.

As conjectured in our analysis on liquidation in Perpetual Protocol V2, we hypothesize that price rise in Bitcoin attract liquidity providers in offering liquidity. To detect empirical evidence to our conjecture, we conduct the Granger Causality Test to see the relationship between the return on Bitcoin and the net change in liquidity in the vBTC-vUSDC pool on Perpetual Protocol V2, where the net change is calculated by subtracting the daily added liquidity by the daily withdrawn liquidity from the pool (measured in Bitcoin). Return on Bitcoin is calculated in daily frequency with the log-form return as follows: $R_t = \log(P_t) - \log(\bar{P}_t)$, where P_t and \bar{P}_t denote the close and open price of Bitcoin on day t . We retrieve data from 15 May 2023 to 15 Oct 2023 on Etherscan (<https://optimistic.etherscan.io/>). The testing results are reported in Table 7. There exists Granger-causality from the return on Bitcoin to the net change in vBTC-vUSDC liquidity pool, indicated by the statistically significant F-statistic with p-value of 0.079. However, the causality in reversed direction (net change in vBTC-vUSDC liquidity pool Granger-cause return on Bitcoin) is not evidenced, with p-value of 0.311, supporting our hypothesis that more liquidity is injected when Bitcoin price jump. The large lag, i.e., 15 days in this case, may suggests that traders need time to adjust positions in the liquidity pool in reaction to the rise in Bitcoin price. The empirical evidence here shows a rough picture of how traders change their deposit in the liquidity pool while future studies are needed with more factors controlled.

The ADF test and regression results of Eq. (29) and Eq. (30) for Binance using data separately before (Period(1)) and after (Period(2)) 1 May 2021 are reported in Table 8. Same with the case in Table 4, coefficients for both expected and unexpected portion of the trading volume are estimated to be significantly positive, and those for open interest are

