Homework 7

Kevin Ha

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Problem 12.31

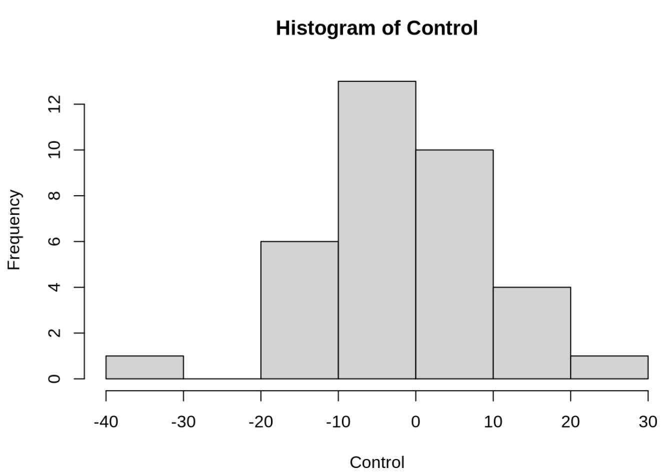
Problem 12.31a

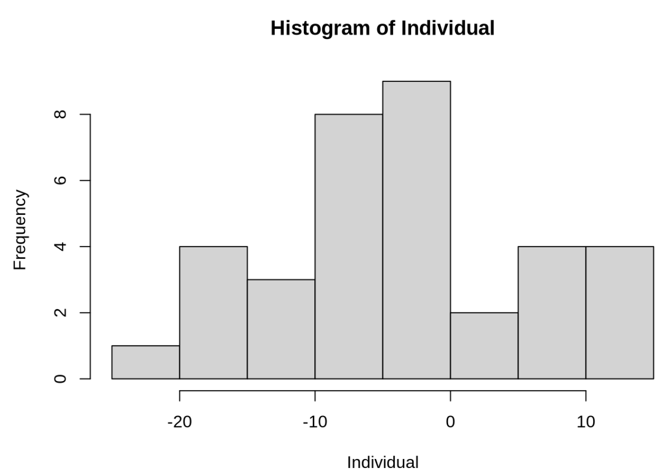
```
##            Sample Size Sample Mean Sample Standard Deviation
## Control          35.00       -1.01                     11.50
## Group            34.00      -10.79                     11.14
## Individual       35.00       -3.71                      9.08
```

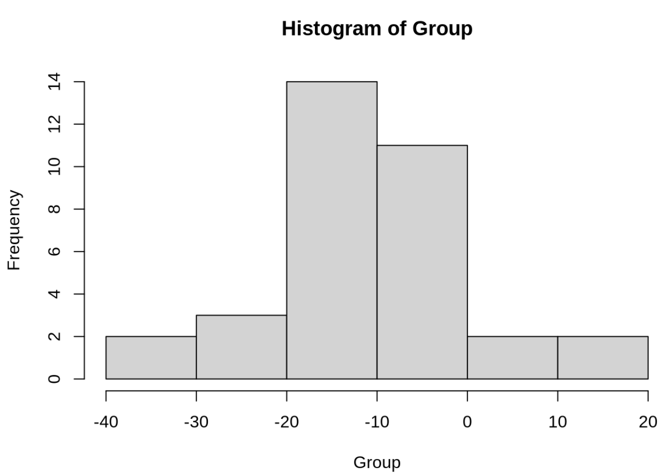
Problem 12.31b

Yes, because the largest standard deviation is less than twice the smallest. $(11.50 < 2(9.08))$

Problem 12.31c







We can be confident that the samples means are approximately normal as most of the groups appear symmetric from the histogram distribution and have a sample size > 15.

