DNE means "Does Not Exist," $\lfloor x \rfloor$ outputs the greatest integer that is less than or equal to x, and NOTA means "None of the above." Good luck!

1. Find $f'(1)$ where $f(x) = x^2 + x^0 + x^2 + x^3$.

   A. 7 B. 8 C. 9 D. 10 E. NOTA

2. Evaluate:

$$\lim_{x \to 0} \frac{\sin(6x)}{2x}$$

   A. 1 B. 2 C. 3 D. 6 E. NOTA

3. Find $f'(\pi)$:

$$f(x) = \int_0^x e^t \cos\left(\frac{t}{3}\right) dt$$

   A. $\frac{e^\pi}{2} - 1$ B. $\frac{e^\pi \sqrt{3}}{2} - 1$ C. $\frac{e^\pi \sqrt{3}}{2}$ D. $\frac{e^\pi}{2}$ E. NOTA

4. Evaluate:

$$\lim_{x \to 0} \frac{e^{2x} - 1}{x}$$

   A. DNE B. 0 C. 1 D. 2 E. NOTA

5. Evaluate:

$$\lim_{x \to 4} \frac{x^2 - 16}{x^2 - 2x - 8}$$

   A. $\frac{4}{3}$ B. 2 C. $\frac{8}{3}$ D. DNE E. NOTA

6. Evaluate:

$$\lim_{x\to 0}\frac{\ln(\cos(x))}{x^2}$$

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. NOTA

7. Evaluate:

$$\lim_{x\to 0}\frac{1-\cos(x)}{\tan^2(x)}$$

A. 0 B. $\frac{1}{2}$ C. 1 D. DNE E. NOTA

8. Let $f^{-1}(x) = g(x)$ and $f(x) = \arctan(\ln(x))$. Then

$$g'\left(\frac{\pi}{4}\right) =$$

A. $\frac{1}{2e}$ B. $\frac{1}{e}$ C. e D. $2e$ E. NOTA

9. Evaluate:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n^2}\sin\left(\frac{i}{n}\pi\right)$$

A. $\frac{1}{\pi}$ B. $\frac{2}{\pi}$ C. $\frac{3}{\pi}$ D. $\frac{4}{\pi}$ E. NOTA

10. At time $t = 0$, Gon and Hisoka are both at the origin. Gon travels along the x-axis according to the equation $x(t) = t^2 - 8t$ and Hisoka travels along the y-axis according to the equation $y(t) = t^3 - 3$. Find the rate at which the distance between Gon and Hisoka is changing at $t = 2$.

A. $\frac{12}{13}$ B. $\frac{36}{13}$ C. $\frac{108}{13}$ D. $\frac{324}{13}$ E. NOTA

11. Let $x(t) = t^2 + 6t + 2$ and $y(t) = -t^3 + 4t$. Evaluate at $t = 1$:

$$\frac{d^2y}{dx^2}$$

A. $-\frac{13}{128}$ B. $-\frac{25}{256}$ C. $-\frac{3}{32}$ D. $-\frac{23}{256}$ E. NOTA

12. Evaluate:

$$\lim_{x \to 0} \frac{\ln^2(1+x) - x^2}{x^3}$$

A. DNE B. -2 C. -1 D. 0 E. NOTA

13. If $y' + y - 2 = xy + 2(x + y)$ and $f(0) = 1$, find $f(2)$.

A. $2e^4 - 1$ B. $3e^6 - 2$ C. $2e^6 - 1$ D. $3e^4 - 2$ E. NOTA

14. Let $g(x) = 4 - x^2$ and $f(x) = \lfloor x \rfloor$. Evaluate:

$$\lim_{x \to 0} f\big(g(x)\big)$$

A. 1 B. 2 C. 3 D. 4 E. NOTA

15. Let $r = \sin(\theta) + \cos^2(\theta)$. Evaluate $\frac{dy}{dx}$ at $\theta = \frac{3\pi}{4}$.

A. -1 B. $-\frac{1}{3}$ C. $\frac{6\sqrt{2}+3}{7}$ D. $\frac{2\sqrt{2}-1}{3}$ E. NOTA

16. A cubic has the property that $f(0) = f'(0) = f''(0) = f'''(0)$. If the roots of this cubic are r_1, r_2, and r_3,

$$\sum_{i=1}^{3} \frac{1}{2 - r_i} = \frac{K}{H}$$

If K and H are relatively prime positive integers, find $K + H$.

A. 7 B. 16 C. 25 D. 34 E. NOTA

17. Let $f(x) = x^3$. Evaluate:

$$\lim_{h\to 0} \frac{f(x+5h) - f(x-3h)}{2h}$$

A. 0 B. $6x^2$ C. $12x^2$ D. $18x^2$ E. NOTA

18. Let $f(x) = \frac{1}{(x-5)^2(x-8)}$. Let $M =$ the sum of the x-coordinates of the critical points of $f(x)$. Find the sum of the digits of M.