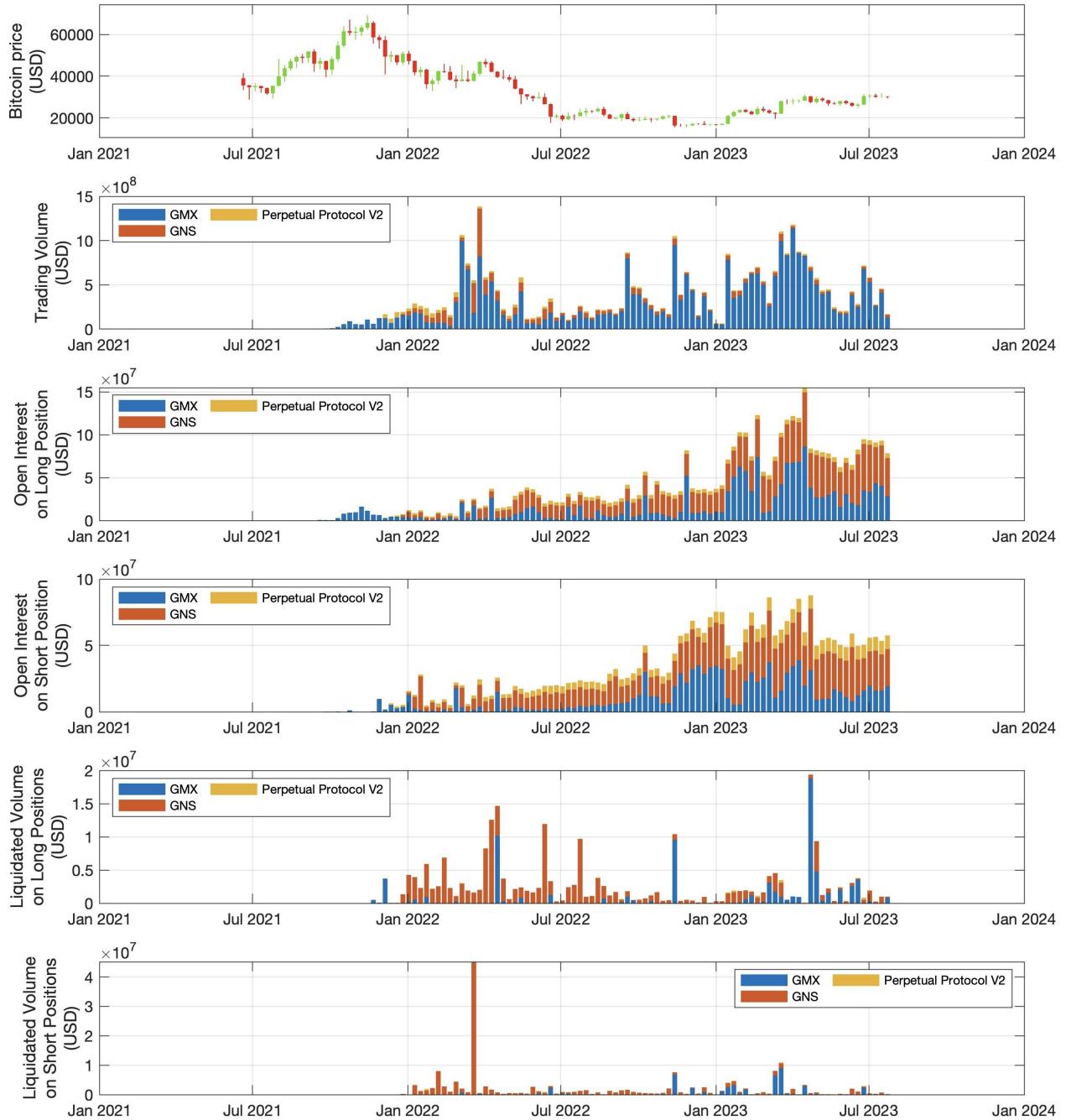


Figure 14: Trading Volumes, Open Interest, and Liquidated Volumes of Perpetual Futures on the Three DEXs.

estimated to be negative, substantiating our notion in Section 5.3 that the increase in open interest mitigates the effects of shocks on price volatility.

Table 8: ADF Test and Regression Results with Truncated Data for Binance

Regression Results Eq. (29) and Eq. (30)			ADF Test		
Variables	Period(1)	Period(2)	Variables	Period(1)	Period(2)
Panel a: Regression Results on Eq. (29):			Panel b: Results of ADF test		
Intercept	0.011 (3.23)***	0.010 (3.90)***	Trading Volume	-2.775*	-3.172**
Lagged volatility	0.145 (4.58)***	0.041 (2.07)**	OI	-1.051	-3.810***
Trading Activity:			Liquidation on Short	-1.430	-4.583***
Expected trading volume	5.57e-08 (5.10)***	1.84e-08 (8.52)***	Liquidation on Long	-4.977***	-4.939***
Unexpected trading volume	7.208e-08 (8.42)***	3.958e-08 (14.24)***	\mathcal{SI}	-2.258	-3.853***
Expected OI	-1.623e-11 (-6.47)***	-1.66e-12 (-2.27)**	$\mathcal{LIQ}_{\text{short}}$	-1.822	-8.152***
Unexpected OI	-5.132e-12 (-0.55)	-1.338e-11 (-5.09)***	$\mathcal{LIQ}_{\text{long}}$	-5.951***	-7.284***
Adjusted R^2	0.82	0.76		-	-
AIC	-1688	-5408		-	-
No. of obs.	269	878		-	-
Liquidation:					
Expected liquidated volume on short	3.859e-10 (5.57)***	8.418e-10 (3.41)***			
Unexpected liquidated volume on short	1.083e-10 (4.52)***	1.367e-09 (8.59)***			
Expected liquidated volume on long	1.694e-10 (4.9)***	1.882e-09 (8.59)***			
Unexpected liquidated volume on long	9.729e-11 (8.46)***	9.686e-10 (9.16)***			
Panel c: Regression Results on Eq. (30):					
Intercept	0.007 (2.30)**	-0.004 (-1.47)			
Lagged volatility	0.176 (5.62)***	0.026 (1.34)			
Trading Activity:					
Trading volume	5.329e-08 (4.27)***	2.725e-08 (5.89)***			
OI	-2.124e-13 (-0.10)	3.563e-12 (3.61)***			
\mathcal{SI} and \mathcal{LIQ} :					
\mathcal{SI}	-0.009 (-1.37)	0.007 (0.59)			
$\mathcal{LIQ}_{\text{short}}$	0.158 (6.03)***	2.188 (11.03)***			
$\mathcal{LIQ}_{\text{long}}$	0.156 (12.09)***	2.374 (8.15)***			
Adjusted R^2	0.81	0.76			
AIC	-1675	-3649			
No. of obs.	269	878	269	878	

This table reports the estimation results of Eq. (29) (Panel a) and Eq. (30) (Panel c), as well as the ADF test (Panel b), for Binance in two periods separately. Period (1) presents the time from 5 August 2020 to 30 April 2021, and Period (2) stands for the time from 1 May 2021 to 26 September 2023. ADF test statistics are for the hypothesis that a series contains a unit root. In each cell of Panel a and Panel c, the t-statistics are in the parentheses. *, **, and *** denote significance at 0.1, 0.05, and 0.01 level, respectively.