Problem 12.32

Problem 12.32a

One way ANOVA:

$$\bar{X} = \frac{(n_1\bar{X_1}+n_2\bar{X_2}+n_3\bar{X_3})}{(n_1+n_2+n_3)} = \frac{(35*-1.01+34*-10.79+35*-3.71)}{(35+34+35)} = -5.12$$

$$SSB = \sum_{i=1}^{3} n_i(\bar{X}_i - \bar{X})^2 = 35(-1.01+5.12)^2 + 34(-10.79+5.12)^2 + 35(-3.71+5.12)^2 = 1753.87$$

$$SSE = \sum_{i=1}^{3} (n_i - 1)S_i^2 = (35-1)(11.50)^2 + (34-1)(11.14)^2 + (35-1)(9.08)^2 = 11394.96$$

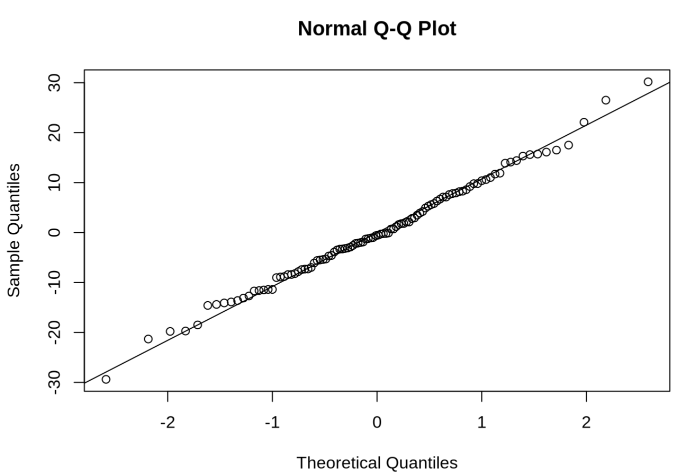
$$f = \frac{MSB}{MSE} = \frac{SSB/(k-1)}{SSE/(n-k)} = \frac{1753.87/(3-1)}{11394.96/(104-3)} = 7.77$$

```
pf(7.77, 2, 104-3, lower.tail = FALSE)
```

```
## [1] 0.0007265093
```

Summary: Because of the p-value is less than 0.05, the null hypothesis can be rejected and thus the group population means are not all equal.

Problem 12.32b



Again, we can feel confident that the sample means are approximately normal as a vast majority of the residuals follow the straight diagonal line of the QQ-plot, indicating strong evidence of normality.

Problem 12.32c

Testing statistic for comparing two means:

$$T_{i,j} = \frac{\bar{X}_i - \bar{X}_j}{\sqrt{S_p^2(\frac{1}{n_i}+\frac{1}{n_j})}}$$

Pooled sample variance:

$$S_p^2 = MSE = \frac{SSE}{n-k} = \frac{11394.96}{(104-3)} = 112.82$$

$$T_{Individual,Group} = \frac{-3.71+10.79}{\sqrt{112.82(\frac{1}{35}+\frac{1}{34})}} = 2.77$$

$$T_{Control,Individual} = \frac{-1.01+3.71}{\sqrt{112.82(\frac{1}{35}+\frac{1}{35})}} = 1.06$$

$$T_{Control,Group} = \frac{-1.01+10.79}{\sqrt{112.82(\frac{1}{35}+\frac{1}{35})}} = 3.82$$

LSD for $T_{Individual,Group}$

```
2*(1-pt(2.77, 104-3))
```

```
## [1] 0.006672348
```

LSD for $T_{Control,Individual}$

```
2*(1-pt(1.06, 104-3))
```

```
## [1] 0.2916729
```

LSD for $T_{Control,Group}$

```
2*(1-pt(3.82, 104-3))
```

```
## [1] 0.0002306425
```

Problem 12.32d

Based on the results of the (a), (b), (c) the null hypothesis ($H_0 : \mu_1 = \mu_2 = \mu_3$) fails to be rejected when comparing the Control and Individual groups as the calculated LSD values is greater than 0.05, while the H_0 is rejected when comparing Control / Group & Individual & Group as both groups have a LSD value < 0.05.

Problem 12.33

Problem 12.33a

```
##            Sample Size Sample Mean Sample Standard Deviation
## Control     35.0000000  -0.4584416                 5.2276025
## Group       34.0000000  -4.9024064                 5.0632505
## Individual  35.0000000  -1.6857143                 4.1265290
```