A. 2 B. 3 C. 4 D. 5 E. NOTA

19. Let $x = c$ satisfy the Mean Value Theorem for Derivatives for $f(x) = \sqrt{x}$ on the interval $(0, a)$. Given that a and c are both positive integers, find the sum of the digits of the minimum possible value of $a^2 + c^2$.

A. 5 B. 6 C. 7 D. 8 E. NOTA

20.

$$f(x) = \sum_{n=2}^{\infty} \frac{x^n}{n(n-1)}$$

Find $\left\lfloor 100 f\left(\frac{1}{2}\right) \right\rfloor$ given that $\ln(2) \approx 0.69$

A. 13 B. 14 C. 15 D. 16 E. NOTA

21. Define the sequence $a_n\{n \geq 0\}$ for integer n recursively as follows:

$$a_n = 2a_{n-1} + 3a_{n-2}$$

Given that $a_0 = 10$ and $a_1 = 14$, find $\lim\limits_{n\to\infty} \frac{a_{n+2}}{a_n}$

A. 3 B. 9 C. e D. e^2 E. NOTA

22. Usain Bolt has come out of retirement to try to break his own world record in the 100-meter dash. Let $v(t)$ be his velocity in meters per second and let $x(t)$ be his position in meters. He runs in a manner that results in the following equation for some constant k:

$$\frac{v(t)}{x(t)} = k\arctan(t)$$

But this time, he decides to cheat. At time $t = 0$, he is at the 1-meter mark. At time $t = 1$, he is at the $\frac{e^{\pi}}{4}$ meter mark. If $x(\sqrt{3})$ can be written in the form $\frac{e^{a\pi}}{b}$ for real numbers a and b, find $\left(\frac{b}{a}\right)^2$. *Hint: k turns out to be a quite friendly number.*

A. 16 B. 24 C. 48 D. 72 E. NOTA

23. Let $f(x) = \cos^4(x)$. Let $g(x)$ be the third-degree Maclaurin series approximation for $f(x)$. Let R be the region bounded by $y = g(x)$ and the x-axis. Find the area of R.

A. $\frac{\sqrt{2}}{3}$ B. $\frac{2}{3}$ C. $\frac{2\sqrt{2}}{3}$ D. $\frac{4}{3}$ E. NOTA

24. A cone has a height of h, a radius of r, and a constant volume of V. Find the ratio of the height of the cone to the radius of the cone $\frac{h}{r}$ that minimizes the lateral surface area of the cone.

A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2 E. NOTA

25. The 6th term of the Maclaurin series for $f(x) = e^{\arctan(x)}$ can be written in the form $\frac{m}{n}x^6$ for relatively prime positive integers m and n. Find $m + n$.

A. 55 B. 173 C. 185 D. 378 E. NOTA

Questions **26-28** rely on the information given below.
Given an $a > 1$, define b as the minimum positive value that results in the following limit equaling zero:

$$\lim_{n\to\infty} \frac{\binom{an}{n}}{b^n}$$

Note that b is to be expressed in terms of a.

26. When $a = 4$, find $\lfloor b \rfloor$.

A. 6 B. 7 C. 8 D. 9 E. NOTA

27. Evaluate:

$$\lim_{a\to\infty}\frac{db}{da}$$

Note that we're treating b as a function of a with the result from #26.

A. e B. $2e$ C. e^2 D. $2e^2$ E. NOTA

28. Given that $\lim_{n\to\infty}\frac{\binom{an}{n}\sqrt{n}}{b^n}=k$ for some non-zero finite value k, the value of $2k$ that satisfies the equation $a=k$ can be written in the form $1+\sqrt{1+\frac{m}{\pi}}$ for some positive integer m. Find m.

A. 1 B. 2 C. 3 D. 4 E. NOTA

For questions 29 and 30, let $f(x)=\frac{x^3}{x^2+bx+c}$ and $g(x)=f'(x)$ for non-zero real numbers b and c.

29. If the equation $g(x)=0$ has two distinct solutions, the maximum value of $b-c$ can be written in the form $\frac{K}{H}$ for relatively prime positive integers K and H. Find $K+H$.

A. 7 B. 8 C. 9 D. 10 E. NOTA

30. Let b and c be randomly chosen on the interval $(0,16)$. The probability that the graph of $y=g(x)$ has 0 vertical asymptotes and intersects the x-axis at 3 points can be written in the form $\frac{K-\sqrt{H}}{E}$ where $K,H,$ and E are positive integers such that H is minimized. Find $K+H+E$.

A. 9 B. 10 C. 11 D. 12 E. NOTA