Algebra HSLU Kevin Häusler

Zahlen und Logik

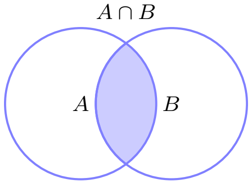
Zahlen

*	Bedeutung	Beispiel
$\mathbb{N}$	`Natürliche Zahlen = ganze Positive`	`{1;2;3;}`
$\mathbb{N}_0$	`Natürliche Zahlen mit 0`	`{0;1;2;}`
$\mathbb{Z}$	`Ganze Zahlen = N + ganze Negative`	`{-1;0;1;}`
$\mathbb{Q}$	`Rationale Zahlen = Bruchzahlen`	$\frac{3}{7}$ $\frac{5}{9}$ $\frac{2}{3}$
	`Irrationale Zahlen = Nachkommastellen`	`0.3281`
$\mathbb{R}$	`Reele Zahlen = Q + Irrationale Zahlen`	`Alle`

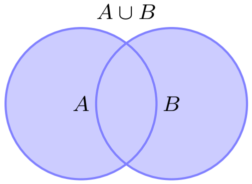
Mengen Operationen

*	Bedeutung
$\varnothing$ oder `{}`	`Leere Menge, enthält keine Elemente`
$x \in A$	`Beschreibt Element x ist in Menge A`
$x \notin A$	`Beschreibt Element x ist nicht in Menge A`
$A \subset B$	`A ist eine Teilmenge von B`
$A \cap B$	`Schnittmenge von A und B`
$A \cup B$	`Vereinigungsgsmenge von A und B`
$A \backslash B$	`Differenzbildung, Menge von A ohne B`

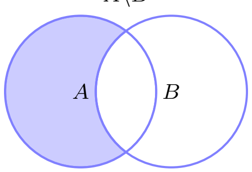
$A \cap B$



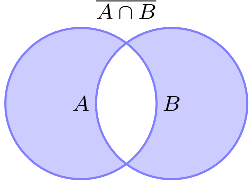
$A \cup B$



$A \backslash B$



$\overline{A \cap B}$



Aussagenlogik

Term: Ein Term ist eine sinnvolle Zusammensetzung von Zahlen, Variablen, Operationszeichen und Klammern. Ein Term hat keinen Wahrheitsgehalt, ist also weder wahr noch falsch.

Aussage: Eine Aussage beschreibt durch Worte oder Zeichen einen Sachverhalt. Eine Aussage ist entweder wahr oder falsch.

Aussageform: Jeder sprachliche oder zeichensymbolische Ausdruck mit wenigstens einer Variablen wenn er durch jede sinnvolle Belegung der Variablen jeweils eine Aussage wird.

*	Bedeutung	Beispiel
`|A|`	`Kardinalität/Mächtigkeit beschreibt` `Anzahl Elemente einer Menge`	`A = {1;2}` `|A| = 2`
$\mathbb{N}_0$	`Natürliche Zahlen mit 0`	`{0;1;2;}`
$\mathbb{Z}$	`Ganze Zahlen = N + ganze Negative`	`{-1;0;1;}`
$\mathbb{Q}$	`Rationale Zahlen = Bruchzahlen`	$\frac{3}{7}$ $\frac{5}{9}$ $\frac{2}{3}$

A	B	A	∧	B	A	∨	B	¬	B	A	∨	¬	B
T	T	T	T	T	T	T	T	F	T	T	T	F	T
T	F	T	F	F	T	T	F	T	F	T	T	T	F
F	T	F	F	T	F	T	T	F	T	F	F	F	T
F	F	F	F	F	F	F	F	T	F	F	T	T	F

Summe und Produkte

Funktionen

LaTeX(usually pronounced "LAY teck," sometimes "LAH teck," and never "LAY tex") is a mathematics typesetting program that is the standard for most professional mathematics writing. It is based on the typesetting program TeX created by Donald Knuth of Stanford University (his first version appeared in 1978). Leslie Lamport was responsible for creating LaTeX a more user friendly version of TeX. A team of LaTeX programmers created the current version, LaTeX 2ε.

Math vs. text vs. functions

In properly typeset mathematics variables appear in italics (e.g., $f(x) = x^2 + 2x - 3$). The exception to this rule is predefined functions (e.g., $\sin(x)$). Thus it is important to **always** treat text, variables, and functions correctly. See the difference between x and x, -1 and -1, and $sin(x)$ and $\sin(x)$.

$$\alpha Z\frac{1}{2}$$

There are two ways to present a mathematical expression—*inline* or as an *equation*.