Problem 12.33b

One way ANOVA:

$$\bar{X} = \frac{(n_1\bar{X}_1+n_2\bar{X}_2+n_3\bar{X}_3)}{(n_1+n_2+n_3)} = \frac{(35*-0.46+34*-4.90+35*-1.69)}{(35+34+35)} = -2.33$$

$$SSB = \sum_{i=1}^{3} n_i(\bar{X}_i - \bar{X})^2 = 35(-0.46+2.33)^2 + 34(-4.90+2.33)^2 + 35(-1.69+2.33)^2 = 361.29`$$

$$SSE = \sum_{i=1}^{3}(n_i-1)S_i^2 = (35-1)(5.23)^2 + (34-1)(5.06)^2 + (35-1)(4.13)^2 = 2354.85`$$

$$f = \frac{MSB}{MSE} = \frac{SSB/(k-1)}{SSE/(n-k)} = \frac{361.29/(3-1)}{2354.85/(104-3)} = 7.75$$

```
pf(7.75, 2, 104-3, lower.tail = FALSE)
```

```
## [1] 0.0007392139
```

Summary: Because of the p-value is less than 0.05, the null hypothesis can be rejected and thus the group population means are not all equal.

Problem 12.41

Problem 12.41a

Asking for average of blue and green eye color: $(\mu_1 + \mu_4)/2$

$\psi_1 = \mu_2 - (\mu_1 + \mu_4)/2$

Problem 12.41b

Looking up are the linear combinations of each eye color: $\mu_1 + \mu_2 + \mu_4$

Taking the average of the combination above and contrasting it with the looking down (μ_3):

$\psi_2 = \frac{(\mu_1+\mu_2+\mu_4)}{3} - \mu_3$

Problem 12.42

Problem 12.42a

Eye Color Contrast:

$H_0 : \psi_1 = 0$

$H_a : \psi_1 \neq 0$

Looking Contrast:

$H_0 : \psi_2 = 0$

$H_a : \psi_2 \neq 0$

Problem 12.42b

```
##       Sample Size Sample Mean Sample Standard Deviation
## Blue        67.00        3.19                      1.75
## Brown       37.00        3.72                      1.72
## Down        41.00        3.11                      1.53
## Green       77.00        3.86                      1.67
```

$c_1 = \mu_2 - (\mu_1 + \mu_4)/2 = 3.72 - (3.19 + 3.86)/2 = 0.195$

$c_2 = \frac{(\mu_1+\mu_2+\mu_4)}{3} - \mu_3 = \frac{(3.19+3.72+3.86)}{3} - 3.11 = 0.48$

Problem 12.42c

Pooled sample variance:

$$S_p^2 = \sqrt{MSE} = \sqrt{\frac{SSE}{n-k}} = \sqrt{\frac{(67-1)(1.75)^2+(37-1)(1.72)^2+(41-1)(1.53)^2+(77-1)(1.67)^2}{(221-4)}} = 2.830506$$

Formula for standard error of c:

$$SE_c = \sqrt{S_p^2 \sum_{i=1}^{3} \frac{a_i^2}{\bar{x}_i}}$$

$$SE_{c_1} = \sqrt{2.83 * (\frac{(-1^2)}{37} + \frac{(1/2)^2}{67} + \frac{(-1/2)^2}{77})} = 0.30$$

$$SE_{c_2} = \sqrt{2.83 * (\frac{(-1^2)}{41} + \frac{(1/3)^2}{67} + \frac{(-1/3)^2}{77} + \frac{(-1/3)^2}{37})} = 0.29$$

Problem 12.42d

$t = \frac{c_\psi}{SE_c}$

For contrast (eye color): $t_1 = \frac{0.195}{0.30} = 0.64$

```
2*pt(0.64, 220 - 4, lower.tail = FALSE)
```

```
## [1] 0.5228509
```

For contrast (looking): $t_1 = \frac{0.48}{0.29} = 1.66$

```
2*pt(1.66, 220 - 4, lower.tail = FALSE)
```

```
## [1] 0.09836549
```

Conclusion:

For contrast (eye-color) we fail to reject the null hypothesis since the calculated p-value > 0.05, for the contrast (looking) we also fail to reject since the p-value is > 0.05.

Problem 12.42e

$df = 222 - 1 = 221$

```
abs(qt(0.05 / 2, 221))
```

```
## [1] 1.970756
```

95% confidence interval for contrast (eye color):

$$\psi_1 : 0.195 \pm (1.97 * 0.30) = (-0.40, 0.79)$$

95% confidence interval for contrast (looking):

$$\psi_2 : 0.48 \pm (1.97 * 0.29) = (-0.09, 1.05)$$