References

- [1] Albert S Kyle. Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society*, pages 1315–1335, 1985.
- [2] Catherine T Shalen. Volume, volatility, and the dispersion of beliefs. *The Review of Financial Studies*, 6(2):405–434, 1993.
- [3] Jonathan Chiu, Emre Ozdenoren, Kathy Yuan, and Shengxing Zhang. On the fragility of defi lending. *Available at SSRN 4328481*, 2022.
- [4] Alfred Lehar and Christine A Parlour. Systemic fragility in decentralized markets. *Available at SSRN*, 2022.
- [5] Thomas J Rivera, Fahad Saleh, and Quentin Vandeweyer. Equilibrium in a defi lending market. *Available at SSRN 4389890*, 2023.
- [6] Aaron Green, Christopher Cammilleri, John S Erickson, Oshani Seneviratne, and Kristin P Bennett. Defi survival analysis: Insights into risks and user behaviors. In *The International Conference on Mathematical Research for Blockchain Economy*, pages 127–141. Springer, 2022.
- [7] Lioba Heimbach and Wenqian Huang. Defi leverage. *Available at SSRN 4459384*, 2023.
- [8] Roman Kozhan and Ganesh Viswanath-Natraj. Decentralized stablecoins and collateral risk. *WBS Finance Group Research Paper*, 2021.
- [9] Daniel Perez, Sam M Werner, Jiahua Xu, and Benjamin Livshits. Liquidations: Defi on a knife-edge. In *Financial Cryptography and Data Security: 25th International Conference, FC 2021, Virtual Event, March 1–5, 2021, Revised Selected Papers, Part II 25*, pages 457–476. Springer, 2021.
- [10] Kaihua Qin, Liyi Zhou, Pablo Gamito, Philipp Jovanovic, and Arthur Gervais. An empirical study of defi liquidations: Incentives, risks, and instabilities. In *Proceedings of the 21st ACM Internet Measurement Conference*, pages 336–350, 2021.
- [11] Matthias Schaible. *Decentralized Lending: Empirical Analysis of Interest and Liquidation Mechanisms*. Springer Nature, 2022.
- [12] Jakub Warmuz, Amit Chaudhary, and Daniele Pinna. Toxic liquidation spirals: Evidence from the bad debt incurred by aave. *arXiv preprint arXiv:2212.07306*, 2022.
- [13] Hugo E Ramirez and Julián Fernando Sanchez. Optimal liquidation with temporary and permanent price impact, an application to cryptocurrencies. *arXiv preprint arXiv:2303.10043*, 2023.
- [14] Katrin Schuler, Matthias Nadler, and Fabian Schär. Contagion and loss redistribution in crypto asset markets: Modelling the intersection of defi and cefi. *Available at SSRN 4499113*, 2023.
- [15] Kaihua Qin, Liyi Zhou, Yaroslav Afonin, Ludovico Lazzaretti, and Arthur Gervais. Cefi vs. defi—comparing centralized to decentralized finance. *arXiv preprint arXiv:2106.08157*, 2021.
- [16] Zhipeng Wang, Kaihua Qin, Duc Vu Minh, and Arthur Gervais. Speculative multipliers on defi: Quantifying on-chain leverage risks. In *International Conference on Financial Cryptography and Data Security*, pages 38–56. Springer, 2022.
- [17] Erdinc Akyildirim, Shaen Corbet, Paraskevi Katsiampa, Neil Kellard, and Ahmet Sensoy. The development of bitcoin futures: Exploring the interactions between cryptocurrency derivatives. *Finance Research Letters*, 34:101234, 2020.
- [18] Carol Alexander, Jaehyuk Choi, Heungju Park, and Sungbin Sohn. Bitmex bitcoin derivatives: Price discovery, informational efficiency, and hedging effectiveness. *Journal of Futures Markets*, 40(1):23–43, 2020.
- [19] Jui-Cheng Hung, Hung-Chun Liu, and J Jimmy Yang. Trading activity and price discovery in bitcoin futures markets. *Journal of Empirical Finance*, 62:107–120, 2021.
- [20] Songrun He, Asaf Manela, Omri Ross, and Victor von Wachter. Fundamentals of perpetual futures. *arXiv preprint arXiv:2212.06888*, 2022.
- [21] Kyle Soska, Jin-Dong Dong, Alex Khodaverdian, Ariel Zetlin-Jones, Bryan Routledge, and Nicolas Christin. Towards understanding cryptocurrency derivatives: a case study of bitmex. In *Proceedings of the Web Conference 2021*, pages 45–57, 2021.
- [22] Carol Alexander, Jun Deng, and Bin Zou. Hedging with automatic liquidation and leverage selection on bitcoin futures. *European Journal of Operational Research*, 306(1):478–493, 2023.
- [23] Jason Milionis, Ciama C Moallemi, Tim Roughgarden, and Anthony Lee Zhang. Automated market making and loss-versus-rebalancing. *arXiv preprint arXiv:2208.06046*, 2022.