Inline mathematical expressions

Inline expressions occur in the middle of a sentence. To produce an inline expression, place the math expression between dollar signs (`$`). For example, typing `$90^{\circ}$ is the same as $\frac{\pi}{2}$ radians` yields 90° is the same as $\frac{\pi}{2}$ radians.

Equations

Equations are mathematical expressions that are given their own line and are centered on the page. These are usually used for important equations that deserve to be showcased on their own line or for large equations that cannot fit inline. To produce an inline expression, place the mathematical expression between the symbols `\[` and `\]`. Typing `\[x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\]` yields

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Displaystyle

To get full-sized inline mathematical expressions use `\displaystyle`. Use this sparingly. Typing `I want this $\displaystyle \sum_{n=1}^{\infty} \frac{1}{n}$, not this $\sum_{n=1}^{\infty} \frac{1}{n}$.` yields

I want this $\displaystyle \sum_{n=1}^{\infty} \frac{1}{n}$, not this $\sum_{n=1}^{\infty} \frac{1}{n}$.

Images

You can put images (pdf, png, jpg, or gif) in your document. They need to be in the same location as your .tex file when you compile the document. Omit `[width=.5in]` if you want the image to be full-sized.

```
\begin{figure}[ht]
\includegraphics[width=.5in]{imagename.jpg}
\caption{The (optional) caption goes here.}
\end{figure}
```

Text decorations

Your text can be *italics* (`\textit{italics}`), **boldface** (`\textbf{boldface}`), or underlined (`\underline{underlined}`).

Your math can contain boldface, $\mathbf{R}$ (`\mathbf{R}`), or blackboard bold, $\mathbb{R}$ (`\mathbb{R}`). You may want to used these to express the sets of real numbers ($\mathbb{R}$ or $\mathbf{R}$), integers ($\mathbb{Z}$ or $\mathbf{Z}$), rational numbers ($\mathbb{Q}$ or $\mathbf{Q}$), and natural numbers ($\mathbb{N}$ or $\mathbf{N}$).

To have text appear in a math expression use `\text`.

`(0,1]=\{x\in\mathbb{R}:x>0\text{ and }x\le 1\}` yields $(0,1]=\{x\in\mathbb{R}:x>0 \text{ and } x\le 1\}$. (Without the `\text` command it treats "and" as three variables: $(0,1]=\{x\in\mathbb{R}:x>0 and x\le 1\}$.)

Spaces and new lines

LaTeX ignores extra spaces and new lines. For example,

```
This   sentence will       look

fine after      it is     compiled.
```

This sentence will look fine after it is compiled.

Leave one full empty line between two paragraphs. Place `\\` at the end of a line to create a new line (but not create a new paragraph).

```
This

compiles



like\\

this.
```

This compiles

like
this.

Use `\noindent` to prevent a paragraph from indenting.

Comments

Use `%` to create a comment. Nothing on the line after the `%` will be typeset. `$f(x)=\sin(x)$ %this is the sine function` yields $f(x)=\sin(x)$

Delimiters

description	*command*	*output*
parentheses	`(x)`	(x)
brackets	`[x]`	[x]
curly braces	`\{x\}`	{x}

To make your delimiters large enough to fit the content, use them together with `\right` and `\left`. For example, `\left\{\sin\left(\frac{1}{n}\right)\right\}_{n}^{\infty}` produces $\left\{\sin\left(\frac{1}{n}\right)\right\}_{n}^{\infty}$.

Curly braces are non-printing characters that are used to gather text that has more than one character. Observe the differences between the four expressions `x^2`, `x^{2}`, `x^2t`, `x^{2t}` when typeset: x^2, x^{2}, x^2t, x^{2t}.

Lists

You can produce ordered and unordered lists.

description	*command*	*output*

unordered list	`\begin{itemize}`
	`\item`
	`Thing 1`
	`\item`
	`Thing 2`
	`\end{itemize}`
		• Thing 1
		• Thing 2

ordered list	`\begin{enumerate}`
	`\item`
	`Thing 1`
	`\item`
	`Thing 2`
	`\end{enumerate}`
		1. Thing 1
		2. Thing 2

Symbols (in *math* mode)

