Homework 4

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Problem 6.58

6.58 (a)

Given $z = 1.77$

$H_a : \mu > \mu_0 = P(Z \geq 1.77) = 1 - P(Z \leq 1.77)$

```
## [1] 0.9616364
```

$1 - P(Z \leq 1.77)$ = 0.0383636

6.58 (b)

$H_a : \mu > \mu_0 = P(Z \leq 1.77)$

$P(Z \leq 1.77)$ = 0.9616364

6.58 (c)

$H_a : \mu \neq \mu_0 = 2P(Z \geq 1.77)$

$2P(Z \geq 1.77)$ = 0.0767271

Problem 6.59

6.59 (a)

Given $z = -1.69$

$H_a : \mu > \mu_0 = P(Z \geq -1.69) = 1 - P(Z \leq -1.69)$

```
p_val <- pnorm(-1.69)
p_val
```

```
## [1] 0.04551398
```

$P(Z \geq -1.69) = 1 - P(Z \leq -1.69)$ = 0.954486

6.59 (b)

$H_a : \mu > \mu_0 = P(Z \leq -1.69)$

$P(Z \leq -1.69)$ = 0.045514

6.59 (c)

$H_a : \mu \neq \mu_0 = 2P(Z \geq |-1.69|)$

```
absp_val <- pnorm(1.69)
absp_val
```

```
## [1] 0.954486
```

$2P(Z \geq 1.69)$ = $1 - P(Z \leq 1.69) = 0.091028$

Problem 6.71

6.71 (a)

Given $\overline{x} = 127.8$ and $\sigma = 30$

Test statistic formula: $z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$

$z = \frac{127.8 - 115}{30/\sqrt{25}} = 2.1333333$

```
p_val = pnorm(2.13)
p_val
```

```
## [1] 0.9834142
```

$H_a : \mu > 115 = 1 - P(Z \leq 2.1\overline{3}) = 0.0165858$

The calculated p-value is very small and thus concludes that older students that have better attitudes towards school do end up getting better SSHA scores.

6.71 (b)

The two important assumptions are that there is a SRS and there is a normal distribution of the data. SRS is more important, as normal distribution doesn't influence since there isn't any assumed skewness or outliers within the data.

Problem 6.73s

6.73 (a)

$H_0 : \mu = 0$ mpg
$H_a : \mu \neq 0$ mpg

6.73 (b)

Given $\sigma = 30$, $n = 20$, $\mu_0 = 0$, $\overline{X} = 2.73$

```
x = c(5.0, 6.5, -0.6, 1.7, 3.7, 4.5, 8.0, 2.2, 4.9, 3.0, 4.4, 0.1, 3.0, 1.1, 1.1, 5.0,
2.1, 3.7, -0.6, -4.2)
mean(x)
```

```
## [1] 2.73
```

$z = \frac{2.73-0}{3.0/\sqrt{20}} = 4.0696437$

```
p_val = pnorm(4.07)
p_val
```

```
## [1] 0.9999765
```

$H_a : 2(1 - P(Z < 4.07)) = 4.7013138 \times 10^{-5}$

The P-value is extremely small which indicates that there is strong evidence against the null hypothesis and indicates that the calculations will be different.

Problem 6.99

6.99 (a)

$n = 100$

$z = \frac{2453.7-2403.7}{880/\sqrt{100}} = 0.5681818$

```
p_val = pnorm(0.57)
p_val
```

```
## [1] 0.7156612
```

$H_a : 1 - P(Z < 0.72) = 0.2843388$

6.99 (b)

$n = 500$

$z = \frac{2453.7-2403.7}{880/\sqrt{100}} = 1.2704932$

```
p_val = pnorm(1.27)
p_val
```

```
## [1] 0.8979577
```

$H_a : 1 - P(Z < 0.72) = 0.1020423$

6.99 (c)

$n = 2500$

$z = \frac{2453.7 - 2403.7}{880/\sqrt{100}} = 2.8409091$

```
p_val = pnorm(2.84)
p_val
```

```
## [1] 0.9977443
```

$H_a : 1 - P(Z < 0.72) = 0.0022557$

Problem 6.120

6.120 (a)

Probability of Type I error:

$P(X \leq 2 \text{ when } p_0 \text{ is correct}) = 0.1 + 0.1 + 0.2 = 0.4$

6.120 (b)

Probability of Type II error:

$P(X > 2 \text{ when } p_1 \text{ is correct}) = 0.1 + 0.1 + 0.1 + 0.1 = 0.4$

Problem 7.22

7.22(a)

Degrees of freedom: $n - 1 = 16 - 1 = 15$

7.22(b)

t is bracketed by $t^* = 2.131$ and $t^* = 2.249$

7.22(c)

The p-value falls in between the range: $0.02 < p < 0.025$

7.22(d)

$t = 2.15$ is **statistically significant** at the 5% level as $p < 0.05$, however it is **not significant** at the 1% level as $p > 0.01$

7.22(e)

```
1 - pt(q = 2.15, df = 15)
```

```
## [1] 0.02413742
```

Problem 7.23

7.23(a)

Degrees of freedom: $n - 1 = 27 - 1 = 26$

7.23(b)

t is bracketed by $t^* = 1.706$ and $t^* = 2.056$

7.23(c)

Two tailed test so $2P(T \geq |t|)$

The p-value falls in between the range: $0.05 < p < 0.10$

7.23(d)

$t = 2.01$ is **not significant** at the 5% level as $p > 0.05$, and is **not significant** at the 1% level as $p > 0.01$

7.23(e)

```
2 * (1 - pt(q = 2.01, df = 26))
```

```
## [1] 0.05491354
```