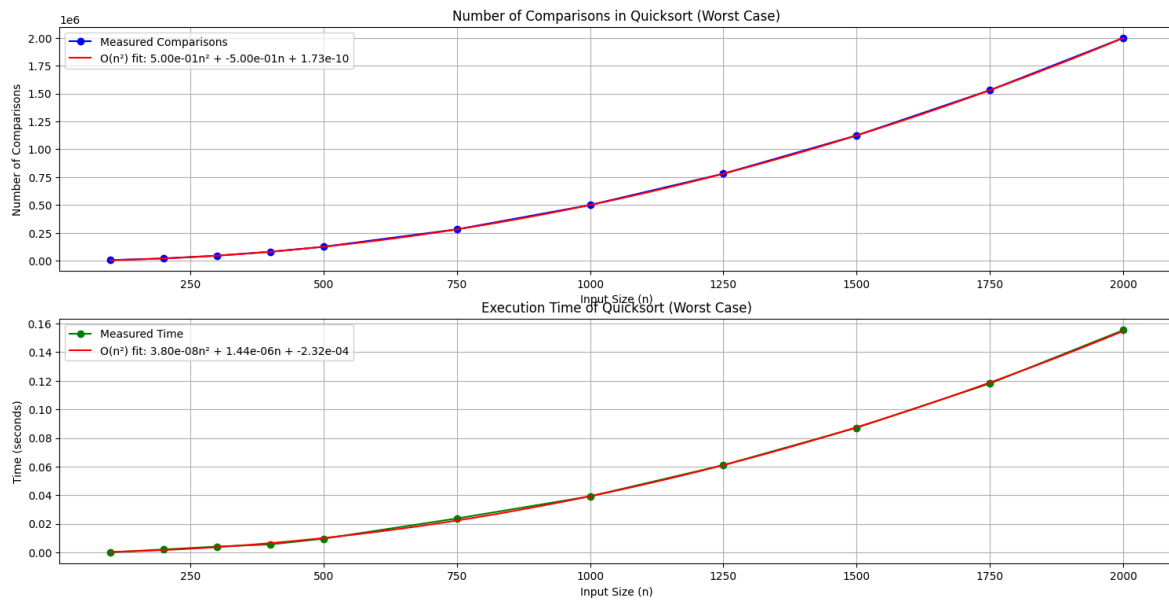


EX4

4.



1.- Let $T(n)$ be the time complexity for sorting an array of size n

- In the worst case, each partition operation divides the array into subarrays of size 0 and $n-1$

- Each partition operation itself takes $O(n)$ time (to compare each element with the pivot)

This gives us the recurrence relation:

$$T(n) = T(n-1) + T(0) + O(n)$$

Since $T(0) = 0$, this simplifies to:

$$T(n) = T(n-1) + O(n)$$

Expanding this recursion:

$$\begin{aligned}
 T(n) &= T(n-1) + O(n) \\
 &= T(n-2) + O(n-1) + O(n) \\
 &= T(n-3) + O(n-2) + O(n-1) + O(n) \\
 &\dots \\
 &= T(1) + O(2) + O(3) + \dots + O(n-1) + O(n) \\
 &= O(1) + O(2) + O(3) + \dots + O(n-1) + O(n) \\
 &= O(1 + 2 + 3 + \dots + n-1 + n) \\
 &= O(n(n+1)/2) \\
 &= O(n^2)
 \end{aligned}$$

Therefore, the worst-case time complexity of quicksort is $O(n^2)$.

3. Derivation of Worst-Case Complexity for Quicksort:

In the worst case scenario for quicksort:

1. The pivot chosen at each step results in the most unbalanced partition possible
2. With last-element pivot selection, this occurs when the array is already sorted