The basics

description	*command*	*output*
addition	`+`	$+$
subtraction	`-`	$-$
plus or minus	`\pm`	$\pm$
multiplication (times)	`\times`	$\times$
multiplication (dot)	`\cdot`	$\cdot$
division symbol	`\div`	$\div$
division (slash)	`/`	$/$
circle plus	`\oplus`	$\oplus$
circle times	`\otimes`	$\otimes$
equal	`=`	$=$
not equal	`\ne`	$\ne$
less than	`<`	$<$
greater than	`>`	$>$
less than or equal to	`\le`	$\le$
greater than or equal to	`\ge`	$\ge$
approximately equal to	`\approx`	$\approx$
infinity	`\infty`	∞
dots	`1,2,3,\ldots`	$1,2,3,\ldots$
dots	`1+2+3+\cdots`	$1+2+3+\cdots$
fraction	`\frac{a}{b}`	$\frac{a}{b}$
square root	`\sqrt{x}`	$\sqrt{x}$
nth root	`\sqrt[n]{x}`	$\sqrt[n]{x}$
exponentiation	`a^b`	a^b
subscript	`a_b`	a_b
absolute value	`|x|`	$\lvert x\rvert$
natural log	`\ln(x)`	$\ln(x)$
logarithms	`\log_{a}b`	$\log_{a}b$
exponential function	`e^x=\exp(x)`	$e^x=\exp(x)$
degree	`\deg(f)`	$\deg(f)$

Functions

<table>
<tr><th>description</th><th>command</th><th>output</th></tr>
<tr><td>maps to</td><td><code>\to</code></td><td>$\to$</td></tr>
<tr><td>composition</td><td><code>\circ</code></td><td>$\circ$</td></tr>
<tr><td>piecewise function</td><td><code>|x|=</code>
<code>\begin{cases}</code>
<code>x & x\ge 0\\</code>
<code>-x & x<0</code>
<code>\end{cases}</code></td><td>$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$</td></tr>
</table>

Greek and Hebrew letters

command	*output*	*command*	*output*
`\alpha`	α	`\tau`	τ
`\beta`	β	`\theta`	θ
`\chi`	χ	`\upsilon`	υ
`\delta`	δ	`\xi`	ξ
`\epsilon`	ϵ	`\zeta`	ζ
`\varepsilon`	ε	`\Delta`	Δ
`\eta`	η	`\Gamma`	Γ
`\gamma`	γ	`\Lambda`	Λ
`\iota`	ι	`\Omega`	Ω
`\kappa`	κ	`\Phi`	Φ
`\lambda`	λ	`\Pi`	Π
`\mu`	μ	`\Psi`	Ψ
`\nu`	ν	`\Sigma`	Σ
`\omega`	ω	`\Theta`	Θ
`\phi`	ϕ	`\Upsilon`	Υ
`\varphi`	φ	`\Xi`	Ξ
`\pi`	π	`\aleph`	$\aleph$
`\psi`	ψ	`\beth`	$\beth$
`\rho`	ρ	`\daleth`	$\daleth$
`\sigma`	σ	`\gimel`	$\gimel$

Set theory

description	*command*	*output*
set brackets	`\{1,2,3\}`	$\{1,2,3\}$
element of	`\in`	$\in$
not an element of	`\not\in`	$\not\in$
subset of	`\subset`	$\subset$
subset of	`\subseteq`	$\subseteq$
not a subset of	`\not\subset`	$\not\subset$
contains	`\supset`	$\supset$
contains	`\supseteq`	$\supseteq$
union	`\cup`	$\cup$
intersection	`\cap`	$\cap$
big union	`\bigcup_{n=1}^{10}A_n`	$\bigcup_{n=1}^{10}A_n$
big intersection	`\bigcap_{n=1}^{10}A_n`	$\bigcap_{n=1}^{10}A_n$
empty set	`\emptyset`	$\emptyset$
power set	`\mathcal{P}`	$\mathcal{P}$
minimum	`\min`	$\min$
maximum	`\max`	$\max$
supremum	`\sup`	$\sup$
infimum	`\inf`	$\inf$
limit superior	`\limsup`	$\limsup$
limit inferior	`\liminf`	$\liminf$
closure	`\overline{A}`	$\overline{A}$

Calculus

description	*command*	*output*
derivative	`\frac{df}{dx}`	$\frac{df}{dx}$
derivative	`\f'`	f'
partial derivative	`\frac{\partial f}` `{\partial x}`	$\frac{\partial f}{\partial x}$
integral	`\int`	$\int$
double integral	`\iint`	$\iint$
triple integral	`\iiint`	$\iiint$
limits	`\lim_{x\to \infty}`	$\lim_{x\to \infty}$
summation	`\sum_{n=1}^{\infty}a_n`	$\sum_{n=1}^{\infty}a_n$
product	`\prod_{n=1}^{\infty}a_n`	$\prod_{n=1}^{\infty}a_n$