- [24] Joel Hasbrouck, Thomas J Rivera, and Fahad Saleh. An economic model of a decentralized exchange with concentrated liquidity. *Available at SSRN 4529513*, 2023.
- [25] A Lehar and C Parlour. Decentralized exchange: The uniswap automated market maker. *Preprint, submitted August, 14, 2021*.
- [26] Agostino Capponi and Ruizhe Jia. The adoption of blockchain-based decentralized exchanges. *arXiv preprint arXiv:2103.08842*, 2021.
- [27] Jianlei Han, Shiyang Huang, and Zhuo Zhong. Trust in defi: an empirical study of the decentralized exchange. *Available at SSRN 3896461*, 2022.
- [28] Robert J Shiller. Measuring asset values for cash settlement in derivative markets: hedonic repeated measures indices and perpetual futures. *The Journal of Finance*, 48(3):911–931, 1993.
- [29] Vitalik Buterin. Ethereum: A next-generation smart contract and decentralized application platform.
- [30] Lewis Gudgeon, Sam Werner, Daniel Perez, and William J Knottenbelt. Defi protocols for loanable funds: Interest rates, liquidity and market efficiency. In *Proceedings of the 2nd ACM Conference on Advances in Financial Technologies*, pages 92–112, 2020.
- [31] Noah Zinsmeister and Dan Robinson. Hayden adams hayden@uniswap.org.
- [32] Manuel Ammann. *Credit risk valuation: methods, models, and applications*. Springer Science & Business Media, 2002.
- [33] Hendrik Bessembinder and Paul J Seguin. Price volatility, trading volume, and market depth: Evidence from futures markets. *Journal of financial and Quantitative Analysis*, 28(1):21–39, 1993.
- [34] Changyun Wang. The effect of net positions by type of trader on volatility in foreign currency futures markets. *Journal of Futures Markets*, 22(5):427–450, 2002.
- [35] Carol Alexander, Jun Deng, and Bin Zou. Hedging with automatic liquidation and leverage selection on bitcoin futures. *European Journal of Operational Research*, 306(1):478–493, 2023.
- [36] Philip Garcia, Raymond M Leuthold, and Hector Zapata. Lead-lag relationships between trading volume and price variability: New evidence. *The Journal of Futures Markets (1986-1998)*, 6(1):1, 1986.
- [37] Mark B Garman and Michael J Klass. On the estimation of security price volatilities from historical data. *Journal of business*, pages 67–78, 1980.
- [38] Ruey S Tsay. *Analysis of financial time series*. John wiley & sons, 2005.
- [39] Riccardo De Blasis and Alexander Webb. Arbitrage, contract design, and market structure in bitcoin futures markets. *Journal of Futures Markets*, 42(3):492–524, 2022.
- [40] GMX announcements.
- [41] Arthur Hayes. Bitmex leverage statistics, 2019.
- [42] Vijay Mohan. Automated market makers and decentralized exchanges: A defi primer. *Financial Innovation*, 8(1):20, 2022.
- [43] Jun Aoyagi and Yuki Ito. Coexisting exchange platforms: Limit order books and automated market makers. *Available at SSRN*, 2021.
- [44] Grant McQueen, Michael Pinegar, and Steven Thorley. Delayed reaction to good news and the cross-autocorrelation of portfolio returns. *The Journal of Finance*, 51(3):889–919, 1996.
- [45] Eugene F Fama. Efficient capital markets: A review of theory and empirical work. *The journal of Finance*, 25(2):383–417, 1970.