Logic

description	*command*	*output*
not	`\sim`	$\sim$
and	`\land`	$\land$
or	`\lor`	$\lor$
if...then	`\to`	$\to$
if and only if	`\leftrightarrow`	$\leftrightarrow$
logical equivalence	`\equiv`	$\equiv$
therefore	`\therefore`	$\therefore$
there exists	`\exists`	$\exists$
for all	`\forall`	$\forall$
implies	`\Rightarrow`	$\Rightarrow$
equivalent	`\Leftrightarrow`	$\Leftrightarrow$

Linear algebra

<table>
<tr><th>description</th><th>command</th><th>output</th></tr>
<tr><td>vector</td><td><code>\vec{v}</code></td><td>$\vec{v}$</td></tr>
<tr><td>vector</td><td><code>\mathbf{v}</code></td><td>$\mathbf{v}$</td></tr>
<tr><td>norm</td><td><code>||\vec{v}||</code></td><td>$||\vec{v}||$</td></tr>
<tr><td>matrix</td><td><code>\left[</code>
<code>\begin{array}{ccc}</code>
<code>1 & 2 & 3 \\</code>
<code>4 & 5 & 6\\</code>
<code>7 & 8 & 0</code>
<code>\end{array}</code>
<code>\right]</code></td><td>$\left[\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{array}\right]$</td></tr>
<tr><td>determinant</td><td><code>\left|</code>
<code>\begin{array}{ccc}</code>
<code>1 & 2 & 3 \\</code>
<code>4 & 5 & 6 \\</code>
<code>7 & 8 & 0</code>
<code>\end{array}</code>
<code>\right|</code></td><td>$\left|\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{array}\right|$</td></tr>
<tr><td>determinant</td><td><code>\det(A)</code></td><td>$\det(A)$</td></tr>
<tr><td>trace</td><td><code>\operatorname{tr}(A)</code></td><td>$\operatorname{tr}(A)$</td></tr>
<tr><td>dimension</td><td><code>\dim(V)</code></td><td>$\dim(V)$</td></tr>
</table>

Number theory

<table>
<tr><th>description</th><th>command</th><th>output</th></tr>
<tr><td>divides</td><td><code>|</code></td><td>$|$</td></tr>
<tr><td>does not divide</td><td><code>\not |</code></td><td>$\not|$</td></tr>
<tr><td>div</td><td><code>\operatorname{div}</code></td><td>div</td></tr>
<tr><td>mod</td><td><code>\mod</code></td><td>$\bmod$</td></tr>
<tr><td>greatest common divisor</td><td><code>\gcd</code></td><td>$\gcd$</td></tr>
<tr><td>ceiling</td><td><code>\lceil x \rceil</code></td><td>$\lceil x \rceil$</td></tr>
<tr><td>floor</td><td><code>\lfloor x \rfloor</code></td><td>$\lfloor x \rfloor$</td></tr>
</table>

Geometry and trigonometry

description	*command*	*output*
angle	`\angle ABC`	$\angle ABC$
degree	`90^{\circ}`	90°
triangle	`\triangle ABC`	$\triangle ABC$
segment	`\overline{AB}`	$\overline{AB}$
sine	`\sin`	$\sin$
cosine	`\cos`	$\cos$
tangent	`\tan`	$\tan$
cotangent	`\cot`	$\cot$
secant	`\sec`	$\sec$
cosecant	`\csc`	$\csc$
inverse sine	`\arcsin`	$\arcsin$
inverse cosine	`\arccos`	$\arccos$
inverse tangent	`\arctan`	$\arctan$

Symbols (in *text* mode)

The followign symbols do **not** have to be surrounded by dollar signs.

description	*command*	*output*
dollar sign	`\$`	$
percent	`\%`	%
ampersand	`\&`	&
pound	`\#`	#
backslash	`\textbackslash`	\
left quote marks	` `` `	“
right quote marks	`''`	”
single left quote	`` ` ``	‘
single right quote	`'`	’
hyphen	`X-ray`	X-ray
en-dash	`pp. 5--15`	pp. 5–15
em-dash	`Yes---or no?`	Yes—or no?

Resources

Great symbol look-up site: Detexify
LaTeX Mathematical Symbols
The Comprehensive LaTeX Symbol List
The Not So Short Introduction to LaTeX 2ε
TUG: The TeX Users Group
CTAN: The Comprehensive TeX Archive Network

LaTeX for the Mac: MacTeX
LaTeX for the PC: TeXnicCenter and MiKTeX
LaTeX online: WriteLaTeX.

Dave Richeson, Dickinson College, http://divisbyzero.com/